1 **a**

$$
\begin{array}{rl}
 & x^2 + x - 2 \\
x+1 & \overline{)\, x^3 + 2x^2 - x - 2} \\
 & \underline{x^3 + x^2} \\
 & x^2 - x \\
 & \underline{x^2 + x} \\
 & -2x - 2 \\
 & \underline{-2x - 2}
\end{array}
$$

quotient: $x^2 + x - 2$

b

$$
\begin{array}{rl}
 & x^2 + 4x - 1 \\
x-2 & \overline{)\, x^3 + 2x^2 - 9x + 2} \\
 & \underline{x^3 - 2x^2} \\
 & 4x^2 - 9x \\
 & \underline{4x^2 - 8x} \\
 & -x + 2 \\
 & \underline{-x + 2}
\end{array}
$$

quotient: $x^2 + 4x - 1$

c

$$
\begin{array}{rl}
 & x^2 - x + 5 \\
x+4 & \overline{)\, x^3 + 3x^2 + x + 20} \\
 & \underline{x^3 + 4x^2} \\
 & -x^2 + x \\
 & \underline{-x^2 - 4x} \\
 & 5x + 20 \\
 & \underline{5x + 20}
\end{array}
$$

quotient: $x^2 - x + 5$

d

$$
\begin{array}{rl}
 & 2x^2 + x - 3 \\
x-1 & \overline{)\, 2x^3 - x^2 - 4x + 3} \\
 & \underline{2x^3 - 2x^2} \\
 & x^2 - 4x \\
 & \underline{x^2 - x} \\
 & -3x + 3 \\
 & \underline{-3x + 3}
\end{array}
$$

quotient: $2x^2 + x - 3$

e

$$
\begin{array}{rl}
 & 6x^2 + 11x - 18 \\
x-5 & \overline{)\, 6x^3 - 19x^2 - 73x + 90} \\
 & \underline{6x^3 - 30x^2} \\
 & 11x^2 - 73x \\
 & \underline{11x^2 - 55x} \\
 & -18x + 90 \\
 & \underline{-18x + 90}
\end{array}
$$

quotient: $6x^2 + 11x - 18$

f

$$
\begin{array}{rl}
 & -x^2 + 7x - 4 \\
x+2 & \overline{)\, -x^3 + 5x^2 + 10x - 8} \\
 & \underline{-x^3 - 2x^2} \\
 & 7x^2 + 10x \\
 & \underline{7x^2 + 14x} \\
 & -4x - 8 \\
 & \underline{-4x - 8}
\end{array}
$$

quotient: $-x^2 + 7x - 4$

g

$$
\begin{array}{rl}
 & x^2 - 3x + 7 \\
x+3 & \overline{)\, x^3 + 0x^2 - 2x + 21} \\
 & \underline{x^3 + 3x^2} \\
 & -3x^2 - 2x \\
 & \underline{-3x^2 - 9x} \\
 & 7x + 21 \\
 & \underline{7x + 21}
\end{array}
$$

quotient: $x^2 - 3x + 7$

h

$$
\begin{array}{rl}
 & 3x^2 - 2x + 12 \\
x+6 & \overline{)\, 3x^3 + 16x^2 + 0x + 72} \\
 & \underline{3x^3 + 18x^2} \\
 & -2x^2 + 0x \\
 & \underline{-2x^2 - 12x} \\
 & 12x + 72 \\
 & \underline{12x + 72}
\end{array}
$$

quotient: $3x^2 - 2x + 12$

2 **a**

$$
\begin{array}{rllll}
 & x^2 & +\ 3x & +\ 2 & \\
\cline{2-5}
x+5\,\big) & x^3 & +\ 8x^2 & +\ 17x & +\ 16 \\
 & x^3 & +\ 5x^2 & & \\
\cline{2-3}
 & & 3x^2 & +\ 17x & \\
 & & 3x^2 & +\ 15x & \\
\cline{3-4}
 & & & 2x & +\ 16 \\
 & & & 2x & +\ 10 \\
\cline{4-5}
 & & & & 6
\end{array}
$$

quotient: $x^2 + 3x + 2$ remainder: 6

b

$$
\begin{array}{rllll}
 & x^2 & -\ 8x & +\ 5 & \\
\cline{2-5}
x-7\,\big) & x^3 & -\ 15x^2 & +\ 61x & -\ 48 \\
 & x^3 & -\ 7x^2 & & \\
\cline{2-3}
 & & -\ 8x^2 & +\ 61x & \\
 & & -\ 8x^2 & +\ 56x & \\
\cline{3-4}
 & & & 5x & -\ 48 \\
 & & & 5x & -\ 35 \\
\cline{4-5}
 & & & & -\ 13
\end{array}
$$

quotient: $x^2 - 8x + 5$ remainder: -13

c

$$
\begin{array}{rllll}
 & 3x^2 & -\ 2x & +\ 4 & \\
\cline{2-5}
x+2\,\big) & 3x^3 & +\ 4x^2 & +\ 0x & +\ 7 \\
 & 3x^3 & +\ 6x^2 & & \\
\cline{2-3}
 & & -\ 2x^2 & +\ 0x & \\
 & & -\ 2x^2 & -\ 4x & \\
\cline{3-4}
 & & & 4x & +\ 7 \\
 & & & 4x & +\ 8 \\
\cline{4-5}
 & & & & -\ 1
\end{array}
$$

