1 Evaluate

a $\sqrt{49}$ **b** $\sqrt{121}$ **c** $\sqrt{\tfrac{1}{9}}$ **d** $\sqrt{\tfrac{4}{25}}$ **e** $\sqrt{0.01}$ **f** $\sqrt{0.09}$

g $\sqrt[3]{8}$ **h** $\sqrt[3]{1000}$ **i** $\sqrt[4]{81}$ **j** $\sqrt{1\tfrac{9}{16}}$ **k** $\sqrt[3]{0.125}$ **l** $\sqrt[3]{15\tfrac{5}{8}}$

2 Simplify

a $\sqrt{7} \times \sqrt{7}$ **b** $4\sqrt{5} \times \sqrt{5}$ **c** $(3\sqrt{3})^2$ **d** $(\sqrt{6})^4$

e $(\sqrt{2})^5$ **f** $(2\sqrt{3})^3$ **g** $\sqrt{2} \times \sqrt{8}$ **h** $2\sqrt{3} \times \sqrt{27}$

i $\dfrac{\sqrt{32}}{\sqrt{2}}$ **j** $\dfrac{\sqrt{3}}{\sqrt{12}}$ **k** $(\sqrt[3]{6})^3$ **l** $(3\sqrt[3]{2})^3$

3 Express in the form $k\sqrt{2}$

a $\sqrt{18}$ **b** $\sqrt{50}$ **c** $\sqrt{8}$ **d** $\sqrt{98}$ **e** $\sqrt{200}$ **f** $\sqrt{162}$

4 Simplify

a $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{27}$ **e** $\sqrt{24}$ **f** $\sqrt{128}$

g $\sqrt{45}$ **h** $\sqrt{40}$ **i** $\sqrt{75}$ **j** $\sqrt{112}$ **k** $\sqrt{99}$ **l** $\sqrt{147}$

m $\sqrt{216}$ **n** $\sqrt{800}$ **o** $\sqrt{180}$ **p** $\sqrt{60}$ **q** $\sqrt{363}$ **r** $\sqrt{208}$

5 Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

d $\sqrt{360} - 2\sqrt{40}$ **e** $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$ **f** $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

6 Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ **b** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ **c** $(1 + \sqrt{3})(2 + \sqrt{3})$

d $(4 + \sqrt{3})(1 + 2\sqrt{3})$ **e** $(3\sqrt{3} - 4)^2$ **f** $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

7 Simplify

a $(\sqrt{5} + 1)(2\sqrt{5} + 3)$ **b** $(1 - \sqrt{2})(4\sqrt{2} - 3)$ **c** $(2\sqrt{7} + 3)^2$

d $(3\sqrt{2} - 1)(2\sqrt{2} + 5)$ **e** $(\sqrt{5} - \sqrt{2})(\sqrt{5} + 2\sqrt{2})$ **f** $(3 - \sqrt{8})(4 + \sqrt{2})$

8 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{2}{\sqrt{3}}$ **c** $\dfrac{1}{\sqrt{8}}$ **d** $\dfrac{14}{\sqrt{7}}$ **e** $\dfrac{3\sqrt{2}}{\sqrt{3}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{15}}$

g $\dfrac{1}{3\sqrt{7}}$ **h** $\dfrac{12}{\sqrt{72}}$ **i** $\dfrac{1}{\sqrt{80}}$ **j** $\dfrac{3}{2\sqrt{54}}$ **k** $\dfrac{4\sqrt{20}}{3\sqrt{18}}$ **l** $\dfrac{3\sqrt{175}}{2\sqrt{27}}$

9 Simplify

a $\sqrt{8} + \dfrac{6}{\sqrt{2}}$ **b** $\sqrt{48} - \dfrac{10}{\sqrt{3}}$ **c** $\dfrac{6-\sqrt{8}}{\sqrt{2}}$

d $\dfrac{\sqrt{45}-5}{\sqrt{20}}$ **e** $\dfrac{1}{\sqrt{18}} + \dfrac{1}{\sqrt{32}}$ **f** $\dfrac{2}{\sqrt{3}} - \dfrac{\sqrt{6}}{\sqrt{72}}$

10 Solve each equation, giving your answers as simply as possible in terms of surds.

a $x(x + 4) = 4(x + 8)$ **b** $x - \sqrt{48} = 2\sqrt{3} - 2x$

c $x\sqrt{18} - 4 = \sqrt{8}$ **d** $x\sqrt{5} + 2 = \sqrt{20}\,(x - 1)$

11 **a** Simplify $(2 - \sqrt{3})(2 + \sqrt{3})$.

b Express $\dfrac{2}{2-\sqrt{3}}$ in the form $a + b\sqrt{3}$.

12 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{2}+1}$ **b** $\dfrac{4}{\sqrt{3}-1}$ **c** $\dfrac{1}{\sqrt{6}-2}$ **d** $\dfrac{3}{2+\sqrt{3}}$

e $\dfrac{1}{2+\sqrt{5}}$ **f** $\dfrac{\sqrt{2}}{\sqrt{2}-1}$ **g** $\dfrac{6}{\sqrt{7}+3}$ **h** $\dfrac{1}{3+2\sqrt{2}}$

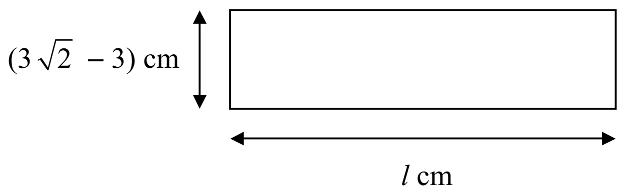
i $\dfrac{1}{4-2\sqrt{3}}$ **j** $\dfrac{3}{3\sqrt{2}+4}$ **k** $\dfrac{2\sqrt{3}}{7-4\sqrt{3}}$ **l** $\dfrac{6}{\sqrt{5}-\sqrt{3}}$

13 Solve the equation

$$3x = \sqrt{5}\,(x + 2),$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

14



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.
Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

15 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$ **b** $\dfrac{1+\sqrt{3}}{2+\sqrt{3}}$ **c** $\dfrac{1+\sqrt{10}}{\sqrt{10}-3}$ **d** $\dfrac{3-\sqrt{2}}{4+3\sqrt{2}}$

e $\dfrac{1-\sqrt{2}}{3-\sqrt{8}}$ **f** $\dfrac{\sqrt{3}-5}{2\sqrt{3}-4}$ **g** $\dfrac{\sqrt{12}+3}{3-\sqrt{3}}$ **h** $\dfrac{3\sqrt{7}-2}{2\sqrt{7}-5}$

1 Evaluate

a 8^2 **b** 6^3 **c** 7^0 **d** $(-5)^4$ **e** $(-3)^5$ **f** $(\frac{1}{2})^4$

g $(\frac{2}{3})^3$ **h** $(-\frac{1}{4})^3$ **i** $(1\frac{1}{3})^2$ **j** $(1\frac{1}{2})^4$ **k** $(0.1)^5$ **l** $(-0.2)^3$

2 Write in the form 2^n

a $2^5 \times 2^3$ **b** 2×2^6 **c** 1 **d** $2^6 \div 2^2$ **e** $2^{15} \div 2^6$ **f** $(2^7)^2$

