

- 1**    **a**  $r = 3$   
 $u_8 = 3 \times 3^7 = 6561$
- b**  $r = \frac{1}{4}$   
 $u_8 = 1024 \times (\frac{1}{4})^7 = \frac{1}{16}$
- c**  $r = -2$   
 $u_8 = 1 \times (-2)^7 = -128$
- 2**    **a**  $a = 1, r = 5$   
 $u_n = 5^{n-1}$
- b**  $a = 3, r = -4$   
 $u_n = 3 \times (-4)^{n-1}$
- c**  $a = 81, r = \frac{2}{3}$   
 $u_n = 81 \times (\frac{2}{3})^{n-1}$
- 3**    **a**  $a = 2, r = 2, n = 12$   
 $S_{12} = \frac{2(2^{12}-1)}{2-1} = 8190$
- b**  $a = 640, r = \frac{1}{2}, n = 12$   
 $S_{12} = \frac{640[1-(\frac{1}{2})^{12}]}{1-\frac{1}{2}} = 1279\frac{11}{16}$
- c**  $a = \frac{1}{6}, r = -3, n = 12$   
 $S_{12} = \frac{\frac{1}{6}[1-(-3)^{12}]}{1-(-3)} = -22\,143\frac{1}{3}$
- 4**    **a**  $S_8 = \frac{4(3^8-1)}{3-1} = 13\,120$
- b**  $S_{14} = \frac{48[1-(\frac{1}{2})^{14}]}{1-\frac{1}{2}} = 95.994$
- c**  $S_{12} = \frac{[-1-(-4)^{12}]}{1-(-4)} = 3\,355\,443$
- d**  $S_{20} = \frac{200[1-(0.7)^{20}]}{1-0.7} = 666.135$
- e**  $S_{15} = \frac{120[1-(-\frac{3}{4})^{15}]}{1-(-\frac{3}{4})} = 69.488$
- f**  $S_{30} = \frac{-25[(1.2)^{30}-1]}{1.2-1} = -29\,547.039$
- 5**    **a** GP:  $a = 3$   
 $r = 3, n = 9$   
 $S_9 = \frac{3(3^9-1)}{3-1} = 29\,523$
- b** GP:  $a = 64$   
 $r = 8, n = 6$   
 $S_6 = \frac{64(8^6-1)}{8-1} = 2\,396\,736$
- c** GP:  $a = 20$   
 $r = 2, n = 10$   
 $S_{10} = \frac{20(2^{10}-1)}{2-1} = 20\,460$
- d** GP:  $a = 0.8$   
 $r = 0.8, n = 8$   
 $S_8 = \frac{0.8[1-(0.8)^8]}{1-0.8} = 3.329 \text{ (3dp)}$
- e** GP:  $a = 2$   
 $r = \frac{1}{6}, n = 10$   
 $S_{10} = \frac{2[1-(\frac{1}{6})^{10}]}{1-\frac{1}{6}} = 2.400 \text{ (3dp)}$
- f** GP:  $a = -4$   
 $r = -4, n = 9$   
 $S_9 = \frac{-4[1-(-4)^9]}{1-(-4)} = -209\,716$
- g** GP:  $a = \frac{1}{16}$   
 $r = \frac{1}{2}, n = 17$   
 $S_{17} = \frac{\frac{1}{16}[1-(\frac{1}{2})^{17}]}{1-\frac{1}{2}} = 0.125 \text{ (3dp)}$
- h** GP:  $a = -54$   
 $r = -3, n = 7$   
 $S_7 = \frac{-54[1-(-3)^7]}{1-(-3)} = -29\,538$
- 6**    **a**  $r = 10 \div 2 = 5$
- b**  $a \times 5 = 2 \therefore a = 0.4$
- c**  $S_8 = \frac{0.4(5^8-1)}{5-1} = 39\,062.4$
- 7**    **a**  $a = 2, ar^3 = 54 \therefore r^3 = 54 \div 2 = 27$   
 $r = \sqrt[3]{27} = 3$
- b**  $u_9 = 2 \times 3^8 = 13\,122$
- 8**    **a**  $r = 8 \div 24 = \frac{1}{3}$
- b**  $a \times (\frac{1}{3})^2 = 24 \therefore a = 216$
- c**  $S_{11} = \frac{216[1-(\frac{1}{3})^{11}]}{1-\frac{1}{3}} = 323.998$
- 9**    **a**  $a = 6, ar^2 = 24 \therefore r^2 = 24 \div 6 = 4$   
 $r = \pm 2$
- b**  $r = 2, S_{15} = \frac{6(2^{15}-1)}{2-1} = 196\,602$
- 10**    **a**  $a = 768, ar^3 = -96$   
 $r^3 = -96 \div 768 = -\frac{1}{8}$   
 $r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
- b**  $u_{10} = 768 \times (-\frac{1}{2})^9 = -1.5$
- 11**    **a**  $ar = 0.5, ar^4 = 32 \therefore r^3 = 32 \div 0.5 = 64$   
 $r = \sqrt[3]{64} = 4, a \times 4 = 0.5 \therefore a = 0.125$
- b**  $0.125 \times 4^{n-1} < 10\,000 \therefore 4^{n-1} < 80\,000$   
 $(n-1) \lg 4 < \lg 80\,000$   
 $n < \frac{\lg 80000}{\lg 4} + 1$   
 $n < 9.14 \therefore 9 \text{ terms}$

