1 $\frac{AC}{\sin 118} = \frac{16}{\sin 26}$

$AC = \frac{16 \times \sin 118}{\sin 26}$

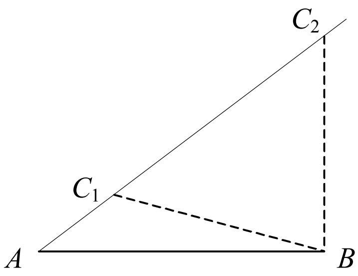
$= 32.2$ cm

2 $\frac{\sin \angle PRQ}{8.2} = \frac{\sin 57}{11.4}$

$\sin \angle PRQ = \frac{8.2 \times \sin 57}{11.4} = 0.6033$

$\angle PRQ = 37.1°$

3



$\frac{\sin \angle ACB}{16.2} = \frac{\sin 37}{12.3}$

$\sin \angle ACB = \frac{16.2 \times \sin 37}{12.3} = 0.7926$

$\angle ACB = 52.4$ or $180 - 52.4 = 52.4$ or 127.6

$\angle ABC = 180 - (37 + \angle ACB) = 90.568$ or 15.432

$\frac{AC}{\sin \angle ABC} = \frac{12.3}{\sin 37}$

$AC = \frac{12.3 \times \sin \angle ABC}{\sin 37} = 20.4$ or 5.4

$\therefore \angle ACB = 52.4°$, $AC = 20.4$ cm or $\angle ACB = 127.6°$, $AC = 5.4$ cm (all 1dp)

4 $XZ^2 = 7.8^2 + 15.3^2$

$- (2 \times 7.8 \times 15.3 \times \cos 31.5°)$

$= 91.422$

$XZ = 9.56$ cm (3sf)

5 $18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$

$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$

$= 0.3032$

$\angle ACB = 72.4°$ (1dp)

6 **a** $\alpha = 180 - (40 + 32) = 108$

$\frac{x}{\sin 108} = \frac{23.1}{\sin 40}$

$x = \frac{23.1 \times \sin 108}{\sin 40}$

$x = 34.2$ cm (3sf)

b $x^2 = 2.7^2 + 3.8^2$

$- (2 \times 2.7 \times 3.8 \times \cos 83)$

$x^2 = 19.229$

$x = 4.39$ m (3sf)

c $\frac{\sin \alpha}{7.6} = \frac{\sin 61}{10.5}$

$\sin \alpha = \frac{7.6 \times \sin 61}{10.5} = 0.6331$

$\alpha = 39.276$

$\beta = 180 - (61 + 39.276) = 79.724$

$\frac{x}{\sin 79.724} = \frac{10.5}{\sin 61}$

$x = \frac{10.5 \times \sin 79.724}{\sin 61}$

$x = 11.8$ cm (3sf)

7 **a** $\frac{\sin \alpha}{67} = \frac{\sin 96.5}{92}$

$\sin \alpha = \frac{67 \times \sin 96.5}{92}$

$\sin \alpha = 0.7236$

$\alpha = 46.351$

$\theta = 180 - 96.5 - \alpha$

$\theta = 37.1°$ (1dp)

b $1.9^2 = 0.8^2 + 1.7^2$

$- (2 \times 0.8 \times 1.7 \times \cos \theta)$

$\cos \theta = \frac{0.8^2 + 1.7^2 - 1.9^2}{2 \times 0.8 \times 1.7}$

$\cos \theta = -0.02941$

$\theta = 91.7°$ (1dp)

c $l^2 = 7.4^2 + 8.7^2$

$- (2 \times 7.4 \times 8.7 \times \cos 43.7)$

$l^2 = 37.3608$, $l = 6.1123$

$\frac{\sin \theta}{7.4} = \frac{\sin 43.7}{6.1123}$

$\sin \theta = \frac{7.4 \times \sin 43.7}{6.1123} = 0.8364$

$\theta = 56.8°$ (1dp)

8 **a** area

$= \frac{1}{2} \times 2.1 \times 3.4 \times \sin 66$

$= 3.26 \text{ m}^2$ (3sf)

b area

$= \frac{1}{2} \times 35 \times 68 \times \sin 116$

$= 1070 \text{ cm}^2$ (3sf)

c $\frac{\sin\alpha}{5.8} = \frac{\sin 72.4}{6.5}$

$\sin\alpha = \frac{5.8 \times \sin 72.4}{6.5} = 0.8505$

$\alpha = 58.270$

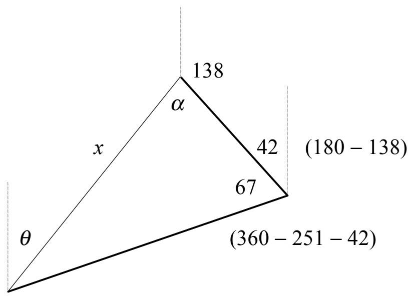
$\beta = 180 - (72.4 + \alpha) = 49.330$

area

$= \frac{1}{2} \times 5.8 \times 6.5 \times \sin 49.330$

$= 14.3 \text{ cm}^2$ (3sf)

9



a $x^2 = 4.2^2 + 7.8^2 - (2 \times 4.2 \times 7.8 \times \cos 67)$

$x^2 = 52.879$

$x = 7.27$ miles (3sf)

b $\frac{\sin\alpha}{7.8} = \frac{\sin 67}{7.2718}$

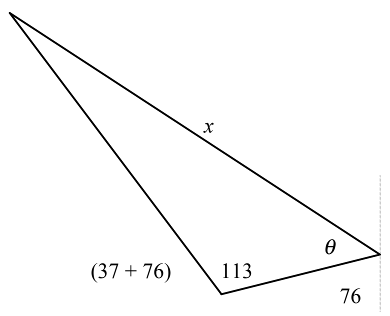
$\sin\alpha = \frac{7.8 \times \sin 67}{7.2718} = 0.9874$

$\alpha = 80.882$

$\theta = 138 + \alpha - 180 = 38.882$

bearing = 039° (nearest degree)

10



$x^2 = 3.2^2 + 6.9^2 - (2 \times 3.2 \times 6.9 \times \cos 113)$

$x^2 = 75.105$

$x = 8.67$ km (3sf)

$\frac{\sin\theta}{6.9} = \frac{\sin 113}{8.666}$

$\sin\theta = \frac{6.9 \times \sin 113}{8.666} = 0.7329$

$\theta = 47.130$

bearing = 180 + 76 + θ = 303° (nearest degree)

11 $9.7^2 = 10.4^2 + 11.0^2 - (2 \times 10.4 \times 11.0 \times \cos \angle BAC)$

$\cos \angle BAC = \frac{10.4^2 + 11.0^2 - 9.7^2}{2 \times 10.4 \times 11.0} = 0.5903$

$\angle BAC = 53.819$

area $= \frac{1}{2} \times 10.4 \times 11.0 \times \sin 53.819 = 46.2 \text{ cm}^2$

12 $\frac{1}{2} \times 22.5 \times YZ \times \sin 34 = 100$

$YZ = \frac{200}{22.5 \times \sin 34} = 15.896$

$XZ^2 = 22.5^2 + 15.896^2 - (2 \times 22.5 \times 15.896 \times \cos 34)$

$= 165.906$

$XZ = 12.9$ cm (3sf)

1 **a** π **b** $\frac{\pi}{6}$ **c** $\frac{\pi}{4}$ **d** 4π **e** $\frac{\pi}{10}$ **f** $\frac{2\pi}{3}$
g $\frac{\pi}{12}$ **h** $\frac{2\pi}{9}$ **i** $\frac{3\pi}{2}$ **j** $\frac{\pi}{24}$ **k** $\frac{4\pi}{5}$ **l** $\frac{11\pi}{9}$

2 **a** 0.17^c **b** 0.66^c **c** 5.08^c **d** 1.11^c **e** 8.85^c **f** 2.20^c

3 **a** $360°$ **b** $60°$ **c** $90°$ **d** $135°$ **e** $10°$ **f** $6°$
g $150°$ **h** $22.5°$ **i** $540°$ **j** $24°$ **k** $420°$ **l** $81°$

4 **a** $114.6°$ **b** $28.6°$ **c** $177.6°$ **d** $81.9°$ **e** $498.5°$ **f** $42.5°$

5 **a** $s = 12 \times \frac{\pi}{4} = 3\pi$ cm
b $60° = \frac{\pi}{3}$
$s = 15 \times \frac{\pi}{3} = 5\pi$ cm
c $s = 9 \times \frac{5\pi}{6} = \frac{15\pi}{2}$ mm

6 **a** $P = (2 \times 5.2) + (5.2 \times 1.2)$
$= 16.6$ cm
b $P = (2 \times 19.6) + (19.6 \times \frac{2\pi}{3})$
$= 80.3$ cm
c $360° - 97° = 263° = 4.5902^c$
$P = (2 \times 8.5) + (8.5 \times 4.5902)$
$= 56.0$ cm