3 Simplify

a $2p^2 \times 4p^5$ **b** $x^2 \times x^3 \times x^5$ **c** $12n^7 \div 2n^2$ **d** $(y^3)^4$

e $(2b)^3 \div 4b^2$ **f** $p^3q \times pq^2$ **g** $x^4y^3 \div xy^2$ **h** $2r^2s \times 3s^2$

i $6x^5y^8 \div 3x^2y$ **j** $6a^4b^5 \times \frac{2}{3}ab^3$ **k** $(5rs^2)^3 \div (10rs)^2$ **l** $3p^4q^3 \div \frac{1}{5}pq^2$

4 Evaluate

a 3^{-2} **b** $(\frac{2}{5})^0$ **c** $(-2)^{-6}$ **d** $(\frac{1}{6})^{-2}$ **e** $(1\frac{1}{2})^{-3}$ **f** $9^{\frac{1}{2}}$

g $16^{\frac{1}{4}}$ **h** $(-27)^{\frac{1}{3}}$ **i** $(\frac{1}{49})^{\frac{1}{2}}$ **j** $125^{\frac{1}{3}}$ **k** $(\frac{4}{9})^{\frac{1}{2}}$ **l** $36^{-\frac{1}{2}}$

m $81^{-\frac{1}{4}}$ **n** $(-64)^{-\frac{1}{3}}$ **o** $(\frac{1}{32})^{-\frac{1}{5}}$ **p** $(-\frac{8}{125})^{\frac{1}{3}}$ **q** $(2\frac{1}{4})^{\frac{1}{2}}$ **r** $(3\frac{3}{8})^{-\frac{1}{3}}$

5 Evaluate

a $4^{\frac{3}{2}}$ **b** $27^{\frac{2}{3}}$ **c** $16^{\frac{3}{4}}$ **d** $(-125)^{\frac{2}{3}}$ **e** $9^{\frac{5}{2}}$ **f** $8^{-\frac{2}{3}}$

g $36^{-\frac{3}{2}}$ **h** $(\frac{1}{8})^{\frac{4}{3}}$ **i** $(\frac{4}{9})^{\frac{3}{2}}$ **j** $(\frac{1}{216})^{-\frac{2}{3}}$ **k** $(\frac{9}{16})^{-\frac{3}{2}}$ **l** $(-\frac{27}{64})^{\frac{4}{3}}$

m $(0.04)^{\frac{1}{2}}$ **n** $(2.25)^{-\frac{3}{2}}$ **o** $(0.064)^{\frac{2}{3}}$ **p** $(1\frac{9}{16})^{-\frac{3}{2}}$ **q** $(5\frac{1}{16})^{\frac{3}{4}}$ **r** $(2\frac{10}{27})^{-\frac{4}{3}}$

6 Work out

a $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$ **b** $16^{\frac{1}{4}} + 25^{\frac{1}{2}}$ **c** $8^{-\frac{1}{3}} \div 36^{\frac{1}{2}}$ **d** $(-64)^{\frac{1}{3}} \times 9^{\frac{3}{2}}$

e $(\frac{1}{3})^{-2} - (-8)^{\frac{1}{3}}$ **f** $(\frac{1}{25})^{\frac{1}{2}} \times (\frac{1}{4})^{-2}$ **g** $81^{\frac{3}{4}} - (\frac{1}{49})^{-\frac{1}{2}}$ **h** $(\frac{1}{27})^{-\frac{1}{3}} \times (\frac{4}{9})^{-\frac{3}{2}}$

i $(\frac{1}{9})^{-\frac{1}{2}} \times (-32)^{\frac{3}{5}}$ **j** $(121)^{0.5} + (32)^{0.2}$ **k** $(100)^{0.5} \div (0.25)^{1.5}$ **l** $(16)^{-0.25} \times (243)^{0.4}$

7 Simplify

a $x^8 \times x^{-6}$ **b** $y^{-2} \times y^{-4}$ **c** $6p^3 \div 2p^7$ **d** $(2x^{-4})^3$

e $y^3 \times y^{-\frac{1}{2}}$ **f** $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$ **g** $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$ **h** $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$ **j** $(3x^{\frac{2}{5}})^2$ **k** $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$ **l** $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\dfrac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$ **n** $\dfrac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$ **o** $\dfrac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$ **p** $\dfrac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

8 Solve each equation.

a $x^{\frac{1}{2}} = 6$ **b** $x^{\frac{1}{3}} = 5$ **c** $x^{-\frac{1}{2}} = 2$ **d** $x^{-\frac{1}{4}} = \frac{1}{3}$

e $x^{\frac{3}{2}} = 8$ **f** $x^{\frac{2}{3}} = 16$ **g** $x^{\frac{4}{3}} = 81$ **h** $x^{-\frac{3}{2}} = 27$

9 Express in the form x^k

a $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$

e $\sqrt{x^3}$ **f** $\sqrt{x} \times \sqrt[3]{x}$ **g** $(\sqrt{x})^5$ **h** $\sqrt[3]{x^2} \times (\sqrt{x})^3$

10 Express each of the following in the form ax^b, where a and b are rational constants.

a $\frac{4}{\sqrt{x}}$ **b** $\frac{1}{2x}$ **c** $\frac{3}{4x^3}$ **d** $\frac{1}{(3x)^2}$ **e** $\frac{2}{5\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt{9x^3}}$

11 Express in the form 2^k

a 8^2 **b** $(\frac{1}{4})^{-2}$ **c** $(\frac{1}{2})^{\frac{1}{3}}$ **d** $16^{-\frac{1}{6}}$ **e** $8^{\frac{2}{5}}$ **f** $(\frac{1}{32})^{-3}$

12 Express each of the following in the form 3^y, where y is a function of x.

a 9^x **b** 81^{x+1} **c** $27^{\frac{x}{4}}$ **d** $(\frac{1}{3})^x$ **e** 9^{2x-1} **f** $(\frac{1}{27})^{x+2}$

13 Given that $y = 2^x$, express each of the following in terms of y.

a 2^{x+1} **b** 2^{x-2} **c** 2^{2x} **d** 8^x **e** 2^{4x+3} **f** $(\frac{1}{2})^{x-3}$

14 Find the value of x such that

a $2^x = 64$ **b** $5^{x-1} = 125$ **c** $3^{x+4} - 27 = 0$ **d** $8^x - 2 = 0$

e $3^{2x-1} = 9$ **f** $16 - 4^{3x-2} = 0$ **g** $9^{x-2} = 27$ **h** $8^{2x+1} = 16$

i $49^{x+1} = \sqrt{7}$ **j** $3^{3x-2} = \sqrt[3]{9}$ **k** $(\frac{1}{6})^{x+3} = 36$ **l** $(\frac{1}{2})^{3x-1} = 8$

15 Solve each equation.

a $2^{x+3} = 4^x$ **b** $5^{3x} = 25^{x+1}$ **c** $9^{2x} = 3^{x-3}$ **d** $16^x = 4^{1-x}$

e $4^{x+2} = 8^x$ **f** $27^{2x} = 9^{3-x}$ **g** $6^{3x-1} = 36^{x+2}$ **h** $8^x = 16^{2x-1}$

i $125^x = 5^{x-3}$ **j** $(\frac{1}{3})^x = 3^{x-4}$ **k** $(\frac{1}{2})^{1-x} = (\frac{1}{8})^{2x}$ **l** $(\frac{1}{4})^{x+1} = 8^x$

16 Expand and simplify

a $x(x^2 - x^{-1})$ **b** $2x^3(x^{-1} + 3)$ **c** $x^{-1}(3x - x^3)$ **d** $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$ **f** $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$ **g** $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$ **h** $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i $(x^2 + 1)(x^4 - 3)$ **j** $(2x^5 + x)(x^4 + 3)$ **k** $(x^2 - 2x^{-1})(x - x^{-2})$ **l** $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