$$12 \quad a \quad \frac{a[(\frac{3}{2})^4 - 1]}{\frac{3}{2} - 1} = 130$$

$$a = 130 \div \frac{65}{8} = 16$$

$$b \quad u_8 = 16 \times (\frac{3}{2})^7 = 273\frac{3}{8}$$

$$c \quad \frac{16[(\frac{3}{2})^n - 1]}{\frac{3}{2} - 1} > 30\,000$$

$$(\frac{3}{2})^n > 938.5$$

$$n \lg \frac{3}{2} > \lg 938.5$$

$$n > \frac{\lg 938.5}{\lg 1.5}$$

$$n > 16.9 \therefore \text{least } n = 17$$

$$13 \quad a \quad a + ar = a(1 + r) = 10.8$$

$$ar^2 + ar^3 = ar^2(1 + r) = 43.2$$

$$\therefore r^2 = 43.2 \div 10.8 = 4$$

$$\text{all terms +ve} \therefore r + \text{ve} \therefore r = 2$$

$$\text{sub. } a = 10.8 \div 3 = 3.6$$

$$b \quad S_{16} = \frac{3.6(2^{16} - 1)}{2 - 1} = 235\,926$$

$$14 \quad a \quad a = 12, r = 0.5$$

$$S_{\infty} = \frac{12}{1 - 0.5} = 24$$

$$b \quad a = 270, r = \frac{1}{3}$$

$$S_{\infty} = \frac{270}{1 - \frac{1}{3}} = 405$$

$$c \quad a = 25, r = -1.2$$

$$\text{no } S_{\infty} \text{ as } r < -1 \therefore \text{diverges}$$

$$d \quad a = 216, r = \frac{2}{3}$$

$$S_{\infty} = \frac{216}{1 - \frac{2}{3}} = 648$$

$$e \quad a = \frac{8}{25}, r = \frac{5}{4}$$

$$\text{no } S_{\infty} \text{ as } r > 1 \therefore \text{diverges}$$

$$f \quad a = 500, r = -0.6$$

$$S_{\infty} = \frac{500}{1 - (-0.6)} = 312.5$$

$$15 \quad a \quad a = 0.9, r = 0.9$$

$$S_{\infty} = \frac{0.9}{1 - 0.9} = 9$$

$$b \quad a = 3, r = \frac{1}{2}$$

$$S_{\infty} = \frac{3}{1 - \frac{1}{2}} = 6$$

$$c \quad a = 1, r = -\frac{3}{4}$$

$$S_{\infty} = \frac{1}{1 - (-\frac{3}{4})} = \frac{4}{7}$$

$$d \quad a = 32, r = 0.8$$

$$S_{\infty} = \frac{32}{1 - 0.8} = 160$$

$$16 \quad a \quad S_{\infty} = \frac{80}{1 - 0.2} = 100$$

$$b \quad S_6 = \frac{80[1 - (0.2)^6]}{1 - 0.2} = 99.9936$$

$$S_{\infty} - S_6 = 0.0064$$

$$17 \quad a \quad 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$$

$$b \quad \text{GP: } a = 1, r = \frac{1}{3}$$

$$S_{\infty} = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$$

$$18 \quad a \quad \frac{a}{1 - 0.55} = 40$$

$$a = 0.45 \times 40 = 18$$

$$b \quad 18 \times (0.55)^{n-1} < 0.001$$

$$(n-1) \lg 0.55 < \lg 0.0000556$$

$$n > \frac{\lg 0.0000556}{\lg 0.55} + 1$$

$$n > 17.4 \therefore \text{smallest } n = 18$$

$$19 \quad a \quad u_1 = S_1 = 2^1 - 1 = 1$$

$$S_5 = 2^5 - 1 = 31, S_4 = 2^4 - 1 = 15$$

$$u_5 = S_5 - S_4 = 31 - 15 = 16$$

$$b \quad S_{n-1} = 2^{n-1} - 1$$

$$u_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1)$$

$$= 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$$

$$20 \quad a \quad \frac{k}{k+10} = \frac{k-6}{k}$$

$$k^2 = (k+10)(k-6)$$

$$4k - 60 = 0$$

$$k = 15$$

$$b \quad u_1 = 25, u_2 = 15 \therefore a = 25, r = 0.6$$

$$S_{\infty} = \frac{25}{1 - 0.6} = 62.5$$

1 a  $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$

$$a \times \left(\frac{3}{4}\right)^2 = 27$$

$$a = \frac{16}{9} \times 27 = 48$$

b  $S_{\infty} = \frac{48}{1 - \frac{3}{4}} = 192$

2 a  $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$

$$(k+4)^2 = (3k+2)(k-8)$$

$$k^2 - 15k - 16 = 0$$

$$(k+1)(k-16) = 0$$

$$k > 0 \therefore k = 16$$

b  $u_1 = 8, u_2 = 20 \therefore a = 8, r = \frac{5}{2}$

$$u_6 = 8 \times \left(\frac{5}{2}\right)^5 = 781\frac{1}{4}$$

c  $S_{10} = \frac{8\left[\left(\frac{5}{2}\right)^{10} - 1\right]}{\frac{5}{2} - 1} = 50\,857.3$

3 a  $ar = 75, ar^4 = 129.6$

$$r^3 = 129.6 \div 75 = 1.728$$

$$r = \sqrt[3]{1.728} = 1.2$$

$$a = 75 \div 1.2 = 62.5$$

b  $u_{10} = 62.5 \times (1.2)^9 = 322.5$

c  $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 a  $S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$$

subtracting,

$$S_n - rS_n = a - ar^n$$

$$(1 - r)S_n = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

b  $\frac{2[1 - (\sqrt{2})^n]}{1 - \sqrt{2}} = 126(\sqrt{2} + 1)$

$$1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$$

$$1 - (\sqrt{2})^n = 63(1 - 2)$$

$$(\sqrt{2})^n = 64$$

$$2^{\frac{1}{2}n} = 2^6$$

$$n = 12$$

5 a  $\frac{18}{1-r} = 15$

$$\therefore 1 - r = \frac{18}{15} = 1.2$$

$$r = -0.2$$

b  $u_3 = 18 \times (-0.2)^2 = 0.72$

c  $S_8 = \frac{18[1 - (-0.2)^8]}{1 - (-0.2)} = 14.9999616$

$$S_{\infty} - S_8 = 0.000\,0384$$

6 a  $S_3 = 5(3^3 - 1) = 130$

$$S_2 = 5(3^2 - 1) = 40$$

$$u_3 = S_3 - S_2 = 90$$

b  $S_{n-1} = 5(3^{n-1} - 1)$

$$u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$$

$$= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$$

7 a  $4 \times (1.25)^7 = 19.1 \text{ mm (3sf)}$

b GP:  $a = 4, r = 1.25$

$$S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8 \text{ mm}$$

$$\therefore \text{length} = 1.37 \text{ m (3sf)}$$

8 a  $ar = 30, ar^3 = 2.7 \therefore r^2 = 2.7 \div 30 = 0.09$

$$r > 0 \therefore r = \sqrt{0.09} = 0.3$$

$$a = 30 \div 0.3 = 100$$

b  $S_{\infty} = \frac{100}{1 - 0.3} = 142.9 \text{ (1dp)}$

9 a GP:  $a = 27, r = 3$

$$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$$

b  $\sum_{r=1}^{15} 2^r$ : GP,  $a = 2, r = 2$

$$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$$

$\sum_{r=1}^{15} 12r$ : AP,  $a = 12, d = 12$

$$S_{15} = \frac{15}{2} [24 + (14 \times 12)] = 1440$$

$$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$$

11 a  $u_8 = 4 \times \left(\frac{1}{2}\right)^7 = \frac{1}{32}$

b  $u_n = 4 \times \left(\frac{1}{2}\right)^{n-1}$

$$= 2^2 \times 2^{1-n}$$

$$= 2^{3-n}$$

c  $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$$= 8(1 - 2^{-n})$$

$$= 8 - (2^3 \times 2^{-n})$$

$$= 8 - 2^{3-n}$$

13 a  $a + ar^2 = a(1 + r^2) = 150$   
 $ar + ar^3 = ar(1 + r^2) = -75$   
 $\therefore r = -75 \div 150 = -\frac{1}{2}$

$$a = 150 \div \frac{5}{4} = 120$$

b  $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

10 a  $a = 64, ar^2 - ar = 20$

$$\therefore 64r^2 - 64r = 20$$

$$16r^2 - 16r - 5 = 0$$

b  $(4r + 1)(4r - 5) = 0$

$$r = -\frac{1}{4} \text{ or } \frac{5}{4}$$

c  $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times \left(-\frac{1}{4}\right)^3 = -1$

