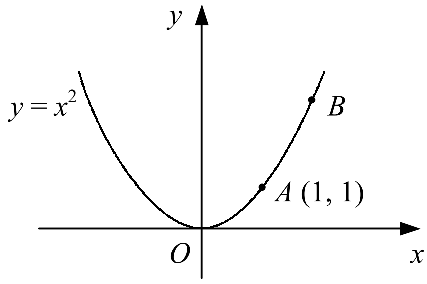
You will need to use a calculator for this worksheet

1



The diagram shows the curve $y = x^2$ which passes through the point A (1, 1) and the point B.

a Copy and complete the table to find the gradient of the chord AB when the x-coordinate of B takes each of the given values.

x-coordinate of B	y-coordinate of B	gradient of AB
2	4	$\frac{4-1}{2-1} = 3$
1.1	1.21	
1.01		
1.001		

b Suggest a value for the gradient of the tangent to the curve $y = x^2$ at the point (1, 1).

c Repeat part **a** using 0, 0.9, 0.99 and 0.999 as the x-coordinates of B and comment on your answer to part **b**.

2 Use a similar table of values to that in question **1** to find a value for the gradient of the tangent to the curve $y = x^2$ at the point A when A has the coordinates

a (2, 4) **b** (4, 16) **c** (1.5, 2.25) **d** (–3, 9)

3 **a** Using your answers to questions **1** and **2**, suggest an expression in terms of x for the gradient of the curve $y = x^2$ at the point (x, y).

b Write down the gradient of the curve $y = x^2$ at the points

i (6, 36) **ii** (2.4, 5.76) **iii** (–3.2, 10.24)

4 By considering the gradient of a suitable sequence of chords, find a value for the gradient of each curve at the given point.

a $y = x^4$ at (1, 1)

b $y = x^2 - 5x + 3$ at (2, –3)

c $y = \sqrt{x}$ at (4, 2)

d $y = \frac{2}{x}$ at (2, 1)

5 **a** By considering the gradient of a suitable sequence of chords, find a value for the gradient of the curve $y = x^3$ at the points

i (1, 1) **ii** (2, 8) **iii** (3, 27)

b Suggest an expression of the form kx^n for the gradient of the curve $y = x^3$ at the point (x, y).

c Find the gradient of the curve $y = x^3$ at the points

i (4, 64) **ii** (–2, –8) **iii** (1.5, 3.375)

1 Differentiate with respect to x

a x^2 **b** x^4 **c** x **d** x^9 **e** x^{-3} **f** x^{-1}

g $4x^2$ **h** $7x$ **i** $2x^5$ **j** 3 **k** $8x^{-2}$ **l** $11x^{-4}$

2 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

a $y = x^5 + x^2$ **b** $y = x + x^3$ **c** $y = x^4 + 2$ **d** $y = x^6 - 2x$

e $y = 6x^3 + 5x^{-2}$ **f** $y = x^2 - 4x + 1$ **g** $y = x^{-1} - x^{-5}$ **h** $y = 4x^3 + 3x^{-4}$

3 Differentiate with respect to t

a t^6 **b** $5t^{-3}$ **c** $t^{\frac{1}{2}}$ **d** $t^{\frac{2}{3}}$ **e** $\frac{3}{4}t^2$ **f** $8t^{\frac{1}{4}}$

g $2t^{\frac{7}{2}}$ **h** $t^{-\frac{1}{5}}$ **i** $\frac{1}{2}t^{\frac{6}{5}}$ **j** $t^{-\frac{3}{2}}$ **k** $12t^{-\frac{5}{4}}$ **l** $\frac{1}{6}t^{\frac{4}{3}}$

