1 **a** $= 7$ **b** $= 11$ **c** $= \frac{1}{3}$

d $= \frac{2}{5}$ **e** $= 0.1$ **f** $= 0.3$

g $= 2$ **h** $= 10$ **i** $= 3$

j $= \sqrt{\frac{25}{16}} = \frac{5}{4}$ or $1\frac{1}{4}$ **k** $= \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ or 0.5 **l** $= \sqrt[3]{\frac{125}{8}} = \frac{5}{2}$ or $2\frac{1}{2}$

2 **a** $= 7$ **b** $= 20$ **c** $= 27$ **d** $= 36$

e $= 4\sqrt{2}$ **f** $= 24\sqrt{3}$ **g** $= \sqrt{16} = 4$ **h** $= 2\sqrt{81} = 18$

i $= \sqrt{16} = 4$ **j** $= \sqrt{\frac{1}{4}} = \frac{1}{2}$ **k** $= 6$ **l** $= 54$

3 **a** $= \sqrt{9}\times\sqrt{2} = 3\sqrt{2}$ **b** $= \sqrt{25}\times\sqrt{2} = 5\sqrt{2}$ **c** $= \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$

d $= \sqrt{49}\times\sqrt{2} = 7\sqrt{2}$ **e** $= \sqrt{100}\times\sqrt{2} = 10\sqrt{2}$ **f** $= \sqrt{81}\times\sqrt{2} = 9\sqrt{2}$

4 **a** $= \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$ **b** $= \sqrt{4}\times\sqrt{7} = 2\sqrt{7}$ **c** $= \sqrt{16}\times\sqrt{5} = 4\sqrt{5}$

d $= \sqrt{9}\times\sqrt{3} = 3\sqrt{3}$ **e** $= \sqrt{4}\times\sqrt{6} = 2\sqrt{6}$ **f** $= \sqrt{64}\times\sqrt{2} = 8\sqrt{2}$

g $= \sqrt{9}\times\sqrt{5} = 3\sqrt{5}$ **h** $= \sqrt{4}\times\sqrt{10} = 2\sqrt{10}$ **i** $= \sqrt{25}\times\sqrt{3} = 5\sqrt{3}$

j $= \sqrt{16}\times\sqrt{7} = 4\sqrt{7}$ **k** $= \sqrt{9}\times\sqrt{11} = 3\sqrt{11}$ **l** $= \sqrt{49}\times\sqrt{3} = 7\sqrt{3}$

m $= \sqrt{36}\times\sqrt{6} = 6\sqrt{6}$ **n** $= \sqrt{400}\times\sqrt{2} = 20\sqrt{2}$ **o** $= \sqrt{36}\times\sqrt{5} = 6\sqrt{5}$

p $= \sqrt{4}\times\sqrt{15} = 2\sqrt{15}$ **q** $= \sqrt{121}\times\sqrt{3} = 11\sqrt{3}$ **r** $= \sqrt{16}\times\sqrt{13} = 4\sqrt{13}$

5 **a** $= 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$ **b** $= 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$ **c** $= 4\sqrt{2} + 6\sqrt{2} = 10\sqrt{2}$

d $= 6\sqrt{10} - 4\sqrt{10} = 2\sqrt{10}$ **e** $= 2\sqrt{5} - 3\sqrt{5} + 6\sqrt{5} = 5\sqrt{5}$ **f** $= 2\sqrt{6} + 5\sqrt{6} - 8\sqrt{6} = -\sqrt{6}$

6 **a** $= 3 + 2\sqrt{3}$

b $= 4 - \sqrt{3} - 2 + 2\sqrt{3}$
$= 2 + \sqrt{3}$

c $= 2 + \sqrt{3} + 2\sqrt{3} + 3$
$= 5 + 3\sqrt{3}$

d $= 4 + 8\sqrt{3} + \sqrt{3} + 6$
$= 10 + 9\sqrt{3}$

e $= 27 - 24\sqrt{3} + 16$
$= 43 - 24\sqrt{3}$

f $= 6\sqrt{3} - 45 + 2 - 5\sqrt{3}$
$= -43 + \sqrt{3}$

7 **a** $= 10 + 3\sqrt{5} + 2\sqrt{5} + 3$
$= 13 + 5\sqrt{5}$

b $= 4\sqrt{2} - 3 - 8 + 3\sqrt{2}$
$= 7\sqrt{2} - 11$

c $= 28 + 12\sqrt{7} + 9$
$= 37 + 12\sqrt{7}$

d $= 12 + 15\sqrt{2} - 2\sqrt{2} - 5$
$= 7 + 13\sqrt{2}$

e $= 5 + 2\sqrt{10} - \sqrt{10} - 4$
$= 1 + \sqrt{10}$

f $= (3 - 2\sqrt{2})(4 + \sqrt{2})$
$= 12 + 3\sqrt{2} - 8\sqrt{2} - 4$
$= 8 - 5\sqrt{2}$

8 **a** $= \frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$ **b** $= \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$ **c** $= \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}\sqrt{2}$

d $= \frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = 2\sqrt{7}$ **e** $= \frac{3\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \sqrt{6}$ **f** $= \frac{\sqrt{5}}{\sqrt{3}\sqrt{5}} = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$

g $= \frac{1}{3\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = \frac{1}{21}\sqrt{7}$ **h** $= \frac{12}{6\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$ **i** $= \frac{1}{4\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{20}\sqrt{5}$

j $= \frac{3}{6\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} = \frac{1}{12}\sqrt{6}$ **k** $= \frac{8\sqrt{5}}{9\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{4}{9}\sqrt{10}$ **l** $= \frac{15\sqrt{7}}{6\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{5}{6}\sqrt{21}$

9 **a** $= 2\sqrt{2} + \frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$= 2\sqrt{2} + 3\sqrt{2}$
$= 5\sqrt{2}$

b $= 4\sqrt{3} - \frac{10}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$
$= 4\sqrt{3} - \frac{10}{3}\sqrt{3}$
$= \frac{2}{3}\sqrt{3}$

c $= \frac{6-2\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{6\sqrt{2}-4}{2}$
$= 3\sqrt{2} - 2$

d $= \frac{3\sqrt{5}-5}{2\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{15-5\sqrt{5}}{10}$
$= \frac{1}{2}(3 - \sqrt{5})$

e $= \frac{1}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{1}{6}\sqrt{2} + \frac{1}{8}\sqrt{2}$
$= \frac{7}{24}\sqrt{2}$

f $= \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}}$
$= \frac{2}{3}\sqrt{3} - \frac{1}{6}\sqrt{3}$
$= \frac{1}{2}\sqrt{3}$

10 **a** $x^2 + 4x = 4x + 32$
$x^2 = 32$
$x = \pm\sqrt{32}$
$x = \pm 4\sqrt{2}$

b $x - 4\sqrt{3} = 2\sqrt{3} - 2x$
$3x = 6\sqrt{3}$
$x = 2\sqrt{3}$

c $3\sqrt{2}\,x - 4 = 2\sqrt{2}$
$6x - 4\sqrt{2} = 4$
$6x = 4 + 4\sqrt{2}$
$x = \frac{2}{3}(1 + \sqrt{2})$

d $\sqrt{5}\,x + 2 = 2\sqrt{5}\,(x - 1)$
$5x + 2\sqrt{5} = 10(x - 1)$
$5x = 10 + 2\sqrt{5}$
$x = 2 + \frac{2}{5}\sqrt{5}$

11 **a** $= 4 - (\sqrt{3})^2 = 4 - 3 = 1$

b $= \frac{2}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2(2+\sqrt{3})}{1} = 4 + 2\sqrt{3}$

12 **a** $= \frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

b $= \frac{4}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1)$

c $= \frac{1}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2)$ or $\frac{1}{2}\sqrt{6}+1$

d $= \frac{3}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3})$

e $= \frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{4-5} = \sqrt{5}-2$

f $= \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} = 2+\sqrt{2}$

g $= \frac{6}{\sqrt{7}+3} \times \frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{6(\sqrt{7}-3)}{7-9} = 3(3-\sqrt{7})$

h $= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

i $= \frac{1}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{4+2\sqrt{3}}{16-12} = \tfrac{1}{2}(2+\sqrt{3})$ or $1+\tfrac{1}{2}\sqrt{3}$

j $= \frac{3}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4} = \frac{3(3\sqrt{2}-4)}{18-16} = \tfrac{3}{2}(3\sqrt{2}-4)$ or $\tfrac{9}{2}\sqrt{2}-6$

k $= \frac{2\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{2\sqrt{3}(7+4\sqrt{3})}{49-48} = 2(7\sqrt{3}+12)$

l $= \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = 3(\sqrt{5}+\sqrt{3})$

13 $3x = \sqrt{5}\,x + 2\sqrt{5}$

$x(3 - \sqrt{5}) = 2\sqrt{5}$

$x = \frac{2\sqrt{5}}{3-\sqrt{5}} = \frac{2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2\sqrt{5}(3+\sqrt{5})}{9-5}$

$x = \frac{6\sqrt{5}+10}{4} = \tfrac{5}{2} + \tfrac{3}{2}\sqrt{5}$

14 $l = \frac{6}{3\sqrt{2}-3} = \frac{6}{3\sqrt{2}-3} \times \frac{3\sqrt{2}+3}{3\sqrt{2}+3} = \frac{6(3\sqrt{2}+3)}{18-9}$

$l = \frac{18(\sqrt{2}+1)}{9} = 2\sqrt{2} + 2$

15 **a** $= \frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}} \times \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}} = \frac{\sqrt{2}(\sqrt{2}-\sqrt{6})}{2-6} = -\tfrac{1}{4}(2-2\sqrt{3}) = \tfrac{1}{2}(\sqrt{3}-1)$

b $= \frac{1+\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{(1+\sqrt{3})(2-\sqrt{3})}{4-3} = 2-\sqrt{3}+2\sqrt{3}-3 = \sqrt{3}-1$

c $= \frac{1+\sqrt{10}}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3} = \frac{(1+\sqrt{10})(\sqrt{10}+3)}{10-9} = \sqrt{10}+3+10+3\sqrt{10} = 13+4\sqrt{10}$

d $= \frac{3-\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{(3-\sqrt{2})(4-3\sqrt{2})}{16-18} = \frac{12-9\sqrt{2}-4\sqrt{2}+6}{-2} = \tfrac{1}{2}(13\sqrt{2}-18)$ or $\tfrac{13}{2}\sqrt{2}-9$

e $= \frac{1-\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{(1-\sqrt{2})(3+2\sqrt{2})}{9-8} = 3+2\sqrt{2}-3\sqrt{2}-4 = -1-\sqrt{2}$

f $= \frac{\sqrt{3}-5}{2\sqrt{3}-4} \times \frac{2\sqrt{3}+4}{2\sqrt{3}+4} = \frac{(\sqrt{3}-5)(2\sqrt{3}+4)}{12-16} = \frac{6+4\sqrt{3}-10\sqrt{3}-20}{-4} = \tfrac{1}{2}(7+3\sqrt{3})$

g $= \frac{2\sqrt{3}+3}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(2\sqrt{3}+3)(3+\sqrt{3})}{9-3} = \frac{6\sqrt{3}+6+9+3\sqrt{3}}{6} = \tfrac{1}{2}(3\sqrt{3}+5)$

h $= \frac{3\sqrt{7}-2}{2\sqrt{7}-5} \times \frac{2\sqrt{7}+5}{2\sqrt{7}+5} = \frac{(3\sqrt{7}-2)(2\sqrt{7}+5)}{28-25} = \frac{42+15\sqrt{7}-4\sqrt{7}-10}{3} = \tfrac{1}{3}(32+11\sqrt{7})$

1 **a** $= 64$ **b** $= 216$ **c** $= 1$

d $= 625$ **e** $= -243$ **f** $= \frac{1}{16}$

g $= \frac{8}{27}$ **h** $= -\frac{1}{64}$ **i** $= (\frac{4}{3})^2 = \frac{16}{9}$ or $1\frac{7}{9}$

j $= (\frac{3}{2})^4 = \frac{81}{16}$ or $5\frac{1}{16}$ **k** $= 0.000\,01$ **l** $= -0.008$

2 **a** 2^8 **b** 2^7 **c** 2^0 **d** 2^4 **e** 2^9 **f** 2^{14}

3 **a** $= 8p^7$ **b** $= x^{10}$ **c** $= 6n^5$ **d** $= y^{12}$

e $= 2b$ **f** $= p^4q^3$ **g** $= x^3y$ **h** $= 6r^2s^3$

i $= 2x^3y^7$ **j** $= 4a^5b^8$ **k** $= 125r^3s^6 \div 100r^2s^2 = \frac{5}{4}rs^4$ **l** $= 15p^3q$

4 **a** $= \frac{1}{3^2} = \frac{1}{9}$ **b** $= 1$ **c** $= \frac{1}{(-2)^6} = \frac{1}{64}$

d $= 6^2 = 36$ **e** $= (\frac{3}{2})^{-3} = (\frac{2}{3})^3 = \frac{8}{27}$ **f** $= \sqrt{9} = 3$

g $= \sqrt[4]{16} = 2$ **h** $= \sqrt[3]{-27} = -3$ **i** $= \sqrt{\frac{1}{49}} = \frac{1}{7}$

j $= \sqrt[3]{125} = 5$ **k** $= \sqrt{\frac{4}{9}} = \frac{2}{3}$ **l** $= \frac{1}{\sqrt{36}} = \frac{1}{6}$

m $= \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$ **n** $= \frac{1}{\sqrt[3]{-64}} = -\frac{1}{4}$ **o** $= \sqrt[5]{32} = 2$

p $= \sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$ **q** $= \sqrt{\frac{9}{4}} = \frac{3}{2}$ or $1\frac{1}{2}$ **r** $= (\frac{27}{8})^{-\frac{1}{3}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

5 **a** $= (\sqrt{4})^3 = 2^3 = 8$ **b** $= (\sqrt[3]{27})^2 = 3^2 = 9$

c $= (\sqrt[4]{16})^3 = 2^3 = 8$ **d** $= (\sqrt[3]{-125})^2 = (-5)^2 = 25$

e $= (\sqrt{9})^5 = 3^5 = 243$ **f** $= \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$

g $= \frac{1}{(\sqrt{36})^3} = \frac{1}{6^3} = \frac{1}{216}$ **h** $= (\sqrt[3]{\frac{1}{8}})^4 = (\frac{1}{2})^4 = \frac{1}{16}$

i $= (\sqrt{\frac{4}{9}})^3 = (\frac{2}{3})^3 = \frac{8}{27}$ **j** $= (\sqrt[3]{216})^2 = 6^2 = 36$

k $= (\sqrt{\frac{16}{9}})^3 = (\frac{4}{3})^3 = \frac{64}{27}$ or $2\frac{10}{27}$ **l** $= (\sqrt[3]{-\frac{27}{64}})^4 = (-\frac{3}{4})^4 = \frac{81}{256}$

m $= \sqrt{\frac{4}{100}} = \frac{2}{10} = \frac{1}{5}$ or 0.2 **n** $= (\frac{9}{4})^{-\frac{3}{2}} = (\sqrt{\frac{4}{9}})^3 = (\frac{2}{3})^3 = \frac{8}{27}$

o $= (\sqrt[3]{\frac{64}{1000}})^2 = (\frac{4}{10})^2 = \frac{4}{25}$ or 0.16 **p** $= (\frac{25}{16})^{-\frac{3}{2}} = (\sqrt{\frac{16}{25}})^3 = (\frac{4}{5})^3 = \frac{64}{125}$

q $= (\frac{81}{16})^{\frac{3}{4}} = (\sqrt[4]{\frac{81}{16}})^3 = (\frac{3}{2})^3 = \frac{27}{8}$ or $3\frac{3}{8}$ **r** $= (\frac{64}{27})^{-\frac{4}{3}} = (\sqrt[3]{\frac{27}{64}})^4 = (\frac{3}{4})^4 = \frac{81}{256}$