quotient: $3x^2 - 2x + 4$ remainder: -1

d

$$
\begin{array}{rllll}
 & -\ x^2 & +\ 3x & -\ 9 & \\
\cline{2-5}
x+8\,\big) & -\ x^3 & -\ 5x^2 & +\ 15x & -\ 50 \\
 & -\ x^3 & -\ 8x^2 & & \\
\cline{2-3}
 & & 3x^2 & +\ 15x & \\
 & & 3x^2 & +\ 24x & \\
\cline{3-4}
 & & & -\ 9x & -\ 50 \\
 & & & -\ 9x & -\ 72 \\
\cline{4-5}
 & & & & 22
\end{array}
$$

quotient: $-x^2 + 3x - 9$ remainder: 22

e

$$
\begin{array}{rllll}
 & 4x^2 & +\ 14x & +\ 26 & \\
\cline{2-5}
x-3\,\big) & 4x^3 & +\ 2x^2 & -\ 16x & +\ 3 \\
 & 4x^3 & -\ 12x^2 & & \\
\cline{2-3}
 & & 14x^2 & -\ 16x & \\
 & & 14x^2 & -\ 42x & \\
\cline{3-4}
 & & & 26x & +\ 3 \\
 & & & 26x & -\ 78 \\
\cline{4-5}
 & & & & 81
\end{array}
$$

quotient: $4x^2 + 14x + 26$ remainder: 81

f

$$
\begin{array}{rllll}
 & -\ 6x^2 & -\ 10x & +\ 20 & \\
\cline{2-5}
x+2\,\big) & -\ 6x^3 & -\ 22x^2 & +\ 0x & +\ 1 \\
 & -\ 6x^3 & -\ 12x^2 & & \\
\cline{2-3}
 & & -\ 10x^2 & +\ 0x & \\
 & & -\ 10x^2 & -\ 20x & \\
\cline{3-4}
 & & & 20x & +\ 1 \\
 & & & 20x & +\ 40 \\
\cline{4-5}
 & & & & -\ 39
\end{array}
$$

quotient: $-6x^2 - 10x + 20$ remainder: -39

3 **a** let $\mathrm{f}(x) \equiv x^3 + 2x^2 - 2x - 1$
$\mathrm{f}(1) = 1 + 2 - 2 - 1 = 0$
$\therefore$ $(x - 1)$ is a factor

b let $\mathrm{f}(x) \equiv x^3 - 5x^2 - 9x + 2$
$\mathrm{f}(-2) = -8 - 20 + 18 + 2 = -8$
$\therefore$ $(x + 2)$ is not a factor

c let $\mathrm{f}(x) \equiv x^3 - x^2 - 14x + 27$
$\mathrm{f}(3) = 27 - 9 - 42 + 27 = 3$
$\therefore$ $(x - 3)$ is not a factor

d let $\mathrm{f}(x) \equiv 2x^3 + 13x^2 + 2x - 24$
$\mathrm{f}(-6) = -432 + 468 - 12 - 24 = 0$
$\therefore$ $(x + 6)$ is a factor

e let $\mathrm{f}(x) \equiv 2x^3 - 5x^2 + 7x - 14$
$\mathrm{f}(-\frac{1}{2}) = -\frac{1}{4} - \frac{5}{4} - \frac{7}{2} - 14 = -19$
$\therefore$ $(2x + 1)$ is not a factor

f let $\mathrm{f}(x) \equiv 2 - 17x + 25x^2 - 6x^3$
$\mathrm{f}(\frac{2}{3}) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$
$\therefore$ $(3x - 2)$ is a factor

4 **a** $f(1) = 1 - 2 - 11 + 12 = 0$

$\therefore (x - 1)$ is a factor of $f(x)$

b

$$\begin{array}{rl} & x^2 - x - 12 \\ x - 1 & \overline{)\, x^3 - 2x^2 - 11x + 12} \\ & \underline{x^3 - x^2} \\ & \quad - x^2 - 11x \\ & \quad \underline{- x^2 + x} \\ & \qquad\quad - 12x + 12 \\ & \qquad\quad \underline{- 12x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 1)(x^2 - x - 12)$

$\equiv (x - 1)(x + 3)(x - 4)$

5 $g(-3) = -54 + 9 + 39 + 6 = 0$

$\therefore (x + 3)$ is a factor of $g(x)$

$$\begin{array}{rl} & 2x^2 - 5x + 2 \\ x + 3 & \overline{)\, 2x^3 + x^2 - 13x + 6} \\ & \underline{2x^3 + 6x^2} \\ & \quad - 5x^2 - 13x \\ & \quad \underline{- 5x^2 - 15x} \\ & \qquad\quad 2x + 6 \\ & \qquad\quad \underline{2x + 6} \end{array}$$