17 Simplify

a $\frac{x^3 + 2x}{x}$ **b** $\frac{4t^5 - 6t^3}{2t^2}$ **c** $\frac{x^{\frac{3}{2}} - 3x}{x^{\frac{1}{2}}}$ **d** $\frac{y^2(y^3 - 6)}{3y}$

e $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$ **f** $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$ **g** $\frac{x + 1}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}}$ **h** $\frac{2t^3 - 4t}{t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}}$

1 Express each of the following in the form $a\sqrt{2} + b\sqrt{3}$, where a and b are integers.

a $\sqrt{27} + 2\sqrt{50}$

b $\sqrt{6}(\sqrt{3} - \sqrt{8})$

2 Given that $x > 0$, find in the form $k\sqrt{3}$ the value of x such that

$$x(x - 2) = 2(6 - x).$$

3 Solve the equation

$$25^x = 5^{4x + 1}.$$

4 **a** Express $\sqrt[3]{24}$ in the form $k\sqrt[3]{3}$.

b Find the integer n such that

$$\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{n}.$$

5 Show that

$$\frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5} - \sqrt{7}}$$

can be written in the form $k\sqrt{7}$, where k is an integer to be found.

6 Showing your method clearly,

a express $\sqrt{37.5}$ in the form $a\sqrt{6}$,

b express $\sqrt{9\tfrac{3}{5}} - \sqrt{6\tfrac{2}{3}}$ in the form $b\sqrt{15}$.

7 Given that $x = 2^{t - 1}$ and $y = 2^{3t}$,

a find expressions in terms of t for

i xy **ii** $2y^2$

b Hence, or otherwise, find the value of t for which

$$2y^2 - xy = 0.$$

8 Solve the equation

$$\sqrt{2}(3x - 1) = 2(2x + 3),$$

giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

9 Given that $6^{y + 1} = 36^{x - 2}$,

a express y in the form $ax + b$,

b find the value of $4^{x - \frac{1}{2}y}$.

10 Express each of the following in the form $a + b\sqrt{2}$, where a and b are integers.

a $(3 - \sqrt{2})(1 + \sqrt{2})$

b $\dfrac{\sqrt{2}}{\sqrt{2} - 1}$

11 Solve the equation

$$16^{x+1} = 8^{2x+1}.$$

12 Given that

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3},$$

find the values of the integers a and b.

13 **a** Find the value of t such that

$$(\tfrac{1}{4})^{t-3} = 8.$$

b Solve the equation

$$(\tfrac{1}{3})^{y} = 27^{y+1}.$$

14 Express each of the following in the form $a + b\sqrt{5}$, where a and b are integers.

a $\sqrt{20}(\sqrt{5} - 3)$

b $(1 - \sqrt{5})(3 + 2\sqrt{5})$

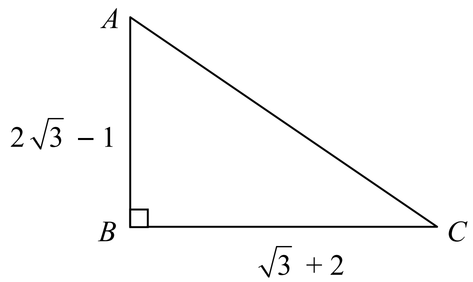
c $\dfrac{1+\sqrt{5}}{\sqrt{5}-2}$

15 Given that $a^{\frac{1}{3}} = b^{\frac{3}{4}}$, and that $a > 0$ and $b > 0$,

a find an expression for $a^{\frac{1}{2}}$ in terms of b,

b find an expression for $b^{\frac{1}{2}}$ in terms of a.

16



In triangle ABC, $AB = 2\sqrt{3} - 1$, $BC = \sqrt{3} + 2$ and $\angle ABC = 90°$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC = 2\sqrt{5}$.

c Show that $\tan(\angle ACB) = 5\sqrt{3} - 8$.

17 **a** Given that $y = 2^x$, express each of the following in terms of y.

i 2^{x+2} **ii** 4^x

b Hence, or otherwise, find the value of x for which

$$4^x - 2^{x+2} = 0.$$

18 Given that the point with coordinates $(1 + \sqrt{3}, 5\sqrt{3})$ lies on the curve with the equation

$$y = 2x^2 + px + q,$$

find the values of the rational constants p and q.

1 Giving your answers in descending powers of x, simplify

a $(x^2 + 3x + 2) + (2x^2 + 5x + 1)$

b $(x^3 + 4x^2 + x - 6) + (x^2 - 3x + 7)$

c $(4 - x + 2x^3) + (3 - x + 6x^2 - 5x^3)$

d $(x^5 + 8x^3 - 5x^2 - 9) + (-x^4 - 4x + 1)$

e $(3x^3 - 7x^2 + 2) - (x^3 + 2x^2 + x - 6)$

f $(x^5 + 3x^4 - x^2 - 3) - (x^4 + 2x^3 - 3x + 2)$

g $(2x^7 - 9x^5 + x^3 + x) - (3x^6 - 4x^3 + x + 5)$

h $2(x^4 + 4x^2 - 3) + (x^4 + 3x^3 - 8)$

i $3(7 + 4x - x^2 - 2x^3) + 5(-2 - 3x + x^3)$

j $6(x^3 + 5x^2 - 2) - 3(2x^3 - x^2 - x)$

k $8(x^4 + 2x^2 - 4x - 1) - 2(5 - 3x + x^3)$

l $7(x^6 + 3x^3 + x^2 - 4) - 4(2x^6 + x^5 - 3x - 7)$

2 Simplify

a $(3y^2 + 2y + 1) + (y^3 - 4y^2 + 7y) + (2y^3 - y^2 - 8y + 5)$

b $3(t^4 - t^3 + 4t) + (6 - t - 3t^3) + 2(t^4 - 2t^2 + 4)$

c $(x^3 - 6x^2 + 8) + (5x^2 - x + 1) - (2x^3 + 3x^2 + x - 7)$

d $2(3 + m + 7m^2 - 3m^5) + 6(1 - m^2 + 2m^4) - 5(m^5 + 3m^3 - m^2 + 2)$

e $\frac{1}{3}(1 - 2u + \frac{3}{5}u^2 + 3u^4) - \frac{1}{2}(2 - u + \frac{2}{3}u^2 - \frac{1}{2}u^3)$