7 **a** $\theta = 11 \div 16 = 0.69^c$ **b** $\theta = 35 \div 7.2 = 4.86^c$ **c** $\theta = 20.3 \div 17.9 = 1.13^c$

8 **a** $78.5° = 1.3701^c$
$OA = 46.2 \div 1.3701 = 33.7$ cm (3sf)
b $P = (2 \times OA) + 46.2 = 114$ cm (3sf)

9 **a** $A = \frac{1}{2} \times 50^2 \times \frac{\pi}{3}$
$= 1309.0$ cm^2
b $94° = 1.6406^c$
$A = \frac{1}{2} \times (14.2)^2 \times 1.6406$
$= 165.4$ cm^2
c $A = \frac{1}{2} \times 7^2 \times 4.3$
$= 105.4$ cm^2

10 **a** $\theta = 12 \div 8 = 1.5^c$ **b** $A = \frac{1}{2} \times 8^2 \times 1.5 = 48$ cm^2

11 **a** $P = (2 \times 11.6) + (11.6 \times 1.4) = 39.4$ cm
b $2\pi - 1.4 = 4.8832$
$P = (2 \times 11.6) + (11.6 \times 4.8832) = 79.8$ cm
c $A = \frac{1}{2} \times (11.6)^2 \times 1.4 = 94.2$ cm^2
d $A = \frac{1}{2} \times (11.6)^2 \times 4.8832 = 329$ cm^2

12 **a** $A = \frac{1}{2} \times 11^2 \times 0.9$
$= 54.45$ cm^2
b $A = \frac{1}{2} \times 11^2 \times \sin 0.9^c$
$= 47.4$ cm^2 (3sf)
c $A = 54.45 - 47.391$
$= 7.06$ cm^2 (3sf)

13 **a** $A = [\frac{1}{2} \times (16.2)^2 \times 1.05]$
$- [\frac{1}{2} \times (16.2)^2 \times \sin 1.05^c]$
$= 137.781 - 113.823$
$= 24.0$ cm^2 (3sf)
b $A = [\frac{1}{2} \times 32^2 \times \frac{\pi}{4}]$
$- [\frac{1}{2} \times 32^2 \times \sin \frac{\pi}{4}]$
$= 402.124 - 362.039$
$= 40.1$ mm^2 (3sf)
c $130.5° = 2.2777^c$
$A = [\frac{1}{2} \times (62.3)^2 \times 2.2777]$
$- [\frac{1}{2} \times (62.3)^2 \times \sin 2.2777^c]$
$= 4420.1 - 1475.7$
$= 2940$ cm^2 (3sf)

1 $(2 \times 12.6) + 12.6\theta = 31.7$

$\theta = 6.5 \div 12.6 = 0.5159^c$

$A = \frac{1}{2} \times (12.6)^2 \times 0.5159 = 40.95 \text{ cm}^2$

2 **a** $\frac{1}{2} \times (7.3)^2 \times \theta = 38.4$

$\theta = 38.4 \div 26.645 = 1.44^c$ (3sf)

b chord $AB = 2 \times 7.3 \sin(\frac{1}{2}\theta) = 9.633$

arc $AB = 7.3\theta = 10.521$

$P = 9.633 + 10.521 = 20.2$ cm (3sf)

3 **a** $\frac{1}{2}r^2\theta = 40 \quad \therefore\ \theta = \frac{80}{r^2}$

$P = 2r + r\theta = 2r + (r \times \frac{80}{r^2})$

$= (2r + \frac{80}{r})$ cm

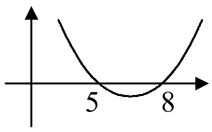
b $2r + \frac{80}{r} < 26$

$2r^2 + 80 < 26r$

$r^2 - 13r + 40 < 0$

$(r - 5)(r - 8) < 0$

$5 < r < 8$



4 **a** $AB^2 = 10^2 = 100,\ AC^2 + BC^2 = 6^2 + 8^2 = 100$

$AB^2 = AC^2 + BC^2$

$\therefore\ \angle ACB = 90°$ (converse of Pythagoras')

$\therefore$ triangle ABC is right-angled

b $\tan(\angle ABC) = \frac{AC}{BC} = \frac{3}{4} \quad \therefore\ \angle ABC = 0.64^c$

c $\angle BAC = \frac{\pi}{2} - 0.6435 = 0.9273$

area of sectors:

centre $A = \frac{1}{2} \times 4^2 \times 0.9273 = 7.4184$

centre $B = \frac{1}{2} \times 6^2 \times 0.6435 = 11.5830$

centre $C = \frac{1}{4} \times \pi \times 2^2 = 3.1416$

area of triangle $ABC = \frac{1}{2} \times AC \times BC = 24$

shaded area

$= 24 - (7.4184 + 11.5830 + 3.1416)$

$= 1.86 \text{ cm}^2$

5 **a** let centre of circle be O

let midpoint of AB be M

$AM^2 = OA^2 - OM^2 = 5^2 - 3^2 = 16$

$AM = 4 \quad \therefore\ AB = 8$ cm

b $\cos(\angle AOM) = \frac{3}{5}$

$\angle AOB = 2 \times \angle AOM = 1.8546^c$

arc $AB = 5 \times 1.8546 = 9.2730$

$P = 2 \times (6 + 14 - 8 + 9.2730) = 42.5$ cm

c area of segment

$= \frac{1}{2} \times 5^2 \times 1.8546 - \frac{1}{2} \times 5^2 \times \sin 1.8546^c$

$= 23.182 - 12 = 11.182$

area of logo $= (6 \times 14) + (2 \times 11.182)$

$= 106 \text{ cm}^2$ (3sf)

6 **a** $OC = (r + 2)$ cm

$A_1 = [\frac{1}{2} \times 8^2 \times \theta] - [\frac{1}{2} \times (r + 2)^2 \times \theta]$

$= \frac{1}{2}\theta[64 - (r^2 + 4r + 4)]$

$= \frac{1}{2}\theta(60 - 4r - r^2) \text{ cm}^2$

b $A_2 = \frac{1}{2}r^2\theta$

$\therefore\ \frac{1}{2}\theta(60 - 4r - r^2) = 7 \times \frac{1}{2}r^2\theta$

$60 - 4r - r^2 = 7r^2$

$2r^2 + r - 15 = 0$

$(2r - 5)(r + 3) = 0$

$r > 0 \therefore\ r = 2.5$

7 let length of wire $= 3l$

area of $A = \frac{1}{2} \times l^2 \times \sin\frac{\pi}{3} = 0.43301l^2$

angle at centre of $B = l \div l = 1^c$

area of $B = \frac{1}{2} \times l^2 \times 1 = 0.5l^2$

% change $= \frac{0.5l^2 - 0.43301l^2}{0.43301l^2} \times 100\%$

$= 15.5\%$, increase

1 **a** 0.755 **b** −0.354 **c** 0.530 **d** −0.255

2
a $= \frac{1}{2}$ **b** $= \frac{1}{\sqrt{2}}$ **c** $= 1$ **d** $= \frac{\sqrt{3}}{2}$

e $= 1$ **f** $= \frac{1}{\sqrt{3}}$ **g** $= -\cos 60° = -\frac{1}{2}$ **h** $= \sin 45° = \frac{1}{\sqrt{2}}$

i $= \tan 30° = \frac{1}{\sqrt{3}}$ **j** $= -\cos 45° = -\frac{1}{\sqrt{2}}$ **k** $= -\sin 60° = -\frac{\sqrt{3}}{2}$ **l** $= -\tan 60° = -\sqrt{3}$

m $= \cos 30° = \frac{\sqrt{3}}{2}$ **n** $= -\tan 30° = -\frac{1}{\sqrt{3}}$ **o** $= \cos 60° = \frac{1}{2}$ **p** $= \sin 45° = \frac{1}{\sqrt{2}}$

q $= -\tan 45° = -1$ **r** $= \sin 60° = \frac{\sqrt{3}}{2}$ **s** $= \tan 30° = \frac{1}{\sqrt{3}}$ **t** $= -\cos 30° = -\frac{\sqrt{3}}{2}$

3 **a** 0.913 **b** −0.851 **c** 0.042 **d** 0.252

4
a $= \frac{1}{2}$ **b** $= 0$ **c** $= \frac{1}{\sqrt{2}}$ **d** $= \sqrt{3}$

e $= \frac{1}{2}$ **f** $= \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **g** $= -\tan \frac{\pi}{4} = -1$ **h** $= -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

i $= -\tan \frac{\pi}{3} = -\sqrt{3}$ **j** $= -\cos \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$ **k** $= -\sin \frac{\pi}{6} = -\frac{1}{2}$ **l** $= \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

m $= \sin 0 = 0$ **n** $= -\tan \frac{\pi}{4} = -1$ **o** $= -\cos \frac{\pi}{3} = -\frac{1}{2}$ **p** $= -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