$\therefore g(x) \equiv (x + 3)(2x^2 - 5x + 2)$

$\equiv (x + 3)(2x - 1)(x - 2)$

$g(x) = 0 \;\Rightarrow\; (x + 3)(2x - 1)(x - 2) = 0$

$x = -3, \tfrac{1}{2}$ or 2

6 $f(4) = 0 \quad \therefore (x - 4)$ is a factor of $f(x)$

$$\begin{array}{rl} & 6x^2 + 17x - 3 \\ x - 4 & \overline{)\, 6x^3 - 7x^2 - 71x + 12} \\ & \underline{6x^3 - 24x^2} \\ & \quad 17x^2 - 71x \\ & \quad \underline{17x^2 - 68x} \\ & \qquad\quad - 3x + 12 \\ & \qquad\quad \underline{- 3x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 4)(6x^2 + 17x - 3)$

$\equiv (x - 4)(6x - 1)(x + 3)$

$f(x) = 0 \;\Rightarrow\; (x - 4)(6x - 1)(x + 3) = 0$

$x = -3, \tfrac{1}{6}$ or 4

7 **a** $g(-2) = 0 \;\therefore (x + 2)$ is a factor of $g(x)$

$$\begin{array}{rl} & x^2 + 5x - 3 \\ x + 2 & \overline{)\, x^3 + 7x^2 + 7x - 6} \\ & \underline{x^3 + 2x^2} \\ & \quad 5x^2 + 7x \\ & \quad \underline{5x^2 + 10x} \\ & \qquad\quad - 3x - 6 \\ & \qquad\quad \underline{- 3x - 6} \end{array}$$

$\therefore g(x) \equiv (x + 2)(x^2 + 5x - 3)$

b other solutions given by $x^2 + 5x - 3 = 0$

$x = \dfrac{-5 \pm \sqrt{25 + 12}}{2} = \dfrac{-5 \pm \sqrt{37}}{2}$

$x = -5.54$ or 0.54

8 **a** $f(1) = 1 + 2 - 11 - 12 = -20$

$f(2) = 8 + 8 - 22 - 12 = -18$

$f(-1) = -1 + 2 + 11 - 12 = 0$

$f(-2) = -8 + 8 + 22 - 12 = 10$

b $(x + 1)$ is a factor of $f(x)$

$$\begin{array}{rl} & x^2 + x - 12 \\ x + 1 & \overline{)\, x^3 + 2x^2 - 11x - 12} \\ & \underline{x^3 + x^2} \\ & \quad x^2 - 11x \\ & \quad \underline{x^2 + x} \\ & \qquad\quad - 12x - 12 \\ & \qquad\quad \underline{- 12x - 12} \end{array}$$

$\therefore f(x) = (x + 1)(x^2 + x - 12)$

$= (x + 1)(x + 4)(x - 3)$

9 **a** let $\mathrm{f}(x) = x^3 - 2x^2 - 5x + 6$
$\mathrm{f}(1) = 0$
$\therefore\ (x - 1)$ is a factor

$$
\begin{array}{r}
x^2 - x - 6 \\
x - 1\,\overline{)\,x^3 - 2x^2 - 5x + 6} \\
\underline{x^3 - x^2 \phantom{{}- 5x + 6}} \\
- x^2 - 5x \phantom{{}+ 6} \\
\underline{- x^2 + x \phantom{{}+ 6}} \\
- 6x + 6 \\
\underline{- 6x + 6}
\end{array}
$$

$\therefore$
$\mathrm{f}(x) = (x - 1)(x^2 - x - 6)$
$= (x - 1)(x + 2)(x - 3)$

b let $\mathrm{f}(x) = x^3 + x^2 - 5x - 2$
$\mathrm{f}(1) = -5,\ \mathrm{f}(2) = 0$
$\therefore\ (x - 2)$ is a factor

$$
\begin{array}{r}
x^2 + 3x + 1 \\
x - 2\,\overline{)\,x^3 + x^2 - 5x - 2} \\
\underline{x^3 - 2x^2 \phantom{{}- 5x - 2}} \\
3x^2 - 5x \phantom{{}- 2} \\
\underline{3x^2 - 6x \phantom{{}- 2}} \\
x - 2 \\
\underline{x - 2}
\end{array}
$$

$\therefore$
$\mathrm{f}(x) = (x - 2)(x^2 + 3x + 1)$

c let $\mathrm{f}(x) = 20 + 11x - 8x^2 + x^3$
$\mathrm{f}(1) = 24,\ \mathrm{f}(2) = 18,\ \mathrm{f}(-1) = 0$
$\therefore\ (x + 1)$ is a factor

$$
\begin{array}{r}
x^2 - 9x + 20 \\
x + 1\,\overline{)\,x^3 - 8x^2 + 11x + 20} \\
\underline{x^3 + x^2 \phantom{{}+ 11x + 20}} \\
- 9x^2 + 11x \phantom{{}+ 20} \\
\underline{- 9x^2 - 9x \phantom{{}+ 20}} \\
20x + 20 \\
\underline{20x + 20}
\end{array}
$$

$\therefore$
$\mathrm{f}(x) = (x + 1)(x^2 - 9x + 20)$
$= (x + 1)(x - 4)(x - 5)$

d let $\mathrm{f}(x) = 3x^3 - 4x^2 - 35x + 12$
$\mathrm{f}(1) = -24,\ \mathrm{f}(2) = -50,$
$\mathrm{f}(-1) = 40,\ \mathrm{f}(-2) = 42$
$\mathrm{f}(3) = -48,\ \mathrm{f}(-3) = 0$
$\therefore\ (x + 3)$ is a factor

$$
\begin{array}{r}
3x^2 - 13x + 4 \\
x + 3\,\overline{)\,3x^3 - 4x^2 - 35x + 12} \\
\underline{3x^3 + 9x^2 \phantom{{}- 35x + 12}} \\
- 13x^2 - 35x \phantom{{}+ 12} \\
\underline{- 13x^2 - 39x \phantom{{}+ 12}} \\
4x + 12 \\
\underline{4x + 12}
\end{array}
$$