6

a $= \sqrt{4} \times \sqrt[3]{27}$
$= 2 \times 3 = 6$

b $= \sqrt[4]{16} + \sqrt{25}$
$= 2 + 5 = 7$

c $= \frac{1}{\sqrt[3]{8}} \div \sqrt{36}$
$= \frac{1}{2} \div 6 = \frac{1}{12}$

d $= \sqrt[3]{-64} \times (\sqrt{9})^3$
$= -4 \times 27 = -108$

e $= 3^2 - \sqrt[3]{-8}$
$= 9 - (-2) = 11$

f $= \sqrt{\frac{1}{25}} \times 4^2$
$= \frac{1}{5} \times 16 = \frac{16}{5}$ or $3\frac{1}{5}$

g $= (\sqrt[4]{81})^3 - \sqrt{49}$
$= 27 - 7 = 20$

h $= \sqrt[3]{27} \times (\sqrt{\frac{9}{4}})^3$
$= 3 \times \frac{27}{8} = \frac{81}{8}$ or $10\frac{1}{8}$

i $= \sqrt{9} \times (\sqrt[5]{-32})^3$
$= 3 \times (-8) = -24$

j $= \sqrt{121} + \sqrt[5]{32}$
$= 11 + 2 = 13$

k $= \sqrt{100} \div (\sqrt{\frac{1}{4}})^3$
$= 10 \div \frac{1}{8} = 80$

l $= \frac{1}{\sqrt[4]{16}} \times (\sqrt[5]{243})^2$
$= \frac{1}{2} \times 9 = \frac{9}{2}$ or $4\frac{1}{2}$

7

a $= x^2$
b $= y^{-6}$
c $= 3p^{-4}$
d $= 8x^{-12}$

e $= y^{\frac{5}{2}}$
f $= 8b^{\frac{2}{3} + \frac{1}{4}} = 8b^{\frac{11}{12}}$
g $= x^{\frac{3}{5} - \frac{1}{3}} = x^{\frac{4}{15}}$
h $= a^{\frac{1}{2} - \frac{4}{3}} = a^{-\frac{5}{6}}$

i $= p^{\frac{1}{4} - (-\frac{1}{5})} = p^{\frac{9}{20}}$
j $= 9x^{\frac{4}{5}}$
k $= y^{1 + \frac{5}{6} - \frac{3}{2}} = y^{\frac{1}{3}}$
l $= \frac{1}{3}t$

m $= b^{2 + \frac{1}{4} - \frac{1}{2}} = b^{\frac{7}{4}}$
n $= y^{\frac{1}{2} + \frac{1}{3} - 1} = y^{-\frac{1}{6}}$
o $= 2x^{\frac{2}{3} + (-\frac{1}{6}) - \frac{3}{4}} = 2x^{-\frac{1}{4}}$
p $= \frac{1}{4}a^{1 + \frac{3}{4} - (-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{4}}$

8

a $x = 6^2 = 36$

b $x = 5^3 = 125$

c $x^{\frac{1}{2}} = \frac{1}{2}$
$x = (\frac{1}{2})^2 = \frac{1}{4}$

d $x^{\frac{1}{4}} = 3$
$x = 3^4 = 81$

e $x^{\frac{1}{2}} = \sqrt[3]{8} = 2$
$x = 2^2 = 4$

f $x^{\frac{1}{3}} = \pm\sqrt{16} = \pm 4$
$x = (\pm 4)^3 = \pm 64$

g $x^{\frac{1}{3}} = \pm\sqrt[4]{81} = \pm 3$
$x = (\pm 3)^3 = \pm 27$

h $x^{\frac{3}{2}} = \frac{1}{27}$
$x^{\frac{1}{2}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$
$x = (\frac{1}{3})^2 = \frac{1}{9}$

9

a $= x^{\frac{1}{2}}$
b $= x^{-\frac{1}{3}}$
c $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$
d $= \dfrac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$

e $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$
f $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$
g $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$
h $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$

10 **a** $4x^{-\frac{1}{2}}$ **b** $\frac{1}{2}x^{-1}$ **c** $\frac{3}{4}x^{-3}$ **d** $\frac{1}{9}x^{-2}$ **e** $\frac{2}{5}x^{-\frac{1}{3}}$ **f** $\frac{1}{3}x^{-\frac{3}{2}}$

11

a $= (2^3)^2 = 2^6$
b $= (2^{-2})^{-2} = 2^4$
c $= (2^{-1})^{\frac{1}{3}} = 2^{-\frac{1}{3}}$

d $= (2^4)^{-\frac{1}{6}} = 2^{-\frac{2}{3}}$
e $= (2^3)^{\frac{2}{5}} = 2^{\frac{6}{5}}$
f $= (2^{-5})^{-3} = 2^{15}$

12

a $= (3^2)^x = 3^{2x}$
b $= (3^4)^{x+1} = 3^{4x+4}$
c $= (3^3)^{\frac{x}{4}} = 3^{\frac{3}{4}x}$

d $= (3^{-1})^x = 3^{-x}$
e $= (3^2)^{2x-1} = 3^{4x-2}$
f $= (3^{-3})^{x+2} = 3^{-3x-6}$

13

a $= 2 \times 2^x = 2y$
b $= 2^{-2} \times 2^x = \frac{1}{4}y$
c $= (2^x)^2 = y^2$

d $= (2^3)^x = 2^{3x} = (2^x)^3 = y^3$
e $= 2^3 \times 2^{4x} = 8y^4$
f $= (2^{-1})^{x-3} = 2^3 \times 2^{-x} = \dfrac{8}{y}$

14

a $2^x = 2^6$
$x = 6$

b $5^{x-1} = 5^3$
$x - 1 = 3$
$x = 4$

c $3^{x+4} = 27 = 3^3$
$x + 4 = 3$
$x = -1$

d $(2^3)^x = 2^{3x} = 2$
$3x = 1$
$x = \frac{1}{3}$

e $3^{2x-1} = 3^2$
$2x - 1 = 2$
$x = \frac{3}{2}$

f $16 = 4^2 = 4^{3x-2}$
$2 = 3x - 2$
$x = \frac{4}{3}$

g $(3^2)^{x-2} = 3^{2x-4} = 3^3$
$2x - 4 = 3$
$x = \frac{7}{2}$

h $(2^3)^{2x+1} = 2^{6x+3} = 2^4$
$6x + 3 = 4$
$x = \frac{1}{6}$

i $(7^2)^{x+1} = 7^{2x+2} = 7^{\frac{1}{2}}$
$2x + 2 = \frac{1}{2}$
$x = -\frac{3}{4}$

j $3^{3x-2} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}}$
$3x - 2 = \frac{2}{3}$
$x = \frac{8}{9}$

k $(6^{-1})^{x+3} = 6^{-x-3} = 6^2$
$-x - 3 = 2$
$x = -5$

l $(2^{-1})^{3x-1} = 2^{1-3x} = 2^3$
$1 - 3x = 3$
$x = -\frac{2}{3}$

15

a $2^{x+3} = (2^2)^x = 2^{2x}$
$x + 3 = 2x$
$x = 3$

b $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$
$3x = 2x + 2$
$x = 2$

c $(3^2)^{2x} = 3^{4x} = 3^{x-3}$
$4x = x - 3$
$x = -1$

d $(4^2)^x = 4^{2x} = 4^{1-x}$
$2x = 1 - x$
$x = \frac{1}{3}$

e $(2^2)^{x+2} = (2^3)^x$
$2^{2x+4} = 2^{3x}$
$2x + 4 = 3x$
$x = 4$

f $(3^3)^{2x} = (3^2)^{3-x}$
$3^{6x} = 3^{6-2x}$
$6x = 6 - 2x$
$x = \frac{3}{4}$

g $6^{3x-1} = (6^2)^{x+2}$
$6^{3x-1} = 6^{2x+4}$
$3x - 1 = 2x + 4$
$x = 5$

h $(2^3)^x = (2^4)^{2x-1}$
$2^{3x} = 2^{8x-4}$
$3x = 8x - 4$
$x = \frac{4}{5}$

i $(5^3)^x = 5^{x-3}$
$5^{3x} = 5^{x-3}$
$3x = x - 3$
$x = -\frac{3}{2}$

j $(3^{-1})^x = 3^{x-4}$
$3^{-x} = 3^{x-4}$
$-x = x - 4$
$x = 2$

k $(2^{-1})^{1-x} = (2^{-3})^{2x}$
$2^{x-1} = 2^{-6x}$
$x - 1 = -6x$
$x = \frac{1}{7}$

l $(2^{-2})^{x+1} = (2^3)^x$
$2^{-2x-2} = 2^{3x}$
$-2x - 2 = 3x$
$x = -\frac{2}{5}$

16

a $= x^3 - 1$

b $= 2x^2 + 6x^3$

c $= 3 - x^2$

d $= 12x^3 + 8x$

e $= 3x^3 + 2x$

f $= 3 - 3x^2$

g $= 5x^{\frac{1}{2}} + x^2$

h $= 3x^2 - x^{-1}$

i $= x^6 + x^4 - 3x^2 - 3$

j $= 2x^9 + 6x^5 + x^5 + 3x$
$= 2x^9 + 7x^5 + 3x$

k $= x^3 - 1 - 2 + 2x^{-3}$
$= x^3 - 3 + 2x^{-3}$

l $= x^3 - x^{\frac{5}{2}} - x^{\frac{5}{2}} + x^2$
$= x^3 - 2x^{\frac{5}{2}} + x^2$

17

a $= x^2 + 2$

b $= 2t^3 - 3t$

c $= x - 3x^{\frac{1}{2}}$

d $= \dfrac{y^5 - 6y^2}{3y}$
$= \frac{1}{3}y^4 - 2y$

e $= p^{\frac{1}{4}} + p^{\frac{3}{4}}$

f $= 2w^{\frac{3}{2}} - \frac{1}{2}w$

g $= \dfrac{x^{\frac{1}{2}}(x+1)}{x+1}$
$= x^{\frac{1}{2}}$

h $= \dfrac{t^{\frac{1}{2}} \times 2t(t^2-2)}{t^2-2}$
$= 2t^{\frac{3}{2}}$

1 **a** $= \sqrt{9}\sqrt{3} + 2\sqrt{25}\sqrt{2}$
$= 10\sqrt{2} + 3\sqrt{3}$

b $= \sqrt{18} - \sqrt{48}$
$= \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{3}$
$= 3\sqrt{2} - 4\sqrt{3}$

2 $x^2 - 2x = 12 - 2x$
$x^2 = 12$
$x = \pm\sqrt{12} = \pm 2\sqrt{3}$
$x > 0 \;\therefore\; x = 2\sqrt{3}$

3 $25^x = (5^2)^x = 5^{4x+1}$
$5^{2x} = 5^{4x+1}$
$2x = 4x + 1$
$x = -\frac{1}{2}$

4 **a** $= \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$

b $\sqrt[3]{81} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$
$\therefore\; \sqrt[3]{24} + \sqrt[3]{81} = 2\sqrt[3]{3} + 3\sqrt[3]{3} = 5\sqrt[3]{3}$
$= \sqrt[3]{125\times 3} = \sqrt[3]{375}$
$\therefore\; n = 375$

5 $\frac{10\sqrt{3}}{\sqrt{15}} = \frac{10\sqrt{3}}{\sqrt{5}\sqrt{3}} = \frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = 2\sqrt{5}$
$\frac{4}{\sqrt{5}-\sqrt{7}}\times\frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} = \frac{4(\sqrt{5}+\sqrt{7})}{5-7} = -2\sqrt{5} - 2\sqrt{7}$
$\therefore\; \frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}} = 2\sqrt{5} - 2\sqrt{5} - 2\sqrt{7}$
$= -2\sqrt{7}$ $[\, k = -2 \,]$

6 **a** $= \sqrt{\frac{75}{2}} = \frac{5\sqrt{3}}{\sqrt{2}} = \frac{5\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{5}{2}\sqrt{6}$

b $= \sqrt{\frac{48}{5}} - \sqrt{\frac{20}{3}} = \frac{4\sqrt{3}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}}$
$= \frac{4\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{4}{5}\sqrt{15} - \frac{2}{3}\sqrt{15}$
$= \frac{2}{15}\sqrt{15}$

7 **a** **i** $xy = 2^{t-1} \times 2^{3t} = 2^{4t-1}$
ii $2y^2 = 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1}$

b $2^{6t+1} - 2^{4t-1} = 0$
$2^{6t+1} = 2^{4t-1}$
$6t + 1 = 4t - 1$
$t = -1$

8 $3x\sqrt{2} - \sqrt{2} = 4x + 6$
$x(3\sqrt{2} - 4) = 6 + \sqrt{2}$
$x = \frac{6+\sqrt{2}}{3\sqrt{2}-4} = \frac{6+\sqrt{2}}{3\sqrt{2}-4}\times\frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{(6+\sqrt{2})(3\sqrt{2}+4)}{18-16}$
$= \frac{1}{2}(18\sqrt{2} + 24 + 6 + 4\sqrt{2})$
$= \frac{1}{2}(30 + 22\sqrt{2})$
$= 15 + 11\sqrt{2}$

9 **a** $6^{y+1} = 36^{x-2} = (6^2)^{x-2}$
$6^{y+1} = 6^{2x-4}$
$y + 1 = 2x - 4$
$y = 2x - 5$

b $x - \frac{1}{2}y = x - \frac{1}{2}(2x - 5) = x - x + \frac{5}{2} = \frac{5}{2}$
$\therefore\; 4^{x - \frac{1}{2}y} = 4^{\frac{5}{2}} = (\sqrt{4})^5 = 2^5 = 32$

10 **a** $= 3 + 3\sqrt{2} - \sqrt{2} - 2$
$= 1 + 2\sqrt{2}$

b $= \frac{\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1}$
$= \sqrt{2}(\sqrt{2} + 1)$
$= 2 + \sqrt{2}$

11 $(2^4)^{x+1} = (2^3)^{2x+1}$

$2^{4x+4} = 2^{6x+3}$

$4x + 4 = 6x + 3$

$x = \frac{1}{2}$

12 $a^2 - 4a\sqrt{3} + 12 = b - 20\sqrt{3}$

a and b integers $\therefore$ $-4a = -20$

$a = 5$

also $a^2 + 12 = b$

$b = 37$

13 **a** $(2^{-2})^{t-3} = 2^3$

$2^{6-2t} = 2^3$

$6 - 2t = 3$

$t = \frac{3}{2}$

b $(3^{-1})^y = (3^3)^{y+1}$

$3^{-y} = 3^{3y+3}$

$-y = 3y + 3$

$y = -\frac{3}{4}$

14 **a** $= 2\sqrt{5}(\sqrt{5} - 3)$

$= 10 - 6\sqrt{5}$

b $= 3 + 2\sqrt{5} - 3\sqrt{5} - 10$

$= -7 - \sqrt{5}$

c $= \frac{1+\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{(1+\sqrt{5})(\sqrt{5}+2)}{5-4}$

$= (1 + \sqrt{5})(\sqrt{5} + 2)$

$= \sqrt{5} + 2 + 5 + 2\sqrt{5}$

$= 7 + 3\sqrt{5}$

15 **a** $a = (b^{\frac{3}{4}})^3 = b^{\frac{9}{4}}$

$a^{\frac{1}{2}} = (b^{\frac{9}{4}})^{\frac{1}{2}} = b^{\frac{9}{8}}$

b $b = (a^{\frac{1}{3}})^{\frac{4}{3}} = a^{\frac{4}{9}}$

$b^{\frac{1}{2}} = (a^{\frac{4}{9}})^{\frac{1}{2}} = a^{\frac{2}{9}}$

16 **a** area $= \frac{1}{2}(2\sqrt{3} - 1)(\sqrt{3} + 2)$

$= \frac{1}{2}(6 + 4\sqrt{3} - \sqrt{3} - 2)$

$= \frac{1}{2}(4 + 3\sqrt{3})$ or $2 + \frac{3}{2}\sqrt{3}$

b $AC^2 = (2\sqrt{3} - 1)^2 + (\sqrt{3} + 2)^2$

$= 12 - 4\sqrt{3} + 1 + 3 + 4\sqrt{3} + 4 = 20$

$\therefore$ $AC = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

c $\tan(\angle ACB) = \frac{2\sqrt{3}-1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{(2\sqrt{3}-1)(\sqrt{3}-2)}{3-4}$