3 Giving your answers in ascending powers of x, simplify

a $x(2 - 3x + x^2) + 4(1 + 2x^2 - x^3)$

b $x(x^4 + 7x^2 - 5x + 9) - 2(x^4 - 4x^3 - 3)$

c $2x(-5 + 4x - x^3) + 7(2 - 3x^2 + x^4)$

d $x^2(8 + 2x + x^2) - 3(5 + 4x^2 + x^3)$

e $3x^2(x + 3) - x(x^3 + 4x^2) + 5(x^3 - 2x)$

f $x^2(6 - x + 5x^2) + 7x(2 - x^3) + 4(1 - 3x - x^2)$

4 Show that

a $(3x + 1)(x^2 - 2x + 4) \equiv 3x^3 - 5x^2 + 10x + 4$

b $(1 + 2x - x^2)(1 - 2x + x^2) \equiv 1 - 4x^2 + 4x^3 - x^4$

c $(3 - x)^3 \equiv 27 - 27x + 9x^2 - x^3$

5 Giving your answers in descending powers of x, expand and simplify

a $(x + 1)(x^2 + 5x - 6)$

b $(2x - 5)(x^2 - 3x + 7)$

c $(4 - 7x)(2 + 5x - x^2)$

d $(3x - 2)^3$

e $(x^2 + 3)(2x^2 - x + 9)$

f $(4x - 1)(x^4 - 3x^2 + 5x + 2)$

g $(x^2 + 2x + 5)(x^2 + 3x + 1)$

h $(x^2 + x - 3)(2x^2 - x + 4)$

i $(3x^2 - 5x + 2)(2x^2 - 4x - 8)$

j $(x^2 + 2x - 6)^2$

k $(x^3 + 4x^2 + 1)(2x^4 + x^2 + 3)$

l $(6 - 2x + x^3)(3 + x^2 - x^3 + 2x^4)$

6 Simplify

a $(p^2 - 1)(p + 4)(2p + 3)$

b $(t + 2)(t^2 + 3t + 5) + (t + 4)(t^2 + t + 7)$

c $2(x^2 - 3)(x^2 + x - 4) + (3x - 1)(4x^3 + 2x^2 - x + 6)$

d $(u^3 - 4u^2 - 3)(u + 2) - (2u^3 + u - 1)(u^2 + 5u - 3)$

1 Factorise

a $x^2 + 4x + 3$ **b** $x^2 + 7x + 10$ **c** $y^2 - 3y + 2$ **d** $x^2 - 6x + 9$

e $y^2 - y - 2$ **f** $a^2 + 2a - 8$ **g** $x^2 - 1$ **h** $p^2 + 9p + 14$

i $x^2 - 2x - 15$ **j** $16 - 10m + m^2$ **k** $t^2 + 3t - 18$ **l** $y^2 - 13y + 40$

m $r^2 - 16$ **n** $y^2 - 2y - 63$ **o** $121 + 22a + a^2$ **p** $x^2 + 6x - 72$

q $26 - 15x + x^2$ **r** $s^2 + 23s + 120$ **s** $p^2 + 14p - 51$ **t** $m^2 - m - 90$

2 Factorise

a $2x^2 + 3x + 1$ **b** $2 + 7p + 3p^2$ **c** $2y^2 - 5y + 3$ **d** $2 - m - m^2$

e $3r^2 - 2r - 1$ **f** $5 - 19y - 4y^2$ **g** $4 - 13a + 3a^2$ **h** $5x^2 - 8x - 4$

i $4x^2 + 8x + 3$ **j** $9s^2 - 6s + 1$ **k** $4m^2 - 25$ **l** $2 - y - 6y^2$

m $4u^2 + 17u + 4$ **n** $6p^2 + 5p - 4$ **o** $8x^2 + 19x + 6$ **p** $12r^2 + 8r - 15$

3 Using factorisation, solve each equation.

a $x^2 - 4x + 3 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 4x - 5 = 0$ **d** $x^2 - 7x = 8$

e $x^2 - 25 = 0$ **f** $x(x - 1) = 42$ **g** $x^2 = 3x$ **h** $27 + 12x + x^2 = 0$

i $60 - 4x - x^2 = 0$ **j** $5x + 14 = x^2$ **k** $2x^2 - 3x + 1 = 0$ **l** $x(x - 1) = 6(x - 2)$

m $3x^2 + 11x = 4$ **n** $x(2x - 3) = 5$ **o** $6 + 23x - 4x^2 = 0$ **p** $6x^2 + 10 = 19x$

q $4x^2 + 4x + 1 = 0$ **r** $3(x^2 + 4) = 13x$ **s** $(2x + 5)^2 = 5 - x$ **t** $3x(2x - 7) = 2(7x + 3)$

4 Factorise fully

a $2y^2 - 10y + 12$ **b** $x^3 + x^2 - 2x$ **c** $p^3 - 4p$ **d** $3m^3 + 21m^2 + 18m$

e $a^4 + 4a^2 + 3$ **f** $t^4 + 3t^2 - 10$ **g** $12 + 20x - 8x^2$ **h** $6r^2 - 9r - 42$

i $6x^3 - 26x^2 + 8x$ **j** $y^4 + 3y^3 - 18y^2$ **k** $m^4 - 1$ **l** $p^5 - 4p^3 + 4p$

5 Sketch each curve showing the coordinates of any points of intersection with the coordinate axes.

a $y = x^2 - 3x + 2$ **b** $y = x^2 + 5x + 6$ **c** $y = x^2 - 9$

d $y = x^2 - 2x$ **e** $y = x^2 - 10x + 25$ **f** $y = 2x^2 - 14x + 20$

g $y = -x^2 + 5x - 4$ **h** $y = 2 + x - x^2$ **i** $y = 2x^2 - 3x + 1$

j $y = 2x^2 + 13x + 6$ **k** $y = 3 - 8x + 4x^2$ **l** $y = 2 + 7x - 4x^2$

m $y = 5x^2 - 17x + 6$ **n** $y = -6x^2 + 7x - 2$ **o** $y = 6x^2 + x - 5$

6 Solve each of the following equations.

a $x - 5 + \dfrac{4}{x} = 0$ **b** $x - \dfrac{10}{x} = 3$ **c** $2x^3 - x^2 - 3x = 0$ **d** $x^2(10 - x^2) = 9$

e $\dfrac{5}{x^2} + \dfrac{4}{x} - 1 = 0$ **f** $\dfrac{x-6}{x-4} = x$ **g** $x + 5 = \dfrac{3}{x+3}$ **h** $x^2 - \dfrac{4}{x^2} = 3$

i $4x^4 + 7x^2 = 2$ **j** $\dfrac{2x}{3-x} = \dfrac{1}{x+2}$ **k** $\dfrac{2x+1}{x+3} = \dfrac{2}{x}$ **l** $\dfrac{7}{x+2} - 3x = 2$

1 Express in the form $(x + a)^2 + b$

a $x^2 + 2x + 4$ **b** $x^2 - 2x + 4$ **c** $x^2 - 4x + 1$ **d** $x^2 + 6x$

e $x^2 + 4x + 8$ **f** $x^2 - 8x - 5$ **g** $x^2 + 12x + 30$ **h** $x^2 - 10x + 25$

i $x^2 + 6x - 9$ **j** $18 - 4x + x^2$ **k** $x^2 + 3x + 3$ **l** $x^2 + x - 1$

m $x^2 - 18x + 100$ **n** $x^2 - x - \frac{1}{2}$ **o** $20 + 9x + x^2$ **p** $x^2 - 7x - 2$

q $5 - 3x + x^2$ **r** $x^2 - 11x + 37$ **s** $x^2 + \frac{2}{3}x + 1$ **t** $x^2 - \frac{1}{2}x - \frac{1}{4}$

2 Express in the form $a(x + b)^2 + c$

a $2x^2 + 4x + 3$ **b** $2x^2 - 8x - 7$ **c** $3 - 6x + 3x^2$ **d** $4x^2 + 24x + 11$

e $-x^2 - 2x - 5$ **f** $1 + 10x - x^2$ **g** $2x^2 + 2x - 1$ **h** $3x^2 - 9x + 5$

i $3x^2 - 24x + 48$ **j** $3x^2 - 15x$ **k** $70 + 40x + 5x^2$ **l** $2x^2 + 5x + 2$

m $4x^2 + 6x - 7$ **n** $-2x^2 + 4x - 1$ **o** $4 - 2x - 3x^2$ **p** $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$

3 Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.