4 Find $\mathrm{f}'(x)$

a $\mathrm{f}(x) = 2x + \frac{1}{3}x^6$ **b** $\mathrm{f}(x) = x^{\frac{3}{2}} - 5$ **c** $\mathrm{f}(x) = x + 4x^{\frac{1}{2}}$ **d** $\mathrm{f}(x) = 6x^{\frac{5}{3}} - x^{-4}$

e $\mathrm{f}(x) = 7 + x^{-\frac{4}{5}}$ **f** $\mathrm{f}(x) = 2x^{\frac{1}{6}} + x^{\frac{3}{4}}$ **g** $\mathrm{f}(x) = 3x^{-1} - 5x^{-\frac{3}{2}}$ **h** $\mathrm{f}(x) = 2 - 7x^{-1} + x^{-\frac{8}{3}}$

5 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

a $y = \sqrt{x}$ **b** $y = 4 - \frac{1}{x}$ **c** $y = 3x^2 + \sqrt[3]{x}$ **d** $y = 9x + \frac{3}{x}$

e $y = \frac{1}{4x} - \frac{1}{x^2}$ **f** $y = \frac{6}{\sqrt[4]{x}}$ **g** $y = \sqrt{x^5}$ **h** $y = 8\sqrt{x} + \frac{4}{3x^2}$

6 Find $\frac{\mathrm{d}s}{\mathrm{d}t}$

a $s = t(t + 3)$ **b** $s = (t - 2)^2$ **c** $s = 5t(t^3 + 4t)$ **d** $s = t^2(7t - t^{-1})$

e $s = (t + 1)(t + 6)$ **f** $s = (t - 4)(t + 2)$ **g** $s = t(t^4 + 3t^2 + 9)$ **h** $s = t(t - 1)(2t - 3)$

7 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

a $y = \sqrt{x}\,(x - 4)$ **b** $y = \frac{x^3 - 2x}{x}$ **c** $y = \frac{4x^3 + x}{x^2}$ **d** $y = \frac{x + 3}{\sqrt{x}}$

e $y = \frac{4 - x^3}{2x}$ **f** $y = \frac{5 + \sqrt{x}}{x^2}$ **g** $y = \frac{9x - 2}{3x}$ **h** $y = \frac{8x + x^3}{4\sqrt{x}}$

8 In each case, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

a $y = 4x^2 - x + 3$ **b** $y = x^3 + 5x^2 + 2x - 6$ **c** $y = 8 - \frac{2}{x}$

d $y = 2x^4 + 3x^2 - 9$ **e** $y = \frac{3x^6 - 4}{x^2}$ **f** $y = 6x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

1 Find the gradient at the point with x-coordinate 3 on each of the following curves.

a $y = x^3$ **b** $y = 4x - x^2$ **c** $y = 2x^2 - 8x + 3$ **d** $y = \frac{3}{x} + 2$

2 Find the gradient of each curve at the given point.

a $y = 3x^2 + x - 5$ $(1, -1)$ **b** $y = x^4 + 2x^3$ $(-2, 0)$

c $y = x(2x - 3)$ $(2, 2)$ **d** $y = x^2 - 2x^{-1}$ $(2, 3)$

e $y = x^2 + 6x + 8$ $(-3, -1)$ **f** $y = 4x + x^{-2}$ $(\frac{1}{2}, 6)$

3 Evaluate $f'(4)$ when

a $f(x) = (x + 1)^2$ **b** $f(x) = x^{\frac{1}{2}}$ **c** $f(x) = x - 4x^{-2}$ **d** $f(x) = 5 - 6x^{\frac{3}{2}}$

4 The curve with equation $y = x^3 - 4x^2 + 3x$ crosses the x-axis at the points A, B and C.

a Find the coordinates of the points A, B and C.

b Find the gradient of the curve at each of the points A, B and C.

5 For the curve with equation $y = 2x^2 - 5x + 1$,

a find $\frac{dy}{dx}$,

b find the value of x for which $\frac{dy}{dx} = 7$.

6 Find the coordinates of the points on the curve with the equation $y = x^3 - 8x$ at which the gradient of the curve is 4.

7 A curve has the equation $y = x^3 + x^2 - 4x + 1$.

a Find the gradient of the curve at the point P $(-1, 5)$.

Given that the gradient at the point Q on the curve is the same as the gradient at the point P,

b find, as exact fractions, the coordinates of the point Q.