$= -(2\sqrt{3} - 1)(\sqrt{3} - 2)$

$= -(6 - 4\sqrt{3} - \sqrt{3} + 2)$

$= -(8 - 5\sqrt{3}) = 5\sqrt{3} - 8$

17 **a** **i** $2^{x+2} = 2^2 \times 2^x = 4y$

ii $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

b $y^2 - 4y = 0$

$y(y - 4) = 0$

$y = 0$ or 4

$2^x = 0$ (no solutions) or $2^x = 4$

$x = 2$

18 $5\sqrt{3} = 2(1 + \sqrt{3})^2 + p(1 + \sqrt{3}) + q$

$5\sqrt{3} = 2 + 4\sqrt{3} + 6 + p + p\sqrt{3} + q$

p, q rational $\therefore$ $5\sqrt{3} = 4\sqrt{3} + p\sqrt{3}$

$p = 1$

and $0 = 2 + 6 + p + q$

$q = -9$

1 **a** $= 3x^2 + 8x + 3$

b $= x^3 + 5x^2 - 2x + 1$

c $= -3x^3 + 6x^2 - 2x + 7$

d $= x^5 - x^4 + 8x^3 - 5x^2 - 4x - 8$

e $= 3x^3 - 7x^2 + 2 - x^3 - 2x^2 - x + 6$
$= 2x^3 - 9x^2 - x + 8$

f $= x^5 + 3x^4 - x^2 - 3 - x^4 - 2x^3 + 3x - 2$
$= x^5 + 2x^4 - 2x^3 - x^2 + 3x - 5$

g $= 2x^7 - 9x^5 + x^3 + x - 3x^6 + 4x^3 - x - 5$
$= 2x^7 - 3x^6 - 9x^5 + 5x^3 - 5$

h $= 2x^4 + 8x^2 - 6 + x^4 + 3x^3 - 8$
$= 3x^4 + 3x^3 + 8x^2 - 14$

i $= 21 + 12x - 3x^2 - 6x^3 - 10 - 15x + 5x^3$
$= -x^3 - 3x^2 - 3x + 11$

j $= 6x^3 + 30x^2 - 12 - 6x^3 + 3x^2 + 3x$
$= 33x^2 + 3x - 12$

k $= 8x^4 + 16x^2 - 32x - 8 - 10 + 6x - 2x^3$
$= 8x^4 - 2x^3 + 16x^2 - 26x - 18$

l $= 7x^6 + 21x^3 + 7x^2 - 28 - 8x^6 - 4x^5 + 12x + 28$
$= -x^6 - 4x^5 + 21x^3 + 7x^2 + 12x$

2 **a** $= 3y^3 - 2y^2 + y + 6$

b $= 3t^4 - 3t^3 + 12t + 6 - t - 3t^3 + 2t^4 - 4t^2 + 8$
$= 5t^4 - 6t^3 - 4t^2 + 11t + 14$

c $= x^3 - 6x^2 + 8 + 5x^2 - x + 1 - 2x^3 - 3x^2 - x + 7$
$= -x^3 - 4x^2 - 2x + 16$

d $= 6 + 2m + 14m^2 - 6m^5 + 6 - 6m^2 + 12m^4 - 5m^5 - 15m^3 + 5m^2 - 10$
$= 2 + 2m + 13m^2 - 15m^3 + 12m^4 - 11m^5$

e $= \frac{1}{3} - \frac{2}{3}u + \frac{1}{5}u^2 + u^4 - 1 + \frac{1}{2}u - \frac{1}{3}u^2 + \frac{1}{4}u^3$
$= -\frac{2}{3} - \frac{1}{6}u - \frac{2}{15}u^2 + \frac{1}{4}u^3 + u^4$

3 **a** $= 2x - 3x^2 + x^3 + 4 + 8x^2 - 4x^3$
$= 4 + 2x + 5x^2 - 3x^3$

b $= x^5 + 7x^3 - 5x^2 + 9x - 2x^4 + 8x^3 + 6$
$= 6 + 9x - 5x^2 + 15x^3 - 2x^4 + x^5$

c $= -10x + 8x^2 - 2x^4 + 14 - 21x^2 + 7x^4$
$= 14 - 10x - 13x^2 + 5x^4$

d $= 8x^2 + 2x^3 + x^4 - 15 - 12x^2 - 3x^3$
$= -15 - 4x^2 - x^3 + x^4$

e $= 3x^3 + 9x^2 - x^4 - 4x^3 + 5x^3 - 10x$
$= -10x + 9x^2 + 4x^3 - x^4$

f $= 6x^2 - x^3 + 5x^4 + 14x - 7x^4 + 4 - 12x - 4x^2$
$= 4 + 2x + 2x^2 - x^3 - 2x^4$

4 **a** LHS $= (3x + 1)(x^2 - 2x + 4)$
$= 3x(x^2 - 2x + 4) + (x^2 - 2x + 4)$
$= 3x^3 - 6x^2 + 12x + x^2 - 2x + 4$
$= 3x^3 - 5x^2 + 10x + 4 =$ RHS

b LHS $= (1 + 2x - x^2)(1 - 2x + x^2)$
$= (1 - 2x + x^2) + 2x(1 - 2x + x^2) - x^2(1 - 2x + x^2)$
$= 1 - 2x + x^2 + 2x - 4x^2 + 2x^3 - x^2 + 2x^3 - x^4$
$= 1 - 4x^2 + 4x^3 - x^4 =$ RHS

c LHS $= (3 - x)^3$
$= (3 - x)(9 - 6x + x^2)$
$= 3(9 - 6x + x^2) - x(9 - 6x + x^2)$
$= 27 - 18x + 3x^2 - 9x + 6x^2 - x^3$
$= 27 - 27x + 9x^2 - x^3 =$ RHS

5 **a** $= x(x^2 + 5x - 6) + (x^2 + 5x - 6)$
$= x^3 + 5x^2 - 6x + x^2 + 5x - 6$
$= x^3 + 6x^2 - x - 6$

b $= 2x(x^2 - 3x + 7) - 5(x^2 - 3x + 7)$
$= 2x^3 - 6x^2 + 14x - 5x^2 + 15x - 35$
$= 2x^3 - 11x^2 + 29x - 35$

c $= 4(2 + 5x - x^2) - 7x(2 + 5x - x^2)$
$= 8 + 20x - 4x^2 - 14x - 35x^2 + 7x^3$
$= 7x^3 - 39x^2 + 6x + 8$

d $= (3x - 2)(3x - 2)^2 = (3x - 2)(9x^2 - 12x + 4)$
$= 3x(9x^2 - 12x + 4) - 2(9x^2 - 12x + 4)$
$= 27x^3 - 36x^2 + 12x - 18x^2 + 24x - 8$
$= 27x^3 - 54x^2 + 36x - 8$

e $= x^2(2x^2 - x + 9) + 3(2x^2 - x + 9)$
$= 2x^4 - x^3 + 9x^2 + 6x^2 - 3x + 27$
$= 2x^4 - x^3 + 15x^2 - 3x + 27$

f $= 4x(x^4 - 3x^2 + 5x + 2) - (x^4 - 3x^2 + 5x + 2)$
$= 4x^5 - 12x^3 + 20x^2 + 8x - x^4 + 3x^2 - 5x - 2$
$= 4x^5 - x^4 - 12x^3 + 23x^2 + 3x - 2$

g $= x^2(x^2 + 3x + 1) + 2x(x^2 + 3x + 1) + 5(x^2 + 3x + 1)$
$= x^4 + 3x^3 + x^2 + 2x^3 + 6x^2 + 2x + 5x^2 + 15x + 5$
$= x^4 + 5x^3 + 12x^2 + 17x + 5$

h $= x^2(2x^2 - x + 4) + x(2x^2 - x + 4) - 3(2x^2 - x + 4)$
$= 2x^4 - x^3 + 4x^2 + 2x^3 - x^2 + 4x - 6x^2 + 3x - 12$
$= 2x^4 + x^3 - 3x^2 + 7x - 12$

i $= 3x^2(2x^2 - 4x - 8) - 5x(2x^2 - 4x - 8) + 2(2x^2 - 4x - 8)$
$= 6x^4 - 12x^3 - 24x^2 - 10x^3 + 20x^2 + 40x + 4x^2 - 8x - 16$
$= 6x^4 - 22x^3 + 32x - 16$

j $= x^2(x^2 + 2x - 6) + 2x(x^2 + 2x - 6) - 6(x^2 + 2x - 6)$
$= x^4 + 2x^3 - 6x^2 + 2x^3 + 4x^2 - 12x - 6x^2 - 12x + 36$
$= x^4 + 4x^3 - 8x^2 - 24x + 36$

k $= x^3(2x^4 + x^2 + 3) + 4x^2(2x^4 + x^2 + 3) + (2x^4 + x^2 + 3)$
$= 2x^7 + x^5 + 3x^3 + 8x^6 + 4x^4 + 12x^2 + 2x^4 + x^2 + 3$
$= 2x^7 + 8x^6 + x^5 + 6x^4 + 3x^3 + 13x^2 + 3$

l $= 6(3 + x^2 - x^3 + 2x^4) - 2x(3 + x^2 - x^3 + 2x^4) + x^3(3 + x^2 - x^3 + 2x^4)$
$= 18 + 6x^2 - 6x^3 + 12x^4 - 6x - 2x^3 + 2x^4 - 4x^5 + 3x^3 + x^5 - x^6 + 2x^7$
$= 2x^7 - x^6 - 3x^5 + 14x^4 - 5x^3 + 6x^2 - 6x + 18$

6 **a** $= (p^2 - 1)(2p^2 + 11p + 12)$
$= p^2(2p^2 + 11p + 12) - (2p^2 + 11p + 12)$
$= 2p^4 + 11p^3 + 12p^2 - 2p^2 - 11p - 12$
$= 2p^4 + 11p^3 + 10p^2 - 11p - 12$

b $= t(t^2 + 3t + 5) + 2(t^2 + 3t + 5) + t(t^2 + t + 7) + 4(t^2 + t + 7)$
$= t^3 + 3t^2 + 5t + 2t^2 + 6t + 10 + t^3 + t^2 + 7t + 4t^2 + 4t + 28$
$= 2t^3 + 10t^2 + 22t + 38$

c $= 2x^2(x^2 + x - 4) - 6(x^2 + x - 4) + 3x(4x^3 + 2x^2 - x + 6) - (4x^3 + 2x^2 - x + 6)$
$= 2x^4 + 2x^3 - 8x^2 - 6x^2 - 6x + 24 + 12x^4 + 6x^3 - 3x^2 + 18x - 4x^3 - 2x^2 + x - 6$
$= 14x^4 + 4x^3 - 19x^2 + 13x + 18$

d $= u(u^3 - 4u^2 - 3) + 2(u^3 - 4u^2 - 3) - 2u^3(u^2 + 5u - 3) - u(u^2 + 5u - 3) + (u^2 + 5u - 3)$
$= u^4 - 4u^3 - 3u + 2u^3 - 8u^2 - 6 - 2u^5 - 10u^4 + 6u^3 - u^3 - 5u^2 + 3u + u^2 + 5u - 3$
$= -2u^5 - 9u^4 + 3u^3 - 12u^2 + 5u - 9$

C1 ALGEBRA — *Answers - Worksheet E*

1

a $(x+1)(x+3)$	**b** $(x+2)(x+5)$	**c** $(y-1)(y-2)$	**d** $(x-3)^2$
e $(y+1)(y-2)$	**f** $(a+4)(a-2)$	**g** $(x+1)(x-1)$	**h** $(p+2)(p+7)$
i $(x+3)(x-5)$	**j** $(m-2)(m-8)$	**k** $(t+6)(t-3)$	**l** $(y-5)(y-8)$
m $(r+4)(r-4)$	**n** $(y+7)(y-9)$	**o** $(a+11)^2$	**p** $(x+12)(x-6)$
q $(x-2)(x-13)$	**r** $(s+8)(s+15)$	**s** $(p+17)(p-3)$	**t** $(m-10)(m+9)$

2

a $(2x+1)(x+1)$	**b** $(3p+1)(p+2)$	**c** $(2y-3)(y-1)$	**d** $(2+m)(1-m)$
e $(3r+1)(r-1)$	**f** $(5+y)(1-4y)$	**g** $(3a-1)(a-4)$	**h** $(5x+2)(x-2)$
i $(2x+1)(2x+3)$	**j** $(3s-1)^2$	**k** $(2m+5)(2m-5)$	**l** $(2+3y)(1-2y)$
m $(4u+1)(u+4)$	**n** $(3p+4)(2p-1)$	**o** $(8x+3)(x+2)$	**p** $(6r-5)(2r+3)$

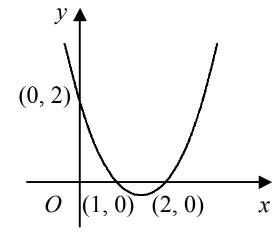
3

a $(x-1)(x-3)=0$ $x=1$ or 3	**b** $(x+4)(x+2)=0$ $x=-4$ or -2	**c** $(x+5)(x-1)=0$ $x=-5$ or 1	**d** $x^2-7x-8=0$ $(x+1)(x-8)=0$ $x=-1$ or 8
e $(x+5)(x-5)=0$ $x=-5$ or 5	**f** $x^2-x-42=0$ $(x+6)(x-7)=0$ $x=-6$ or 7	**g** $x^2-3x=0$ $x(x-3)=0$ $x=0$ or 3	**h** $(x+9)(x+3)=0$ $x=-9$ or -3
i $x^2+4x-60=0$ $(x+10)(x-6)=0$ $x=-10$ or 6	**j** $x^2-5x-14=0$ $(x+2)(x-7)=0$ $x=-2$ or 7	**k** $(2x-1)(x-1)=0$ $x=\frac{1}{2}$ or 1	**l** $x^2-x=6x-12$ $x^2-7x+12=0$ $(x-3)(x-4)=0$ $x=3$ or 4
m $3x^2+11x-4=0$ $(3x-1)(x+4)=0$ $x=-4$ or $\frac{1}{3}$	**n** $2x^2-3x-5=0$ $(2x-5)(x+1)=0$ $x=-1$ or $\frac{5}{2}$	**o** $4x^2-23x-6=0$ $(4x+1)(x-6)=0$ $x=-\frac{1}{4}$ or 6	**p** $6x^2-19x+10=0$ $(3x-2)(2x-5)=0$ $x=\frac{2}{3}$ or $\frac{5}{2}$
q $(2x+1)^2=0$ $x=-\frac{1}{2}$	**r** $3x^2-13x+12=0$ $(3x-4)(x-3)=0$ $x=\frac{4}{3}$ or 3	**s** $4x^2+20x+25=5-x$ $4x^2+21x+20=0$ $(4x+5)(x+4)=0$ $x=-4$ or $-\frac{5}{4}$	**t** $6x^2-21x=14x+6$ $6x^2-35x-6=0$ $(6x+1)(x-6)=0$ $x=-\frac{1}{6}$ or 6

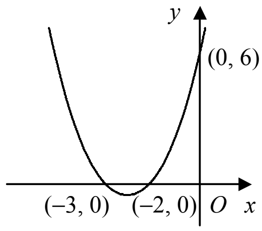
4

a $=2(y^2-5y+6)$ $=2(y-3)(y-2)$	**b** $=x(x^2+x-2)$ $=x(x-1)(x+2)$	**c** $=p(p^2-4)$ $=p(p+2)(p-2)$	**d** $=3m(m^2+7m+6)$ $=3m(m+1)(m+6)$
e $=(a^2+1)(a^2+3)$	**f** $=(t^2+5)(t^2-2)$	**g** $=4(3+5x-2x^2)$ $=4(3-x)(1+2x)$	**h** $=3(2r^2-3r-14)$ $=3(2r-7)(r+2)$
i $=2x(3x^2-13x+4)$ $=2x(3x-1)(x-4)$	**j** $=y^2(y^2+3y-18)$ $=y^2(y+6)(y-3)$	**k** $=(m^2+1)(m^2-1)$ $=(m^2+1)(m+1)(m-1)$	**l** $=p(p^4-4p^2+4)$ $=p(p^2-2)^2$

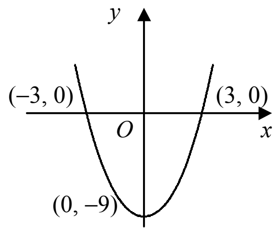
5 **a** $x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
$x = 1$ or 2



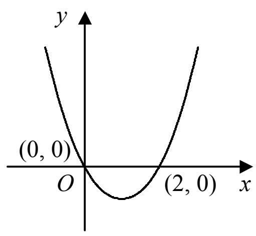
b $x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or -2



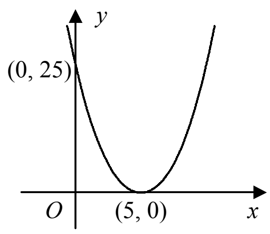
c $x^2 - 9 = 0$
$(x + 3)(x - 3) = 0$
$x = -3$ or 3



