1 Integrate with respect to x

a x^2 **b** x^6 **c** x **d** x^{-4} **e** 5 **f** $3x^2$

g $4x^7$ **h** $6x^{-2}$ **i** $8x^5$ **j** $\frac{1}{3}x$ **k** $2x^{-9}$ **l** $\frac{3}{4}x^{-3}$

2 Find

a $\int (2x + 3)\ \mathrm{d}x$ **b** $\int (12x^3 - 4x)\ \mathrm{d}x$ **c** $\int (7 - x^2)\ \mathrm{d}x$ **d** $\int (x^2 + x + 1)\ \mathrm{d}x$

e $\int (x^4 + 5x^2)\ \mathrm{d}x$ **f** $\int x(x^2 - 3)\ \mathrm{d}x$ **g** $\int (x - 2)^2\ \mathrm{d}x$ **h** $\int (3x^4 + x^2 - 6)\ \mathrm{d}x$

i $\int (2 + \frac{1}{x^2})\ \mathrm{d}x$ **j** $\int (x - \frac{1}{x^3})\ \mathrm{d}x$ **k** $\int x^2(\frac{2}{x^4} - 3)\ \mathrm{d}x$ **l** $\int (x - \frac{4}{x})^2\ \mathrm{d}x$

3 Integrate with respect to y

a $y^{\frac{1}{2}}$ **b** $y^{\frac{5}{2}}$ **c** $y^{-\frac{1}{2}}$ **d** $4y^{\frac{1}{3}}$ **e** $y^{\frac{3}{4}}$ **f** $5y^{-\frac{2}{3}}$

g $\sqrt[4]{y}$ **h** $\frac{7}{\sqrt{y}}$ **i** $\frac{1}{2y^2}$ **j** $\sqrt{y^3}$ **k** $\frac{5}{2y^4}$ **l** $\frac{1}{3\sqrt{y}}$

4 Find

a $\int (3t^{\frac{1}{2}} - 1)\ \mathrm{d}t$ **b** $\int (2r + \sqrt{r})\ \mathrm{d}r$ **c** $\int (3p - 1)^2\ \mathrm{d}p$ **d** $\int (4x + x^{\frac{1}{3}})\ \mathrm{d}x$

e $\int (\frac{1}{y^3} + y)\ \mathrm{d}y$ **f** $\int (\frac{1}{2}x^2 - x^{\frac{3}{2}})\ \mathrm{d}x$ **g** $\int \frac{t^3 + 2t}{t}\ \mathrm{d}t$ **h** $\int (r^{\frac{5}{3}} - r^{\frac{2}{3}})\ \mathrm{d}r$

i $\int \frac{4p^4 - p^2}{2p}\ \mathrm{d}p$ **j** $\int (4 - y^{\frac{7}{4}})\ \mathrm{d}y$ **k** $\int \frac{1 + 6x^2}{3x^2}\ \mathrm{d}x$ **l** $\int \frac{2t + 3}{\sqrt{t}}\ \mathrm{d}t$

5 Find $\int y\ \mathrm{d}x$ when

a $y = 3x^2 - x + 6$ **b** $y = x^6 - x^3 + 2x - 5$ **c** $y = x(x - 2)(x + 1)$

d $y = (x^{\frac{1}{2}} + 2)^2$ **e** $y = (x^2 - 4)(2x + 3)$ **f** $y = x^3 - 2x^{\frac{4}{3}} + \frac{7}{x^2}$

g $y = \frac{1}{4x^3} - \frac{2}{3x^2}$ **h** $y = (1 - \frac{2}{x^2})^2$ **i** $y = (x^{\frac{5}{2}} - 1)(x^{\frac{3}{2}} + 1)$

6 Find a general expression for y given that

a $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x + 3$ **b** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^3 - x^2$ **c** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{3x^3}$

d $\frac{\mathrm{d}y}{\mathrm{d}x} = (x + 1)^3$ **e** $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - \frac{3}{\sqrt{x}}$ **f** $\frac{\mathrm{d}y}{\mathrm{d}x} = x^{\frac{3}{2}} - 2x^{-\frac{3}{2}}$

g $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3 - x^2}{2x^2}$ **h** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{x^3}(5 - x)$ **i** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9x - 2}{3\sqrt{x}}$

1 **a** Find $\int (2x + 1)\ \mathrm{d}x$.

b Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 1$ and that $y = 5$ when $x = 1$, find an expression for y in terms of x.