a $y^2 - 4y + 2 = 0$ **b** $p^2 + 2p - 2 = 0$ **c** $x^2 - 6x + 4 = 0$ **d** $7 + 10r + r^2 = 0$

e $x^2 - 2x = 11$ **f** $a^2 - 12a - 18 = 0$ **g** $m^2 - 3m + 1 = 0$ **h** $9 - 7t + t^2 = 0$

i $u^2 + 7u = 44$ **j** $2y^2 - 4y + 1 = 0$ **k** $3p^2 + 18p = -23$ **l** $2x^2 + 12x = 9$

m $-m^2 + m + 1 = 0$ **n** $4x^2 + 49 = 28x$ **o** $1 - t - 3t^2 = 0$ **p** $2a^2 - 7a + 4 = 0$

4 By completing the square, find the maximum or minimum value of y and the value of x for which this occurs. State whether your value of y is a maximum or a minimum in each case.

a $y = x^2 - 2x + 7$ **b** $y = x^2 + 2x - 3$ **c** $y = 1 - 6x + x^2$

d $y = x^2 + 10x + 35$ **e** $y = -x^2 + 4x + 4$ **f** $y = x^2 + 3x - 2$

g $y = 2x^2 + 8x + 5$ **h** $y = -3x^2 + 6x$ **i** $y = 7 - 5x - x^2$

j $y = 4x^2 - 12x + 9$ **k** $y = 4x^2 + 20x - 8$ **l** $y = 17 - 2x - 2x^2$

5 Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the y-axis.

a $y = x^2 - 4x + 3$ **b** $y = x^2 + 2x - 24$ **c** $y = x^2 - 2x + 5$

d $y = 30 + 8x + x^2$ **e** $y = x^2 + 2x + 1$ **f** $y = 8 + 2x - x^2$

g $y = -x^2 + 8x - 7$ **h** $y = -x^2 - 4x - 7$ **i** $y = x^2 - 5x + 4$

j $y = x^2 + 3x + 3$ **k** $y = 3 + 8x + 4x^2$ **l** $y = -2x^2 + 8x - 15$

m $y = 1 - x - 2x^2$ **n** $y = 25 - 20x + 4x^2$ **o** $y = 3x^2 - 4x + 2$

6 **a** Express $x^2 - 4\sqrt{2}\,x + 5$ in the form $a(x + b)^2 + c$.

b Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}\,x + 5$.

7 $$f(x) \equiv x^2 + 2kx - 3.$$

By completing the square, find the roots of the equation $f(x) = 0$ in terms of the constant k.

1 By completing the square, show that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2 Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate.

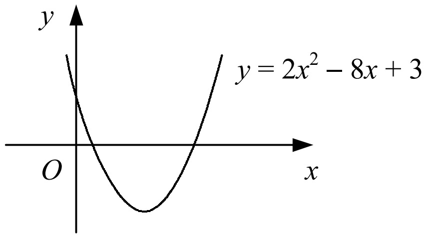
a $x^2 + 4x + 1 = 0$ **b** $4 + 8t - t^2 = 0$ **c** $y^2 - 20y + 91 = 0$ **d** $r^2 + 2r - 7 = 0$

e $6 + 18a + a^2 = 0$ **f** $m(m - 5) = 5$ **g** $x^2 + 11x + 27 = 0$ **h** $2u^2 + 6u + 3 = 0$

i $5 - y - y^2 = 0$ **j** $2x^2 - 3x = 2$ **k** $3p^2 + 7p + 1 = 0$ **l** $t^2 - 14t = 14$

m $0.1r^2 + 1.4r = 0.9$ **n** $6u^2 + 4u = 1$ **o** $\frac{1}{2}y^2 - 3y = \frac{2}{3}$ **p** $4x(x - 3) = 11 - 4x$

3



The diagram shows the curve with equation $y = 2x^2 - 8x + 3$.

Find and simplify the exact coordinates of the points where the curve crosses the x-axis.

4 State the condition for which the roots of the equation $ax^2 + bx + c = 0$ are

a real and distinct **b** real and equal **c** not real

5 Sketch the curve $y = ax^2 + bx + c$ and the x-axis in the cases where

a $a > 0$ and $b^2 - 4ac > 0$ **b** $a < 0$ and $b^2 - 4ac < 0$

c $a > 0$ and $b^2 - 4ac = 0$ **d** $a < 0$ and $b^2 - 4ac > 0$

6 By evaluating the discriminant, determine whether the roots of each equation are real and distinct, real and equal or not real.

a $x^2 + 2x - 7 = 0$ **b** $x^2 + x + 3 = 0$ **c** $x^2 - 4x + 5 = 0$ **d** $x^2 - 6x + 3 = 0$

e $x^2 + 14x + 49 = 0$ **f** $x^2 - 9x + 17 = 0$ **g** $x^2 + 3x = 11$ **h** $2 + 3x + 2x^2 = 0$

i $5x^2 + 8x + 3 = 0$ **j** $3x^2 - 7x + 5 = 0$ **k** $9x^2 - 12x + 4 = 0$ **l** $13x^2 + 19x + 7 = 0$

m $4 - 11x + 8x^2 = 0$ **n** $x^2 + \frac{2}{3}x = \frac{1}{4}$ **o** $x^2 - \frac{3}{4}x + \frac{1}{8} = 0$ **p** $\frac{2}{5}x^2 + \frac{3}{5}x + \frac{1}{3} = 0$

7 Find the value of the constant p such that the equation $x^2 + x + p = 0$ has equal roots.

8 Given that $q \neq 0$, find the value of the constant q such that the equation $x^2 + 2qx - q = 0$ has a repeated root.

9 Given that the x-axis is a tangent to the curve with the equation

$$y = x^2 + rx - 2x + 4,$$

find the two possible values of the constant r.

1 **a** Factorise fully the expression

$$20x - 2x^2 - 6x^3.$$

b Hence, find all solutions to the equation

$$20x - 2x^2 - 6x^3 = 0.$$

2 A is the point $(-2, 1)$ and B is the point $(6, k)$.

a Show that $AB^2 = k^2 - 2k + 65$.

Given also that $AB = 10$,

b find the possible values of k.

3 Solve the equations

a $x - \dfrac{5}{x} = 4$

b $\dfrac{9}{5-x} - 1 = 2x$

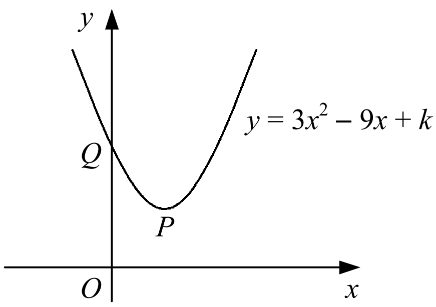
4 **a** Find the coordinates of the turning point of the curve with equation $y = 3 - 5x - 2x^2$.

b Sketch the curve $y = 3 - 5x - 2x^2$, showing the coordinates of any points of intersection with the coordinate axes.

5 Find in the form $k\sqrt{2}$ the solutions of the equation

$$2x^2 + 5\sqrt{2}\,x - 6 = 0.$$

6



The diagram shows the curve with equation $y = 3x^2 - 9x + k$ where k is a constant.

a Find the x-coordinate of the turning point of the curve, P.

Given that the y-coordinate of P is $\frac{17}{4}$,

b find the coordinates of the point Q where the curve crosses the y-axis.

7 By letting $y = 2^x$, or otherwise, solve the equation

$$2^{2x} - 10(2^x) + 16 = 0.$$

8 Given that the equation

$$kx^2 - 2x + 3 - 2k = 0$$

has equal roots, find the possible values of the constant k.