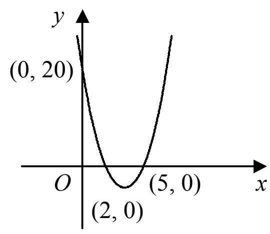
d $x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ or 2



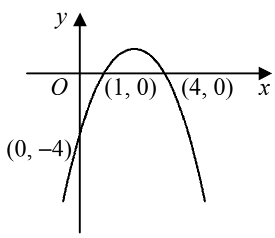
e $x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x = 5$



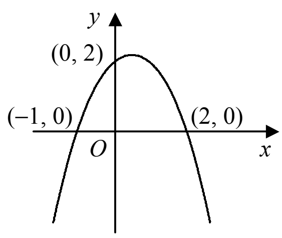
f $2x^2 - 14x + 20 = 0$
$2(x - 2)(x - 5) = 0$
$x = 2$ or 5



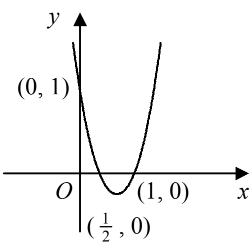
g $-x^2 + 5x - 4 = 0$
$x^2 - 5x + 4 = 0$
$(x - 1)(x - 4) = 0$
$x = 1$ or 4



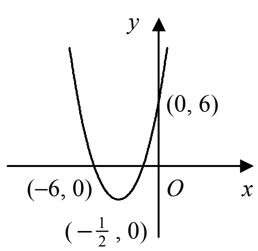
h $2 + x - x^2 = 0$
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2



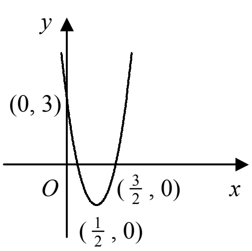
i $2x^2 - 3x + 1 = 0$
$(2x - 1)(x - 1) = 0$
$x = \frac{1}{2}$ or 1



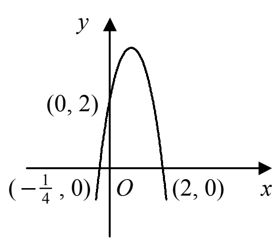
j $2x^2 + 13x + 6 = 0$
$(2x + 1)(x + 6) = 0$
$x = -6$ or $-\frac{1}{2}$



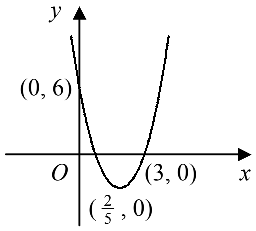
k $3 - 8x + 4x^2 = 0$
$(2x - 1)(2x - 3) = 0$
$x = \frac{1}{2}$ or $\frac{3}{2}$



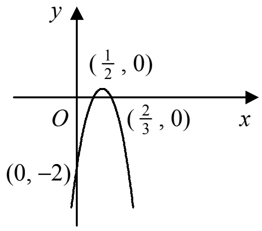
l $2 + 7x - 4x^2 = 0$
$4x^2 - 7x - 2 = 0$
$(4x + 1)(x - 2) = 0$
$x = -\frac{1}{4}$ or 2



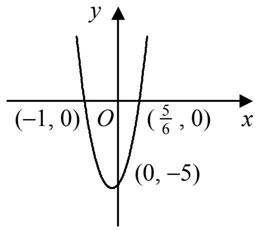
m $5x^2 - 17x + 6 = 0$
$(5x - 2)(x - 3) = 0$
$x = \frac{2}{5}$ or 3



n $-6x^2 + 7x - 2 = 0$
$6x^2 - 7x + 2 = 0$
$(2x - 1)(3x - 2) = 0$
$x = \frac{1}{2}$ or $\frac{2}{3}$



o $6x^2 + x - 5 = 0$
$(6x - 5)(x + 1) = 0$
$x = -1$ or $\frac{5}{6}$



6 **a** $x^2 - 5x + 4 = 0$
$(x - 1)(x - 4) = 0$
$x = 1$ or 4

b $x^2 - 10 = 3x$
$x^2 - 3x - 10 = 0$
$(x + 2)(x - 5) = 0$
$x = -2$ or 5

c $x(2x^2 - x - 3) = 0$
$x(2x - 3)(x + 1) = 0$
$x = -1, 0$ or $\frac{3}{2}$

d $10x^2 - x^4 = 9$
$x^4 - 10x^2 + 9 = 0$
$(x^2 - 1)(x^2 - 9) = 0$
$x^2 = 1$ or 9
$x = \pm 1$ or ± 3

e $5 + 4x - x^2 = 0$
$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$x = -1$ or 5

f $x - 6 = x(x - 4)$
$x - 6 = x^2 - 4x$
$x^2 - 5x + 6 = 0$
$(x - 2)(x - 3) = 0$
$x = 2$ or 3

g $(x + 5)(x + 3) = 3$
$x^2 + 8x + 15 = 3$
$x^2 + 8x + 12 = 0$
$(x + 6)(x + 2) = 0$
$x = -6$ or -2

h $x^4 - 4 = 3x^2$
$x^4 - 3x^2 - 4 = 0$
$(x^2 + 1)(x^2 - 4) = 0$
$x^2 = -1$ (no sol's) or 4
$x = \pm 2$

i $4x^4 + 7x^2 - 2 = 0$
$(4x^2 - 1)(x^2 + 2) = 0$
$x^2 = -2$ (no sol's) or $\frac{1}{4}$
$x = \pm \frac{1}{2}$

j $2x(x + 2) = 3 - x$
$2x^2 + 4x = 3 - x$
$2x^2 + 5x - 3 = 0$
$(2x - 1)(x + 3) = 0$
$x = -3$ or $\frac{1}{2}$

k $x(2x + 1) = 2(x + 3)$
$2x^2 + x = 2x + 6$
$2x^2 - x - 6 = 0$
$(2x + 3)(x - 2) = 0$
$x = -\frac{3}{2}$ or 2

l $7 - 3x(x + 2) = 2(x + 2)$
$7 - 3x^2 - 6x = 2x + 4$
$3x^2 + 8x - 3 = 0$
$(3x - 1)(x + 3) = 0$
$x = -3$ or $\frac{1}{3}$

1

a $$\begin{aligned} &= (x+1)^2 - 1 + 4 \\ &= (x+1)^2 + 3 \end{aligned}$$

b $$\begin{aligned} &= (x-1)^2 - 1 + 4 \\ &= (x-1)^2 + 3 \end{aligned}$$

c $$\begin{aligned} &= (x-2)^2 - 4 + 1 \\ &= (x-2)^2 - 3 \end{aligned}$$

d $$= (x+3)^2 - 9$$

e $$\begin{aligned} &= (x+2)^2 - 4 + 8 \\ &= (x+2)^2 + 4 \end{aligned}$$

f $$\begin{aligned} &= (x-4)^2 - 16 - 5 \\ &= (x-4)^2 - 21 \end{aligned}$$

g $$\begin{aligned} &= (x+6)^2 - 36 + 30 \\ &= (x+6)^2 - 6 \end{aligned}$$

h $$\begin{aligned} &= (x-5)^2 - 25 + 25 \\ &= (x-5)^2 \end{aligned}$$

i $$\begin{aligned} &= (x+3)^2 - 9 - 9 \\ &= (x+3)^2 - 18 \end{aligned}$$

j $$\begin{aligned} &= (x-2)^2 - 4 + 18 \\ &= (x-2)^2 + 14 \end{aligned}$$

k $$\begin{aligned} &= (x+\tfrac{3}{2})^2 - \tfrac{9}{4} + 3 \\ &= (x+\tfrac{3}{2})^2 + \tfrac{3}{4} \end{aligned}$$

l $$\begin{aligned} &= (x+\tfrac{1}{2})^2 - \tfrac{1}{4} - 1 \\ &= (x+\tfrac{1}{2})^2 - \tfrac{5}{4} \end{aligned}$$

m $$\begin{aligned} &= (x-9)^2 - 81 + 100 \\ &= (x-9)^2 + 19 \end{aligned}$$

n $$\begin{aligned} &= (x-\tfrac{1}{2})^2 - \tfrac{1}{4} - \tfrac{1}{2} \\ &= (x-\tfrac{1}{2})^2 - \tfrac{3}{4} \end{aligned}$$

o $$\begin{aligned} &= (x+\tfrac{9}{2})^2 - \tfrac{81}{4} + 20 \\ &= (x+\tfrac{9}{2})^2 - \tfrac{1}{4} \end{aligned}$$

p $$\begin{aligned} &= (x-\tfrac{7}{2})^2 - \tfrac{49}{4} - 2 \\ &= (x-\tfrac{7}{2})^2 - \tfrac{57}{4} \end{aligned}$$

q $$\begin{aligned} &= (x-\tfrac{3}{2})^2 - \tfrac{9}{4} + 5 \\ &= (x-\tfrac{3}{2})^2 + \tfrac{11}{4} \end{aligned}$$

r $$\begin{aligned} &= (x-\tfrac{11}{2})^2 - \tfrac{121}{4} + 37 \\ &= (x-\tfrac{11}{2})^2 + \tfrac{27}{4} \end{aligned}$$

s $$\begin{aligned} &= (x+\tfrac{1}{3})^2 - \tfrac{1}{9} + 1 \\ &= (x+\tfrac{1}{3})^2 + \tfrac{8}{9} \end{aligned}$$

t $$\begin{aligned} &= (x-\tfrac{1}{4})^2 - \tfrac{1}{16} - \tfrac{1}{4} \\ &= (x-\tfrac{1}{4})^2 - \tfrac{5}{16} \end{aligned}$$

2

a $$\begin{aligned} &= 2[x^2 + 2x] + 3 \\ &= 2[(x+1)^2 - 1] + 3 \\ &= 2(x+1)^2 + 1 \end{aligned}$$

b $$\begin{aligned} &= 2[x^2 - 4x] - 7 \\ &= 2[(x-2)^2 - 4] - 7 \\ &= 2(x-2)^2 - 15 \end{aligned}$$

c $$\begin{aligned} &= 3[x^2 - 2x] + 3 \\ &= 3[(x-1)^2 - 1] + 3 \\ &= 3(x-1)^2 \end{aligned}$$

d $$\begin{aligned} &= 4[x^2 + 6x] + 11 \\ &= 4[(x+3)^2 - 9] + 11 \\ &= 4(x+3)^2 - 25 \end{aligned}$$

e $$\begin{aligned} &= -[x^2 + 2x] - 5 \\ &= -[(x+1)^2 - 1] - 5 \\ &= -(x+1)^2 - 4 \end{aligned}$$

f $$\begin{aligned} &= -[x^2 - 10x] + 1 \\ &= -[(x-5)^2 - 25] + 1 \\ &= -(x-5)^2 + 26 \end{aligned}$$

g $$\begin{aligned} &= 2[x^2 + x] - 1 \\ &= 2[(x+\tfrac{1}{2})^2 - \tfrac{1}{4}] - 1 \\ &= 2(x+\tfrac{1}{2})^2 - \tfrac{3}{2} \end{aligned}$$

h $$\begin{aligned} &= 3[x^2 - 3x] + 5 \\ &= 3[(x-\tfrac{3}{2})^2 - \tfrac{9}{4}] + 5 \\ &= 3(x-\tfrac{3}{2})^2 - \tfrac{7}{4} \end{aligned}$$

i $$\begin{aligned} &= 3[x^2 - 8x] + 48 \\ &= 3[(x-4)^2 - 16] + 48 \\ &= 3(x-4)^2 \end{aligned}$$

j $$\begin{aligned} &= 3[x^2 - 5x] \\ &= 3[(x-\tfrac{5}{2})^2 - \tfrac{25}{4}] \\ &= 3(x-\tfrac{5}{2})^2 - \tfrac{75}{4} \end{aligned}$$

k $$\begin{aligned} &= 5[x^2 + 8x] + 70 \\ &= 5[(x+4)^2 - 16] + 70 \\ &= 5(x+4)^2 - 10 \end{aligned}$$

l $$\begin{aligned} &= 2[x^2 + \tfrac{5}{2}x] + 2 \\ &= 2[(x+\tfrac{5}{4})^2 - \tfrac{25}{16}] + 2 \\ &= 2(x+\tfrac{5}{4})^2 - \tfrac{9}{8} \end{aligned}$$

m $$\begin{aligned} &= 4[x^2 + \tfrac{3}{2}x] - 7 \\ &= 4[(x+\tfrac{3}{4})^2 - \tfrac{9}{16}] - 7 \\ &= 4(x+\tfrac{3}{4})^2 - \tfrac{37}{4} \end{aligned}$$

n $$\begin{aligned} &= -2[x^2 - 2x] - 1 \\ &= -2[(x-1)^2 - 1] - 1 \\ &= -2(x-1)^2 + 1 \end{aligned}$$

o $$\begin{aligned} &= -3[x^2 + \tfrac{2}{3}x] + 4 \\ &= -3[(x+\tfrac{1}{3})^2 - \tfrac{1}{9}] + 4 \\ &= -3(x+\tfrac{1}{3})^2 + \tfrac{13}{3} \end{aligned}$$

p $$\begin{aligned} &= \tfrac{1}{3}[x^2 + \tfrac{3}{2}x] - \tfrac{1}{4} \\ &= \tfrac{1}{3}[(x+\tfrac{3}{4})^2 - \tfrac{9}{16}] - \tfrac{1}{4} \\ &= \tfrac{1}{3}(x+\tfrac{3}{4})^2 - \tfrac{7}{16} \end{aligned}$$

3

a $$\begin{aligned} &(y-2)^2 - 4 + 2 = 0 \\ &(y-2)^2 = 2 \\ &y - 2 = \pm\sqrt{2} \\ &y = 2 \pm \sqrt{2} \end{aligned}$$

b $$\begin{aligned} &(p+1)^2 - 1 - 2 = 0 \\ &(p+1)^2 = 3 \\ &p + 1 = \pm\sqrt{3} \\ &p = -1 \pm \sqrt{3} \end{aligned}$$

c $$\begin{aligned} &(x-3)^2 - 9 + 4 = 0 \\ &(x-3)^2 = 5 \\ &x - 3 = \pm\sqrt{5} \\ &x = 3 \pm \sqrt{5} \end{aligned}$$

d $$\begin{aligned} &(r+5)^2 - 25 + 7 = 0 \\ &(r+5)^2 = 18 \\ &r + 5 = \pm\sqrt{18} = \pm 3\sqrt{2} \\ &r = -5 \pm 3\sqrt{2} \end{aligned}$$

e $$\begin{aligned} &(x-1)^2 - 1 = 11 \\ &(x-1)^2 = 12 \\ &x - 1 = \pm\sqrt{12} = \pm 2\sqrt{3} \\ &x = 1 \pm 2\sqrt{3} \end{aligned}$$

f $$\begin{aligned} &(a-6)^2 - 36 - 18 = 0 \\ &(a-6)^2 = 54 \\ &a - 6 = \pm\sqrt{54} = \pm 3\sqrt{6} \\ &a = 6 \pm 3\sqrt{6} \end{aligned}$$

g $$\begin{aligned} &(m-\tfrac{3}{2})^2 - \tfrac{9}{4} + 1 = 0 \\ &(m-\tfrac{3}{2})^2 = \tfrac{5}{4} \\ &m - \tfrac{3}{2} = \pm\tfrac{\sqrt{5}}{2} \\ &m = \tfrac{1}{2}(3 \pm \sqrt{5}) \end{aligned}$$

h $$\begin{aligned} &(t-\tfrac{7}{2})^2 - \tfrac{49}{4} + 9 = 0 \\ &(t-\tfrac{7}{2})^2 = \tfrac{13}{4} \\ &t - \tfrac{7}{2} = \pm\tfrac{\sqrt{13}}{2} \\ &t = \tfrac{1}{2}(7 \pm \sqrt{13}) \end{aligned}$$