2 Use the given boundary conditions to find an expression for y in each case.

a $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 6x$, $y = 1$ at $x = 2$

b $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - x$, $y = 41$ at $x = 4$

c $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 4x + 1$, $y = 4$ at $x = -3$

d $\frac{\mathrm{d}y}{\mathrm{d}x} = 7 - 5x - x^3$, $y = 0$ at $x = 2$

e $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - \frac{2}{x^2}$, $y = -1$ at $x = \frac{1}{2}$

f $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - \sqrt{x}$, $y = 8$ at $x = 4$

3 The curve $y = \mathrm{f}(x)$ passes through the point (3, 5).

Given that $\mathrm{f}'(x) = 3 + 2x - x^2$, find an expression for $\mathrm{f}(x)$.

4 Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}},$$

and that $y = 7$ when $x = 0$, find the value of y when $x = 4$.

5 The curve $y = \mathrm{f}(x)$ passes through the point $(-1, 4)$. Given that $\mathrm{f}'(x) = 2x^3 - x - 8$,

a find an expression for $\mathrm{f}(x)$,

b find an equation of the tangent to the curve at the point on the curve with x-coordinate 2.

6 The curve $y = \mathrm{f}(x)$ passes through the origin.

Given that $\mathrm{f}'(x) = 3x^2 - 8x - 5$, find the coordinates of the other points where the curve crosses the x-axis.

7 Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x + \frac{2}{x^2},$$

a find an expression for y in terms of x.

Given also that $y = 8$ when $x = 2$,

b find the value of y when $x = \frac{1}{2}$.

8 The curve C with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + kx,$$

where k is a constant.

Given that C passes through the points (1, 6) and (2, 1),

a find the value of k,

b find an equation of the curve.

C1 INTEGRATION — Worksheet C

1 Find

$$\int (x^2 + 6\sqrt{x} - 3)\ \mathrm{d}x.$$ **(3)**

2 The curve $y = \mathrm{f}(x)$ passes through the point $(1, -2)$.

Given that

$$\mathrm{f}'(x) = 1 - \frac{6}{x^3},$$

a find an expression for $\mathrm{f}(x)$. **(4)**

The point A on the curve $y = \mathrm{f}(x)$ has x-coordinate 2.

b Show that the normal to the curve $y = \mathrm{f}(x)$ at A has the equation

$$16x + 4y - 19 = 0.$$ **(5)**

3 The curve $y = \mathrm{f}(x)$ passes through the point $(3, 22)$.

Given that

$$\mathrm{f}'(x) = 3x^2 + 2x - 5,$$

a find an expression for $\mathrm{f}(x)$. **(4)**

Given also that

$$\mathrm{g}(x) = (x + 3)(x - 1)^2,$$

b show that $\mathrm{g}(x) = \mathrm{f}(x) + 2$, **(3)**

c sketch the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ on the same set of axes. **(3)**

4 Given that

$$y = x^2 - \frac{3}{x^2},$$

find

a $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, **(2)**

b $\int y\ \mathrm{d}x$. **(3)**

5 The curve C with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 1.$$

Given that the tangent to the curve at the point P with x-coordinate 2 passes through the origin, find an equation for the curve. **(7)**

6 A curve with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sqrt{x} - \frac{2}{\sqrt{x}}, \quad x > 0.$$

a Find the gradient of the curve at the point where $x = 2$, giving your answer in its simplest form. **(2)**

Given also that the curve passes through the point $(4, 7)$,

b find the y-coordinate of the point on the curve where $x = 3$, giving your answer in the form $a\sqrt{3} + b$, where a and b are integers. **(6)**

7 Find

a $\int (x+2)^2 \, dx$, **(3)**

b $\int \frac{1}{4\sqrt{x}} \, dx$. **(3)**

8 The curve C has the equation $y = f(x)$ and crosses the x-axis at the point P $(-2, 0)$.

Given that

$$f'(x) = 3x^2 - 2x - 3,$$

a find an expression for $f(x)$, **(4)**

b show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 5 - 2x.$$ **(3)**

9 Given that

$$\frac{dy}{dx} = 2x - \frac{3}{x^2}, \quad x \neq 0,$$

and that $y = 0$ at $x = 1$,

a find an expression for y in terms of x, **(4)**

b show that for all non-zero values of x

$$x^2 \frac{d^2y}{dx^2} - 2y = k,$$

where k is a constant to be found. **(4)**

10 Integrate with respect to x

a $\frac{1}{x^3}$, **(2)**

b $\frac{(x-1)^2}{\sqrt{x}}$. **(5)**

11 The curve $y = f(x)$ passes through the point $(2, -5)$.

Given that

$$f'(x) = 4x^3 - 8x,$$

a find an expression for $f(x)$, **(4)**

b find the coordinates of the points where the curve crosses the x-axis. **(4)**

12 The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = k - x^{-\frac{1}{2}}, \quad x > 0,$$

where k is a constant.

Given that C passes through the points $(1, -2)$ and $(4, 5)$,

a find the value of k, **(5)**

b show that the normal to C at the point $(1, -2)$ has the equation

$$x + 2y + 3 = 0.$$ **(4)**