9 $\mathrm{f}(x) \equiv 3 + 4x - x^2.$

a Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$.

b State the coordinates of the turning point of the curve $y = \mathrm{f}(x)$.

c Solve the equation $\mathrm{f}(x) = 2$, giving your answers in the form $d + e\sqrt{5}$.

10 Giving your answers in terms of surds, solve the equations

a $3x^2 - 5x + 1 = 0$

b $\dfrac{x}{x+2} = \dfrac{3}{x-1}$

11 **a** By completing the square, find, in terms of k, the solutions of the equation

$$x^2 - 4kx + 6 = 0.$$

b Using your answers to part **a**, solve the equation

$$x^2 - 12x + 6 = 0.$$

12 **a** Find in the form $a + b\sqrt{3}$, where a and b are integers, the values of x such that

$$2x^2 - 12x = 6.$$

b Solve the equation

$$2y^3 + y^2 - 15y = 0.$$

13 Labelling the coordinates of any points of intersection with the coordinate axes, sketch the curves

a $y = (x + 1)(x - p)$ where $p > 0$,

b $y = (x + q)^2$ where $q < 0$.

14 $\mathrm{f}(x) \equiv 2x^2 - 6x + 5.$

a Find the values of A, B and C such that

$$\mathrm{f}(x) \equiv A(x + B)^2 + C.$$

b Hence deduce the minimum value of $\mathrm{f}(x)$.

15 **a** Given that $t = x^{\frac{1}{3}}$ express $x^{\frac{2}{3}}$ in terms of t.

b Hence, or otherwise, solve the equation

$$2x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0.$$

16 **a** Express $k^2 - 8k + 20$ in the form $a(k + b)^2 + c$, where a, b and c are constants.

b Hence prove that the equation

$$x^2 - kx + 2k = 5$$

has real and distinct roots for all real values of k.

17 **a** Show that

$$(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^4 - x^3 - 13x^2 + x + 12.$$

b Hence solve the equation

$$x^4 - x^3 - 13x^2 + x + 12 = 0.$$

1 Solve each pair of simultaneous equations.

a $y = 3x$
$y = 2x + 1$

b $y = x - 6$
$y = \frac{1}{2}x - 4$

c $y = 2x + 6$
$y = 3 - 4x$

d $x + y - 3 = 0$
$x + 2y + 1 = 0$

e $x + 2y + 11 = 0$
$2x - 3y + 1 = 0$

f $3x + 3y + 4 = 0$
$5x - 2y - 5 = 0$

2 Find the coordinates of the points of intersection of the given straight line and curve in each case.

a $y = x + 2$
$y = x^2 - 4$

b $y = 4x + 11$
$y = x^2 + 3x - 1$

c $y = 2x - 1$
$y = 2x^2 + 3x - 7$

3 Solve each pair of simultaneous equations.

a $x^2 - y + 3 = 0$
$x - y + 5 = 0$

b $2x^2 - y - 8x = 0$
$x + y + 3 = 0$

c $x^2 + y^2 = 25$
$2x - y = 5$

d $x^2 + 2xy + 15 = 0$
$2x - y + 10 = 0$

e $x^2 - 2xy - y^2 = 7$
$x + y = 1$

f $3x^2 - x - y^2 = 0$
$x + y - 1 = 0$

g $2x^2 + xy + y^2 = 22$
$x + y = 4$

h $x^2 - 4y - y^2 = 0$
$x - 2y = 0$

i $x^2 + xy = 4$
$3x + 2y = 6$

j $2x^2 + y - y^2 = 8$
$2x - y = 3$

k $x^2 - xy + y^2 = 13$
$2x - y = 7$

l $x^2 - 5x + y^2 = 0$
$3x + y = 5$

m $3x^2 - xy + y^2 = 36$
$x - 2y = 10$

n $2x^2 + x - 4y = 6$
$3x - 2y = 4$

o $x^2 + x + 2y^2 - 52 = 0$
$x - 3y + 17 = 0$

4 Solve each pair of simultaneous equations.

a $x - \frac{1}{y} - 4y = 0$
$x - 6y - 1 = 0$

b $xy = 6$
$x - y = 5$

c $\frac{3}{x} - 2y + 4 = 0$
$4x + y - 7 = 0$

5 The line $y = 5 - x$ intersects the curve $y = x^2 - 3x + 2$ at the points P and Q.
Find the length PQ in the form $k\sqrt{2}$.

6 Solve the simultaneous equations

$$3^{x-1} = 9^{2y}$$
$$8^{x-2} = 4^{1+y}$$

7 Given that

$$(A + 2\sqrt{3})(B - \sqrt{3}) \equiv 9\sqrt{3} - 1,$$

find the values of the integers A and B.

1 Find the set of values of x for which

a $2x + 1 < 7$ **b** $3x - 1 \geq 20$ **c** $2x - 5 > 3$ **d** $6 + 3x \leq 42$

e $5x + 17 \geq 2$ **f** $\frac{1}{3}x + 7 < 8$ **g** $9x - 4 \geq 50$ **h** $3x + 11 < 7$

i $18 - x > 4$ **j** $10 + 4x \leq 0$ **k** $12 - 3x < 10$ **l** $9 - \frac{1}{2}x \geq 4$

2 Solve each inequality.

a $2y - 3 > y + 4$ **b** $5p + 1 \leq p + 3$ **c** $x - 2 < 3x - 8$

d $a + 11 \geq 15 - a$ **e** $17 - 2u < 2 + u$ **f** $5 - b \geq 14 - 3b$

g $4x + 23 < x + 5$ **h** $12 + 3y \geq 2y - 1$ **i** $16 - 3p \leq 36 + p$

j $5(r - 2) > 30$ **k** $3(1 - 2t) \leq t - 4$ **l** $2(3 + x) \geq 4(6 - x)$

m $7(y + 3) - 2(3y - 1) < 0$ **n** $4(5 - 2x) > 3(7 - 2x)$ **o** $3(4u - 1) - 5(u - 3) < 9$

3 Find the set of values of x for which

a $x^2 - 4x + 3 < 0$ **b** $x^2 - 4 \leq 0$ **c** $15 + 8x + x^2 < 0$ **d** $x^2 + 2x \leq 8$

e $x^2 - 6x + 5 > 0$ **f** $x^2 + 4x > 12$ **g** $x^2 + 10x + 21 \geq 0$ **h** $22 + 9x - x^2 > 0$

i $63 - 2x - x^2 \leq 0$ **j** $x^2 + 11x + 30 > 0$ **k** $30 + 7x - x^2 > 0$ **l** $x^2 + 91 \geq 20x$

4 Solve each inequality.

a $2x^2 - 9x + 4 \leq 0$ **b** $2r^2 - 5r - 3 < 0$ **c** $2 - p - 3p^2 \geq 0$

d $2y^2 + 9y - 5 > 0$ **e** $4m^2 + 13m + 3 < 0$ **f** $9x - 2x^2 \leq 10$

g $a^2 + 6 < 8a - 9$ **h** $x(x + 4) \leq 7 - 2x$ **i** $y(y + 9) > 2(y - 5)$

j $x(2x + 1) > x^2 + 6$ **k** $u(5 - 6u) < 3 - 4u$ **l** $2t + 3 \geq 3t(t - 2)$

m $(y - 2)^2 \leq 2y - 1$ **n** $(p + 2)(p + 3) \geq 20$ **o** $2(13 + 2x) < (6 + x)(1 - x)$

5 Giving your answers in terms of surds, find the set of values of x for which

a $x^2 + 2x - 1 < 0$ **b** $x^2 - 6x + 4 > 0$ **c** $11 - 6x - x^2 > 0$ **d** $x^2 + 4x + 1 \geq 0$

6 Find the value or set of values of k such that

a the equation $x^2 - 6x + k = 0$ has equal roots,

b the equation $x^2 + 2x + k = 0$ has real and distinct roots,

c the equation $x^2 - 3x + k = 0$ has no real roots,

d the equation $x^2 + kx + 4 = 0$ has real roots,

e the equation $kx^2 + x - 1 = 0$ has equal roots,

f the equation $x^2 + kx - 3k = 0$ has no real roots,

g the equation $x^2 + 2x + k - 2 = 0$ has real and distinct roots,

h the equation $2x^2 - kx + k = 0$ has equal roots,

i the equation $x^2 + kx + 2k - 3 = 0$ has no real roots,

j the equation $3x^2 + kx - x + 3 = 0$ has real roots.