i $(u + \frac{7}{2})^2 - \frac{49}{4} = 44$

$(u + \frac{7}{2})^2 = \frac{225}{4}$

$u + \frac{7}{2} = \pm\frac{15}{2}$

$u = -\frac{7}{2} \pm \frac{15}{2}$

$u = -11$ or 4

j $y^2 - 2y + \frac{1}{2} = 0$

$(y - 1)^2 - 1 + \frac{1}{2} = 0$

$(y - 1)^2 = \frac{1}{2}$

$y - 1 = \pm\frac{1}{\sqrt{2}} = \pm\frac{1}{2}\sqrt{2}$

$y = 1 \pm \frac{1}{2}\sqrt{2}$

k $p^2 + 6p = -\frac{23}{3}$

$(p + 3)^2 - 9 = -\frac{23}{3}$

$(p + 3)^2 = \frac{4}{3}$

$p + 3 = \pm\frac{2}{\sqrt{3}} = \pm\frac{2}{3}\sqrt{3}$

$p = -3 \pm \frac{2}{3}\sqrt{3}$

l $x^2 + 6x = \frac{9}{2}$

$(x + 3)^2 - 9 = \frac{9}{2}$

$(x + 3)^2 = \frac{27}{2}$

$x + 3 = \pm\sqrt{\frac{27}{2}} = \pm\frac{3}{2}\sqrt{6}$

$x = -3 \pm \frac{3}{2}\sqrt{6}$

m $m^2 - m = 1$

$(m - \frac{1}{2})^2 - \frac{1}{4} = 1$

$(m - \frac{1}{2})^2 = \frac{5}{4}$

$m - \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$

$m = \frac{1}{2}(1 \pm \sqrt{5})$

n $4x^2 - 28x + 49 = 0$

$x^2 - 7x + \frac{49}{4} = 0$

$(x - \frac{7}{2})^2 - \frac{49}{4} + \frac{49}{4} = 0$

$(x - \frac{7}{2})^2 = 0$

$x = \frac{7}{2}$

o $t^2 + \frac{1}{3}t = \frac{1}{3}$

$(t + \frac{1}{6})^2 - \frac{1}{36} = \frac{1}{3}$

$(t + \frac{1}{6})^2 = \frac{13}{36}$

$t + \frac{1}{6} = \pm\frac{\sqrt{13}}{6}$

$t = \frac{1}{6}(-1 \pm \sqrt{13})$

p $a^2 - \frac{7}{2}a + 2 = 0$

$(a - \frac{7}{4})^2 - \frac{49}{16} + 2 = 0$

$(a - \frac{7}{4})^2 = \frac{17}{16}$

$a - \frac{7}{4} = \frac{\sqrt{17}}{4}$

$a = \frac{1}{4}(7 \pm \sqrt{17})$

4 **a** $y = (x - 1)^2 - 1 + 7$

$y = (x - 1)^2 + 6$

$y = 6$ at $x = 1$, minimum

b $y = (x + 1)^2 - 1 - 3$

$y = (x + 1)^2 - 4$

$y = -4$ at $x = -1$, minimum

c $y = (x - 3)^2 - 9 + 1$

$y = (x - 3)^2 - 8$

$y = -8$ at $x = 3$, minimum

d $y = (x + 5)^2 - 25 + 35$

$y = (x + 5)^2 + 10$

$y = 10$ at $x = -5$, minimum

e $y = -[x^2 - 4x] + 4$

$y = -[(x - 2)^2 - 4] + 4$

$y = -(x - 2)^2 + 8$

$y = 8$ at $x = 2$, maximum

f $y = (x + \frac{3}{2})^2 - \frac{9}{4} - 2$

$y = (x + \frac{3}{2})^2 - \frac{17}{4}$

$y = -\frac{17}{4}$ at $x = -\frac{3}{2}$, minimum

g $y = 2[x^2 + 4x] + 5$

$y = 2[(x + 2)^2 - 4] + 5$

$y = 2(x + 2)^2 - 3$

$y = -3$ at $x = -2$, minimum

h $y = -3[x^2 - 2x]$

$y = -3[(x - 1)^2 - 1]$

$y = -3(x - 1)^2 + 3$

$y = 3$ at $x = 1$, maximum

i $y = -[x^2 + 5x] + 7$

$y = -[(x + \frac{5}{2})^2 - \frac{25}{4}] + 7$

$y = -(x + \frac{5}{2})^2 + \frac{53}{4}$

$y = \frac{53}{4}$ at $x = -\frac{5}{2}$, maximum

j $y = 4[x^2 - 3x] + 9$

$y = 4[(x - \frac{3}{2})^2 - \frac{9}{4}] + 9$

$y = 4(x - \frac{3}{2})^2$

$y = 0$ at $x = \frac{3}{2}$, minimum

k $y = 4[x^2 + 5x] - 8$

$y = 4[(x + \frac{5}{2})^2 - \frac{25}{4}] - 8$

$y = 4(x + \frac{5}{2})^2 - 33$

$y = -33$ at $x = -\frac{5}{2}$, minimum

l $y = -2[x^2 + x] + 17$

$y = -2[(x + \frac{1}{2})^2 - \frac{1}{4}] + 17$

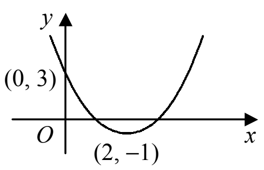
$y = -2(x + \frac{1}{2})^2 + \frac{35}{2}$

$y = \frac{35}{2}$ at $x = -\frac{1}{2}$, maximum

5 **a** $y = (x - 2)^2 - 4 + 3$

$y = (x - 2)^2 - 1$

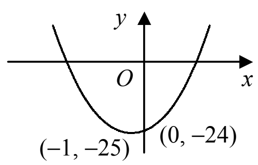
minimum $(2, -1)$



b $y = (x + 1)^2 - 1 - 24$

$y = (x + 1)^2 - 25$

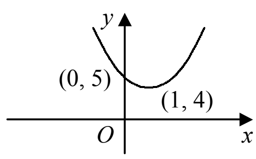
minimum $(-1, -25)$



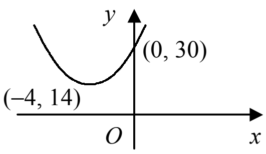
c $y = (x - 1)^2 - 1 + 5$

$y = (x - 1)^2 + 4$

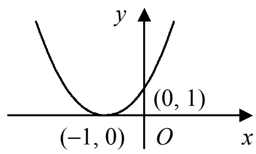
minimum $(1, 4)$



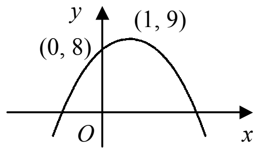
d $y = (x + 4)^2 - 16 + 30$
$y = (x + 4)^2 + 14$
minimum $(-4, 14)$



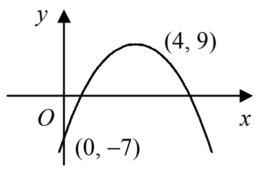
e $y = (x + 1)^2 - 1 + 1$
$y = (x + 1)^2$
minimum $(-1, 0)$



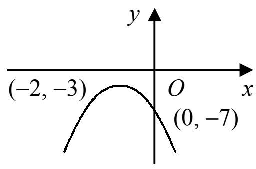
f $y = -[x^2 - 2x] + 8$
$y = -[(x - 1)^2 - 1] + 8$
$y = -(x - 1)^2 + 9$
maximum $(1, 9)$



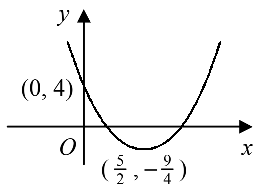
g $y = -[x^2 - 8x] - 7$
$y = -[(x - 4)^2 - 16] - 7$
$y = -(x - 4)^2 + 9$
maximum $(4, 9)$



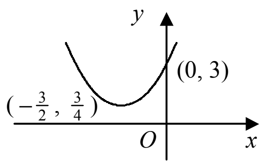
h $y = -[x^2 + 4x] - 7$
$y = -[(x + 2)^2 - 4] - 7$
$y = -(x + 2)^2 - 3$
maximum $(-2, -3)$



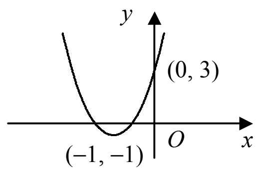
i $y = (x - \frac{5}{2})^2 - \frac{25}{4} + 4$
$y = (x - \frac{5}{2})^2 - \frac{9}{4}$
minimum $(\frac{5}{2}, -\frac{9}{4})$



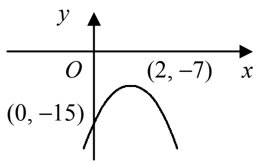
j $y = (x + \frac{3}{2})^2 - \frac{9}{4} + 3$
$y = (x + \frac{3}{2})^2 + \frac{3}{4}$
minimum $(-\frac{3}{2}, \frac{3}{4})$



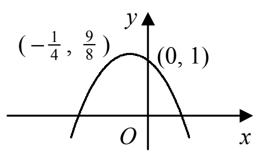
k $y = 4[x^2 + 2x] + 3$
$y = 4[(x + 1)^2 - 1] + 3$
$y = 4(x + 1)^2 - 1$
minimum $(-1, -1)$



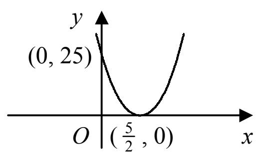
l $y = -2[x^2 - 4x] - 15$
$y = -2[(x - 2)^2 - 4] - 15$
$y = -2(x - 2)^2 - 7$
maximum $(2, -7)$



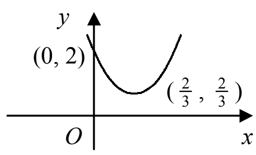
m $y = -2[x^2 + \frac{1}{2}x] + 1$
$y = -2[(x + \frac{1}{4})^2 - \frac{1}{16}] + 1$
$y = -2(x + \frac{1}{4})^2 + \frac{9}{8}$
maximum $(-\frac{1}{4}, \frac{9}{8})$



n $y = 4[x^2 - 5x] + 25$
$y = 4[(x - \frac{5}{2})^2 - \frac{25}{4}] + 25$
$y = 4(x - \frac{5}{2})^2$
minimum $(\frac{5}{2}, 0)$



o $y = 3[x^2 - \frac{4}{3}x] + 2$
$y = 3[(x - \frac{2}{3})^2 - \frac{4}{9}] + 2$
$y = 3(x - \frac{2}{3})^2 + \frac{2}{3}$
minimum $(\frac{2}{3}, \frac{2}{3})$



6 **a** $= (x - 2\sqrt{2})^2 - 8 + 5$
$= (x - 2\sqrt{2})^2 - 3$

b $x = 2\sqrt{2}$

7 $x^2 + 2kx - 3 = 0$
$(x + k)^2 - k^2 - 3 = 0$
$(x + k)^2 = k^2 + 3$
$x + k = \pm\sqrt{k^2 + 3}$
$x = -k \pm \sqrt{k^2 + 3}$

1 $ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2 **a** $x = \frac{-4 \pm \sqrt{16 - 4}}{2}$

$x = \frac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$

b $t = \frac{-8 \pm \sqrt{64 + 16}}{-2}$

$t = \frac{-8 \pm 4\sqrt{5}}{-2}$

$t = 4 \pm 2\sqrt{5}$

c $y = \frac{20 \pm \sqrt{400 - 364}}{2}$

$y = \frac{20 \pm 6}{2}$

$y = 7$ or 13

d $r = \frac{-2 \pm \sqrt{4 + 28}}{2}$

$r = \frac{-2 \pm 4\sqrt{2}}{2}$

$r = -1 \pm 2\sqrt{2}$

e $a = \frac{-18 \pm \sqrt{324 - 24}}{2}$

$a = \frac{-18 \pm 10\sqrt{3}}{2}$

$a = -9 \pm 5\sqrt{3}$

f $m^2 - 5m - 5 = 0$

$m = \frac{5 \pm \sqrt{25 + 20}}{2}$

$m = \frac{1}{2}(5 \pm 3\sqrt{5})$

g $x = \frac{-11 \pm \sqrt{121 - 108}}{2}$

$x = \frac{1}{2}(-11 \pm \sqrt{13})$

h $u = \frac{-6 \pm \sqrt{36 - 24}}{4}$

$u = \frac{-6 \pm 2\sqrt{3}}{4}$

$u = \frac{1}{2}(-3 \pm \sqrt{3})$

i $y = \frac{1 \pm \sqrt{1 + 20}}{-2}$

$y = -\frac{1}{2}(1 \pm \sqrt{21})$

j $2x^2 - 3x - 2 = 0$

$x = \frac{3 \pm \sqrt{9 + 16}}{4}$

$x = \frac{3 \pm 5}{4}$

$x = -\frac{1}{2}$ or 2

k $p = \frac{-7 \pm \sqrt{49 - 12}}{6}$

$p = \frac{1}{6}(-7 \pm \sqrt{37})$

l $t^2 - 14t - 14 = 0$

$t = \frac{14 \pm \sqrt{196 + 56}}{2}$

$t = \frac{14 \pm 6\sqrt{7}}{2}$

$t = 7 \pm 3\sqrt{7}$

m $r^2 + 14r - 9 = 0$

$r = \frac{-14 \pm \sqrt{196 + 36}}{2}$

$r = \frac{-14 \pm 2\sqrt{58}}{2}$

$r = -7 \pm \sqrt{58}$

n $6u^2 + 4u - 1 = 0$

$u = \frac{-4 \pm \sqrt{16 + 24}}{12}$

$u = \frac{-4 \pm 2\sqrt{10}}{12}$

$u = \frac{1}{6}(-2 \pm \sqrt{10})$

o $3y^2 - 18y - 4 = 0$

$y = \frac{18 \pm \sqrt{324 + 48}}{6}$

$y = \frac{18 \pm 2\sqrt{93}}{6}$

$y = 3 \pm \frac{1}{3}\sqrt{93}$

p $4x^2 - 8x - 11 = 0$

$x = \frac{8 \pm \sqrt{64 + 176}}{8}$

$x = \frac{8 \pm 4\sqrt{15}}{8}$

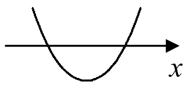
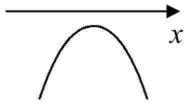
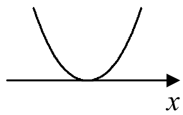
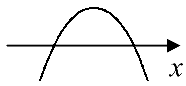
$x = 1 \pm \frac{1}{2}\sqrt{15}$

3 $2x^2 - 8x + 3 = 0$

$x = \frac{8 \pm \sqrt{64 - 24}}{4} = \frac{8 \pm 2\sqrt{10}}{4} = 2 \pm \frac{1}{2}\sqrt{10}$

$\therefore$ $(2 - \frac{1}{2}\sqrt{10}, 0)$ and $(2 + \frac{1}{2}\sqrt{10}, 0)$

4 **a** $b^2 - 4ac > 0$ **b** $b^2 - 4ac = 0$ **c** $b^2 - 4ac < 0$

5 **a** 
b 
c 
d 

6 **a** $b^2 - 4ac = 32$ $\therefore$ real and distinct
b $b^2 - 4ac = -11$ $\therefore$ not real
c $b^2 - 4ac = -4$ $\therefore$ not real
d $b^2 - 4ac = 24$ $\therefore$ real and distinct

e $b^2 - 4ac = 0$ $\therefore$ real and equal
f $b^2 - 4ac = 13$ $\therefore$ real and distinct
g $b^2 - 4ac = 53$ $\therefore$ real and distinct
h $b^2 - 4ac = -7$ $\therefore$ not real

i $b^2 - 4ac = 4$ $\therefore$ real and distinct
j $b^2 - 4ac = -11$ $\therefore$ not real
k $b^2 - 4ac = 0$ $\therefore$ real and equal
l $b^2 - 4ac = -3$ $\therefore$ not real

m $b^2 - 4ac = -7$ $\therefore$ not real
n $b^2 - 4ac = \frac{13}{9}$ $\therefore$ real and distinct
o $b^2 - 4ac = \frac{1}{16}$ $\therefore$ real and distinct
p $b^2 - 4ac = -\frac{13}{75}$ $\therefore$ not real

7 equal roots
$\therefore$ $b^2 - 4ac = 0$
$1 - 4p = 0$
$p = \frac{1}{4}$

8 repeated root
$\therefore$ $b^2 - 4ac = 0$
$4q^2 + 4q = 0$
$4q(q + 1) = 0$
$q \neq 0$ $\therefore$ $q = -1$