$\therefore$
$\mathrm{f}(x) = (x + 3)(3x^2 - 13x + 4)$
$= (x + 3)(3x - 1)(x - 4)$

e let $\mathrm{f}(x) = x^3 + 8$
$\mathrm{f}(1) = 9,\ \mathrm{f}(2) = 16$
$\mathrm{f}(-1) = 7,\ \mathrm{f}(-2) = 0$
$\therefore\ (x + 2)$ is a factor

$$
\begin{array}{r}
x^2 - 2x + 4 \\
x + 2\,\overline{)\,x^3 + 0x^2 + 0x + 8} \\
\underline{x^3 + 2x^2 \phantom{{}+ 0x + 8}} \\
- 2x^2 + 0x \phantom{{}+ 8} \\
\underline{- 2x^2 - 4x \phantom{{}+ 8}} \\
4x + 8 \\
\underline{4x + 8}
\end{array}
$$

$\therefore$
$\mathrm{f}(x) = (x + 2)(x^2 - 2x + 4)$

f let $\mathrm{f}(x) = 12 + 29x + 8x^2 - 4x^3$
$\mathrm{f}(1) = 45,\ \mathrm{f}(2) = 70,$
$\mathrm{f}(-1) = -5,\ \mathrm{f}(-2) = 18$
$\mathrm{f}(3) = 63,\ \mathrm{f}(-3) = 105$
$\mathrm{f}(4) = 0$
$\therefore\ (x - 4)$ is a factor

$$
\begin{array}{r}
-4x^2 - 8x - 3 \\
x - 4\,\overline{)\,-4x^3 + 8x^2 + 29x + 12} \\
\underline{-4x^3 + 16x^2 \phantom{{}+ 29x + 12}} \\
- 8x^2 + 29x \phantom{{}+ 12} \\
\underline{- 8x^2 + 32x \phantom{{}+ 12}} \\
- 3x + 12 \\
\underline{- 3x + 12}
\end{array}
$$

$\therefore$
$\mathrm{f}(x) = (x - 4)(-4x^2 - 8x - 3)$
$= -(x - 4)(4x^2 + 8x + 3)$
$= (4 - x)(2x + 1)(2x + 3)$

10 **a** let $f(x) = x^3 - x^2 - 10x - 8$
$f(1) = -18$, $f(2) = -24$,
$f(-1) = 0$
$\therefore\ (x + 1)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & -2x & -8 & \\ \hline x+1 & x^3 & -x^2 & -10x & -8 \\ & x^3 & +x^2 & & \\ \hline & & -2x^2 & -10x & \\ & & -2x^2 & -2x & \\ \hline & & & -8x & -8 \\ & & & -8x & -8 \\ \hline \end{array}$$

$\therefore$
$(x + 1)(x^2 - 2x - 8) = 0$
$(x + 1)(x + 2)(x - 4) = 0$
$x = -2, -1, 4$

b let $f(x) = x^3 + 2x^2 - 9x - 18$
$f(1) = -24$, $f(2) = -20$
$f(-1) = -8$, $f(-2) = 0$
$\therefore\ (x + 2)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & +0x & -9 & \\ \hline x+2 & x^3 & +2x^2 & -9x & -18 \\ & x^3 & +2x^2 & & \\ \hline & & 0x^2 & -9x & \\ & & 0x^2 & +0x & \\ \hline & & & -9x & -18 \\ & & & -9x & -18 \\ \hline \end{array}$$

$\therefore$
$(x + 2)(x^2 - 9) = 0$
$(x + 2)(x + 3)(x - 3) = 0$
$x = -3, -2, 3$

c let $f(x) = 4x^3 - 12x^2 + 9x - 2$
$f(1) = -1$, $f(2) = 0$
$\therefore\ (x - 2)$ is a factor

$$\begin{array}{r|rrrr} & 4x^2 & -4x & +1 & \\ \hline x-2 & 4x^3 & -12x^2 & +9x & -2 \\ & 4x^3 & -8x^2 & & \\ \hline & & -4x^2 & +9x & \\ & & -4x^2 & +8x & \\ \hline & & & x & -2 \\ & & & x & -2 \\ \hline \end{array}$$

$\therefore$
$(x - 2)(4x^2 - 4x + 1) = 0$
$(x - 2)(2x - 1)^2 = 0$
$x = \frac{1}{2}, 2$

d let $f(x) = x^3 - 5x^2 + 3x + 1$
$f(1) = 0$
$\therefore\ (x - 1)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & -4x & -1 & \\ \hline x-1 & x^3 & -5x^2 & +3x & +1 \\ & x^3 & -x^2 & & \\ \hline & & -4x^2 & +3x & \\ & & -4x^2 & +4x & \\ \hline & & & -x & +1 \\ & & & -x & +1 \\ \hline \end{array}$$