8 Find an equation of the tangent to each curve at the given point.

a $y = x^2$ $(2, 4)$ **b** $y = x^2 + 3x + 4$ $(-1, 2)$

c $y = 2x^2 - 6x + 8$ $(1, 4)$ **d** $y = x^3 - 4x^2 + 2$ $(3, -7)$

9 Find an equation of the tangent to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = 3 - x^2$ $(-3, -6)$ **b** $y = \frac{2}{x}$ $(2, 1)$

c $y = 2x^2 + 5x - 1$ $(\frac{1}{2}, 2)$ **d** $y = x - 3\sqrt{x}$ $(4, -2)$

10 Find an equation of the normal to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = x^2 - 4$ $(1, -3)$ **b** $y = 3x^2 + 7x + 7$ $(-2, 5)$

c $y = x^3 - 8x + 4$ $(2, -4)$ **d** $y = x - \frac{6}{x}$ $(3, 1)$

11 Find, in the form $y = mx + c$, an equation of

a the tangent to the curve $y = 3x^2 - 5x + 2$ at the point on the curve with x-coordinate 2,

b the normal to the curve $y = x^3 + 5x^2 - 12$ at the point on the curve with x-coordinate -3.

12 A curve has the equation $y = x^3 + 3x^2 - 16x + 2$.

a Find an equation of the tangent to the curve at the point P $(2, -10)$.

The tangent to the curve at the point Q is parallel to the tangent at the point P.

b Find the coordinates of the point Q.

13 A curve has the equation $y = x^2 - 3x + 4$.

a Find an equation of the normal to the curve at the point A $(2, 2)$.

The normal to the curve at A intersects the curve again at the point B.

b Find the coordinates of the point B.

14 $$\text{f}(x) \equiv x^3 + 4x^2 - 18.$$

a Find $\text{f}'(x)$.

b Show that the tangent to the curve $y = \text{f}(x)$ at the point on the curve with x-coordinate -3 passes through the origin.

15 The curve C has the equation $y = 6 + x - x^2$.

a Find the coordinates of the point P, where C crosses the positive x-axis, and the point Q, where C crosses the y-axis.

b Find an equation of the tangent to C at P.

c Find the coordinates of the point where the tangent to C at P meets the tangent to C at Q.

16 The straight line l is a tangent to the curve $y = x^2 - 5x + 3$ at the point A on the curve.

Given that l is parallel to the line $3x + y = 0$,

a find the coordinates of the point A,

b find the equation of the line l in the form $y = mx + c$.

17 The line with equation $y = 2x + k$ is a normal to the curve with equation $y = \frac{16}{x^2}$.

Find the value of the constant k.

18 A ball is thrown vertically downwards from the top of a cliff. The distance, s metres, of the ball from the top of the cliff after t seconds is given by $s = 3t + 5t^2$.

Find the rate at which the distance the ball has travelled is increasing when

a $t = 0.6$,

b $s = 54$.

19 Water is poured into a vase such that the depth, h cm, of the water in the vase after t seconds is given by $h = kt^{\frac{1}{3}}$, where k is a constant. Given that when $t = 1$, the depth of the water in the vase is increasing at the rate of 3 cm per second,

a find the value of k,

b find the rate at which h is increasing when $t = 8$.

1 $\mathrm{f}(x) = (x + 1)(x - 2)^2.$

a Sketch the curve $y = \mathrm{f}(x)$, showing the coordinates of any points where the curve meets the coordinate axes. **(3)**

b Find $\mathrm{f}'(x)$. **(4)**

c Show that the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = 1$ has the equation

$$y = 5 - 3x.$$ **(3)**

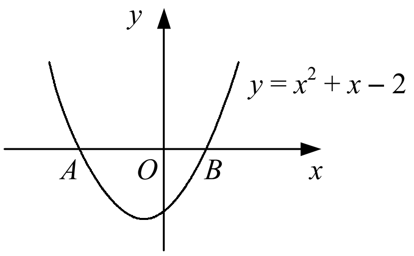
2 The curve C has the equation $y = x - 3x^{\frac{1}{2}} + 3$ and passes through the point $P\,(4, 1)$.