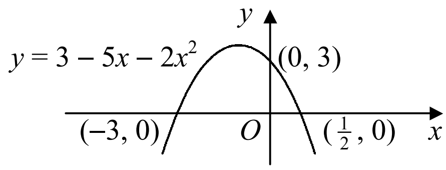
9 $x^2 + rx - 2x + 4 = 0$ has equal roots
$\therefore$ $b^2 - 4ac = 0$
$(r - 2)^2 - 16 = 0$
$r^2 - 4r - 12 = 0$
$(r + 2)(r - 6) = 0$
$r = -2$ or 6

1 **a** $= 2x(10 - x - 3x^2)$
$= 2x(2 + x)(5 - 3x)$
b $2x(2 + x)(5 - 3x) = 0$
$x = -2, 0 \text{ or } \frac{5}{3}$

2 **a** $AB^2 = (6 + 2)^2 + (k - 1)^2 = 64 + k^2 - 2k + 1$
$= k^2 - 2k + 65$
b $k^2 - 2k + 65 = 10^2 = 100$
$k^2 - 2k - 35 = 0$
$(k + 5)(k - 7) = 0$
$k = -5 \text{ or } 7$

3 **a** $x^2 - 5 = 4x$
$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$x = -1 \text{ or } 5$
b $9 - (5 - x) = 2x(5 - x)$
$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$x = \frac{1}{2} \text{ or } 4$

4 **a** $y = -2[x^2 + \frac{5}{2}x] + 3$
$= -2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 3$
$= -2(x + \frac{5}{4})^2 + \frac{49}{8}$
$\therefore$ turning point is $(-\frac{5}{4}, \frac{49}{8})$
b $3 - 5x - 2x^2 = 0$
$2x^2 + 5x - 3 = 0$
$(2x - 1)(x + 3) = 0,\ x = -3 \text{ or } \frac{1}{2}$



5 $x = \dfrac{-5\sqrt{2} \pm \sqrt{50 + 48}}{4}$
$= \dfrac{-5\sqrt{2} \pm \sqrt{98}}{4}$
$= \dfrac{-5\sqrt{2} \pm 7\sqrt{2}}{4}$
$= -3\sqrt{2} \text{ or } \frac{1}{2}\sqrt{2}$

6 **a** $y = 3[x^2 - 3x] + k = 3[(x - \frac{3}{2})^2 - \frac{9}{4}] + k$
$= 3(x - \frac{3}{2})^2 - \frac{27}{4} + k$
$\therefore$ x-coordinate of $P = \frac{3}{2}$
b y-coord of $P = k - \frac{27}{4} = \frac{17}{4}\ \therefore k = 11$
$\therefore$ curve is $y = 3x^2 - 9x + 11$
$\therefore$ coordinates of Q are $(0, 11)$

7 $y^2 - 10y + 16 = 0$
$(y - 2)(y - 8) = 0$
$y = 2^x = 2 \text{ or } 8$
$x = 1 \text{ or } 3$

8 equal roots $\therefore b^2 - 4ac = 0$
$4 - 4k(3 - 2k) = 0$
$2k^2 - 3k + 1 = 0$
$(2k - 1)(k - 1) = 0$
$k = \frac{1}{2} \text{ or } 1$

9 **a** $f(x) = -[x^2 - 4x] + 3$
$= -[(x - 2)^2 - 4] + 3$
$= -(x - 2)^2 + 7$
b turning point is $(2, 7)$
c $-(x - 2)^2 + 7 = 2$
$(x - 2)^2 = 5$
$x = 2 \pm \sqrt{5}$

10 **a** $x = \dfrac{5 \pm \sqrt{25 - 12}}{6}$
$= \frac{1}{6}(5 \pm \sqrt{13})$
b $x(x - 1) = 3(x + 2)$
$x^2 - 4x - 6 = 0$
$x = \dfrac{4 \pm \sqrt{16 + 24}}{2} = \dfrac{4 \pm 2\sqrt{10}}{2}$
$= 2 \pm \sqrt{10}$

11 **a** $(x - 2k)^2 - 4k^2 + 6 = 0$

$(x - 2k)^2 = 4k^2 - 6$

$x - 2k = \pm\sqrt{4k^2 - 6}$

$x = 2k \pm \sqrt{4k^2 - 6}$

b $k = 3$

$\therefore\ x = 6 \pm \sqrt{36 - 6}$

$= 6 \pm \sqrt{30}$

12 **a** $x^2 - 6x - 3 = 0$

$x = \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm 4\sqrt{3}}{2}$

$= 3 \pm 2\sqrt{3}$

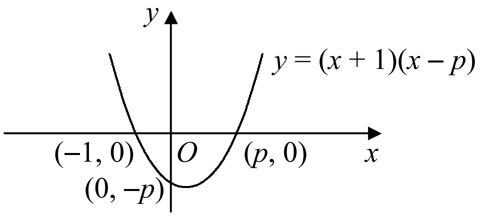
b $y(2y^2 + y - 15) = 0$

$y(2y - 5)(y + 3) = 0$

$y = -3,\ 0$ or $\frac{5}{2}$

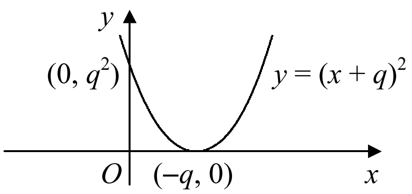
13 **a** $x = 0 \ \Rightarrow\ y = -p$

$y = 0 \ \Rightarrow\ x = -1$ or p



b $x = 0 \ \Rightarrow\ y = q^2$

$y = 0 \ \Rightarrow\ x = -q \quad [-q > 0]$



14 **a** $f(x) = 2[x^2 - 3x] + 5$

$= 2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 5$

$= 2(x - \frac{3}{2})^2 + \frac{1}{2}$

$\therefore\ A = 2,\ B = -\frac{3}{2},\ C = \frac{1}{2}$

b minimum value of $f(x) = \frac{1}{2}$

15 **a** $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = t^2$

b let $t = x^{\frac{1}{3}} \ \Rightarrow\ 2t^2 + t - 6 = 0$

$(2t - 3)(t + 2) = 0$

$t = -2$ or $\frac{3}{2}$

but $x = t^3 \quad \therefore \quad x = -8$ or $\frac{27}{8}$

16 **a** $= (k - 4)^2 - 16 + 20$

$= (k - 4)^2 + 4$

b $x^2 - kx + 2k - 5 = 0$

discriminant $= b^2 - 4ac$

$= k^2 - 4(2k - 5)$

$= k^2 - 8k + 20$

using **a** $= (k - 4)^2 + 4$

for all real k, $(k - 4)^2 \geq 0$

$\therefore$ discriminant > 0

$\therefore$ real and distinct roots for all real k

17 **a** $(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^2(x^2 - 3x - 4) + 2x(x^2 - 3x - 4) - 3(x^2 - 3x - 4)$

$\equiv x^4 - 3x^3 - 4x^2 + 2x^3 - 6x^2 - 8x - 3x^2 + 9x + 12$

$\equiv x^4 - x^3 - 13x^2 + x + 12$

b $(x^2 + 2x - 3)(x^2 - 3x - 4) = 0$

$(x + 3)(x - 1)(x + 1)(x - 4) = 0$

$x = -3,\ -1,\ 1$ or 4

1 **a** $3x = 2x + 1$
$x = 1$
$\therefore\ x = 1, y = 3$

b $x - 6 = \frac{1}{2}x - 4$
$x = 4$
$\therefore\ x = 4, y = -2$

c $2x + 6 = 3 - 4x$
$x = -\frac{1}{2}$
$\therefore\ x = -\frac{1}{2}, y = 5$

d subtracting
$y + 4 = 0$
$y = -4$
$\therefore\ x = 7, y = -4$

e $2x + 4y + 22 = 0$
$2x - 3y + 1 = 0$
subtracting
$7y + 21 = 0$
$y = -3$
$\therefore\ x = -5, y = -3$

f $6x + 6y + 8 = 0$
$15x - 6y - 15 = 0$
adding
$21x - 7 = 0$
$x = \frac{1}{3}$
$\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$

2 **a** $x + 2 = x^2 - 4$
$x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x = -2$ or 3
$\therefore\ (-2, 0)$ and $(3, 5)$

b $4x + 11 = x^2 + 3x - 1$
$x^2 - x - 12 = 0$
$(x + 3)(x - 4) = 0$
$x = -3$ or 4
$\therefore\ (-3, -1)$ and $(4, 27)$

c $2x - 1 = 2x^2 + 3x - 7$
$2x^2 + x - 6 = 0$
$(2x - 3)(x + 2) = 0$
$x = -2$ or $\frac{3}{2}$
$\therefore\ (-2, -5)$ and $(\frac{3}{2}, 2)$

3 **a** subtracting
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2
$\therefore\quad x = -1, y = 4$
or $\quad x = 2, y = 7$

b adding
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or 3
$\therefore\quad x = \frac{1}{2}, y = -\frac{7}{2}$
or $\quad x = 3, y = -6$

c $y = 2x - 5$
sub
$x^2 + (2x - 5)^2 = 25$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or 4
$\therefore\quad x = 0, y = -5$
or $\quad x = 4, y = 3$

d $y = 2x + 10$
sub.
$x^2 + 2x(2x + 10) + 15 = 0$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ or -1
$\therefore\quad x = -3, y = 4$
or $\quad x = -1, y = 8$

e $y = 1 - x$
sub.
$x^2 - 2x(1 - x) - (1 - x)^2 = 7$
$x^2 = 4$
$x = \pm 2$
$\therefore\quad x = -2, y = 3$
or $\quad x = 2, y = -1$

f $y = 1 - x$
sub.
$3x^2 - x - (1 - x)^2 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$x = -1$ or $\frac{1}{2}$
$\therefore\quad x = -1, y = 2$
or $\quad x = \frac{1}{2}, y = \frac{1}{2}$

g $y = 4 - x$
sub.
$2x^2 + x(4 - x) + (4 - x)^2 = 22$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ or 3
$\therefore\quad x = -1, y = 5$
or $\quad x = 3, y = 1$

h $x = 2y$
sub.
$(2y)^2 - 4y - y^2 = 0$
$3y^2 - 4y = 0$
$y(3y - 4) = 0$
$y = 0$ or $\frac{4}{3}$
$\therefore\quad x = 0, y = 0$
or $\quad x = \frac{8}{3}, y = \frac{4}{3}$

i $y = 3 - \frac{3}{2}x$
sub.
$x^2 + x(3 - \frac{3}{2}x) = 4$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x = 2$ or 4
$\therefore\quad x = 2, y = 0$
or $\quad x = 4, y = -3$

j $y = 2x - 3$
sub.
$2x^2 + (2x-3) - (2x-3)^2 = 8$
$x^2 - 7x + 10 = 0$
$(x-2)(x-5) = 0$
$x = 2$ or 5
$\therefore \quad x = 2, y = 1$
or $\quad x = 5, y = 7$

k $y = 2x - 7$
sub.
$x^2 - x(2x-7) + (2x-7)^2 = 13$
$x^2 - 7x + 12 = 0$
$(x-3)(x-4) = 0$
$x = 3$ or 4
$\therefore \quad x = 3, y = -1$
or $\quad x = 4, y = 1$

l $y = 5 - 3x$
sub.
$x^2 - 5x + (5-3x)^2 = 0$
$2x^2 - 7x + 5 = 0$
$(2x-5)(x-1) = 0$
$x = 1$ or $\frac{5}{2}$
$\therefore \quad x = 1, y = 2$
or $\quad x = \frac{5}{2}, y = -\frac{5}{2}$

m $x = 2y + 10$
sub.
$3(2y+10)^2 - y(2y+10) + y^2 = 36$
$y^2 + 10y + 24 = 0$
$(y+6)(y+4) = 0$
$y = -6$ or -4
$\therefore \quad x = -2, y = -6$
or $\quad x = 2, y = -4$

n $y = \frac{3}{2}x - 2$
sub.
$2x^2 + x - 4(\frac{3}{2}x - 2) = 6$
$2x^2 - 5x + 2 = 0$
$(2x-1)(x-2) = 0$
$x = \frac{1}{2}$ or 2
$\therefore \quad x = \frac{1}{2}, y = -\frac{5}{4}$
or $\quad x = 2, y = 1$

o $x = 3y - 17$
sub.
$(3y-17)^2 + (3y-17) + 2y^2 - 52 = 0$
$y^2 - 9y + 20 = 0$
$(y-4)(y-5) = 0$
$y = 4$ or 5
$\therefore \quad x = -5, y = 4$
or $\quad x = -2, y = 5$

4 **a** subtracting
$-\frac{1}{y} + 2y + 1 = 0$
$-1 + 2y^2 + y = 0$
$2y^2 + y - 1 = 0$
$(2y-1)(y+1) = 0$
$y = -1$ or $\frac{1}{2}$
$\therefore \quad x = -5, y = -1$
or $\quad x = 4, y = \frac{1}{2}$

b $y = x - 5$
sub.
$x(x-5) = 6$
$x^2 - 5x - 6 = 0$
$(x+1)(x-6) = 0$
$x = -1$ or 6
$\therefore \quad x = -1, y = -6$
or $\quad x = 6, y = 1$

c $y = 7 - 4x$
sub.
$\frac{3}{x} - 2(7-4x) + 4 = 0$
$3 - 2x(7-4x) + 4x = 0$
$8x^2 - 10x + 3 = 0$
$(4x-3)(2x-1) = 0$
$x = \frac{1}{2}$ or $\frac{3}{4}$
$\therefore \quad x = \frac{1}{2}, y = 5$
or $\quad x = \frac{3}{4}, y = 4$

5 $5 - x = x^2 - 3x + 2$
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x = -1$ or 3
P and Q are the points $(-1, 6)$ and $(3, 2)$
$PQ^2 = (3+1)^2 + (2-6)^2$
$PQ = \sqrt{32} = 4\sqrt{2}$

6 $3^{x-1} = (3^2)^{2y} \quad \therefore\ x - 1 = 4y$
$(2^3)^{x-2} = (2^2)^{1+y} \quad \therefore\ 3x - 6 = 2 + 2y$
$6x - 16 = 4y$
$\Rightarrow \quad 6x - 16 = x - 1$
$x = 3$
$\therefore \quad x = 3, y = \frac{1}{2}$

7 $AB - A\sqrt{3} + 2B\sqrt{3} - 6 \equiv 9\sqrt{3} - 1$
A and B integers $\therefore \quad AB - 6 = -1 \quad (1) \quad$ and $\quad -A + 2B = 9 \quad (2)$
$(2) \quad \Rightarrow \quad A = 2B - 9$
sub. (1) $\quad B(2B-9) - 6 = -1 \quad \Rightarrow \quad 2B^2 - 9B - 5 = 0$
$(2B+1)(B-5) = 0$
$B = -\frac{1}{2}$ or 5
B integer $\therefore\ B = 5 \quad \Rightarrow \quad A = 1, B = 5$