5
a (0, 0), (180, 0), (360, 0), (540, 0), (720, 0)

b (90, 1), (270, −1), (450, 1), (630, −1)

6
a (0, 0), (180, 0), (360, 0), (540, 0), (720, 0)

b $x = 90$, $x = 270$, $x = 450$, $x = 630$

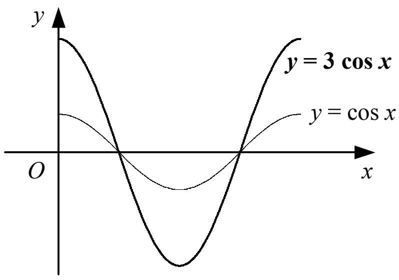
7
a stretch by a factor of 3 in the y-direction about the x-axis

b stretch by a factor of $\frac{1}{4}$ in the x-direction about the y-axis

c translation by 60 units in the negative x-direction

d reflection in the y-axis

8 **a**



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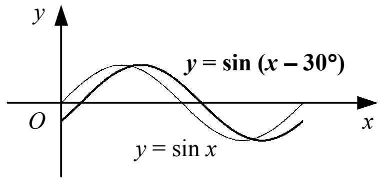

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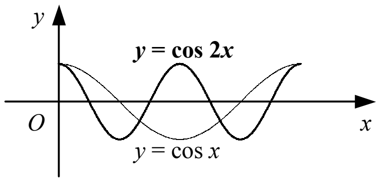

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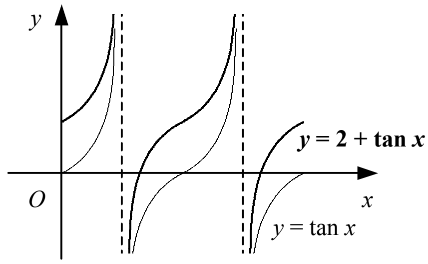

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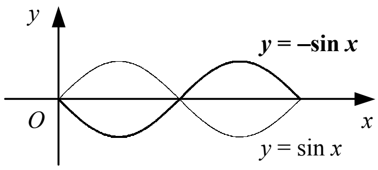

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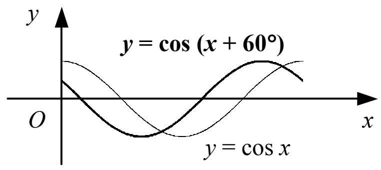

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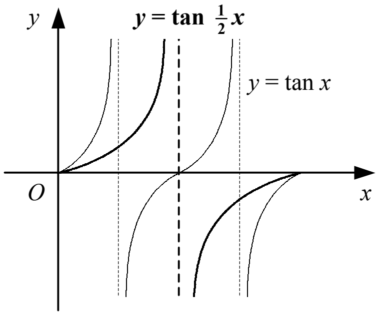

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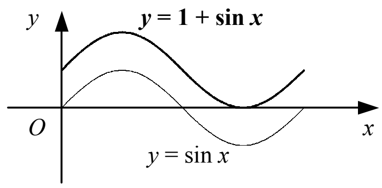


9 **a** (–90°, –2), (90°, 2)

b (–180°, 1), (0, 3), (180°, 1)

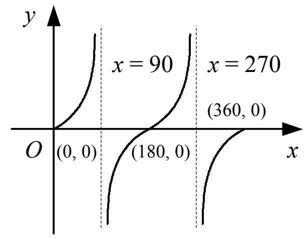
c (–150°, –1), (–90°, 1), (–30°, –1), (30°, 1), (90°, –1), (150°, 1)

d (–135°, –1), (45°, 1)

10 **a** 360°   **b** 180°   **c** 360°

d 180°   **e** 180°   **f** 1080°

11 **a**



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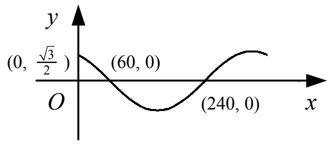

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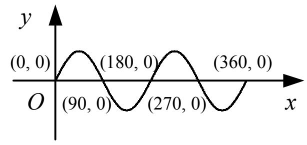

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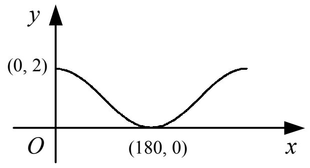

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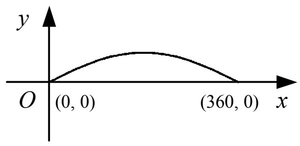

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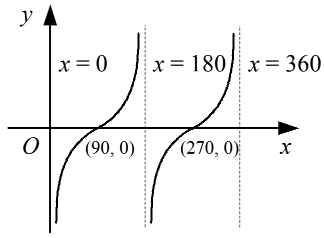

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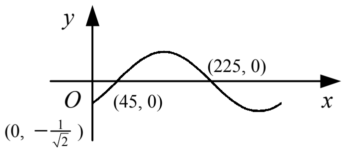

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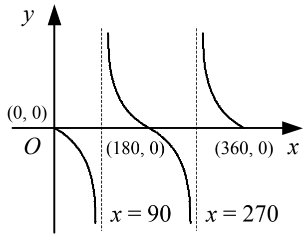
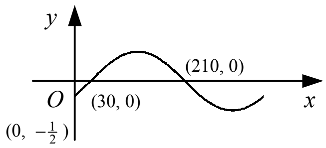
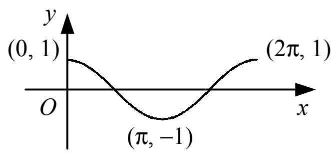


i



12 **a**



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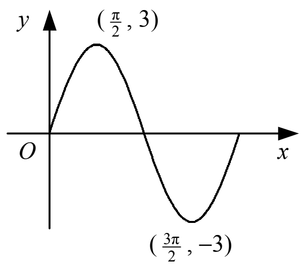

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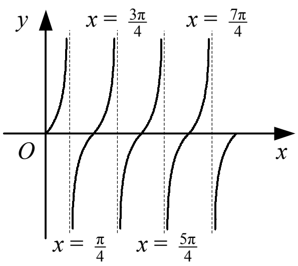
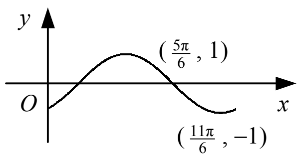
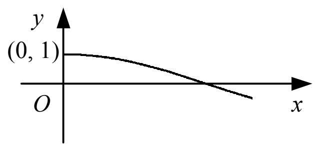