a Show that the tangent to C at P passes through the origin. **(5)**

The normal to C at P crosses the y-axis at the point Q.

b Find the area of triangle OPQ, where O is the origin. **(4)**

3



The diagram shows the curve $y = x^2 + x - 2$. The curve crosses the x-axis at the points $A\,(a, 0)$ and $B\,(b, 0)$ where $a < b$.

a Find the values of a and b. **(3)**

b Show that the normal to the curve at A has the equation

$$x - 3y + 2 = 0.$$ **(5)**

The tangent to the curve at B meets the normal to the curve at A at the point C.

c Find the exact coordinates of C. **(4)**

4 Given that $y = \dfrac{x^2 - 6x - 3}{3x^{\frac{1}{2}}}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $\dfrac{(x+a)^2}{bx^{\frac{3}{2}}}$, where a and b are integers to be found. **(6)**

5 The point A lies on the curve $y = \dfrac{12}{x^2}$ and the x-coordinate of A is 2.

a Find an equation of the tangent to the curve at A. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

b Verify that the point where the tangent at A intersects the curve again has the coordinates $(-1, 12)$. **(3)**

6 A curve has the equation $y = 2 + 3x + kx^2 - x^3$ where k is a constant.

Given that the gradient of the curve is -6 at the point P where $x = -1$,

a find the value of k. **(4)**

Given also that the tangent to the curve at the point Q is parallel to the tangent at P,

b find the length PQ, giving your answer in the form $k\sqrt{5}$. **(5)**

7 Differentiate $x^2 + \frac{1}{2x}$ with respect to x. **(3)**

8 A curve has the equation $y = 2x^2 - 7x + 1$ and the point A on the curve has x-coordinate 2.

a Find an equation of the tangent to the curve at A. **(4)**

The normal to the curve at the point B is parallel to the tangent at A.

b Find the coordinates of B. **(3)**

9 $$y = x^2 + 3x^{\frac{1}{2}}.$$

a Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

b Show that $2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} - 6x = 0$. **(4)**

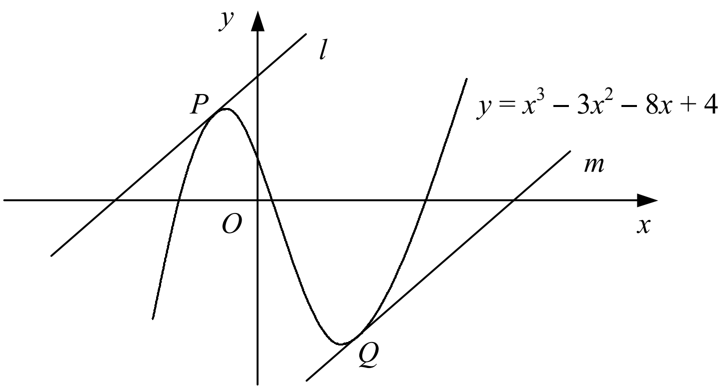
10 A curve has the equation $y = 2 + \frac{4}{x}$.

a Find an equation of the normal to the curve at the point M (4, 3). **(5)**

The normal to the curve at M intersects the curve again at the point N.

b Find the coordinates of the point N. **(5)**

11



The diagram shows the curve with equation $y = x^3 - 3x^2 - 8x + 4$.

The straight line l is the tangent to the curve at the point P (−1, 8).

a Find an equation of line l. **(4)**

The straight line m is parallel to l and is the tangent to the curve at the point Q.

b Find an equation of line m. **(4)**

c Find an equation of the normal to the curve at P. **(2)**

d Hence, or otherwise, show that the distance between lines l and m is $16\sqrt{2}$. **(4)**

12 A curve has the equation $y = \sqrt{x}\,(k - x)$, where k is a constant.

Given that the gradient of the curve is $\sqrt{2}$ at the point P where $x = 2$,

a find the value of k, **(5)**

b show that the normal to the curve at P has the equation

$$x + \sqrt{2}\,y = c,$$

where c is an integer to be found. **(5)**