$\therefore$
$(x - 1)(x^2 - 4x - 1) = 0$
$x = 1$ or $\dfrac{4 \pm \sqrt{16+4}}{2}$
$x = 1, 2 \pm \sqrt{5}$

e let $f(x) = x^3 + 4x^2 - 9x - 6$
$f(1) = -10$, $f(2) = 0$
$\therefore\ (x - 2)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & +6x & +3 & \\ \hline x-2 & x^3 & +4x^2 & -9x & -6 \\ & x^3 & -2x^2 & & \\ \hline & & 6x^2 & -9x & \\ & & 6x^2 & -12x & \\ \hline & & & 3x & -6 \\ & & & 3x & -6 \\ \hline \end{array}$$

$\therefore$
$(x - 2)(x^2 + 6x + 3) = 0$
$x = 2$ or $\dfrac{-6 \pm \sqrt{36-12}}{2}$
$x = 2, -3 \pm \sqrt{6}$

f let $f(x) = x^3 - 14x + 15$
$f(1) = 2$, $f(2) = -5$, $f(-1) = 28$,
$f(-2) = 35$, $f(3) = 0$
$\therefore\ (x - 3)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & +3x & -5 & \\ \hline x-3 & x^3 & +0x^2 & -14x & +15 \\ & x^3 & -3x^2 & & \\ \hline & & 3x^2 & -14x & \\ & & 3x^2 & -9x & \\ \hline & & & -5x & +15 \\ & & & -5x & +15 \\ \hline \end{array}$$

$\therefore$
$(x - 3)(x^2 + 3x - 5) = 0$
$x = 3$ or $\dfrac{-3 \pm \sqrt{9+20}}{2}$
$x = 3, \frac{1}{2}(-3 \pm \sqrt{29})$

11 **a** $f(2) = 0$
$\therefore\ 16 - 4 - 30 + c = 0$
$c = 18$

b

$$\begin{array}{r|rrrr} & 2x^2 & +3x & -9 & \\ \hline x-2 & 2x^3 & -x^2 & -15x & +18 \\ & 2x^3 & -4x^2 & & \\ \hline & & 3x^2 & -15x & \\ & & 3x^2 & -6x & \\ \hline & & & -9x & +18 \\ & & & -9x & +18 \\ \hline \end{array}$$

$\therefore\ f(x) \equiv (x - 2)(2x^2 + 3x - 9)$
$\equiv (x - 2)(2x - 3)(x + 3)$

12 **a** $g(-1) = 0$
$\therefore\ -1 + p + 13 + q = 0$
$p + q + 12 = 0$ (1)
$g(3) = 0$
$\therefore\ 27 + 9p - 39 + q = 0$
$9p + q - 12 = 0$ (2)
$(2) - (1) \Rightarrow 8p - 24 = 0 \Rightarrow p = 3$
sub (1) $\Rightarrow 3 + q + 12 = 0 \Rightarrow q = -15$

b $(x + 1)(x - 3)(ax + b) \equiv x^3 + 3x^2 - 13x - 15$
by inspection
$g(x) \equiv (x + 1)(x - 3)(x + 5)$
$g(x) = 0 \Rightarrow (x + 1)(x - 3)(x + 5) = 0$
$x = -5, -1$ or 3

13 **a** $= f(2) = 8 + 16 - 2 + 6 = 28$

b $= f(-1) = -1 - 2 - 7 + 1 = -9$

c $= f(-5) = -250 + 25 - 45 + 17 = -163$

d $= f(\frac{1}{2}) = 1 + 1 - 3 - 3 = -4$

e $= f(-\frac{1}{2}) = -\frac{1}{4} - \frac{3}{4} + 10 - 7 = 2$

f $= f(\frac{2}{3}) = \frac{8}{9} - \frac{8}{3} + \frac{4}{3} - 7 = -7\frac{4}{9}$

14 $f(2) = 5$

$\therefore\ 8 - 16 + 10 + c = 5$

$c = 3$

15 $f(\frac{1}{2}) = -2$

$\therefore\ \frac{1}{4} - \frac{9}{4} + \frac{1}{2}k + 5 = -2$

$k = -10$

16 **a** $f(-3) = 22$

$\therefore\ -54 + 9a + 13 = 22$

$a = 7$

b $f(x) = 2x^3 + 7x^2 + 13$

remainder $= f(4)$

$= 128 + 112 + 13$

$= 253$

17 **a** $f(-1) = 0$

$\therefore\ -p + q - q + 3 = 0$

$p = 3$

b $f(x) = 3x^3 + qx^2 + qx + 3$

$f(2) = 15$

$\therefore\ 24 + 4q + 2q + 3 = 15$

$q = -2$

18 **a** $p(3) = 0$

$\therefore\ 27 + 9a + 27 + b = 0$

$9a + b = -54 \quad (1)$

b $p(-2) = -30$

$\therefore\ -8 + 4a - 18 + b = -30$

$4a + b = -4 \quad (2)$

$(1) - (2) \Rightarrow\ 5a = -50$

$\therefore\ a = -10,\ b = 36$

19 $f(-1) = 3$

$\therefore\ -4 - 6 - m + n = 3$

$n - m = 13 \quad (1)$

$f(\frac{1}{2}) = 15$

$\therefore\ \frac{1}{2} - \frac{3}{2} + \frac{1}{2}m + n = 15$

$n + \frac{1}{2}m = 16 \quad (2)$

$(2) - (1) \Rightarrow\ \frac{3}{2}m = 3$

$\therefore\ m = 2,\ n = 15$

20 **a** $g(4) = 39$

$\therefore\ 64 + 4c + 3 = 39$

$c = -7$

b $g(x) = x^3 - 7x + 3$

$$\begin{array}{r|l}
 & x^2 - 2x - 3 \\
\hline
x + 2 & x^3 + 0x^2 - 7x + 3 \\
 & x^3 + 2x^2 \\
\hline
 & -2x^2 - 7x \\
 & -2x^2 - 4x \\
\hline
 & -3x + 3 \\
 & -3x - 6 \\
\hline
 & 9
\end{array}$$