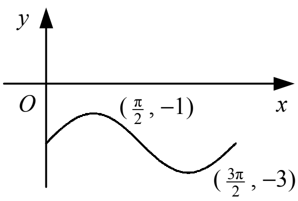
d



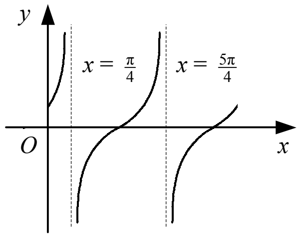
e



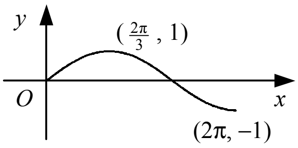
f



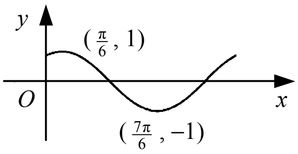
g



h



i



C2 TRIGONOMETRY — Answers - Worksheet E

1

a $x = 30,\ 180 - 30$
$x = 30°,\ 150°$

b $x = 60,\ 180 + 60$
$x = 60°,\ 240°$

c $x = 90°,\ 270°$

d $x = 270°$

e $x = 30,\ 360 - 30$
$x = 30°,\ 330°$

f $x = 45,\ 180 - 45$
$x = 45°,\ 135°$

g $x = 180 - 45,\ 360 - 45$
$x = 135°,\ 315°$

h $x = 180 - 60,\ 180 + 60$
$x = 120°,\ 240°$

i $x = 180 + 60,\ 360 - 60$
$x = 240°,\ 300°$

j $x = 30,\ 180 + 30$
$x = 30°,\ 210°$

k $x = 180 - 45,\ 180 + 45$
$x = 135°,\ 225°$

l $x = 180 - 60,\ 360 - 60$
$x = 120°,\ 300°$

2

a $\theta = 66.4,\ 360 - 66.4$
$\theta = 66.4°,\ 293.6°$

b $\theta = 15.7,\ 180 - 15.7$
$\theta = 15.7°,\ 164.3°$

c $\theta = 58.0,\ 180 + 58.0$
$\theta = 58.0°,\ 238.0°$

d $\theta = 54.4,\ 180 - 54.4$
$\theta = 54.4°,\ 125.6°$

e $\theta = 5.7,\ 180 + 5.7$
$\theta = 5.7°,\ 185.7°$

f $\theta = 79.3,\ 360 - 79.3$
$\theta = 79.3°,\ 280.7°$

g $\theta = 180 + 36.9,\ 360 - 36.9$
$\theta = 216.9°,\ 323.1°$

h $\theta = 180 - 35.0,\ 360 - 35.0$
$\theta = 145.0°,\ 325.0°$

i $\theta = 180 - 67.0,\ 180 + 67.0$
$\theta = 113.0°,\ 247.0°$

j $\theta = 180 - 73.6,\ 360 - 73.6$
$\theta = 106.4°,\ 286.4°$

k $\theta = 180 - 50.5,\ 180 + 50.5$
$\theta = 129.5°,\ 230.5°$

l $\theta = 180 + 11.7,\ 360 - 11.7$
$\theta = 191.7°,\ 348.3°$

3

a $x - 60 = 30,\ 180 - 30$
$= 30,\ 150$
$x = 90,\ 210$

b $x + 30 = 45,\ 180 + 45$
$= 45,\ 225$
$x = 15,\ 195$

c $x - 45 = 78.5,\ 360 - 78.5$
$= 78.5,\ 281.5$
$x = 123.5,\ 326.5$

d $x + 30 = 38.0,\ 180 + 38.0$
$= 38.0,\ 218.0$
$x = 8.0,\ 188.0$

e $x + 45 = 180 - 60,\ 180 + 60$
$= 120,\ 240$
$x = 75,\ 195$

f $x - 60 = 180 + 62.9,\ 360 - 62.9$
$= 242.9,\ 297.1$
$x = 302.9,\ 357.1$

g $x + 45 = 360 - 25.8,\ 360 + 25.8$
$= 334.2,\ 385.8$
$x = 289.2,\ 340.8$

h $x + 30 = 180 - 8.0,\ 360 + 8.0$
$= 172.0,\ 368.0$
$x = 142.0,\ 338.0$

i $x - 60 = -53.1,\ 53.1$
$x = 6.9,\ 113.1$

j $x - 30 = -17.5,\ 180 + 17.5$
$= -17.5,\ 197.5$
$x = 12.5,\ 227.5$

k $x - 60 = -51.6,\ 180 - 51.6$
$= -51.6,\ 128.4$
$x = 8.4,\ 188.4$

l $2x = 30,\ 180 - 30,\ 360 + 30,\ 540 - 30$
$= 30,\ 150,\ 390,\ 510$
$x = 15,\ 75,\ 195,\ 255$

m $2x = 50.208,\ 360 - 50.208,\ 360 + 50.208,\ 720 - 50.208$
$= 50.208,\ 309.792,\ 410.208,\ 669.792$
$x = 25.1,\ 154.9,\ 205.1,\ 334.9$

n $2x = 180 + 10.370,\ 360 - 10.370,\ 540 + 10.370,\ 720 - 10.370$
$= 190.370,\ 349.630,\ 550.370,\ 709.630$
$x = 95.2,\ 174.8,\ 275.2,\ 354.8$

o $2x = 180 - 69.950,\ 360 - 69.950,\ 540 - 69.950,\ 720 - 69.950$
$= 110.050,\ 290.050,\ 470.050,\ 650.050$
$x = 55.0,\ 145.0,\ 235.0,\ 325.0$

p $\frac{1}{2}x = 44.668,\ 180 - 44.668$
$= 44.668,\ 135.332$
$x = 89.3,\ 270.7$

q $3x = 30.583,\ 180 + 30.583,\ 360 + 30.583,\ 540 + 30.583,\ 720 + 30.583,\ 900 + 30.583$
$= 30.583,\ 210.583,\ 390.583,\ 570.583,\ 750.583,\ 930.583$
$x = 10.2,\ 70.2,\ 130.2$
$190.2,\ 250.2,\ 310.2$

r $2x = 180 - 65.481,\ 180 + 65.481,\ 540 - 65.481,\ 540 + 65.481$
$= 114.519,\ 245.481,\ 474.519,\ 605.481$
$x = 57.3,\ 122.7,\ 237.3,\ 302.7$

4 **a** $x = 0, \pi, 2\pi$

b $x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$
$x = \frac{\pi}{3}, \frac{5\pi}{3}$

c $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$
$x = \frac{\pi}{4}, \frac{5\pi}{4}$

d $x = \pi$

e $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

f $x = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

g $x + \frac{\pi}{6} = \frac{\pi}{3}, \pi + \frac{\pi}{3}$
$= \frac{\pi}{3}, \frac{4\pi}{3}$
$x = \frac{\pi}{6}, \frac{7\pi}{6}$

h $x - \frac{\pi}{4} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
$= \frac{\pi}{6}, \frac{5\pi}{6}$
$x = \frac{5\pi}{12}, \frac{13\pi}{12}$

i $x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$
$= \frac{5\pi}{6}, \frac{7\pi}{6}$
$x = \frac{\pi}{2}, \frac{5\pi}{6}$

j $x + \frac{\pi}{3} = \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$
$= \frac{3\pi}{4}, \frac{9\pi}{4}$
$x = \frac{5\pi}{12}, \frac{23\pi}{12}$

k $2x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4},$
$3\pi - \frac{\pi}{4}, 3\pi + \frac{\pi}{4}$
$= \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$
$x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

l $3x = \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6},$
$3\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 5\pi + \frac{\pi}{6}$
$= \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}$
$x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

5 **a** $\theta = -90°, 90°$

b $\tan 2\theta = -1$
$2\theta = 180 - 45, 360 - 45$
$-45, -45 - 180$
$= -225, -45, 135, 315$
$\theta = -112.5°, -22.5°,$
$67.5°, 157.5°$

c $\theta + 60 = 16.9, 180 - 16.9$
$= 16.9, 163.1$
$\theta = -43.1°, 103.1°$

d $\tan(\theta - 15) = 1.85$
$\theta - 15 = 61.6, 61.6 - 180$
$= -118.4, 61.6$
$\theta = -103.4°, 76.6°$

e $\sin 2\theta = 0.3$
$2\theta = 17.458, 180 - 17.458,$
$17.458 - 360,$
$-17.458 - 180$
$= -342.542, -197.458,$
$17.458, 162.542$
$\theta = -171.3°, -98.7°$
$8.7°, 81.3°$

f $\cos 3\theta = 0.5$
$3\theta = 60, 360 - 60, 360 + 60,$
$-60, 60 - 360, -60 - 360$
$= -420, -300, -60,$
$60, 300, 420$
$\theta = -140°, -100°, -20°$
$20°, 100°, 140°$

g $\sin(\theta + 110) = -1$
$\theta + 110 = 270$
$\theta = 160°$

h $\cos(\theta - 27) = 0.6$
$\theta - 27 = 53.1, -53.1$
$\theta = -26.1°, 80.1°$

i $\tan\theta = \frac{7}{3}$
$\theta = 66.8, 66.8 - 180$
$\theta = -113.2°, 66.8°$

j $\cos 2\theta = -0.375$
$2\theta = 180 - 67.976,$
$180 + 67.976,$
$67.976 - 180,$
$-67.976 - 180$
$= -247.976, -112.024,$
$112.024, 247.976$
$\theta = -124.0°, -56.0°,$
$56.0°, 124.0°$

k $\tan(\theta + 92) = -\frac{1}{3}$
$\theta + 92 = 180 - 18.4, -18.4$
$= -18.4, 161.6$
$\theta = -110.4°, 69.6°$

l $\sin\frac{1}{3}\theta = 0.25$
$\frac{1}{3}\theta = 14.478$
$\theta = 43.4°$

6

a $2x + 30 = 45,\ 180 + 45$
$= 45,\ 225$
$2x = 15,\ 195$
$x = 7.5°,\ 97.5°$

b $2x - 15 = 0,\ 180$
$2x = 15,\ 195$
$x = 7.5°,\ 97.5°$

c $2x + 70 = 360 - 60,\ 360 + 60$
$= 300,\ 420$
$2x = 230,\ 350$
$x = 115°,\ 175°$

d $2x + 210 = 360 + 15.070,\ 540 - 15.070$
$= 375.070,\ 524.930$
$2x = 165.070,\ 314.930$
$x = 82.5°,\ 157.5°$

e $2x - 38 = 180 - 50.208,\ 180 + 50.208$
$= 129.792,\ 230.208$
$2x = 167.792,\ 268.208$
$x = 83.9°,\ 134.1°$

f $2x - 56 = 180 - 17.745,\ -17.745$
$= -17.745,\ 162.256$
$2x = 38.256,\ 218.256$
$x = 19.1°,\ 109.1°$