$$r = \frac{5}{4} \Rightarrow u_4 = 64 \times \left(\frac{5}{4}\right)^3 = 125$$

d  $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

12 a  $u_6 = 4 \times 3^6 = 2916$

b GP:  $a = 12, r = 3$

$$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$$

$$\therefore 6(3^t - 1) > 10^{25}$$

$$3^t > \frac{10^{25}}{6} + 1$$

$$t \lg 3 > \lg \left( \frac{10^{25}}{6} + 1 \right)$$

$$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$$

$$t > 50.8 \therefore \text{smallest } t = 51$$

14 a  $b - a = (3a + 4) - b$   
 $2b = 4a + 4$

$$b = 2a + 2$$

b  $\frac{2a+2}{a} = \frac{6a+1}{2a+2}$

$$(2a + 2)^2 = a(6a + 1)$$

$$2a^2 - 7a - 4 = 0$$

$$(2a + 1)(a - 4) = 0$$

$$a \text{ integer } \therefore a = 4$$

$$\text{sub. } b = 10$$

15 a after 4<sup>th</sup> bounce,  
reaches  $3 \times (0.6)^4 = 0.3888$  m

b total distance

$$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \dots]$$

$$= h + 2 \times S_\infty \text{ of GP, } a = 0.6h, r = 0.6$$

$$= h + \frac{2 \times 0.6h}{1 - 0.6}$$

$$= h + 3h = 4h \text{ metres}$$

- 1**
- a**  $= 1 + 4x + 6x^2 + 4x^3 + x^4$
- b**  $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$
- c**  $= 1 + 3(4x) + 3(4x)^2 + (4x)^3$   
 $= 1 + 12x + 48x^2 + 64x^3$
- d**  $= 1 + 3(-2y) + 3(-2y)^2 + (-2y)^3$   
 $= 1 - 6y + 12y^2 - 8y^3$
- e**  $= 1 + 4(\frac{1}{2}x) + 6(\frac{1}{2}x)^2 + 4(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$   
 $= 1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$
- f**  $= 1 + 3(\frac{1}{3}y) + 3(\frac{1}{3}y)^2 + (\frac{1}{3}y)^3$   
 $= 1 + y + \frac{1}{3}y^2 + \frac{1}{27}y^3$
- g**  $= 1 + 5(x^2) + 10(x^2)^2 + 10(x^2)^3 + 5(x^2)^4 + (x^2)^5$   
 $= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}$
- h**  $= 1 + 4(-\frac{3}{2}x) + 6(-\frac{3}{2}x)^2 + 4(-\frac{3}{2}x)^3 + (-\frac{3}{2}x)^4$   
 $= 1 - 6x + \frac{27}{2}x^2 - \frac{27}{2}x^3 + \frac{81}{16}x^4$
- 2**
- a**  $= x^3 + 3x^2y + 3xy^2 + y^3$
- b**  $= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
- c**  $= x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$   
 $= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$
- d**  $= 2^3 + 3(2^2)y + 3(2)y^2 + y^3$   
 $= 8 + 12y + 6y^2 + y^3$
- e**  $= 3^3 + 3(3^2)(-x) + 3(3)(-x)^2 + (-x)^3$   
 $= 27 - 27x + 9x^2 - x^3$
- f**  $= 5^4 + 4(5^3)(2x) + 6(5^2)(2x)^2 + 4(5)(2x)^3 + (2x)^4$   
 $= 625 + 1000x + 600x^2 + 160x^3 + 16x^4$
- g**  $= 3^5 + 5(3^4)(-4y) + 10(3^3)(-4y)^2 + 10(3^2)(-4y)^3 + 5(3)(-4y)^4 + (-4y)^5$   
 $= 243 - 1620y + 4320y^2 - 5760y^3 + 3840y^4 - 1024y^5$
- h**  $= 3^4 + 4(3^3)(\frac{1}{2}x) + 6(3^2)(\frac{1}{2}x)^2 + 4(3)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$   
 $= 81 + 54x + \frac{27}{2}x^2 + \frac{3}{2}x^3 + \frac{1}{16}x^4$
- 3**
- a**  $= 1 + 10x + \frac{10 \times 9}{2}x^2 + \frac{10 \times 9 \times 8}{3 \times 2}x^3 + \dots$   
 $= 1 + 10x + 45x^2 + 120x^3 + \dots$
- b**  $= 1 + 6(-x) + \frac{6 \times 5}{2}(-x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(-x)^3 + \dots$   
 $= 1 - 6x + 15x^2 - 20x^3 + \dots$
- c**  $= 1 + 8(2x) + \frac{8 \times 7}{2}(2x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(2x)^3 + \dots$   
 $= 1 + 16x + 112x^2 + 448x^3 + \dots$
- d**  $= 1 + 7(-\frac{1}{2}x) + \frac{7 \times 6}{2}(-\frac{1}{2}x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(-\frac{1}{2}x)^3 + \dots$   
 $= 1 - \frac{7}{2}x + \frac{21}{4}x^2 - \frac{35}{8}x^3 + \dots$
- e**  $= 1 + 6(x^3) + \frac{6 \times 5}{2}(x^3)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(x^3)^3 + \dots$   
 $= 1 + 6x^3 + 15x^6 + 20x^9 + \dots$
- f**  $= 2^9 + 9(2^8)x + \frac{9 \times 8}{2}(2^7)x^2 + \frac{9 \times 8 \times 7}{3 \times 2}(2^6)x^3 + \dots$   
 $= 512 + 2304x + 4608x^2 + 5376x^3 + \dots$
- g**  $= 3^7 + 7(3^6)(-x) + \frac{7 \times 6}{2}(3^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(3^4)(-x)^3 + \dots$   
 $= 2187 - 5103x + 5103x^2 - 2835x^3 + \dots$
- h**  $= 2^{10} + 10(2^9)(5x) + \frac{10 \times 9}{2}(2^8)(5x)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(2^7)(5x)^3 + \dots$   
 $= 1024 + 25\,600x + 288\,000x^2 + 1\,920\,000x^3 + \dots$
- 4**
- a**  $= \binom{20}{3} = 1140$
- b**  $= \binom{14}{4} \times (-1)^4 = 1001$
- c**  $= \binom{9}{2} \times 4^2 = 576$
- d**  $= \binom{14}{3} \times (-3)^3 = -9828$
- e**  $= \binom{12}{4} \times (-\frac{1}{3})^4 = \frac{55}{9}$  or  $6\frac{1}{9}$
- f**  $= \binom{16}{5} \times (-\frac{1}{2})^5 = -136.5$
- g**  $= \binom{15}{2} \times (\frac{2}{5})^2 = \frac{84}{5}$  or  $16.8$
- h**  $= \binom{8}{3} = 56$