quotient $= x^2 - 2x - 3$

remainder $= 9$

1 **a** $f(-2) = 0 \Rightarrow\ -8 - 20 - 2a + b = 0$

$\Rightarrow\ -2a + b = 28 \quad (1)$

$f(3) = 0 \ \Rightarrow\ 27 - 45 + 3a + b = 0$

$\Rightarrow\ 3a + b = 18 \quad (2)$

$(2) - (1)\ \ 5a = -10 = 0 \ \Rightarrow\ a = -2$

sub. (1) $\Rightarrow\ b = 24$

b $f(x) \equiv x^3 - 5x^2 - 2x + 24$

$(x + 2)(x - 3)(ax + b) \equiv x^3 - 5x^2 - 2x + 24$

by inspection

$f(x) \equiv (x + 2)(x - 3)(x - 4)$

2 $f(k) = 8f(\tfrac{1}{2}k)$

$8k^3 - k^2 + 7 = 8(k^3 - \tfrac{1}{4}k^2 + 7)$

$8k^3 - k^2 + 7 = 8k^3 - 2k^2 + 56$

$k^2 = 49$

$k = \pm 7$

3 **a** $f(2) = 24 - 4 - 24 + 4 = 0$

$\therefore\ (x - 2)$ is a factor of $f(x)$

b

$$
\begin{array}{r|l}
 & 3x^2 + 5x - 2 \\
\hline
x - 2 & 3x^3 - x^2 - 12x + 4 \\
 & 3x^3 - 6x^2 \\
\hline
 & 5x^2 - 12x \\
 & 5x^2 - 10x \\
\hline
 & -2x + 4 \\
 & -2x + 4 \\
\hline
\end{array}
$$

$\therefore\ f(x) = (x - 2)(3x^2 + 5x - 2)$

$= (x - 2)(3x - 1)(x + 2)$

$f(x) = 0 \ \Rightarrow\ (x - 2)(3x - 1)(x + 2) = 0$

$x = -2,\ \tfrac{1}{3}$ or 2

4 $6 + 7x - x^3 = 0$

let $f(x) = 6 + 7x - x^3$

$f(1) = 12,\ f(2) = 12,\ f(-1) = 0$

$\therefore\ (x + 1)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & -x^2 + x + 6 \\
\hline
x + 1 & -x^3 + 0x^2 + 7x + 6 \\
 & -x^3 - x^2 \\
\hline
 & x^2 + 7x \\
 & x^2 + x \\
\hline
 & 6x + 6 \\
 & 6x + 6 \\
\hline
\end{array}
$$

$\therefore\ (x + 1)(-x^2 + x + 6) = 0$

$-(x + 1)(x - 3)(x + 2) = 0$

$x = -2, -1, 3$

$\therefore\ (-2, 0), (-1, 0)$ and $(3, 0)$

5 **a** $f(-1) = -4$

$\therefore\ -3 + p - 8 + q = -4$

$p + q = 7 \qquad (1)$

$f(2) = 80$

$\therefore\ 24 + 4p + 16 + q = 80$

$4p + q = 40 \qquad (2)$

$(2) - (1) \Rightarrow 3p = 33$

$\therefore\ p = 11,\ q = -4$

b $f(x) \equiv 3x^3 + 11x^2 + 8x - 4$

$f(-2) = -24 + 44 - 16 - 4 = 0$

$\therefore\ (x + 2)$ is a factor

c

$$\begin{array}{r|rrrr} & 3x^2 & +\ 5x & -\ 2 & \\ \hline x+2 & 3x^3 & +\ 11x^2 & +\ 8x & -\ 4 \\ & 3x^3 & +\ 6x^2 & & \\ \hline & & 5x^2 & +\ 8x & \\ & & 5x^2 & +\ 10x & \\ \hline & & & -\ 2x & -\ 4 \\ & & & -\ 2x & -\ 4 \\ \hline \end{array}$$

$\therefore\ f(x) = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

$\therefore\ f(x) = 0 \Rightarrow x = -2$ or $\frac{1}{3}$

6 **a** let $f(x) = x^3 - 4x^2 - 7x + 10$

$f(1) = 1 - 4 - 7 + 10 = 0$

$\therefore\ (x - 1)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & -\ 3x & -\ 10 & \\ \hline x-1 & x^3 & -\ 4x^2 & -\ 7x & +\ 10 \\ & x^3 & -\ x^2 & & \\ \hline & & -\ 3x^2 & -\ 7x & \\ & & -\ 3x^2 & +\ 3x & \\ \hline & & & -\ 10x & +\ 10 \\ & & & -\ 10x & +\ 10 \\ \hline \end{array}$$