g $3x - 24 = 42.862,\ 360 - 42.862,\ 360 + 42.862$
$= 42.862,\ 317.138,\ 402.862$
$3x = 66.862,\ 341.138,\ 426.862$
$x = 22.3°,\ 113.7°,\ 142.3°$

h $3x + 60 = 180 - 62.241,\ 360 - 62.241,\ 540 - 62.241$
$= 117.759,\ 297.759,\ 477.759$
$3x = 57.759,\ 237.759,\ 417.759$
$x = 19.3°,\ 79.3°,\ 139.3°$

i $\frac{1}{2}x + 18 = 34.890$
$\frac{1}{2}x = 16.890$
$x = 33.8°$

7

a $x = 0.48,\ \pi + 0.4795$
$x = 0.48^c,\ 3.62^c$

b $2x = 1.2503,\ 2\pi - 1.2503,\ 2\pi + 1.2503,\ 4\pi - 1.25032$
$= 1.2503,\ 5.0328,\ 7.5335,\ 11.3160$
$x = 0.63^c,\ 2.52^c,\ 3.77^c,\ 5.66^c$

c $x + \frac{\pi}{4} = \pi - 0.7754,\ 2\pi + 0.7754$
$= 2.3662,\ 7.0586$
$x = 1.58^c,\ 6.27^c$

d $\cos x = -\frac{1}{3}$
$x = \pi - 1.2310,\ \pi + 1.2310$
$= 1.91^c,\ 4.37^c$

e $\frac{1}{2}x = 0.0901,\ \pi - 0.0901$
$= 0.0901,\ 3.0515$
$x = 0.18^c,\ 6.10^c$

f $2x = \pi - 0.2213,\ 2\pi - 0.2213,\ 3\pi - 0.2213,\ 4\pi - 0.2213$
$= 2.9203,\ 6.0619,\ 9.2035,\ 12.3451$
$x = 1.46^c,\ 3.03^c,\ 4.60^c,\ 6.17^c$

g $\sin(x - \frac{\pi}{3}) = 0.75$
$x - \frac{\pi}{3} = 0.8481,\ \pi - 0.8481$
$= 0.8481,\ 2.2935$
$x = 1.90^c,\ 3.34^c$

h $2x + \frac{\pi}{6} = 1.1071,\ \pi + 1.1071,\ 2\pi + 1.1071,\ 3\pi + 1.1071$
$= 1.1071,\ 4.2487,\ 7.3903,\ 10.5319$
$2x = 0.5835,\ 3.7251,\ 6.8667,\ 10.0083$
$x = 0.29^c,\ 1.86^c,\ 3.43^c,\ 5.00^c$

i $3x = \pi - 0.6266,\ \pi + 0.6266,\ 3\pi - 0.6266,\ 3\pi + 0.6266,\ 5\pi - 0.6266,\ 5\pi + 0.6266$
$= 2.5149,\ 3.7682,\ 8.7981,\ 10.0514,\ 15.0813,\ 16.3346$
$x = 0.84^c,\ 1.26^c,\ 2.93^c,\ 3.35^c,\ 5.03^c,\ 5.44^c$

j $\tan x = -\frac{5}{3}$
$x = \pi - 1.0304,\ 2\pi - 1.0304$
$x = 2.11^c,\ 5.25^c$

k $2x - \frac{\pi}{2} = \pi - 1.2239,\ \pi + 1.2239,\ 3\pi - 1.2239,\ 3\pi + 1.2239$
$= 1.9177,\ 4.3655,\ 8.2009,\ 10.6487$
$2x = 3.4885,\ 5.9363,\ 9.7717,\ 12.2195$
$x = 1.74^c,\ 2.97^c,\ 4.89^c,\ 6.11^c$

l $\sin 2x = -\frac{1}{6}$
$2x = \pi + 0.1674,\ 2\pi - 0.1674,\ 3\pi + 0.1674,\ 4\pi - 0.1674$
$= 3.3090,\ 6.1157,\ 9.5922,\ 12.3989$
$x = 1.65^c,\ 3.06^c,\ 4.80^c,\ 6.20^c$

8 **a** $(2y - 1)(y - 1) = 0$
$y = \frac{1}{2}, 1$

b $\sin x = \frac{1}{2}$ or 1
$x = 30, 180 - 30$ or 90
$x = 30°, 90°, 150°$

9 **a** $\sin\theta = \pm\frac{\sqrt{3}}{2}$
$\theta = 60, 180 - 60$ or $180 + 60, 360 - 60$
$\theta = 60, 120, 240, 300$

b $\tan\theta = \pm 1$
$\theta = 45, 180 + 45$ or $180 - 45, 360 - 45$
$\theta = 45, 135, 225, 315$

c $\cos\theta(2\cos\theta + 1) = 0$
$\cos\theta = 0$ or -0.5
$\theta = 90, 270$ or $180 - 60, 180 + 60$
$\theta = 90, 120, 240, 270$

d $\sin\theta = 0$ or $\cos\theta = 0.25$
$\theta = 0, 180, 360$ or $75.5, 360 - 75.5$
$\theta = 0, 75.5, 180, 284.5, 360$

e $\sin\theta(4 - \tan\theta) = 0$
$\sin\theta = 0$ or $\tan\theta = 4$
$\theta = 0, 180, 360$ or $76.0, 180 + 76.0$
$\theta = 0, 76.0, 180, 256.0, 360$

f $\cos\theta = -1$ or 0.5
$\theta = 180$ or $60, 360 - 60$
$\theta = 60, 180, 300$

g $(\tan\theta - 1)(\tan\theta - 2) = 0$
$\tan\theta = 1$ or 2
$\theta = 45, 180 + 45$ or $63.4, 180 + 63.4$
$\theta = 45, 63.4, 225, 243.4$

h $(3\sin\theta - 1)(\sin\theta - 2) = 0$
$\sin\theta = \frac{1}{3}$ or 2 [no solutions]
$\theta = 19.5, 180 - 19.5$
$\theta = 19.5, 160.5$

i $\tan^2\theta - \tan\theta - 6 = 0$
$(\tan\theta + 2)(\tan\theta - 3) = 0$
$\tan\theta = -2$ or 3
$\theta = 180-63.4, 360-63.4$ or $71.6, 180+71.6$
$\theta = 71.6, 116.6, 251.6, 296.6$

j $(3\cos\theta - 2)(2\cos\theta + 1) = 0$
$\cos\theta = -0.5$ or $\frac{2}{3}$
$\theta = 180 - 60, 180 + 60$ or $48.2, 360 - 48.2$
$\theta = 48.2, 120, 240, 311.8$

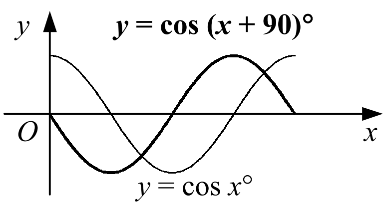
k $4\sin^2\theta - 8\sin\theta + 3 = 0$
$(2\sin\theta - 1)(2\sin\theta - 3) = 0$
$\sin\theta = 0.5$ or 1.5 [no solutions]
$\theta = 30, 180 - 30$
$\theta = 30, 150$

l $\cos\theta = \frac{-2 \pm \sqrt{4+4}}{2}$
$\cos\theta = -1 + \sqrt{2}$ or $-1 - \sqrt{2}$ [no solutions]
$\theta = 65.5, 360 - 65.5$
$\theta = 65.5, 294.5$

m $\tan\theta = \frac{-3 \pm \sqrt{9+4}}{2}$
$\tan\theta = \frac{1}{2}(-3 \pm \sqrt{13})$
$\theta = 180-73.2, 360-73.2$ or $16.8, 180+16.8$
$\theta = 16.8, 106.8, 196.8, 286.8$

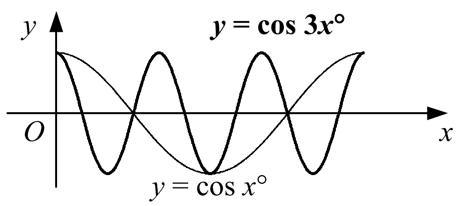
n $3\sin^2\theta + \sin\theta - 1 = 0$
$\sin\theta = \frac{-1 \pm \sqrt{1+12}}{6} = \frac{1}{6}(-1 \pm \sqrt{13})$
$\theta = 180+50.1, 360-50.1$ or $25.7, 180-25.7$
$\theta = 25.7, 154.3, 230.1, 309.9$