- 5    **a**  $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3$   
 $= 1 + 3\sqrt{5} + 15 + 5\sqrt{5}$   
 $= 16 + 8\sqrt{5}$
- b**  $= 1 + 4(-\sqrt{3}) + 6(-\sqrt{3})^2 + 4(-\sqrt{3})^3 + (-\sqrt{3})^4$   
 $= 1 - 4\sqrt{3} + 18 - 12\sqrt{3} + 9$   
 $= 28 - 16\sqrt{3}$
- c**  $= 2^3 + 3(2^2)(\sqrt{2}) + 3(2)(\sqrt{2})^2 + (\sqrt{2})^3$   
 $= 8 + 12\sqrt{2} + 12 + 2\sqrt{2}$   
 $= 20 + 14\sqrt{2}$
- d**  $= 1 + 4(2\sqrt{3}) + 6(2\sqrt{3})^2 + 4(2\sqrt{3})^3 + (2\sqrt{3})^4$   
 $= 1 + 8\sqrt{3} + 72 + 96\sqrt{3} + 144$   
 $= 217 + 104\sqrt{3}$
- 6    **a**  $= 1 + 6x + \frac{6 \times 5}{2}x^2 + \frac{6 \times 5 \times 4}{3 \times 2}x^3 + \dots$   
 $= 1 + 6x + 15x^2 + 20x^3 + \dots$
- b i** let  $x = 0.02$   
 $1.02^6 \approx 1 + 6(0.02) + 15(0.02)^2 + 20(0.02)^3$   
 $= 1 + 0.12 + 0.0060 + 0.000160$   
 $= 1.1262 \text{ (4dp)}$
- ii** let  $x = -0.01$   
 $0.99^6 \approx 1 + 6(-0.01) + 15(-0.01)^2 + 20(-0.01)^3$   
 $= 1 - 0.06 + 0.0015 - 0.000020$   
 $= 0.9415 \text{ (4dp)}$
- 7    **a**  $= 1 + 8(2y) + \frac{8 \times 7}{2}(2y)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(2y)^3 + \dots$   
 $= 1 + 16y + 112y^2 + 448y^3 + \dots$
- b i** let  $y = -0.01$   
 $0.98^8 \approx 1 + 16(-0.01) + 112(-0.01)^2 + 448(-0.01)^3$   
 $= 1 - 0.16 + 0.0112 - 0.000448$   
 $= 0.8508 \text{ (4dp)}$
- ii** let  $y = 0.005$   
 $1.01^8 \approx 1 + 16(0.005) + 112(0.005)^2 + 448(0.005)^3$   
 $= 1 + 0.080 + 0.002800 + 0.000056000$   
 $= 1.0829 \text{ (4dp)}$
- 8    **a**  $= 1 + 4x + 6x^2 + 4x^3 + x^4 + (1 - 4x + 6x^2 - 4x^3 + x^4)$   
 $= 2 + 12x^2 + 2x^4$
- b**  $= 1 + 3(-\frac{1}{3}x) + 3(-\frac{1}{3}x)^2 + (-\frac{1}{3}x)^3 - [1 + 3(\frac{1}{3}x) + 3(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3]$   
 $= 1 - x + \frac{1}{3}x^2 - \frac{1}{27}x^3 - (1 + x + \frac{1}{3}x^2 + \frac{1}{27}x^3)$   
 $= -2x - \frac{2}{27}x^3$
- 9    **a**  $6(ax)^2 = 24x^2$   
 $a^2 = 4$   
 $a < 0 \therefore a = -2$
- b**  $= 4a^3 = -32$

- 1
- a**  $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$   
 $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$
- b**  $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$   
 $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$
- c**  $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$   
 $= 27 + 270x^2 + 900x^4 + 1000x^6$
- d**  $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$   
 $= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$
- e**  $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$   
 $= x^6 - 3x^4y + 3x^2y^2 - y^3$
- f**  $= 5^4 + 4(5^3)(\frac{1}{2}x) + 6(5^2)(\frac{1}{2}x)^2 + 4(5)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$   
 $= 625 + 250x + \frac{75}{2}x^2 + \frac{5}{2}x^3 + \frac{1}{16}x^4$
- g**  $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$   
 $= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
- h**  $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$   
 $= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$
- 2
- a**  $= 1 + 6(3x) + \frac{6 \times 5}{2}(3x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(3x)^3 + \dots$   
 $= 1 + 18x + 135x^2 + 540x^3 + \dots$
- b**  $= 1 + 8(-\frac{1}{4}x) + \frac{8 \times 7}{2}(-\frac{1}{4}x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(-\frac{1}{4}x)^3 + \dots$   
 $= 1 - 2x + \frac{7}{4}x^2 - \frac{7}{8}x^3 + \dots$
- c**  $= 5^7 + 7(5^6)(-x) + \frac{7 \times 6}{2}(5^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(5^4)(-x)^3 + \dots$   
 $= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \dots$
- d**  $= 3^{10} + 10(3^9)(2x^2) + \frac{10 \times 9}{2}(3^8)(2x^2)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(3^7)(2x^2)^3 + \dots$   
 $= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \dots$
- 3
- a**  $= \binom{15}{3} = 455$
- b**  $= \binom{12}{4} \times (-2)^4 = 7920$
- c**  $= \binom{7}{2} \times 3^5 = 5103$
- d**  $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$
- e**  $= \binom{8}{5} \times 2^3 = 448$
- f**  $= \binom{9}{3} \times (-1)^3 = -84$
- 4
- a**  $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$   
 $= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$   
 $= 89 - 28\sqrt{10}$
- b**  $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$   
 $= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \frac{1}{9}\sqrt{3}$   
 $= 3\sqrt{2} + \frac{19}{9}\sqrt{3}$