$\therefore\ (x - 1)(x^2 - 3x - 10) = 0$

$(x - 1)(x + 2)(x - 5) = 0$

$x = -2, 1, 5$

b $y^2 = x$ in part **a**

$y^2 = 1, 5$ or -2 [no solutions]

$y = \pm 1, \pm\sqrt{5}$

7 **a** $= f(-1) = -1 + 7 - 14 + 3 = -5$

b

$$\begin{array}{r|rrrr} & n^2 & +\ 6n & +\ 8 & \\ \hline n+1 & n^3 & +\ 7n^2 & +\ 14n & +\ 3 \\ & n^3 & +\ n^2 & & \\ \hline & & 6n^2 & +\ 14n & \\ & & 6n^2 & +\ 6n & \\ \hline & & & 8n & +\ 3 \\ & & & 8n & +\ 8 \\ \hline & & & & -\ 5 \end{array}$$

$\therefore\ f(n) = (n + 1)(n^2 + 6n + 8) - 5$

$f(n) = (n + 1)(n + 2)(n + 4) - 5$

c $(n + 1)$ and $(n + 2)$ are consecutive integers

$\therefore$ either $(n + 1)$ or $(n + 2)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4) - 5$ is odd

1 **a** $f(-2) = -8 + 4 + 44 - 40 = 0$

$\therefore\ (x + 2)$ is a factor of $f(x)$

b

$$
\begin{array}{r|l}
 & x^2 - x - 20 \\ \hline
x + 2 & x^3 + x^2 - 22x - 40 \\
 & x^3 + 2x^2 \\ \hline
 & \quad - x^2 - 22x \\
 & \quad - x^2 - 2x \\ \hline
 & \qquad - 20x - 40 \\
 & \qquad - 20x - 40 \\ \hline
\end{array}
$$

$\therefore\ f(x) \equiv (x + 2)(x^2 - x - 20)$

$\equiv (x + 2)(x + 4)(x - 5)$

c $f(x) = 0 \ \Rightarrow\ (x + 2)(x + 4)(x - 5) = 0$

$x = -4, -2$ or 5

2 **a** $f(2) = f(-3)$

$\therefore\ 8 - 8 + 2k + 1 = -27 - 18 - 3k + 1$

$k = -9$

b $= f(-2) = -8 - 8 + 18 + 1 = 3$

3 **a** $= p(-2) = -16 - 36 + 4 + 11 = -37$

b

$$
\begin{array}{r|l}
 & 2x^2 - x - 6 \\ \hline
x - 4 & 2x^3 - 9x^2 - 2x + 11 \\
 & 2x^3 - 8x^2 \\ \hline
 & \quad - x^2 - 2x \\
 & \quad - x^2 + 4x \\ \hline
 & \qquad - 6x + 11 \\
 & \qquad - 6x + 24 \\ \hline
 & \qquad\qquad - 13
\end{array}
$$

$\therefore$ quotient $= 2x^2 - x - 6$

remainder $= -13$

4 **a** A is $(0, 12)$

b $x = 1$ is a root of $y = 0$

$\therefore\ (x - 1)$ is a factor of y

$$
\begin{array}{r|l}
 & x^2 - 4x - 12 \\ \hline
x - 1 & x^3 - 5x^2 - 8x + 12 \\
 & x^3 - x^2 \\ \hline
 & \quad - 4x^2 - 8x \\
 & \quad - 4x^2 + 4x \\ \hline
 & \qquad - 12x + 12 \\
 & \qquad - 12x + 12 \\ \hline
\end{array}
$$

$\therefore\ y = (x - 1)(x^2 - 4x - 12)$

$= (x - 1)(x + 2)(x - 6)$

$\therefore\ y = 0$ when $x = -2, 1$ or 6

$\therefore\ B$ is $(-2, 0)$ and D is $(6, 0)$

5 **a** $f(1) = 0$

$\therefore\ 1 - 3 + k + 8 = 0$

$k = -6$

b

$$
\begin{array}{r|l}
 & x^2 - 2x - 8 \\ \hline
x - 1 & x^3 - 3x^2 - 6x + 8 \\
 & x^3 - x^2 \\ \hline
 & \quad - 2x^2 - 6x \\
 & \quad - 2x^2 + 2x \\ \hline
 & \qquad - 8x + 8 \\
 & \qquad - 8x + 8 \\ \hline
\end{array}
$$

$\therefore\ f(x) = (x - 1)(x^2 - 2x - 8)$

$= (x - 1)(x + 2)(x - 4)$

$f(x) = 0 \quad \Rightarrow \quad x = -2, 1, 4$

6 let $f(x) = 2x^3 + x^2 - 13x + 6$

$f(1) = -4, \ f(2) = 0$

$\therefore\ (x - 2)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 5x - 3 \\ \hline
x - 2 & 2x^3 + x^2 - 13x + 6 \\
 & 2x^3 - 4x^2 \\ \hline
 & \quad 5x^2 - 13x \\
 & \quad 5x^2 - 10x \\ \hline
 & \qquad - 3x + 6 \\
 & \qquad - 3x + 6 \\ \hline
\end{array}
$$