10 **a, b**



c $x = 135, 315$

11 **a**



b $x = 0, 90, 180, 270, 360$
c $x = 0, 45, 90, 135, 180$

1 **a** $4 \sin x = -\cos x$

$\frac{\sin x}{\cos x} = -\frac{1}{4}$

$\tan x = -\frac{1}{4}$

b $x = 180 - 14.0,\ 360 - 14.0$

$x = 166.0°,\ 346.0°$

2 **a** LHS $= 5 \sin^2 x + 5 \sin x + 4(1 - \sin^2 x)$

$= \sin^2 x + 5 \sin x + 4$

$=$ RHS

b $(\sin x + 4)(\sin x + 1) = 0$

$\sin x = -1$ or -4 [no solutions]

$x = 270°$

3 **a** $2 \sin x = \cos x$

$\tan x = 0.5$

$x = 26.6,\ 180 + 26.6$

$x = 26.6°,\ 206.6°$

b $\tan x = \frac{4}{3}$

$x = 53.1,\ 180 + 53.1$

$x = 53.1°,\ 233.1°$

c $1 - \sin^2 x + 3 \sin x - 3 = 0$

$\sin^2 x - 3 \sin x + 2 = 0$

$(\sin x - 1)(\sin x - 2) = 0$

$\sin x = 1$ or 2 [no solutions]

$x = 90°$

d $3 \cos^2 x - (1 - \cos^2 x) = 2$

$4 \cos^2 x = 3$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = 30,\ 360 - 30$ or $180 - 30,\ 180 + 30$

$x = 30°,\ 150°,\ 210°,\ 330°$

e $2(1 - \cos^2 x) + 3 \cos x = 3$

$2 \cos^2 x - 3 \cos x + 1 = 0$

$(2 \cos x - 1)(\cos x - 1) = 0$

$\cos x = 0.5$ or 1

$x = 60,\ 360 - 60$ or $0,\ 360$

$x = 0,\ 60°,\ 300°,\ 360°$

f $3(1 - \sin^2 x) = 5(1 - \sin x)$

$3 \sin^2 x - 5 \sin x + 2 = 0$

$(3 \sin x - 2)(\sin x - 1) = 0$

$\sin x = \frac{2}{3}$ or 1

$x = 41.8,\ 180 - 41.8$ or 90

$x = 41.8°,\ 90°,\ 138.2°$

g $3 \sin^2 x = 8 \cos x$

$3(1 - \cos^2 x) = 8 \cos x$

$3 \cos^2 x + 8 \cos x - 3 = 0$

$(3 \cos x - 1)(\cos x + 3) = 0$

$\cos x = \frac{1}{3}$ or -3 [no solutions]

$x = 70.5,\ 360 - 70.5$

$x = 70.5°,\ 289.5°$

h $\cos^2 x = 3 \sin x$

$1 - \sin^2 x = 3 \sin x$

$\sin^2 x + 3 \sin x - 1 = 0$

$\sin x = \frac{-3 \pm \sqrt{9+4}}{2}$

$\sin x = \frac{1}{2}(-3 + \sqrt{13})$ or $\frac{1}{2}(-3 - \sqrt{13})$ [no sols]

$x = 17.6,\ 180 - 17.6$

$x = 17.6°,\ 162.4°$

i $3(1 - \cos^2 x) - 5 \cos x + 2 \cos^2 x = 0$

$\cos^2 x + 5 \cos x - 3 = 0$

$\cos x = \frac{-5 \pm \sqrt{25+12}}{2}$

$\cos x = \frac{1}{2}(-5 + \sqrt{37})$ or $\frac{1}{2}(-5 - \sqrt{37})$ [no sols]

$x = 57.2,\ 360 - 57.2$

$x = 57.2°,\ 302.8°$

j $2 \sin^2 x + 7 \sin x - 2(1 - \sin^2 x) = 0$

$4 \sin^2 x + 7 \sin x - 2 = 0$

$(4 \sin x - 1)(\sin x + 2) = 0$

$\sin x = 0.25$ or -2 [no solutions]

$x = 14.5,\ 180 - 14.5$

$x = 14.5°,\ 165.5°$

k $3 \sin x = 2 \tan x$

$3 \sin x \cos x = 2 \sin x$

$\sin x\,(3 \cos x - 2) = 0$

$\sin x = 0$ or $\cos x = \frac{2}{3}$

$x = 0,\ 180,\ 360$ or $48.2,\ 360 - 48.2$

$x = 0,\ 48.2°,\ 180°,\ 311.8°,\ 360°$

l $(1 - \cos^2 x) - 9 \cos x - \cos^2 x = 5$

$2 \cos^2 x + 9 \cos x + 4 = 0$

$(2 \cos x + 1)(\cos x + 4) = 0$

$\cos x = -0.5$ or -4 [no solutions]

$x = 180 - 60,\ 180 + 60$

$x = 120°,\ 240°$

4 **a** $\cos\theta = \pm 0.5$

$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$

$\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

b $(2\sin\theta + 1)^2 = 0$

$\sin\theta = -0.5$

$\theta = -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}$

c $(\cos\theta + 3)(\cos\theta - 1) = 0$

$\cos\theta = 1$ or -3 [no solutions]

$\theta = 0$

d $3\sin^2\theta - (1 - \sin^2\theta) = 0$

$4\sin^2\theta = 1$

$\sin\theta = \pm 0.5$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

e $4\sin^2\theta - 5\sin\theta + 2(1 - \sin^2\theta) = 0$

$2\sin^2\theta - 5\sin\theta + 2 = 0$

$(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta = 0.5$ or 2 [no solutions]

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

f $(1 - \cos^2\theta) - 3\cos\theta - \cos^2\theta = 2$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -0.5$ or -1

$\theta = \pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$ or $-\pi, \pi$

$\theta = -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi$

5 **a**

$$\begin{aligned}\text{LHS} &= \sin^2 x + 2\sin x\cos x + \cos^2 x \\ &= (\sin^2 x + \cos^2 x) + 2\sin x\cos x \\ &= 1 + 2\sin x\cos x \\ &= \text{RHS}\end{aligned}$$

b

$$\begin{aligned}\text{LHS} &= \frac{1-\cos^2 x}{\cos x} \\ &= \frac{\sin^2 x}{\cos x} \\ &= \sin x \times \frac{\sin x}{\cos x} \\ &= \sin x\tan x \\ &= \text{RHS}\end{aligned}$$

c

$$\begin{aligned}\text{LHS} &= \frac{1-\sin^2 x}{1-\sin x} \\ &= \frac{(1+\sin x)(1-\sin x)}{1-\sin x} \\ &= 1 + \sin x \\ &= \text{RHS}\end{aligned}$$

d

$$\begin{aligned}\text{LHS} &= \frac{(1+\sin x)(1-\sin x)}{\cos x(1-\sin x)} \\ &= \frac{1-\sin^2 x}{\cos x(1-\sin x)} \\ &= \frac{\cos^2 x}{\cos x(1-\sin x)} \\ &= \frac{\cos x}{1-\sin x} \\ &= \text{RHS}\end{aligned}$$

6 **a**

$$\begin{aligned}\text{LHS} &= \cos^2 x - 2\cos x\tan x + \tan^2 x \\ &\quad + \sin^2 x + 2\sin x + 1 \\ &= \cos^2 x - 2\sin x + \tan^2 x \\ &\quad + \sin^2 x + 2\sin x + 1 \\ &= (\cos^2 x + \sin^2 x) + \tan^2 x + 1 \\ &= 2 + \tan^2 x = \text{RHS}\end{aligned}$$

b $2 + \tan^2 x = 3$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

7 **a**

$$\begin{aligned}\text{f}(x) &= (1 - \sin^2 x) + 2\sin x \\ &= 2 - (\sin^2 x - 2\sin x + 1) \\ &= 2 - (\sin x - 1)^2\end{aligned}$$

b max. value of $\text{f}(x) = 2$

occurs when $\sin x = 1 \ \therefore\ x = \frac{\pi}{2}$

1 **a** $\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

b $\cos(x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

$x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$= \frac{5\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{2}, \frac{5\pi}{6}$

2 **a** $\cos^2 A = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$

$\sin^2 A = 1 - \cos^2 A = 2\sqrt{3} - 3$

b $\tan^2 A = \frac{\sin^2 A}{\cos^2 A}$

$= \frac{2\sqrt{3}-3}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{(2\sqrt{3}-3)(4+2\sqrt{3})}{16-12}$

$= \frac{8\sqrt{3}+12-12-6\sqrt{3}}{4} = \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

3 **a** $45° = \frac{\pi}{4}$

$P = (2 \times 8) + (8 \times \frac{\pi}{4}) = 22.3$ cm

b area of sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi$

area of triangle $= \frac{1}{2} \times 8^2 \times \sin\frac{\pi}{4}$

$= 32 \times \frac{1}{\sqrt{2}} = 16\sqrt{2}$

area of segment $= 8\pi - 16\sqrt{2}$

$= 8(\pi - 2\sqrt{2})$ cm^2

4 $2\sin^2\theta + \sin\theta - (1 - \sin^2\theta) = 2$

$3\sin^2\theta + \sin\theta - 3 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{1+36}}{6}$