$$\begin{aligned}
 \mathbf{c} &= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3] \\
 &= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}] \\
 &= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}] \\
 &= 16\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{5} \quad \mathbf{a} &= 1 + 10\left(\frac{x}{2}\right) + \frac{10 \times 9}{2}\left(\frac{x}{2}\right)^2 + \frac{10 \times 9 \times 8}{3 \times 2}\left(\frac{x}{2}\right)^3 + \dots \\
 &= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.01 \\
 1.005^{10} &\approx 1 + 0.05 + 0.001\,125 + 0.000\,015 \\
 &= 1.051\,14 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.008 \\
 0.996^{10} &\approx 1 - 0.040 + 0.000\,720 - 0.000\,007\,680 \\
 &= 0.960\,71 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6} \quad \mathbf{a} &= 3^8 + 8(3^7)x + \frac{8 \times 7}{2}(3^6)x^2 + \frac{8 \times 7 \times 6}{3 \times 2}(3^5)x^3 + \dots \\
 &= 6561 + 17\,496x + 20\,412x^2 + 13\,608x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.001 \\
 3.001^8 &\approx 6561 + 17.496 + 0.020\,412 + 0.000\,013\,608 \\
 &= 6578.516 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.005 \\
 2.995^8 &\approx 6561 - 87.480 + 0.510\,300 - 0.001\,701\,000 \\
 &= 6474.029 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad \mathbf{a} \quad (1 + 10x)^4 &= 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4 \\
 &= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 \\
 \therefore (1 + 10x)^4 + (1 - 10x)^4 &= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10\,000x^4) \\
 &= 2 + 1200x^2 + 20\,000x^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad (2 + \tfrac{1}{3}x)^3 &= 2^3 + 3(2^2)(\tfrac{1}{3}x) + 3(2)(\tfrac{1}{3}x)^2 + (\tfrac{1}{3}x)^3 \\
 &= 8 + 4x + \tfrac{2}{3}x^2 + \tfrac{1}{27}x^3 \\
 \therefore (2 - \tfrac{1}{3}x)^3 - (2 + \tfrac{1}{3}x)^3 &= 8 - 4x + \tfrac{2}{3}x^2 - \tfrac{1}{27}x^3 - (8 + 4x + \tfrac{2}{3}x^2 + \tfrac{1}{27}x^3) \\
 &= -8x - \tfrac{2}{27}x^3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1 + 4y)(1 + 3y + 3y^2 + y^3) \\
 &= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4 \\
 &= 1 + 7y + 15y^2 + 13y^3 + 4y^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1 - x)(1 + \tfrac{3}{x} + \tfrac{3}{x^2} + \tfrac{1}{x^3}) \\
 &= 1 + \tfrac{3}{x} + \tfrac{3}{x^2} + \tfrac{1}{x^3} - x - 3 - \tfrac{3}{x} - \tfrac{1}{x^2} \\
 &= -x - 2 + \tfrac{2}{x^2} + \tfrac{1}{x^3}
 \end{aligned}$$



$$\begin{aligned}
 8 \quad \mathbf{a} &= (1+x^2)[1+10(-3x)+\frac{10 \times 9}{2}(-3x)^2+\frac{10 \times 9 \times 8}{3 \times 2}(-3x)^3+\dots] \\
 &= (1+x^2)[1-30x+405x^2-3240x^3+\dots] \\
 &= 1-30x+405x^2-3240x^3+x^2-30x^3+\dots \\
 &= 1-30x+406x^2-3270x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= (1-2x)[1+8x+\frac{8 \times 7}{2}x^2+\frac{8 \times 7 \times 6}{3 \times 2}x^3+\dots] \\
 &= (1-2x)[1+8x+28x^2+56x^3+\dots] \\
 &= 1+8x+28x^2+56x^3-2x-16x^2-56x^3+\dots \\
 &= 1+6x+12x^2+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1+x+x^2)[1+6(-x)+\frac{6 \times 5}{2}(-x)^2+\frac{6 \times 5 \times 4}{3 \times 2}(-x)^3+\dots] \\
 &= (1+x+x^2)[1-6x+15x^2-20x^3+\dots] \\
 &= 1-6x+15x^2-20x^3+x-6x^2+15x^3+x^2-6x^3+\dots \\
 &= 1-5x+10x^2-11x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1+3x-x^2)[1+7(2x)+\frac{7 \times 6}{2}(2x)^2+\frac{7 \times 6 \times 5}{3 \times 2}(2x)^3+\dots] \\
 &= (1+3x-x^2)[1+14x+84x^2+280x^3+\dots] \\
 &= 1+14x+84x^2+280x^3+3x+42x^2+252x^3-x^2-14x^3+\dots \\
 &= 1+17x+125x^2+518x^3+\dots
 \end{aligned}$$

$$9 \quad \mathbf{a} = \binom{8}{4} \times y^4 \times \left(\frac{1}{y}\right)^4 = 70$$

$$\mathbf{b} = \binom{12}{6} \times (2y)^6 \times \left(-\frac{1}{2y}\right)^6 = 924$$

$$\mathbf{c} = \binom{6}{2} \times \left(\frac{1}{y}\right)^4 \times (y^2)^2 = 15$$

$$\mathbf{d} = \binom{9}{3} \times (3y)^6 \times \left(-\frac{1}{y^2}\right)^3 = -61\,236$$

$$10 \quad \mathbf{a} \quad \frac{n(n-1)}{2} \times \left(\frac{2}{5}\right)^2 = 1.6$$

$$n(n-1) = \frac{25}{2} \times 1.6 = 20$$

$$n^2 - n - 20 = 0$$

$$(n+4)(n-5) = 0$$

$$n > 0 \quad \therefore n = 5$$

$$\mathbf{b} = 5 \times \left(\frac{2}{5}\right)^4 = \frac{16}{125} \text{ or } 0.128$$

$$11 \quad \mathbf{a} \quad y_1 = (1-2x)[1+10x+\frac{10 \times 9}{2}x^2+\dots]$$

$$= 1+10x+45x^2-2x-20x^2+\dots$$

$$= 1+8x+25x^2+\dots$$

$$\therefore a = 25, b = 8, c = 1$$

$$\mathbf{b} \quad x = 0.2: y_1 = 0.6 \times (1.2)^{10} = 3.71504$$

$$y_2 = (25 \times 0.04) + (8 \times 0.2) + 1 = 3.6$$

$$\% \text{ error} = \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\% \text{ (2sf)}$$

$$12 \quad \mathbf{a} \quad (1+px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \dots$$

$$\therefore pq = -12 \text{ and } \frac{1}{2}p^2q(q-1) = 60$$

$$\text{sub. } p = -\frac{12}{q}$$

$$\Rightarrow \frac{72}{q}(q-1) = 60$$

$$72(q-1) = 60q$$

$$q = 6, p = -2$$

$$\mathbf{b} = \frac{6 \times 5 \times 4}{3 \times 2} \times (-2)^3 = -160$$

$$13 \quad \mathbf{a} \quad = 3^{12} + 12(3^{11})\left(-\frac{x}{3}\right) + \frac{12 \times 11}{2}(3^{10})\left(-\frac{x}{3}\right)^2 + \frac{12 \times 11 \times 10}{3 \times 2}(3^9)\left(-\frac{x}{3}\right)^3 + \dots$$

$$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \dots$$

$$\mathbf{b} \quad \text{let } \frac{x}{3} = 0.002 \quad \therefore x = 0.006$$

$$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$$

$$= 527\,205.03 \text{ (2dp)}$$

$$14 \quad \mathbf{a} \quad = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$$\mathbf{b} \quad = 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$$

$$\mathbf{c} \quad \mathbf{i} \quad = [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$$

$$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$$

$$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$$

$$= 76 - 44\sqrt{3}$$

$$\mathbf{ii} \quad = (\sqrt{3} + 1)(76 - 44\sqrt{3})$$

$$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$$

$$= -56 + 32\sqrt{3}$$

$$15 \quad \mathbf{a} \quad = 1 + 9\left(\frac{x}{2}\right) + \frac{9 \times 8}{2}\left(\frac{x}{2}\right)^2 + \frac{9 \times 8 \times 7}{3 \times 2}\left(\frac{x}{2}\right)^3 + \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2}\left(\frac{x}{2}\right)^4 + \dots$$