1 Solve each of the following inequalities.

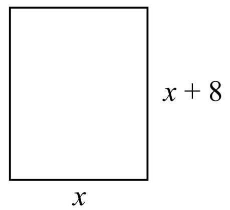
a $\frac{1}{2}y + 3 > 2y - 1$

b $x^2 - 8x + 12 \geq 0$

2 Find the set of integers, n, for which

$$2n^2 - 5n < 12.$$

3



The diagram shows a rectangular birthday card which is x cm wide and $(x + 8)$ cm tall.

Given that the height of the card is to be at least 50% more than its width,

a show that $x \leq 16$.

Given also that the area of the front of the card is to be at least 180 cm^2,

b find the set of possible values of x.

4 Find the set of values of x for which

$$(3x - 1)^2 < 5x - 1.$$

5 Given that $x - y = 8$,

and that $xy \leq 240$,

find the maximum value of $(x + y)$.

6 Solve the inequality

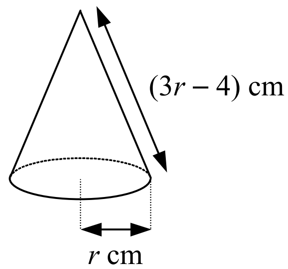
$$(3t + 1)(t - 4) \geq 2t(t - 7).$$

7 Given that the equation $2x(x + 1) = kx - 8$ has real and distinct roots,

a show that $k^2 - 4k - 60 > 0$,

b find the set of possible values of k.

8



A party hat is designed in the shape of a right circular cone of base radius r cm and slant height $(3r - 4)$ cm.

Given that the height of the cone must not be more than 24 cm, find the maximum value of r.

1 **a** Find the value of x such that

$$2^{x-1} = 16.$$ **(3)**

b Find the value of y such that

$$2(3^y - 10) = 34.$$ **(2)**

2 **a** Express $x^2 - 6x + 11$ in the form $(x + a)^2 + b$. **(2)**

b Sketch the curve $y = x^2 - 6x + 11$, and show the coordinates of the turning point of the curve. **(3)**

3 **a** Express $(12\tfrac{1}{4})^{-\frac{1}{2}}$ as an exact fraction in its simplest form. **(2)**

b Solve the equation

$$3x^{-3} = 7\tfrac{1}{9}.$$ **(3)**

4 Solve the equation

$$x\sqrt{12} + 9 = x\sqrt{3},$$

giving your answer in the form $k\sqrt{3}$, where k is an integer. **(4)**

5 **a** Solve the equation

$$x^2 + 10x + 13 = 0,$$

giving your answers in the form $a + b\sqrt{3}$, where a and b are integers. **(4)**

b Hence find the set of values of x for which

$$x^2 + 10x + 13 > 0.$$ **(2)**

6 Solve the equations

a $7(6x - 7) = 9x^2$ **(3)**

b $\dfrac{2}{y+1} + 1 = 2y$ **(4)**

7 Solve the simultaneous equations

$$x - y + 3 = 0$$
$$3x^2 - 2xy + y^2 - 17 = 0$$ **(6)**

8 **a** Find the value of x such that

$$x^{\frac{3}{2}} = 64.$$ **(2)**

b Given that

$$\frac{\sqrt{3}+1}{2\sqrt{3}-3} \equiv a + b\sqrt{3},$$

find the values of the rational constants a and b. **(4)**

9 The point P $(2k, k)$ lies within a circle of radius 3, centre $(2, 4)$.

a Show that $5k^2 - 16k + 11 < 0$. **(4)**

b Hence find the set of possible values of k. **(3)**

10 Solve each of the following inequalities.

a $4x - 1 \leq 2x + 6$ **(2)**

b $x(2x + 1) < 1$ **(4)**

11 $$f(x) = 2x^2 - 8x + 5.$$

a Express f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers. **(3)**

b Write down the coordinates of the turning point of the curve $y = f(x)$. **(1)**

c Solve the equation $f(x) = 0$, giving your answers in the form $p + q\sqrt{6}$, where p and q are rational. **(3)**

12 Simplify

a $\sqrt{12} - \frac{5}{\sqrt{3}}$ **(3)**

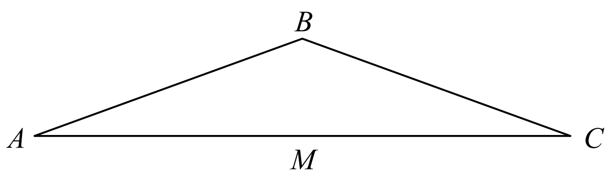
b $\frac{(4\sqrt{x})^3}{16x}$ **(2)**

13 Given that the equation

$$x^2 - 2kx + k + 6 = 0$$

has no real roots, find the set of possible values of the constant k. **(6)**

14



The diagram shows triangle ABC in which $AB = BC = 4 + \sqrt{3}$ and $AC = 4 + 4\sqrt{3}$.

Given that M is the mid-point of AC,

a find the exact length BM, **(4)**

b show that the area of triangle ABC is $6 + 2\sqrt{3}$. **(2)**

15 Solve the equation

$$4^{2y+7} = 8^{y+3}.$$ **(4)**

16 Show that

$$(x^2 - x + 3)(2x^2 - 3x - 9) \equiv Ax^4 + Bx^3 + C,$$

where A, B and C are constants to be found. **(4)**

17 $$f(x) = x^2 + 4x + k.$$

a By completing the square, find in terms of the constant k the roots of the equation $f(x) = 0$. **(4)**

b State the set of values of k for which the equation $f(x) = 0$ has real roots. **(1)**

c Use your answers to part **a** to solve the equation

$$x^2 + 4x - 4 = 0,$$

giving your answers in the form $a + b\sqrt{2}$, where a and b are integers. **(2)**

1 Solve the inequality

$$(x + 1)(x + 2) \leq 12.$$ **(5)**

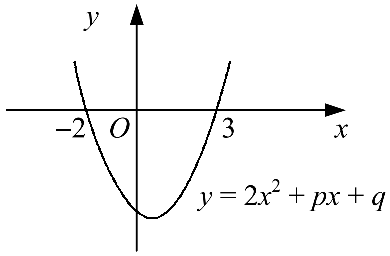
2 **a** Express $2^{\frac{7}{2}} - 2^{\frac{3}{2}}$ in the form $k\sqrt{2}$. **(2)**

b Show that

$$(\sqrt{x} + 6)^2 + (2\sqrt{x} - 3)^2$$

can be written in the form $ax + b$ where a and b are integers to be found. **(3)**