$\sin\theta = \frac{1}{6}(-1+\sqrt{37})$

or $\frac{1}{6}(-1-\sqrt{37})$ [no solutions]

$\theta = 57.9, 180 - 57.9$

$\theta = 57.9°, 122.1°$

5 $3\sin^2 x + 4\sin x - 4 = 0$

$(3\sin x - 2)(\sin x + 2) = 0$

$\sin x = \frac{2}{3}$ or -2 [no solutions]

$x = 0.73, \pi - 0.7297$

$x = 0.73^c, 2.41^c$

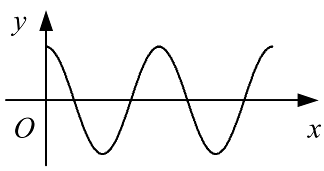
6 $2\sin x = 3\cos x$

$\tan x = 1.5$

$x = 0.98, \pi + 0.9828 = 0.98, 4.12$

$\therefore$ (0.98, 1.66), (4.12, –1.66)

7 **a**



b $2x = 180 - 60, 180 + 60,$

$540 - 60, 540 + 60$

$= 120, 240, 480, 600$

$x = 60, 120, 240, 300$

8 $12\cos^2\theta = 7\sin\theta$

$12(1 - \sin^2\theta) = 7\sin\theta$

$12\sin^2\theta + 7\sin\theta - 12 = 0$

$(4\sin\theta - 3)(3\sin\theta + 4) = 0$

$\sin\theta = 0.75$ or $-\frac{4}{3}$ [no solutions]

$\theta = 48.6, 180 - 48.6$

$\theta = 48.6, 131.4$

9 **a** $\tan 15° = \frac{\sqrt{3}-1}{1+(\sqrt{3}\times 1)} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{(\sqrt{3}-1)(1-\sqrt{3})}{1-3}$

$= -\frac{1}{2}(\sqrt{3}-3-1+\sqrt{3})$

$= 2 - \sqrt{3}$

b $\tan 345° = -\tan 15° = \sqrt{3} - 2$

10 $(1 - \cos^2 x) + 5\cos x - 3\cos^2 x = 2$

$4\cos^2 x - 5\cos x + 1 = 0$

$(4\cos x - 1)(\cos x - 1) = 0$

$\cos x = 0.25$ or 1

$x = 75.5, 360 - 75.5$ or $0, 360$

$x = 0, 75.5°$ (1dp), $284.5°$ (1dp), $360°$

11 **a** $\angle ABC = 180 - (41 + 26) = 113$

$\frac{BC}{\sin 41} = \frac{18}{\sin 113}$

$BC = \frac{18 \times \sin 41}{\sin 113} = 12.8$ cm

b $= \frac{1}{2} \times 18 \times 12.829 \times \sin 26$

$= 50.6$ cm^2

12 $6\cos^2\theta + 5\cos\theta - 4 = 0$

$(3\cos\theta + 4)(2\cos\theta - 1) = 0$

$\cos\theta = 0.5$ or $-\frac{4}{3}$ [no solutions]

$\theta = 60, 360 - 60$

$\theta = 60°, 300°$

13 $\sin^2 x + 5\sin x = 2(1 - \sin^2 x)$

$3\sin^2 x + 5\sin x - 2 = 0$

$(3\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{3}$ or -2 [no solutions]

$x = 19.5, 180 - 19.5$

$x = 19.5°, 160.5°$

14 **a** LHS $= (1 - \cos^2\theta)^2 - 2(1 - \cos^2\theta)$

$= 1 - 2\cos^2\theta + \cos^4\theta - 2 + 2\cos^2\theta$

$= \cos^4\theta - 1$

= RHS

b LHS $= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

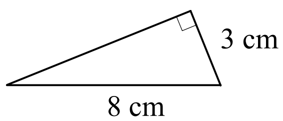
$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta}$

= RHS

15 **a**



$\cos(\angle PQR) = \frac{3}{8}$

$\therefore \angle PQR = 1.186^c$

b $RS^2 = 8^2 - 3^2 = 55$

$RS = \sqrt{55} = 7.42$ cm (3sf)

c obtuse $\angle SPU = 2 \times \angle PQR = 2.3728$

reflex $\angle RQT = 2\pi - \angle SPU = 3.9104$

length of rubber band

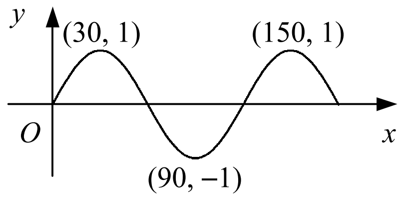
$= (2 \times 7.4162) + (2 \times 2.3728) + (5 \times 3.9104)$

$= 39.1$ cm (3sf)

1 **a** $\theta + \frac{\pi}{4} = \pi - 0.4115,\ 2\pi + 0.4115$
$= 2.7301,\ 6.6947$
$\theta = 1.94^c,\ 5.91^c$

b $\cos 2\theta = \frac{1}{3}$
$2\theta = 1.2310,\ 2\pi - 1.2310$
$2\pi + 1.2310,\ 4\pi - 1.2310$
$= 1.2310,\ 5.0522,\ 7.5141,\ 11.3354$
$\theta = 0.62^c,\ 2.53^c,\ 3.76^c,\ 5.67^c$

2 **a**



b $(\tan\theta + 1)(\tan\theta - 3) = 0$
$\tan\theta = -1$ or 3
$\theta = 180 - 45,\ 360 - 45$ or $71.6,\ 180 + 71.6$
$\theta = 71.6°$ (1dp), $135°$, $251.6°$ (1dp), $315°$

3 **a** $260° = \frac{260}{180}\pi = 4.538$ radians

b $P = (2 \times 6.4) + (6.4 \times 4.538)$
$= 41.8$ cm (3sf)

c $A = \frac{1}{2} \times (6.4)^2 \times 4.538$
$= 92.9$ cm^2 (3sf)

4 $3\cos^2\theta + 6\cos\theta = 2(1 - \cos^2\theta) + 6$
$5\cos^2\theta + 6\cos\theta - 8 = 0$
$(5\cos\theta - 4)(\cos\theta + 2) = 0$
$\cos\theta = 0.8$ or -2 [no solutions]
$\theta = 36.9,\ 360 - 36.9$
$\theta = 36.9°,\ 323.1°$

5 **a** area $= \frac{1}{2} \times 4 \times 5 \times \sin 60°$
$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ cm^2

b $AB^2 = 4^2 + 5^2 - (2 \times 4 \times 5 \times \cos 60°)$
$= 16 + 25 - (40 \times \frac{1}{2}) = 21$
$\therefore AB = \sqrt{21}$ cm

c $\frac{\sin(\angle ABC)}{4} = \frac{\sin 60°}{\sqrt{21}}$
$\therefore \sin(\angle ABC) = \frac{4 \times \frac{\sqrt{3}}{2}}{\sqrt{3}\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{2}{7}\sqrt{7}$

6 $2x + 15 = 63.435,\ 180 + 63.435,$
$360 + 63.435,\ 540 + 63.435$
$= 63.435,\ 243.435,\ 423.435,\ 603.435$
$2x = 48.435,\ 228.435,\ 408.435,\ 588.435$
$x = 24.2,\ 114.2,\ 204.2,\ 294.2$

7 $\sin^2\theta - \cos^2\theta = \cos\theta$
$(1 - \cos^2\theta) - \cos^2\theta = \cos\theta$
$2\cos^2\theta + \cos\theta - 1 = 0$
$(2\cos\theta - 1)(\cos\theta + 1) = 0$
$\cos\theta = 0.5$ or -1
$\theta = 60,\ 360 - 60$ or 180
$\theta = 60°,\ 180°,\ 300°$

8 **a** $(x - 5)^2 - 25 + (y - 1)^2 - 1 - 3 = 0$
$(x - 5)^2 + (y - 1)^2 = 29$
$\therefore$ centre $(5, 1)$ radius $\sqrt{29}$

b sub. $x^2 + 36 - 10x - 12 - 3 = 0$
$x^2 - 10x + 21 = 0$
$(x - 3)(x - 7) = 0$
$x = 3,\ 7$
$\therefore$ $(3, 6)$ and $(7, 6)$

c mid-point of chord $= (5, 6)$
angle of sector $= 2 \times \tan^{-1}\frac{2}{5} = 0.761^c$
area $= \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{29}{2}(0.761 - \sin 0.761) = 1.03$ (3sf)