$\therefore\ (x - 2)(2x^2 + 5x - 3) = 0$

$(x - 2)(2x - 1)(x + 3) = 0$

$x = -3, \frac{1}{2}, 2$

7 **a** $p(-1) = 3$

$\therefore\ -b + a + 10 + b = 3$

$a = -7$

b $p(\tfrac{1}{3}) = -1$

$\therefore\ \tfrac{1}{27}b - \tfrac{7}{9} - \tfrac{10}{3} + b = -1$

$b - 21 - 90 + 27b = -27$

$b = 3$

8 **a** $= f(-1) = -1 - 7 - 1 + 10 = 1$

b $x^3 - 7x^2 + x + 10 = 1$

$x^3 - 7x^2 + x + 9 = 0$

$x = -1$ is solution $\therefore$ $(x + 1)$ is factor

$$\begin{array}{r l}
 & x^2 - 8x + 9 \\
x+1 & \overline{)\, x^3 - 7x^2 + x + 9} \\
 & \underline{x^3 + x^2} \\
 & -8x^2 + x \\
 & \underline{-8x^2 - 8x} \\
 & 9x + 9 \\
 & \underline{9x + 9}
\end{array}$$

$\therefore\ (x + 1)(x^2 - 8x + 9) = 0$

$x = -1,\ \dfrac{8 \pm \sqrt{64 - 36}}{2} = -1,\ 4 \pm \sqrt{7}$

9 $f(\tfrac{2}{3}) = 6$

$\therefore\ \tfrac{8}{9} + \tfrac{4}{9}k - \tfrac{14}{3} + 2k = 6$

$8 + 4k - 42 + 18k = 54$

$22k = 88$

$k = 4$

10 **a** $f(3) = 54 - 63 + 12 - 3 = 0$

$\therefore\ (x - 3)$ is a factor of $f(x)$

b

$$\begin{array}{r l}
 & 2x^2 - x + 1 \\
x-3 & \overline{)\, 2x^3 - 7x^2 + 4x - 3} \\
 & \underline{2x^3 - 6x^2} \\
 & -x^2 + 4x \\
 & \underline{-x^2 + 3x} \\
 & x - 3 \\
 & \underline{x - 3}
\end{array}$$

$\therefore\ f(x) \equiv (x - 3)(2x^2 - x + 1)$

c $f(x) = 0 \Rightarrow (x - 3)(2x^2 - x + 1) = 0$

$x = 3$ or $2x^2 - x + 1 = 0$

for $2x^2 - x + 1 = 0$, $b^2 - 4ac = -7$

$b^2 - 4ac < 0 \quad \Rightarrow \quad$ no real roots

$\therefore$ only one real solution

11 **a** $f(2) = 0$

$\therefore\ 8 + 2p + q = 0$

$q = -2p - 8$

b $f(-1) = -15$

$\therefore\ -1 - p + q = -15$

$q = p - 14$

$\therefore\ p - 14 = -2p - 8$

$p = 2,\ q = -12$

12 $f(-3) = 0$ $\therefore$ $(x + 3)$ is a factor of $f(x)$

$$\begin{array}{r l}
 & x^2 + x - 3 \\
x+3 & \overline{)\, x^3 + 4x^2 + 0x - 9} \\
 & \underline{x^3 + 3x^2} \\
 & x^2 + 0x \\
 & \underline{x^2 + 3x} \\
 & -3x - 9 \\
 & \underline{-3x - 9}
\end{array}$$

$\therefore\ f(x) = (x + 3)(x^2 + x - 3)$

other solutions given by $x^2 + x - 3 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 12}}{2} = \dfrac{-1 \pm \sqrt{13}}{2}$

$x = -2.30$ or 1.30

13 **a** $f(-2) = -7$

$\therefore\ (-2 + k)^3 - 8 = -7$

$(k - 2)^3 = 1$

$k = 3$

b $f(x) \equiv (x + 3)^3 - 8$

$\therefore\ f(-1) = 2^3 - 8 = 0$

$\therefore\ (x + 1)$ is a factor

14 **a** $= f(-2) = -8 - 16 + 14 + 8 = -2$

b $c = 2$

c $g(x) \equiv x^3 - 4x^2 - 7x + 10$

$$\begin{array}{r|l} & x^2 - 6x + 5 \\ x+2 & x^3 - 4x^2 - 7x + 10 \\ & x^3 + 2x^2 \\ \hline & \quad - 6x^2 - 7x \\ & \quad - 6x^2 - 12x \\ \hline & \qquad\quad 5x + 10 \\ & \qquad\quad 5x + 10 \\ \hline \end{array}$$

$\therefore\ g(x) = (x + 2)(x^2 - 6x + 5)$

$= (x + 2)(x - 1)(x - 5)$

$g(x) = 0 \quad\Rightarrow\quad x = -2, 1, 5$

15 **a** $f(\frac{1}{2}k) = 4$

$\therefore\ \frac{1}{8}k^3 - 2k + 1 = 4$

$k^3 - 16k + 8 = 32$

$k^3 - 16k - 24 = 0 \qquad (1)$

b $f(-k) = 1$

$\therefore\ -k^3 + 4k + 1 = 1$

$k^3 = 4k$

sub (1) $\quad\Rightarrow\quad 4k - 16k - 24 = 0$

$12k = -24$

$k = -2$