$$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \dots$$

$$\mathbf{b} \quad = \frac{21}{2} - \left(-\frac{21}{2}\right) = 21$$

$$\mathbf{c} \quad = \left(1 \times \frac{63}{8}\right) + \left(2 \times \frac{21}{2}\right) = 28\frac{7}{8}$$

$$16 \quad 10(x^3)^2\left(\frac{a}{x^2}\right)^3 = -80$$

$$a^3 = -8$$

$$a = -2$$

$$17 \quad \mathbf{a} \quad \left(1 + \frac{x}{k}\right)^n = 1 + n\left(\frac{x}{k}\right) + \frac{n(n-1)}{2}\left(\frac{x}{k}\right)^2 + \frac{n(n-1)(n-2)}{3 \times 2}\left(\frac{x}{k}\right)^3 + \dots$$

$$\therefore \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$$

$$kn(n-1) = n(n-1)(n-2)$$

$$n(n-1)[k - (n-2)] = 0$$

$$n > 1 \quad \therefore k - (n-2) = 0$$

$$k = n - 2$$

$$\mathbf{b} \quad k = 7 - 2 = 5$$

$$\left(1 + \frac{x}{5}\right)^7 = 1 + 7\left(\frac{x}{5}\right) + \frac{7 \times 6}{2}\left(\frac{x}{5}\right)^2 + \frac{7 \times 6 \times 5}{3 \times 2}\left(\frac{x}{5}\right)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2}\left(\frac{x}{5}\right)^4 + \dots$$

$$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \dots$$

$$1 \quad = 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4 \\ = 1 + 16x + 96x^2 + 256x^3 + 256x^4$$

$$2 \quad \text{a } u_5 = 3 \times (-2)^4 = 48 \\ \text{b } S_{10} = \frac{3[1 - (-2)^{10}]}{1 - (-2)} = -1023 \\ \text{c positive terms form GP:} \\ a = 3, r = (-2)^2 = 4 \\ S_8 = \frac{3(4^8 - 1)}{4 - 1} = 65\,535$$

$$3 \quad \text{a } = 1 + 7(3x) + \frac{7 \times 6}{2} (3x)^2 \\ + \frac{7 \times 6 \times 5}{3 \times 2} (3x)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2} (3x)^4 + \dots \\ = 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \dots \\ \text{b let } x = 0.01 \\ 1.03^7 \approx 1 + 0.21 + 0.0189 \\ + 0.000\,945 + 0.000\,028\,35 \\ = 1.229\,87 \text{ (5dp)}$$

$$4 \quad \text{GP: } a = 8, r = 2, n = 10 \\ S_{10} = \frac{8(2^{10} - 1)}{2 - 1} = 8184$$

$$5 \quad \text{a } = 2^5 + 5(2^4)x + 10(2^3)x^2 \\ + 10(2^2)x^3 + 5(2)x^4 + x^5 \\ = 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5 \\ \text{b } = 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5 \\ \text{c } (2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2 \\ + 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5 \\ = 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5} \\ = 682 + 305\sqrt{5} \\ \therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 \\ = (682 + 305\sqrt{5}) - (682 - 305\sqrt{5}) \\ = 610\sqrt{5}, k = 610$$

$$6 \quad \text{a amount in account after 3rd payment in} \\ = 200 + (1.005 \times 200) + (1.005^2 \times 200) \\ = 603.005 \\ \text{interest paid at end of 3rd month} \\ = 0.005 \times 603.005 = \text{£}3.02 \text{ (nearest penny)} \\ \text{b amount paid in} = 12 \times 200 = \text{£}2400 \\ \text{amount in account after 12 months} \\ = 200(1.005 + 1.005^2 + \dots + 1.005^{12}) \\ = 200 \times S_{12} \text{ [GP: } a = 1.005, r = 1.005] \\ = 200 \times \frac{1.005(1.005^{12} - 1)}{1.005 - 1} = 2479.45 \\ \text{total interest} = 2479.45 - 2400 = \text{£}79.45$$

$$7 \quad = 1 + 8(-3x) + \frac{8 \times 7}{2} (-3x)^2 \\ + \frac{8 \times 7 \times 6}{3 \times 2} (-3x)^3 + \dots \\ = 1 - 24x + 252x^2 - 1512x^3 + \dots$$

$$8 \quad \text{a } S_n = a + ar + ar^2 + \dots + ar^{n-1} \\ rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n \\ \text{subtracting, } S_n - rS_n = a - ar^n \\ S_n(1 - r) = a(1 - r^n) \\ S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\text{b } r = 6 \div 3 = 2 \\ a \times 2^3 = 3 \therefore a = \frac{3}{8} \\ S_{16} = \frac{\frac{3}{8}(2^{16} - 1)}{2 - 1} = 24\,575\frac{5}{8}$$

$$\begin{aligned} 9 \quad \mathbf{a} \quad &= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \dots \\ &= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \dots \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad &\frac{1}{2}a^2n(n-1) = 3an \\ &a^2n(n-1) = 6an \\ &an[a(n-1) - 6] = 0 \\ &n \neq 0 \quad \therefore a(n-1) - 6 = 0 \\ &\quad \quad \quad an - a = 6 \\ &\quad \quad \quad n = \frac{6+a}{a} \end{aligned}$$

$$\mathbf{c} \quad n = 10 \quad \therefore \text{coeff. of } x^3 = \frac{10 \times 9 \times 8}{3 \times 2} \times \left(\frac{2}{3}\right)^3 = 35\frac{5}{9}$$

$$11 \quad \mathbf{a} \quad \frac{162}{1-r} = 486$$

$$1-r = \frac{162}{486} = \frac{1}{3} \quad \therefore r = \frac{2}{3}$$

$$\mathbf{b} \quad u_6 = 162 \times \left(\frac{2}{3}\right)^5 = \frac{64}{3} \text{ or } 21\frac{1}{3}$$

$$\mathbf{c} \quad S_{10} = \frac{162[1 - (\frac{2}{3})^{10}]}{1 - \frac{2}{3}} = 477.572$$

$$13 \quad \mathbf{a} \quad \text{time} = 120 \times (0.9)^3 = 87.48 \text{ seconds}$$

$$\mathbf{b} \quad \text{GP: } a = 120, r = 0.9, n = 12$$

$$\begin{aligned} S_{12} &= \frac{120[1 - (0.9)^{12}]}{1 - 0.9} \\ &= 861.08 \text{ seconds} \\ &= 14 \text{ mins } 21 \text{ secs (nearest sec.)} \end{aligned}$$