9 $5\sin^2\theta + 5\sin\theta + 2(1 - \sin^2\theta) = 0$
$3\sin^2\theta + 5\sin\theta + 2 = 0$
$(3\sin\theta + 2)(\sin\theta + 1) = 0$
$\sin\theta = -\frac{2}{3}$ or -1
$\theta = 180 + 41.8,\ 360 - 41.8$ or 270
$\theta = 221.8°$ (1dp), $270°$, $318.2°$ (1dp)

10 **a** $(158°, 0)$, $(338°, 0)$
b $(0, \tan 22°) = (0, 0.404)$ [y-coord to 3sf]
c $x = 68°$ and $x = 248°$

11 **a** $\tan x = 0.4$
$x = 21.8,\ 180 + 21.8$
$x = 21.8°,\ 201.8°$

b $2\sin^2 y - \sin y - 1 = 0$
$(2\sin y + 1)(\sin y - 1) = 0$
$\sin y = -0.5$ or 1
$y = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$
$y = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

12 $3\cos^2\theta - 5\cos\theta + 2(1 - \cos^2\theta) = 0$
$\cos^2\theta - 5\cos\theta + 2 = 0$
$\cos\theta = \frac{5 \pm \sqrt{25-8}}{2}$
$\cos\theta = \frac{1}{2}(5 - \sqrt{17})$ or $\frac{1}{2}(5 + \sqrt{17})$ [no sols]
$\theta = -64.0°,\ 64.0°$

13 **a** $60° = \frac{\pi}{3}$
area $= \frac{1}{2} \times a^2 \times \frac{\pi}{3} = \frac{1}{6}\pi a^2$

b $OC = OA\cos 60° = \frac{1}{2}a$

c area of triangle $OAC = \frac{1}{2} \times a \times \frac{1}{2}a \times \sin 60°$
$= \frac{1}{4}a^2 \times \frac{\sqrt{3}}{2} = \frac{1}{8}a^2\sqrt{3}$

shaded area $= \frac{1}{6}\pi a^2 - \frac{1}{8}a^2\sqrt{3}$
$= \frac{1}{24}a^2(4\pi - 3\sqrt{3})$

1 **a** $x + 40 = \pm 72.5$

$x = -112.5°, 32.5°$

b $\tan 2x = -2$

$2x = 180 - 63.435, 360 - 63.435,$

$-63.435, -180 - 63.435$

$= -243.435, -63.435, 116.565, 296.565$

$x = -121.7°, -31.7°, 58.3°, 148.3°$

2 $\tan x = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \frac{1}{2}\sqrt{2}$

$x = 59.6, 180 + 59.6$ or $16.3, 180 + 16.3$

$x = 16.3, 59.6, 196.3, 239.6$

3 **a** $15\theta = 32.1$

$\theta = 32.1 \div 15 = 2.14$

b $A = \frac{1}{2} \times 15^2 \times 2.14$

$= 240.75 \text{ cm}^2$

4 $2x - \frac{\pi}{3} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}$

$2x = \frac{\pi}{2}, \frac{7\pi}{6}$

$x = \frac{\pi}{4}, \frac{7\pi}{12}$

5 **a** $\sin^2 A = (1 - \sqrt{2})^2$

$= 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$

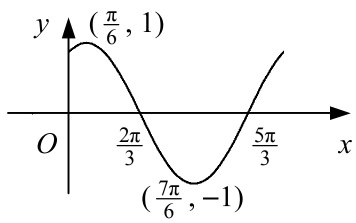
$\cos^2 A = 1 - \sin^2 A = 2\sqrt{2} - 2$

$\therefore \cos^2 A + 2 \sin A$

$= 2\sqrt{2} - 2 + 2(1 - \sqrt{2})$

$\therefore \cos^2 A + 2 \sin A = 0$

b



6 $2 \sin^2 x + \sin x + 1 = 1 - \sin^2 x$

$3 \sin^2 x + \sin x = 0$

$\sin x (3 \sin x + 1) = 0$

$\sin x = 0$ or $-\frac{1}{3}$

$x = 0, 180, 360$ or $180 + 19.5, 360 - 19.5$

$x = 0, 180°, 199.5°$ (1dp), $340.5°$ (1dp), $360°$

7 **a** $\frac{\sin(\angle PRQ)}{10} = \frac{\sin 0.7}{14}$

$\sin(\angle PRQ) = \frac{10 \times \sin 0.7}{14} = 0.4602$

$\angle PRQ = 0.48^c$

b $\angle PQR = \pi - (0.7 + 0.4782) = 1.963$

area of $\Delta = \frac{1}{2} \times 10 \times 14 \times \sin 1.963$

$= 64.67$

area of sector $= \frac{1}{2} \times 10^2 \times 0.7$

$= 35$

shaded area $= 64.67 - 35$

$= 29.7 \text{ cm}^2$ (3sf)

8 **a** **i** $\cos^2 A = 1 - \sin^2 A = 1 - \frac{5}{9} = \frac{4}{9}$

$\cos A = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$0 < A < 90 \;\therefore \cos A = \frac{2}{3}$

ii $\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{5}}{3} \div \frac{2}{3} = \frac{1}{2}\sqrt{5}$

b $\cos x (5 \sin x + 1) = 0$

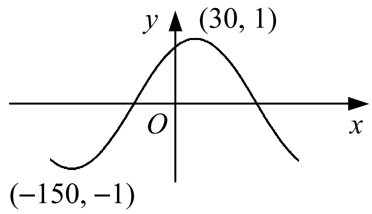
$\cos x = 0$ or $\sin x = -0.2$

$x = 90, 270$ or $180 + 11.5, 360 - 11.5$

$x = 90°, 191.5°$ (1dp), $270°, 348.5°$ (1dp)

9 $2\theta + 30 = 180 - 60,\ 180 + 60$
$= 120,\ 240$
$2\theta = 90,\ 210$
$\theta = 45,\ 105$

10 **a**



b $\cos (x - 30) = 0.2$
$x - 30 = \pm\ 78.5$
$x = -48.5,\ 108.5$

11 $4\cos^2 x - \cos x - 2(1 - \cos^2 x) = 0$
$6\cos^2 x - \cos x - 2 = 0$
$(3\cos x - 2)(2\cos x + 1) = 0$
$\cos x = \frac{2}{3}$ or -0.5
$x = 48.2,\ 360 - 48.2$ or $180 - 60,\ 180 + 60$
$x = 48.2°$ (1dp), $120°$, $240°$, $311.8°$ (1dp)

12 **a** area of sector $= \frac{1}{2} \times r^2 \times \theta$
area of triangle $= \frac{1}{2} \times r^2 \times \sin\theta$
$A_1 = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$
$= \frac{1}{2}r^2(\theta - \sin\theta)$ cm^2

b $\theta = \frac{5\pi}{6}\quad \therefore A_1 = \frac{1}{2}r^2(\frac{5\pi}{6} - \frac{1}{2})$
$= \frac{1}{12}r^2(5\pi - 3)$
$A_2 = \pi r^2 - A_1 = \pi r^2 - (\frac{5}{12}\pi r^2 - \frac{1}{4}r^2)$
$= \frac{7}{12}\pi r^2 + \frac{1}{4}r^2$
$= \frac{1}{12}r^2(7\pi + 3)$
$\therefore A_1 : A_2 = \frac{1}{12}r^2(5\pi - 3) : \frac{1}{12}r^2(7\pi + 3)$
$= (5\pi - 3) : (7\pi + 3)$

13 $3\sin x - 2\cos^2 x = 0$
$3\sin x - 2(1 - \sin^2 x) = 0$
$2\sin^2 x + 3\sin x - 2 = 0$
$(2\sin x - 1)(\sin x + 2) = 0$
$\sin x = 0.5$ or -2 [no solutions]
$x = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$
$x = \frac{\pi}{6},\ \frac{5\pi}{6}$

14 **a** $7^2 = 5^2 + 8^2 - [2 \times 5 \times 8 \times \cos(\angle ABC)]$
$\cos(\angle ABC) = \frac{25 + 64 - 49}{80}$
$= \frac{1}{2}$

b $\sin(\angle ABC) = \sqrt{1 - (\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$
area $= \frac{1}{2} \times 5 \times 8 \times \frac{\sqrt{3}}{2}$
$= 10\sqrt{3}$

15 **a** LHS $= 2 + 2\tan^2\theta + \cos^2\theta + \sin^2\theta$
$= 2 + 2\tan^2\theta + 1$
$= 3 + 2\tan^2\theta$
$=$ RHS

b $3 + 2\tan^2\theta = 7$
$\tan^2\theta = 2$
$\tan\theta = \pm\sqrt{2}$
$\theta = 54.7,\ 180 + 54.7$
or $180 - 54.7,\ 360 - 54.7$
$\theta = 54.7°,\ 125.3°,\ 234.7°,\ 305.3°$ (1dp)