Indices and Surds Solutions

1)

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{6\sqrt{3}}{3}$$
$$= 2\sqrt{3}$$
$$a = 2$$

2)

$$\frac{3}{\sqrt{6}} = \frac{3}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{6}}$$
$$= \frac{3\sqrt{6}}{6}$$
$$= \frac{\sqrt{6}}{2}$$

3)

$$3^{2y} - 3^{y+2} - 3^y + 9 = 0$$
$$(3^y)^2 - 3^y \times 3^2 - 3^y + 9 = 0$$
$$(3^y)^2 - 9 \times 3^y - 3^y + 9 = 0$$
$$(3^y)^2 - 10 \times 3^y + 9 = 0$$

Let $x = 3^y$

$$\Rightarrow \quad x^2 - 10x + 9 = 0$$
$$(x-1)(x-9) = 0$$

$x-1$	$=$	0	or	$x-9$	$=$	0
x	$=$	1	or	x	$=$	9
3^y	$=$	1	or	3^y	$=$	9
3^y	$=$	3^0	or	3^y	$=$	3^2
y	$=$	0	or	y	$=$	2

4)

$$2^{2y} - 2^{y+3} - 2^{y+2} + 32 = 0$$
$$(2^y)^2 - 2^y \times 2^3 - 2^y \times 2^2 + 32 = 0$$
$$(2^y)^2 - 8 \times 2^y - 4 \times 2^y + 32 = 0$$
$$(2^y)^2 - 12 \times 2^y + 32 = 0$$

Let $x = 2^y$

$$x^2 - 12x + 32 = 0$$
$$(x-8)(x-4) = 0$$

$x-8$	$=$	0	or	$x-4$	$=$	0
x	$=$	8	or	x	$=$	4
$\Rightarrow$ 2^y	$=$	8	or	2^y	$=$	4
2^y	$=$	2^3	or	2^y	$=$	2^2
$\Rightarrow$ y	$=$	3	or	y	$=$	2

5)

$$2^y + \frac{16}{2^y} = 17$$

$\times 2^y$ $\quad (2^y)^2 + 16 = 17 \times 2^y$

$$(2^y)^2 - 17 \times 2^y + 16 = 0$$

Let $x = 2^y$

$$x^2 - 17x + 16 = 0$$

$$(x-1)(x-16) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x - 16 = 0$$

$$x = 1 \quad \text{or} \quad x = 16$$

$$2^y = 1 \quad \text{or} \quad 2^y = 16$$

$$2^y = 2^0 \quad \text{or} \quad 2^y = 2^4$$

$\Rightarrow \quad y = 0 \quad \text{or} \quad y = 4$

6)

$$2x^{\frac{1}{3}} = x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}}$$

Multiply both sides by $x^{\frac{2}{3}}$

$$2x^{\frac{1}{3}}x^{\frac{2}{3}} = x^0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

7)

$$12x^{\frac{1}{4}} - 27x^{-\frac{3}{4}} = x^{\frac{5}{4}}$$

$$12x^{\frac{1}{4}} - \frac{27}{x^{\frac{3}{4}}} = x^{\frac{5}{4}}$$

Multiply both sides by $x^{\frac{3}{4}}$

$$12x^{\frac{1}{4}}x^{\frac{3}{4}} - 27x^0 = x^{\frac{5}{4}}x^{\frac{3}{4}}$$

$$12x - 27 = x^2$$

$$x^2 - 12x + 27 = 0$$

$$(x-3)(x-9) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 9 = 0$$

$$x = 3 \quad \text{or} \quad x = 9$$

8)

$$y^{\frac{1}{3}} + 12y^{-\frac{1}{3}} = 7$$

$$y^{\frac{1}{3}} + \frac{12}{y^{\frac{1}{3}}} = 7$$

Multiply both sides by $y^{\frac{1}{3}}$

$$y^{\frac{2}{3}} + 12y^{0} = 7y^{\frac{1}{3}}$$

$$\left(y^{\frac{1}{3}}\right)^2 - 7y^{\frac{1}{3}} + 12 = 0$$

Let $x = y^{\frac{1}{3}}$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$x-4$	$=$	0	or	$x-3$	$=$	0
x	$=$	4	or	x	$=$	3
$y^{\frac{1}{3}}$	$=$	4	or	$y^{\frac{1}{3}}$	$=$	3
y	$=$	4^3	or	y	$=$	3^3
y	$=$	64	or	y	$=$	27

9)

$$x^3(x^3 - 26) = 27$$

$$x^6 - 26x^3 = 27$$

$$x^6 - 26x^3 - 27 = 0$$

$$(x^3)^2 - 26x^3 - 27 = 0$$

Let $y = x^3$

$$y^2 - 26y - 27 = 0$$

$$(y-27)(y+1) = 0$$

$y-27$	$=$	0	or	$y+1$	$=$	0
y	$=$	27	or	y	$=$	-1
x^3	$=$	27	or	x^3	$=$	-1
x	$=$	$\sqrt[3]{27}$	or	x	$=$	$\sqrt[3]{-1}$
x	$=$	3	or	x	$=$	-1

10)

$$5 + 6\sqrt{5} - \frac{10}{\sqrt{5}} + \frac{1}{\sqrt{5}} = 5 + 6\sqrt{5} - \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} + \frac{1}{\left(\sqrt{5}\right)} \times \frac{\left(\sqrt{5}\right)}{\sqrt{5}}$$

$$= 5 + 6\sqrt{5} - \frac{10\sqrt{5}}{5} + \frac{\sqrt{5}}{5}$$

$$= 5 + 6\sqrt{5} - 2\sqrt{5} + \frac{\sqrt{5}}{5}$$

$$5 + 6\sqrt{5} - \frac{10}{\sqrt{5}} + \frac{1}{\sqrt{5}} = 5 + \frac{21}{5}\sqrt{5}$$