$$15 \quad \mathbf{a} \quad 6, 12, 24, 48$$

$$\mathbf{b} \quad \text{GP: } a = 6, r = 2, n = 10$$

$$S_{10} = \frac{6(2^{10} - 1)}{2 - 1} = 6138$$

$$17 \quad \mathbf{a} \quad a \times (1.5)^2 = 18$$

$$a = 18 \div 2.25 = 8$$

$$\mathbf{b} \quad S_6 = \frac{8[(1.5)^6 - 1]}{1.5 - 1} = 166.25$$

$$\mathbf{c} \quad 8 \times (1.5)^{k-1} > 8000$$

$$(k-1) \lg 1.5 > \lg 1000$$

$$k > \frac{\lg 1000}{\lg 1.5} + 1$$

$$k > 18.04 \quad \therefore \text{smallest } k = 19$$

$$\begin{aligned} 10 \quad &= 2^6 + 6(2^5)(5x) + \frac{6 \times 5}{2}(2^4)(5x)^2 + \dots \\ &= 64 + 960x + 6000x^2 + \dots \end{aligned}$$

$$12 \quad \mathbf{a} \quad = 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$$

$$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$$

$$\mathbf{b} \quad \text{term in } x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$$

$$\text{coefficient of } x^2 = 54 + 48 - 1 = 101$$

$$14 \quad = [1 + 8(\frac{x}{2}) + \frac{8 \times 7}{2}(\frac{x}{2})^2 + \dots][1 + 6(-x) + \frac{6 \times 5}{2}(-x)^2 + \dots]$$

$$= [1 + 4x + 7x^2 + \dots][1 - 6x + 15x^2 + \dots]$$

$$= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \dots$$

$$= 1 - 2x - 2x^2 + \dots$$

$$\therefore A = -2, B = -2$$

$$16 \quad \mathbf{a} \quad = 1 + 4x + 6x^2 + 4x^3 + x^4$$

$$\mathbf{b} \quad = 1 - 4x + 6x^2 - 4x^3 + x^4$$

$$\mathbf{c} \quad (1 + 4x + 6x^2 + 4x^3 + x^4)$$

$$+ (1 - 4x + 6x^2 - 4x^3 + x^4) = 82$$

$$2 + 12x^2 + 2x^4 = 82$$

$$x^4 + 6x^2 - 40 = 0$$

$$(x^2 + 10)(x^2 - 4) = 0$$

$$x^2 = -10 \text{ [no real solutions] or } x^2 = 4$$

$$x = \pm 2$$

$$18 \quad (1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$$

$$= 1 + 10(\frac{ax}{2}) + \frac{10 \times 9}{2}(\frac{ax}{2})^2 + \dots$$

$$+ 1 + 10(bx) + \frac{10 \times 9}{2}(bx)^2 + \dots$$

$$= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \dots$$

$$\therefore 5a + 10b = 0 \quad \Rightarrow a = -2b$$

$$\text{and } \frac{45}{4}a^2 + 45b^2 = 90 \quad \Rightarrow a^2 + 4b^2 = 8$$

$$\text{sub. } (-2b)^2 + 4b^2 = 8$$

$$b^2 = 1$$

$$a < b \quad \therefore b = 1, a = -2$$

1 a  $a = 108, ar^3 = 32$   
 $\therefore r^3 = 32 \div 108 = \frac{8}{27}$   
 $r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$   
 $u_3 = 108 \times \left(\frac{2}{3}\right)^2 = 48$   
b  $S_\infty = \frac{108}{1 - \frac{2}{3}} = 324$

3 a new subscribers in 4<sup>th</sup> week  
 $= 200 \times (1.15)^3 = 304.175$   
 $= 304$  (nearest unit)  
b new subscribers: GP,  $a = 200, r = 1.15$   
 $S_{10} = \frac{200[(1.15)^{10} - 1]}{1.15 - 1} = 4060.74$   
total no. of subscribers  $= 3600 + S_{10}$   
 $= 7661$  (nearest unit)

5 a  $= 1 + 2n\left(\frac{x}{k}\right) + \frac{2n(2n-1)}{2}\left(\frac{x}{k}\right)^2$   
 $+ \frac{2n(2n-1)(2n-2)}{3 \times 2}\left(\frac{x}{k}\right)^3 + \dots$   
 $= 1 + \frac{2n}{k}x + \frac{n(2n-1)}{k^2}x^2 + \frac{2n(n-1)(2n-1)}{3k^3}x^3 + \dots$   
b  $\frac{2n(n-1)(2n-1)}{3k^3} = \frac{1}{2} \times \frac{n(2n-1)}{k^2}$   
 $4n(n-1)(2n-1) = 3kn(2n-1)$   
 $n(2n-1)[4(n-1) - 3k] = 0$   
 $n > 1 \therefore 4(n-1) - 3k = 0$   
 $3k = 4(n-1)$   
c  $\frac{2n}{k} = 2 \therefore n = k$   
 $\therefore 3k = 4k - 4$   
 $k = 4, n = 4$

7  $\sum_{r=1}^9 3^r$ : GP,  $a = 3, r = 3$   
 $S_9 = \frac{3(3^9 - 1)}{3 - 1} = 29523$   
 $\therefore \sum_{r=1}^9 (3^r - 1) = 29523 - 9$   
 $= 29514$

2  $= 1 + 5(-2x) + 10(-2x)^2$   
 $+ 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$   
 $= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

4 a  $= 1 + 7(4x) + \frac{7 \times 6}{2}(4x)^2 + \dots$   
 $= 1 + 28x + 336x^2 + \dots$   
b  $(1 + 2x)^2(1 + 4x)^7$   
 $= (1 + 4x + 4x^2)(1 + 28x + 336x^2 + \dots)$   
term in  $x^2$   
 $= (1)(336x^2) + (4x)(28x) + (4x^2)(1)$   
coefficient of  $x^2 = 336 + 112 + 4 = 452$

6 a  $r = 3\sqrt{2} \div \sqrt{6} = \sqrt{3}$   
 $a = \sqrt{6} \div \sqrt{3} = \sqrt{2}$   
b  $S_8 = \frac{\sqrt{2}[(\sqrt{3})^8 - 1]}{\sqrt{3} - 1}$   
 $= \frac{80\sqrt{2}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$   
 $= \frac{80\sqrt{2}(\sqrt{3} + 1)}{3 - 1}$   
 $= 40\sqrt{2}(\sqrt{3} + 1)$