1

a $2x < 6$
$x < 3$

b $3x \geq 21$
$x \geq 7$

c $2x > 8$
$x > 4$

d $3x \leq 36$
$x \leq 12$

e $5x \geq -15$
$x \geq -3$

f $\frac{1}{3}x < 1$
$x < 3$

g $9x \geq 54$
$x \geq 6$

h $3x < -4$
$x < -\frac{4}{3}$

i $x < 14$

j $4x \leq -10$
$x \leq -\frac{5}{2}$

k $2 < 3x$
$x > \frac{2}{3}$

l $5 \geq \frac{1}{2}x$
$x \leq 10$

2

a $y > 7$

b $4p \leq 2$
$p \leq \frac{1}{2}$

c $6 < 2x$
$x > 3$

d $2a \geq 4$
$a \geq 2$

e $15 < 3u$
$u > 5$

f $2b \geq 9$
$b \geq \frac{9}{2}$

g $3x < -18$
$x < -6$

h $y \geq -13$

i $-20 \leq 4p$
$p \geq -5$

j $r - 2 > 6$
$r > 8$

k $3 - 6t \leq t - 4$
$7 \leq 7t$
$t \geq 1$

l $6 + 2x \geq 24 - 4x$
$6x \geq 18$
$x \geq 3$

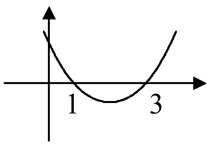
m $7y + 21 - 6y + 2 < 0$
$y < -23$

n $20 - 8x > 21 - 6x$
$-1 > 2x$
$x < -\frac{1}{2}$

o $12u - 3 - 5u + 15 < 9$
$7u < -3$
$u < -\frac{3}{7}$

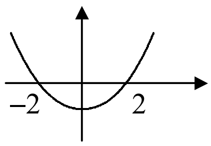
3

a $(x - 1)(x - 3) < 0$



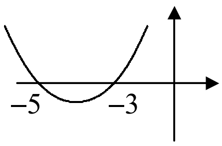
$\therefore\ 1 < x < 3$

b $(x + 2)(x - 2) \leq 0$



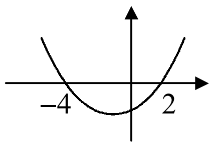
$\therefore\ -2 \leq x \leq 2$

c $(x + 5)(x + 3) < 0$



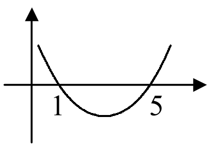
$\therefore\ -5 < x < -3$

d $x^2 + 2x - 8 \leq 0$
$(x + 4)(x - 2) \leq 0$



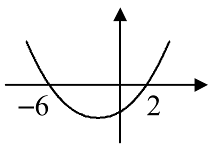
$\therefore\ -4 \leq x \leq 2$

e $(x - 1)(x - 5) > 0$



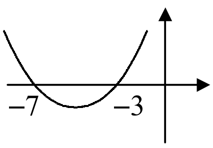
$\therefore\ x < 1$ or $x > 5$

f $x^2 + 4x - 12 > 0$
$(x + 6)(x - 2) > 0$



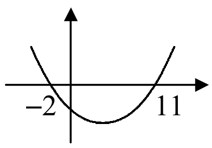
$\therefore\ x < -6$ or $x > 2$

g $(x + 7)(x + 3) \geq 0$



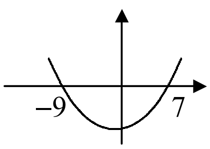
$\therefore\ x \leq -7$ or $x \geq -3$

h $x^2 - 9x - 22 < 0$
$(x + 2)(x - 11) < 0$



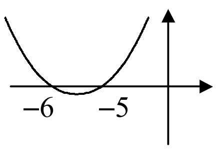
$\therefore\ -2 < x < 11$

i $x^2 + 2x - 63 \geq 0$
$(x + 9)(x - 7) \geq 0$



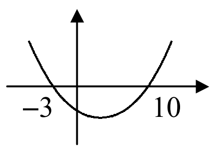
$\therefore\ x \leq -9$ or $x \geq 7$

j $(x + 6)(x + 5) > 0$



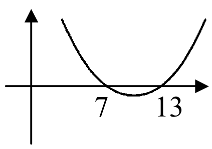
$\therefore\ x < -6$ or $x > -5$

k $x^2 - 7x - 30 < 0$
$(x + 3)(x - 10) < 0$



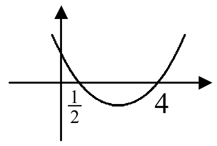
$\therefore\ -3 < x < 10$

l $x^2 - 20x + 91 \geq 0$
$(x - 7)(x - 13) \geq 0$



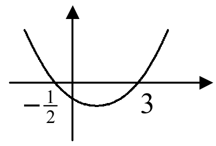
$\therefore\ x \leq 7$ or $x \geq 13$

4 **a** $(2x - 1)(x - 4) \leq 0$



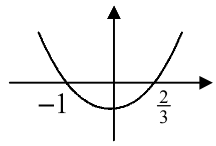
$\therefore\ \frac{1}{2} \leq x \leq 4$

b $(2r + 1)(r - 3) < 0$



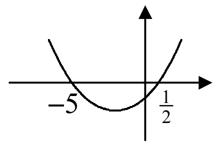
$\therefore\ -\frac{1}{2} < r < 3$

c $3p^2 + p - 2 \leq 0$
$(3p - 2)(p + 1) \leq 0$



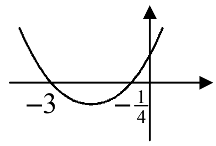
$\therefore\ -1 \leq p \leq \frac{2}{3}$

d $(2y - 1)(y + 5) > 0$



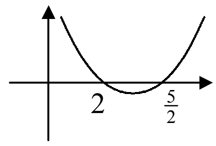
$\therefore\ y < -5$ or $y > \frac{1}{2}$

e $(4m + 1)(m + 3) < 0$



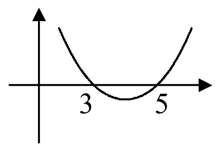
$\therefore\ -3 < m < -\frac{1}{4}$

f $2x^2 - 9x + 10 \geq 0$
$(2x - 5)(x - 2) \geq 0$



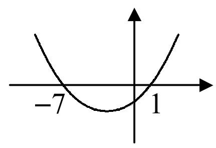
$\therefore\ x \leq 2$ or $x \geq \frac{5}{2}$

g $a^2 - 8a + 15 < 0$
$(a - 3)(a - 5) < 0$



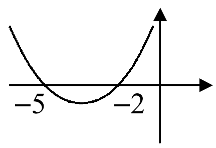
$\therefore\ 3 < a < 5$

h $x^2 + 4x \leq 7 - 2x$
$x^2 + 6x - 7 \leq 0$
$(x + 7)(x - 1) \leq 0$



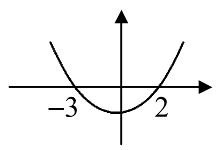
$\therefore\ -7 \leq x \leq 1$

i $y^2 + 9y > 2y - 10$
$y^2 + 7y + 10 > 0$
$(y + 5)(y + 2) > 0$



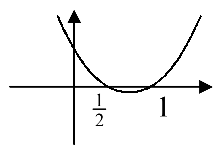
$\therefore\ y < -5$ or $y > -2$

j $2x^2 + x > x^2 + 6$
$x^2 + x - 6 > 0$
$(x + 3)(x - 2) < 0$



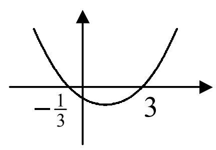
$\therefore\ -3 < x < -2$

k $5u - 6u^2 < 3 - 4u$
$2u^2 - 3u + 1 > 0$
$(2u - 1)(u - 1) > 0$



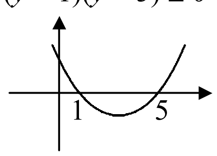
$\therefore\ u < \frac{1}{2}$ or $u > 1$

l $2t + 3 \geq 3t^2 - 6t$
$3t^2 - 8t - 3 \leq 0$
$(3t + 1)(t - 3) \leq 0$



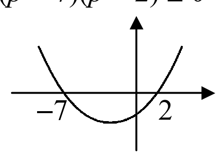
$\therefore\ -\frac{1}{3} \leq t \leq 3$

m $y^2 - 4y + 4 \leq 2y - 1$
$y^2 - 6y + 5 \leq 0$
$(y - 1)(y - 5) \leq 0$



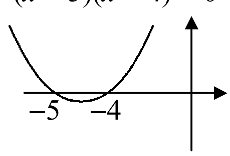
$\therefore\ 1 \leq y \leq 5$

n $p^2 + 5p + 6 \geq 20$
$p^2 + 5p - 14 \geq 0$
$(p + 7)(p - 2) \geq 0$



$\therefore\ p \leq -7$ or $p \geq 2$

o $26 + 4x < 6 - 5x - x^2$
$x^2 + 9x + 20 < 0$
$(x + 5)(x + 4) < 0$



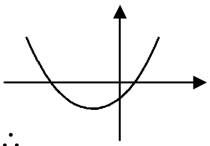
$\therefore\ -5 < x < -4$

5 **a** for critical values

$x = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$



$\therefore$

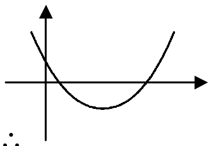
$-1-\sqrt{2} < x < -1+\sqrt{2}$

b for critical values

$x = \dfrac{6 \pm \sqrt{36-16}}{2}$

$x = \dfrac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$



$\therefore$

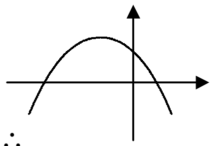
$x < 3-\sqrt{5}$ or $x > 3+\sqrt{5}$

c for critical values

$x = \dfrac{6 \pm \sqrt{36+44}}{-2}$

$x = \dfrac{6 \pm 4\sqrt{5}}{-2}$

$x = -3 \pm 2\sqrt{5}$



$\therefore$

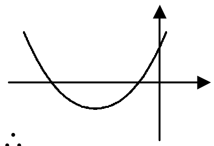
$-3-2\sqrt{5} < x < -3+2\sqrt{5}$

d for critical values

$x = \dfrac{-4 \pm \sqrt{16-4}}{2}$

$x = \dfrac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$



$\therefore$

$x \le -2-\sqrt{3}$ or $x \ge -2+\sqrt{3}$

6 **a** equal roots

$\therefore\ b^2 - 4ac = 0$

$36 - 4k = 0$

$k = 9$

b real and distinct roots

$\therefore\ b^2 - 4ac > 0$

$4 - 4k > 0$

$4 > 4k$

$k < 1$

c no real roots

$\therefore\ b^2 - 4ac < 0$

$9 - 4k < 0$

$9 < 4k$

$k > \frac{9}{4}$

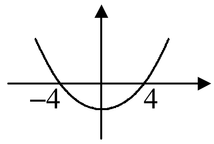
d real roots

$\therefore\ b^2 - 4ac \ge 0$

$k^2 - 16 \ge 0$

$(k+4)(k-4) \ge 0$

$k \le -4$ or $k \ge 4$



e equal roots

$\therefore\ b^2 - 4ac = 0$

$1 + 4k = 0$

$k = -\frac{1}{4}$

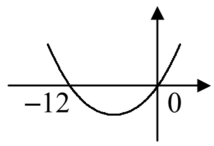
f no real roots

$\therefore\ b^2 - 4ac < 0$

$k^2 + 12k < 0$

$k(k+12) < 0$

$-12 < k < 0$



g real and distinct roots

$\therefore\ b^2 - 4ac > 0$

$4 - 4(k-2) > 0$

$12 > 4k$

$k < 3$

h equal roots

$\therefore\ b^2 - 4ac = 0$

$k^2 - 8k = 0$

$k(k-8) = 0$

$k = 0$ or 8

i no real roots

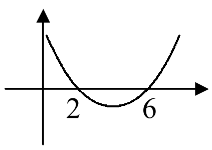
$\therefore\ b^2 - 4ac < 0$

$k^2 - 4(2k-3) < 0$

$k^2 - 8k + 12 < 0$

$(k-2)(k-6) < 0$

$2 < k < 6$



j real roots

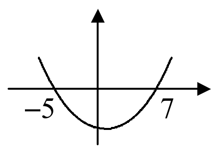
$\therefore\ b^2 - 4ac \ge 0$

$(k-1)^2 - 36 \ge 0$

$k^2 - 2k - 35 \ge 0$

$(k+5)(k-7) \ge 0$

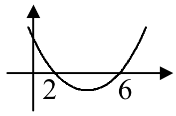
$k \le -5$ or $k \ge 7$



1 **a** $4 > \frac{3}{2}y$

$y < \frac{8}{3}$

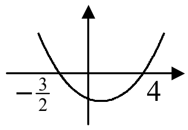
b $(x - 2)(x - 6) \geq 0$



$\therefore\ x \leq 2$ or $x \geq 6$

2 $2n^2 - 5n - 12 < 0$

$(2n + 3)(n - 4) < 0$



$-\frac{3}{2} < n < 4$

n integer $\therefore\ n = -1, 0, 1, 2, 3$

3 **a** $(x + 8) \geq 1.5 \times x$

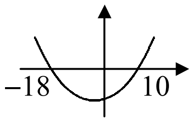
$8 \geq 0.5x$

$x \leq 16$

b $x(x + 8) \geq 180$

$x^2 + 8x - 180 \geq 0$

$(x + 18)(x - 10) \geq 0$



$x \leq -18$ or $x \geq 10$

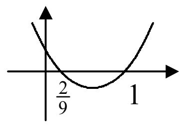
but $x > 0$ (width > 0)

and $x \leq 16$ $\therefore$ $10 \leq x \leq 16$

4 $9x^2 - 6x + 1 < 5x - 1$

$9x^2 - 11x + 2 < 0$

$(9x - 2)(x - 1) < 0$



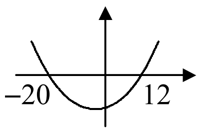
$\frac{2}{9} < x < 1$

5 $x = y + 8$

sub. $y(y + 8) \leq 240$

$y^2 + 8y - 240 \leq 0$

$(y + 20)(y - 12) \leq 0$



$-20 \leq y \leq 12$

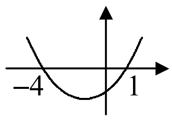
$x + y = y + 8 + y = 2y + 8$

$\therefore$ max value of $(x + y) = 2(12) + 8 = 32$

6 $3t^2 - 11t - 4 \geq 2t^2 - 14t$

$t^2 + 3t - 4 \geq 0$

$(t + 4)(t - 1) \geq 0$



$t \leq -4$ or $t \geq 1$

7 **a** $2x^2 + 2x - kx + 8 = 0$

real and distinct roots

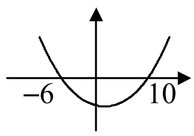
$\therefore\ b^2 - 4ac > 0$

$(2 - k)^2 - 64 > 0$

$4 - 4k + k^2 - 64 > 0$

$k^2 - 4k - 60 > 0$

b $(k + 6)(k - 10) > 0$



$k < -6$ or $k > 10$

8 let height be h $\therefore\ h^2 = (3r - 4)^2 - r^2$

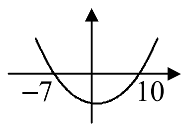
but $h \leq 24$

$\therefore\ h^2 \leq 24^2$

$(3r - 4)^2 - r^2 \leq 576$

$r^2 - 3r - 70 \leq 0$

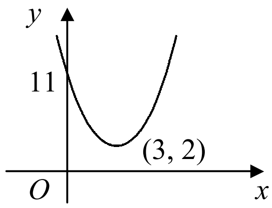
$(r + 7)(r - 10) \leq 0$



$-7 \leq r \leq 10$

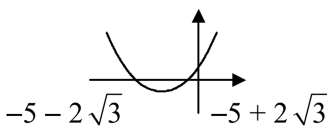
$\therefore$ maximum value of $r = 10$

1 **a** $2^{x-1} = 2^4$
$x - 1 = 4$
$x = 5$
b $3^y - 10 = 17$
$3^y = 27$
$y = 3$

2 **a** $= (x - 3)^2 - 9 + 11$
$= (x - 3)^2 + 2$
b


3 **a** $= (\frac{49}{4})^{-\frac{1}{2}} = \sqrt{\frac{4}{49}} = \frac{2}{7}$
b $3x^{-3} = \frac{64}{9}$
$x^3 = \frac{27}{64}$
$x = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$

4 $2x\sqrt{3} + 9 = x\sqrt{3}$
$x\sqrt{3} = -9$
$x = \frac{-9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -3\sqrt{3}$

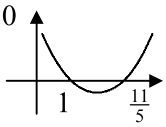
5 **a** $x = \frac{-10 \pm \sqrt{100 - 52}}{2}$
$= \frac{-10 \pm \sqrt{48}}{2}$
$= \frac{-10 \pm 4\sqrt{3}}{2}$
$= -5 \pm 2\sqrt{3}$
b


$x < -5 - 2\sqrt{3}$ or $x > -5 + 2\sqrt{3}$

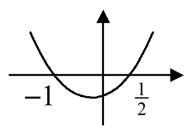
6 **a** $42x - 49 = 9x^2$
$9x^2 - 42x + 49 = 0$
$(3x - 7)^2 = 0$
$x = \frac{7}{3}$
b $2 + (y + 1) = 2y(y + 1)$
$2y^2 + y - 3 = 0$
$(2y + 3)(y - 1) = 0$
$y = -\frac{3}{2}$ or 1

7 $y = x + 3$
sub.
$3x^2 - 2x(x + 3) + (x + 3)^2 - 17 = 0$
$x^2 = 4$
$x = \pm 2$
$\therefore\ x = -2, y = 1$ or $x = 2, y = 5$

8 **a** $x^{\frac{1}{2}} = \sqrt[3]{64} = 4$
$x = 4^2 = 16$
b $\frac{\sqrt{3}+1}{2\sqrt{3}-3} = \frac{\sqrt{3}+1}{2\sqrt{3}-3} \times \frac{2\sqrt{3}+3}{2\sqrt{3}+3} = \frac{(\sqrt{3}+1)(2\sqrt{3}+3)}{12-9}$
$= \frac{1}{3}(6 + 3\sqrt{3} + 2\sqrt{3} + 3)$
$= 3 + \frac{5}{3}\sqrt{3}$
$\therefore\ a = 3,\ b = \frac{5}{3}$