3



The diagram shows the curve with equation $y = 2x^2 + px + q$, where p and q are constants, which crosses the x-axis at the points with coordinates $(-2, 0)$ and $(3, 0)$.

a Show that $p = -2$ and find the value of q. **(4)**

b Find the coordinates of the turning point of the curve. **(3)**

4 Solve the equation

$$2(x - \sqrt{32}) = \sqrt{98} - x,$$

giving your answer in the form $k\sqrt{2}$. **(4)**

5 Given that the equation

$$kx^2 - 4kx + 3 = 0,$$

where k is a constant, has real and distinct roots,

a show that $k(4k - 3) > 0$, **(3)**

b find the set of possible values of k. **(2)**

6 Solve the simultaneous equations

$$4^{2x} = 2^{y-1}$$
$$9^{4x} = 3^{y+1}$$ **(7)**

7 **a** Find the values of the constants a and b such that

$$x^2 - 7x + 9 \equiv (x + a)^2 + b.$$ **(3)**

b Hence, write down an equation of the line of symmetry of the curve $y = x^2 - 7x + 9$. **(1)**

8 **a** Solve the inequality

$$y^2 - 2y - 15 < 0.$$ **(3)**

b Find the exact values of x for which

$$\frac{x}{x-3} = \frac{4}{2-x}.$$ **(5)**

9 Solve the equation

$$2^{x^2+2} = 8^x. \qquad \textbf{(5)}$$

10 Giving your answers in terms of surds, solve the equations

a $t(1 - 2t) = 3(t - 5)$ **(4)**

b $x^4 - x^2 - 6 = 0$ **(4)**

11 Find the set of values of x for which

$$21 - 4x - x^2 \leq 0. \qquad \textbf{(4)}$$

12 **a** Given that $y = 3^x$ express 3^{2x+2} in terms of y. **(2)**

b Hence, or otherwise, solve the equation

$$3^{2x+2} - 10(3^x) + 1 = 0. \qquad \textbf{(4)}$$

13 **a** Express $5\sqrt{3}$ in the form $\sqrt{k}$. **(2)**

b Hence find the integer n such that

$$n < 5\sqrt{3} < n + 1. \qquad \textbf{(3)}$$

14 Solve the simultaneous equations

$$2x^2 - y^2 - 7 = 0$$
$$2x - 3y + 7 = 0 \qquad \textbf{(8)}$$

15 Express each of the following in the form $a + b\sqrt{2}$, where a and b are integers.

a $\dfrac{\sqrt{48} - \sqrt{600}}{\sqrt{12}}$ **(3)**

b $\dfrac{\sqrt{2}}{4 + 3\sqrt{2}}$ **(4)**

16 Given that $5^{x+1} = 25^{y-3}$,

a find an expression for y in terms of x. **(4)**

Given also that $16^{x-1} = 4^z$,

b find an expression for z in terms of y. **(4)**

17 **a** By completing the square, find in terms of the constant k the roots of the equation

$$x^2 - 2kx - k = 0. \qquad \textbf{(4)}$$

b Hence, find the set of values of k for which the equation has real roots. **(3)**

18 **a** Given that $y = x^{\frac{1}{5}}$, show that the equation

$$x^{-\frac{1}{5}} - x^{\frac{1}{5}} = \tfrac{3}{2}$$

can be written as

$$2y^2 + 3y - 2 = 0. \qquad \textbf{(3)}$$

b Hence find the values of x for which

$$x^{-\frac{1}{5}} - x^{\frac{1}{5}} = \tfrac{3}{2}. \qquad \textbf{(4)}$$

1 **a** Express $(\frac{2}{3})^{-2}$ as an exact fraction in its simplest form. **(2)**

b Solve the equation

$$x^{\frac{3}{2}} - 27 = 0.$$ **(3)**

2 Solve the simultaneous equations

$$x + 3y = 16$$
$$x^2 - xy + 2y^2 = 46$$ **(7)**

3 Simplify

a $\sqrt{192} - 2\sqrt{12} + \sqrt{75}$ **(4)**

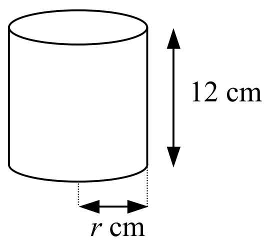
b $(2 + \sqrt{3})(5 - 2\sqrt{3})$ **(3)**

4 $$f(x) \equiv x^2 - 4\sqrt{2}\,x + 11.$$

a Express f(x) in the form $a(x + b)^2 + c$ stating the exact values of the constants a, b and c. **(4)**

b Sketch the curve $y = f(x)$, showing the coordinates of the turning point of the curve and of any points of intersection of the curve with the coordinate axes. **(3)**

5



A sealed metal container for food is a cylinder of height 12 cm and base radius r cm.

Given that the surface area of the container must be at most 128π cm^2,

a show that $r^2 + 12r - 64 \le 0.$ **(3)**

b Hence find the maximum value of r. **(4)**

6 Find the non-zero value of x for which

$$(2\sqrt{x})^3 = 4x.$$ **(4)**

7 **a** Write down the value of x such that $2^x = 32$. **(1)**

b Solve the equation

$$32^{y+1} = 4^y.$$ **(3)**

8 **a** Given that $t = \sqrt{x}$, express $x - 5\sqrt{x}$ in terms of t. **(1)**

b Hence, or otherwise, solve the equation

$$x - 5\sqrt{x} + 6 = 0.$$ **(4)**

9 Prove, by completing the square, that there is no real value of the constant k for which the equation $x^2 + kx + 3 + k^2 = 0$ has real roots. **(6)**

10 **a** Find the value of x such that

$$8^{2x-1} = 32.$$ **(3)**

b Find the value of y such that

$$(\tfrac{1}{3})^{y-2} = 81.$$ **(3)**

11 Solve the inequality

$$x(2x - 7) < (x - 2)^2.$$ **(5)**

12 Express

$$\frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1}$$

in the form $a + b\sqrt{2}$, where a and b are integers. **(6)**

13 **a** Solve the equation

$$6y^2 + 25y - 9 = 0.$$ **(3)**

b Find the values of the constant k such that the equation

$$x^2 + kx + 16 = 0$$

has equal roots. **(3)**

14 **a** Given that $y = 2^x$,

i show that $4^x = y^2$,

ii express 2^{x-1} in terms of y. **(4)**

b By using your answers to part **a**, or otherwise, find the values of x for which

$$4^x - 9(2^{x-1}) + 2 = 0.$$ **(4)**

15 Find the pairs of values (x, y) which satisfy the simultaneous equations

$$x^2 + 2xy + y^2 = 9$$

$$x - 3y = 1$$ **(7)**

16 **a** Prove, by completing the square, that the roots of the equation $x^2 + ax + b = 0$ are given by

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}.$$ **(6)**

b Hence, find an expression for b in terms of a such that the equation $x^2 + ax + b = 0$ has a repeated root. **(2)**

17 $$\mathrm{f}(x) \equiv 2x^2 - 12x + 19.$$

a Prove that $\mathrm{f}(x) \geq 1$ for all real values of x. **(5)**

b Find the set of values of x for which $\mathrm{f}(x) < 9$. **(4)**

18 **a** Express $(1 - \sqrt{5})^2$ in the form $a + b\sqrt{5}$. **(2)**

b Hence, or otherwise, solve the equation

$$y^2 = 3 - \sqrt{5},$$

giving your answers in the form $c\sqrt{2} + d\sqrt{10}$, where c and d are exact fractions. **(6)**