8 a  $= 1 + 9(2x) + \frac{9 \times 8}{2}(2x)^2 + \frac{9 \times 8 \times 7}{3 \times 2}(2x)^3 + \dots$   
 $= 1 + 18x + 144x^2 + 672x^3 + \dots$

b  $(1 - 2x)^9 = 1 - 18x + 144x^2 - 672x^3 + \dots$   
 $\therefore (1 + 2x)^9 + (1 - 2x)^9$   
 $= (1 + 18x + 144x^2 + 672x^3 + \dots)$   
 $+ (1 - 18x + 144x^2 - 672x^3 + \dots)$   
 $= 2 + 288x^2$  (ignoring terms in  $x^4$  and higher)

c let  $x = 0.001$   
 $\therefore 1.002^9 + 0.998^9 \approx 2 + 0.000288$   
 $= 2.000288$  (7sf)

$$\begin{aligned}
 9 \quad (k-x)^9 &= k^9 + 9(k^8)(-x) + \frac{9 \times 8}{2}(k^7)(-x)^2 + \dots \\
 &= k^9 - 9k^8x + 36k^7x^2 + \dots \\
 \therefore -b &= -9k^8 \text{ and } b = 36k^7 \\
 9k^8 &= 36k^7 \\
 9k^7(k-4) &= 0 \\
 k \neq 0 \therefore k &= 4 \\
 a = k^9 &= 262\,144 \\
 b = 9k^8 &= 589\,824
 \end{aligned}$$

$$\begin{aligned}
 11 \quad a \quad \frac{t}{1-r} &= 3t \\
 1-r &= \frac{t}{3t} = \frac{1}{3} \therefore r = \frac{2}{3} \\
 b \quad \frac{t[1-(\frac{2}{3})^4]}{1-\frac{2}{3}} &= 130 \\
 t = (\frac{1}{3} \times 80) \div \frac{65}{81} &= 54
 \end{aligned}$$

$$\begin{aligned}
 13 \quad a &= 12000 \times (0.75)^4 \\
 &= 3796.875 \\
 &= \text{£}3800 \text{ (3sf)} \\
 b \quad \text{GP: } a &= 12000, r = 0.75 \\
 S_8 &= \frac{12000[1-(0.75)^8]}{1-0.75} \\
 &= \text{£}43\,200 \text{ (3sf)}
 \end{aligned}$$

$$\begin{aligned}
 10 \quad &= 3^4 + 4(3)^3(2x) + 6(3)^2(2x)^2 \\
 &\quad + 4(3)(2x)^3 + (2x)^4 \\
 &= 81 + 216x + 216x^2 + 96x^3 + 16x^4
 \end{aligned}$$

$$\begin{aligned}
 12 \quad a &= 1 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3 + (-2x)^4 \\
 &= 1 - 8x + 24x^2 - 32x^3 + 16x^4
 \end{aligned}$$

$$\begin{aligned}
 b \quad \text{let } x &= y^2 - 2y \\
 (1 + 4y - 2y^2)^4 & \\
 &= 1 - 8(y^2 - 2y) + 24(y^2 - 2y)^2 + \dots \\
 \text{term in } y^2 &= -8y^2 + 24(-2y)^2 \\
 \text{coefficient of } y^2 &= -8 + 96 = 88
 \end{aligned}$$

$$\begin{aligned}
 14 \quad a \quad p(-2) &= 1^4 - (-1)^4 = 1 - 1 = 0 \\
 \therefore (x+2) &\text{ is a factor of } p(x) \\
 b \quad p(x) &= [x^4 + 4(x^3)(3) + 6(x^2)(3^2) + 4(x)(3^3) + 3^4] \\
 &\quad - [x^4 + 4x^3 + 6x^2 + 4x + 1] \\
 &= 8x^3 + 48x^2 + 104x + 80 \\
 &= 8(x^3 + 6x^2 + 13x + 10) \\
 x+2 &\overline{) \begin{array}{r} x^3 + 4x^2 + 13x + 10 \\ x^3 + 2x^2 \\ \hline 4x^2 + 13x \\ 4x^2 + 8x \\ \hline 5x + 10 \\ 5x + 10 \\ \hline 0 \end{array}}
 \end{aligned}$$

$$\begin{aligned}
 p(x) &= 8(x+2)(x^2 + 4x + 5) \\
 c \quad 8(x+2)(x^2 + 4x + 5) &= 0 \\
 x = -2 \text{ or } (x^2 + 4x + 5) &= 0 \\
 b^2 - 4ac &= 16 - 20 = -4 \\
 b^2 - 4ac < 0 \therefore &\text{ no real sols to } (x^2 + 4x + 5) = 0 \\
 \therefore &\text{ only one real solution to } p(x) = 0
 \end{aligned}$$

15 a  $(1-x)(1+2x)^n$

$$= (1-x)[1 + n(2x) + \frac{n(n-1)}{2}(2x)^2 + \dots]$$

$$= (1-x)[1 + 2nx + 2n(n-1)x^2 + \dots]$$

$$\therefore 2n(n-1) - 2n = 198$$

$$n^2 - 2n - 99 = 0$$

$$(n+9)(n-11) = 0$$

$$n \geq 0 \therefore n = 11$$

b  $(1-x)(1+2x)^{11}$

$$= (1-x)[\dots + \frac{11 \times 10}{2}(2x)^2 + \frac{11 \times 10 \times 9}{3 \times 2}(2x)^3 + \dots]$$

$$= (1-x)[\dots + 220x^2 + 1320x^3 + \dots]$$

$$\therefore \text{coefficient of } x^3 = 1320 - 220 = 1100$$

17 a  $S_4 = 3^4 - 1 = 80$

$$S_3 = 3^3 - 1 = 26$$

$$u_4 = S_4 - S_3 = 80 - 26 = 54$$

b  $S_{n-1} = 3^{n-1} - 1$

$$u_n = S_n - S_{n-1}$$

$$= (3^n - 1) - (3^{n-1} - 1)$$

$$= 3^n - 3^{n-1}$$

$$= 3^n(1 - \frac{1}{3}) = \frac{2}{3}(3^n) \quad [k = \frac{2}{3}]$$

c  $u_{n-1} = \frac{2}{3}(3^{n-1})$

$$u_n \div u_{n-1} = \frac{2}{3}(3^n) \div \frac{2}{3}(3^{n-1}) = 3$$

$$u_n \div u_{n-1} \text{ is constant } \therefore \text{geometric}$$

16  $= (\frac{3}{x})^4 + 4(\frac{3}{x})^3(-x) + 6(\frac{3}{x})^2(-x)^2$

$$+ 4(\frac{3}{x})(-x)^3 + (-x)^4$$

$$= x^4 - 12x^2 + 54 - \frac{108}{x^2} + \frac{81}{x^4}$$

18 a  $3(x-3) = y-3$

$$y = 3x - 6$$

b  $\left(\frac{x}{3}\right)^3 = \frac{y}{3}$

$$x^3 = 9y = 9(3x-6)$$

$$x^3 - 27x + 54 = 0$$

c trying  $x = 1, 2$  etc.  $\Rightarrow x = 3$  is a solution

$$\therefore (x-3) \text{ is a factor}$$

$$\begin{array}{r} x^2 + 3x - 18 \\ x-3 \overline{) x^3 + 0x^2 - 27x + 54} \\ \underline{x^3 - 3x^2} \phantom{+ 54} \\ 3x^2 - 27x \phantom{+ 54} \\ \underline{3x^2 - 9x} \phantom{+ 54} \\ -18x + 54 \phantom{+ 54} \\ \underline{-18x + 54} \phantom{+ 54} \end{array}$$

$$(x-3)(x^2 + 3x - 18) = 0$$

$$(x-3)(x+6)(x-3) = 0$$

$$x = -6 \text{ or } 3$$