9 **a** let A be $(2, 4)$
$\therefore\ AP^2 = (2k - 2)^2 + (k - 4)^2$
$AP < 3\ \therefore\ (2k - 2)^2 + (k - 4)^2 < 9$
$5k^2 - 16k + 11 < 0$
b $(5k - 11)(k - 1) < 0$


$1 < k < \frac{11}{5}$

10 **a** $2x \le 7$
$x \le \frac{7}{2}$
b $2x^2 + x < 1$
$2x^2 + x - 1 < 0$
$(2x - 1)(x + 1) < 0$


$-1 < x < \frac{1}{2}$

11 **a** $f(x) = 2[x^2 - 4x] + 5$

$= 2[(x - 2)^2 - 4] + 5$

$= 2(x - 2)^2 - 3$

b $(2, -3)$

c $2(x - 2)^2 - 3 = 0$

$x - 2 = \pm\sqrt{\frac{3}{2}}$

$x = 2 \pm \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$x = 2 \pm \frac{1}{2}\sqrt{6}$

12 **a** $= 2\sqrt{3} - \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 2\sqrt{3} - \frac{5}{3}\sqrt{3}$

$= \frac{1}{3}\sqrt{3}$

b $= \frac{64x\sqrt{x}}{16x}$

$= 4\sqrt{x}$

13 no real roots

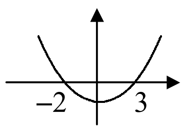
$\therefore\ b^2 - 4ac < 0$

$4k^2 - 4(k + 6) < 0$

$k^2 - k - 6 < 0$

$(k + 2)(k - 3) < 0$

$-2 < k < 3$



14 **a** $AM = \frac{1}{2}AC = 2 + 2\sqrt{3}$

$BM^2 = AB^2 - AM^2$

$= (4 + \sqrt{3})^2 - (2 + 2\sqrt{3})^2$

$= 16 + 8\sqrt{3} + 3 - (4 + 8\sqrt{3} + 12) = 3$

$\therefore\ BM = \sqrt{3}$

b $= \frac{1}{2} \times AC \times BM$

$= \frac{1}{2} \times (4 + 4\sqrt{3}) \times \sqrt{3}$

$= \frac{1}{2}(4\sqrt{3} + 12) = 6 + 2\sqrt{3}$

15 $(2^2)^{2y + 7} = (2^3)^{y + 3}$

$4y + 14 = 3y + 9$

$y = -5$

16 LHS $= x^2(2x^2 - 3x - 9) - x(2x^2 - 3x - 9)$

$+ 3(2x^2 - 3x - 9)$

$= 2x^4 - 3x^3 - 9x^2 - 2x^3 + 3x^2 + 9x$

$+ 6x^2 - 9x - 27$

$= 2x^4 - 5x^3 - 27$

$\therefore\ A = 2, B = -5$ and $C = -27$

17 **a** $x^2 + 4x + k = 0$

$(x + 2)^2 - 4 + k = 0$

$x + 2 = \pm\sqrt{4 - k}$

$x = -2 \pm \sqrt{4 - k}$

b real roots only if $4 - k \geq 0$

$\therefore\ k \leq 4$

c $k = -4$

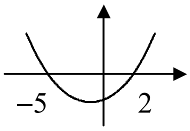
$\therefore\ x = -2 \pm \sqrt{8}$

$x = -2 \pm 2\sqrt{2}$

1 $x^2 + 3x + 2 \le 12$

$x^2 + 3x - 10 \le 0$

$(x + 5)(x - 2) \le 0$



$-5 \le x \le 2$

2 **a** $= 8\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$

b $= x + 12\sqrt{x} + 36 + 4x - 12\sqrt{x} + 9$

$= 5x + 45$

3 **a** $(-2, 0) \Rightarrow 0 = 8 - 2p + q$ (1)

$(3, 0) \Rightarrow 0 = 18 + 3p + q$ (2)

$(2) - (1)$ $0 = 10 + 5p \Rightarrow p = -2$

sub. $\Rightarrow q = -12$

b x-coord $= \frac{-2+3}{2} = \frac{1}{2}$

$\therefore y = -\frac{25}{2} \Rightarrow (\frac{1}{2}, -\frac{25}{2})$

4 $2x - 2\sqrt{32} = \sqrt{98} - x$

$3x = 2\sqrt{32} + \sqrt{98}$

$3x = 8\sqrt{2} + 7\sqrt{2}$

$3x = 15\sqrt{2}$

$x = 5\sqrt{2}$

5 **a** real and distinct roots

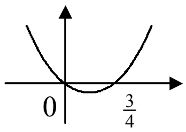
$\therefore b^2 - 4ac > 0$

$16k^2 - 12k > 0$

$4k^2 - 3k > 0$

$k(4k - 3) > 0$

b



$k < 0$ or $k > \frac{3}{4}$

6 $(2^2)^{2x} = 2^{y-1}$

$4x = y - 1$ (1)

$(3^2)^{4x} = 3^{y+1}$

$8x = y + 1$ (2)

(1) and (2) $\Rightarrow$ $y = 4x + 1 = 8x - 1$

$4x = 2$

$x = \frac{1}{2}, y = 3$

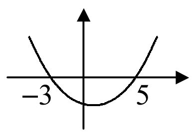
7 **a** LHS $= (x - \frac{7}{2})^2 - \frac{49}{4} + 9$

$= (x - \frac{7}{2})^2 - \frac{13}{4}$

$\therefore a = -\frac{7}{2}, b = -\frac{13}{4}$

b $x = \frac{7}{2}$

8 **a** $(y + 3)(y - 5) < 0$



$-3 < y < 5$

b $x(2 - x) = 4(x - 3)$

$x^2 + 2x - 12 = 0$

$x = \frac{-2 \pm \sqrt{4+48}}{2} = \frac{-2 \pm 2\sqrt{13}}{2}$

$x = -1 \pm \sqrt{13}$

9 $2^{x^2+2} = (2^3)^x$

$x^2 + 2 = 3x$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1$ or 2

10 **a** $t - 2t^2 = 3t - 15$

$2t^2 + 2t - 15 = 0$

$t = \frac{-2 \pm \sqrt{4+120}}{4} = \frac{-2 \pm \sqrt{124}}{4} = \frac{-2 \pm 2\sqrt{31}}{4}$

$t = \frac{1}{2}(-1 \pm \sqrt{31})$

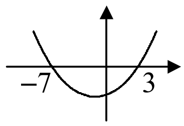
b $(x^2 + 2)(x^2 - 3) = 0$

$x^2 = -2$ [no solutions] or 3

$x = \pm\sqrt{3}$

11 $x^2 + 4x - 21 \geq 0$

$(x + 7)(x - 3) \geq 0$



$x \leq -7$ or $x \geq 3$

12 **a** $3^{2x+2} = 3^2(3^x)^2 = 9y^2$

b $9y^2 - 10y + 1 = 0$

$(9y - 1)(y - 1) = 0$

$y = 3^x = \frac{1}{9}, 1$

$\therefore\ x = -2, 0$

13 **a** $= \sqrt{25 \times 3} = \sqrt{75}$

b $\sqrt{64} < \sqrt{75} < \sqrt{81}$

$\therefore\ 8 < 5\sqrt{3} < 9$

$\therefore\ n = 8$

14 $y = \dfrac{2x+7}{3}$

sub. $2x^2 - (\dfrac{2x+7}{3})^2 - 7 = 0$

$18x^2 - (2x + 7)^2 - 63 = 0$

$x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

$x = -2$ or 4

$\therefore\ x = -2, y = 1$ or $x = 4, y = 5$

15 **a** $= \sqrt{\frac{48}{12}} - \sqrt{\frac{600}{12}}$

$= \sqrt{4} - \sqrt{50}$

$= 2 - 5\sqrt{2}$

b $= \dfrac{\sqrt{2}}{4+3\sqrt{2}} \times \dfrac{4-3\sqrt{2}}{4-3\sqrt{2}} = \dfrac{\sqrt{2}(4-3\sqrt{2})}{16-18}$

$= -\frac{1}{2}(4\sqrt{2} - 6)$

$= 3 - 2\sqrt{2}$

16 **a** $5^{x+1} = (5^2)^{y-3}$

$x + 1 = 2y - 6$

$y = \dfrac{x+7}{2}$

b $(4^2)^{x-1} = 4^z$

$2x - 2 = z$

$x = 2y - 7 \quad \therefore\ z = 2(2y - 7) - 2$

$z = 4y - 16$

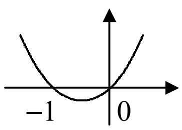
17 **a** $(x - k)^2 - k^2 - k = 0$

$x - k = \pm\sqrt{k^2 + k}$

$x = k \pm \sqrt{k^2 + k}$

b real roots $\therefore\ k^2 + k \geq 0$

$k(k + 1) \geq 0$



$k \leq -1$ or $k \geq 0$

18 **a** $\dfrac{1}{y} - y = \frac{3}{2}$

$2 - 2y^2 = 3y$

$2y^2 + 3y - 2 = 0$

b $(2y - 1)(y + 2) = 0$

$y = -2, \frac{1}{2}$

$x = y^5 = -32, \frac{1}{32}$

1 **a** $= (\frac{3}{2})^2 = \frac{9}{4}$ or $2\frac{1}{4}$

b $x^{\frac{3}{2}} = 27$

$x^{\frac{1}{2}} = \sqrt[3]{27} = 3$

$x = 3^2 = 9$

2 $x = 16 - 3y$

sub. $(16 - 3y)^2 - y(16 - 3y) + 2y^2 = 46$

$y^2 - 8y + 15 = 0$

$(y - 3)(y - 5) = 0$

$y = 3$ or 5

$\therefore\ x = 1, y = 5$ or $x = 7, y = 3$

3 **a** $= 8\sqrt{3} - 4\sqrt{3} + 5\sqrt{3}$

$= 9\sqrt{3}$

b $= 10 - 4\sqrt{3} + 5\sqrt{3} - 6$

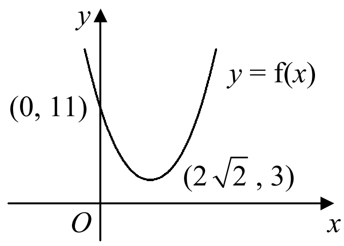
$= 4 + \sqrt{3}$

4 **a** $f(x) = (x - 2\sqrt{2})^2 - 8 + 11$

$= (x - 2\sqrt{2})^2 + 3$

$\therefore\ a = 1, b = -2\sqrt{2}$ and $c = 3$

b turning point is $(2\sqrt{2}, 3)$



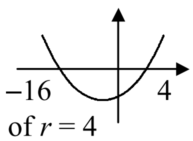
5 **a** $S.A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 24\pi r$

$S.A \leq 128\pi\ \therefore\ 2\pi r^2 + 24\pi r \leq 128\pi$

$r^2 + 12r \leq 64$

$r^2 + 12r - 64 \leq 0$

b $(r + 16)(r - 4) \leq 0$



$-16 \leq r \leq 4$

$\therefore$ maximum value of $r = 4$

6 $8x\sqrt{x} = 4x$

$4x(2\sqrt{x} - 1) = 0$

$x \neq 0\ \therefore\ \sqrt{x} = \frac{1}{2}$

$x = (\frac{1}{2})^2 = \frac{1}{4}$

7 **a** $x = 5$

b $(2^5)^{y+1} = (2^2)^y$

$5y + 5 = 2y$

$y = -\frac{5}{3}$

8 **a** $t^2 - 5t$

b $t^2 - 5t + 6 = 0$

$(t - 2)(t - 3) = 0$

$t = 2, 3$

$x = t^2 = 4, 9$

9 $x^2 + kx + 3 + k^2 = 0$

$\Rightarrow\ (x + \frac{1}{2}k)^2 - \frac{1}{4}k^2 + 3 + k^2 = 0$

$\Rightarrow\ x + \frac{1}{2}k = \pm\sqrt{-\frac{3}{4}k^2 - 3}$

$\Rightarrow\ x = -\frac{1}{2}k \pm \sqrt{-\frac{3}{4}k^2 - 3}$

real $k \Rightarrow\ k^2 \geq 0$

$\Rightarrow\ -\frac{3}{4}k^2 - 3 < 0$

$\therefore$ no real roots

10 **a** $(2^3)^{2x-1} = 2^5$

$6x - 3 = 5$

$x = \frac{4}{3}$

b $(3^{-1})^{y-2} = 3^4$

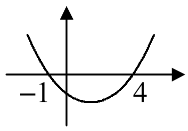
$-y + 2 = 4$

$y = -2$

11 $2x^2 - 7x < x^2 - 4x + 4$

$x^2 - 3x - 4 < 0$

$(x + 1)(x - 4) < 0$



$-1 < x < 4$

12 $\frac{2}{3\sqrt{2}-4} = \frac{2}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{2(3\sqrt{2}+4)}{18-16} = 3\sqrt{2} + 4$

$\frac{3-\sqrt{2}}{\sqrt{2}+1} = \frac{3-\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(3-\sqrt{2})(\sqrt{2}-1)}{2-1}$

$= (3 - \sqrt{2})(\sqrt{2} - 1)$

$= 3\sqrt{2} - 3 - 2 + \sqrt{2} = 4\sqrt{2} - 5$

$\therefore \frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1} = 3\sqrt{2} + 4 - (4\sqrt{2} - 5)$

$= 9 - \sqrt{2}$

13 **a** $(3y - 1)(2y + 9) = 0$

$y = -\frac{9}{2}$ or $\frac{1}{3}$

b equal roots

$\therefore\ b^2 - 4ac = 0$

$k^2 - 64 = 0$

$k = \pm 8$

14 **a** **i** $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

ii $2^{x-1} = 2^{-1} \times 2^x = \frac{1}{2}y$

b let $y = 2^x \Rightarrow$ $y^2 - 9(\frac{1}{2}y) + 2 = 0$

$2y^2 - 9y + 4 = 0$

$(2y - 1)(y - 4) = 0$

$y = 2^x = \frac{1}{2}$ or 4

$x = -1$ or 2

15 $x = 3y + 1$

sub.

$(3y + 1)^2 + 2y(3y + 1) + y^2 = 9$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = -1$ or $\frac{1}{2}$

$\therefore (-2, -1)$ and $(\frac{5}{2}, \frac{1}{2})$

16 **a** $(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + b = 0$

$(x + \frac{1}{2}a)^2 = \frac{1}{4}a^2 - b = \frac{a^2 - 4b}{4}$

$x + \frac{1}{2}a = \pm\sqrt{\frac{a^2-4b}{4}} = \pm\frac{\sqrt{a^2-4b}}{2}$

$x = -\frac{1}{2}a \pm \frac{\sqrt{a^2-4b}}{2}$

$x = \frac{-a \pm \sqrt{a^2-4b}}{2}$

b for repeated root, $a^2 - 4b = 0$

$\Rightarrow\ b = \frac{1}{4}a^2$

17 **a** $\text{f}(x) = 2[x^2 - 6x] + 19$

$= 2[(x - 3)^2 - 9] + 19$

$= 2(x - 3)^2 + 1$

real $x \Rightarrow (x - 3)^2 \geq 0$

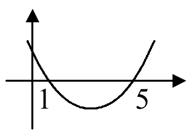
$\Rightarrow 2(x - 3)^2 + 1 \geq 1$

$\Rightarrow \text{f}(x) \geq 1$

b $2x^2 - 12x + 19 < 9$

$x^2 - 6x + 5 < 0$

$(x - 1)(x - 5) < 0$



$1 < x < 5$

18 **a** $= 1 - 2\sqrt{5} + 5$

$= 6 - 2\sqrt{5}$

b $y^2 = \frac{1}{2}(6 - 2\sqrt{5}) = \frac{1}{2}(1 - \sqrt{5})^2$

$y = \pm\frac{1}{\sqrt{2}}(1 - \sqrt{5})$

$y = \pm\frac{1}{2}\sqrt{2}(1 - \sqrt{5})$

$y = \pm\frac{1}{2}(\sqrt{2} - \sqrt{10})$

$y = \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{10}$ or $-\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{10}$