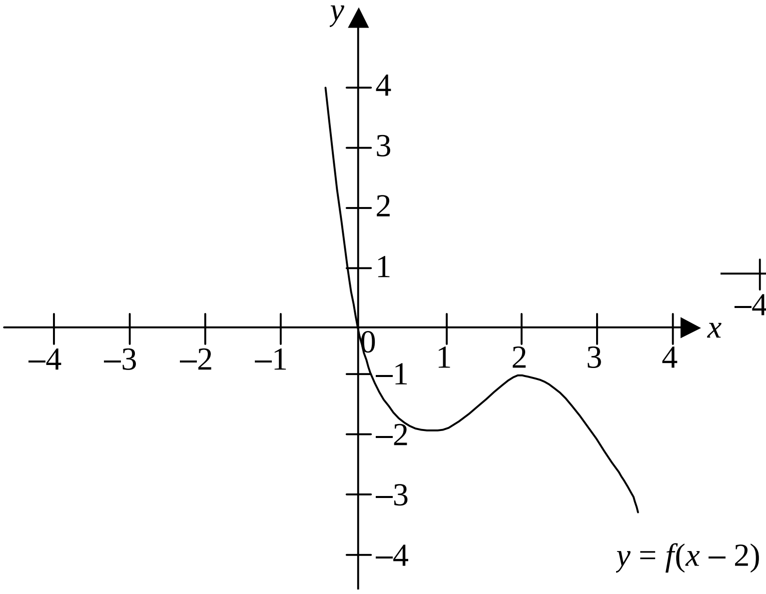
Transformations solutions

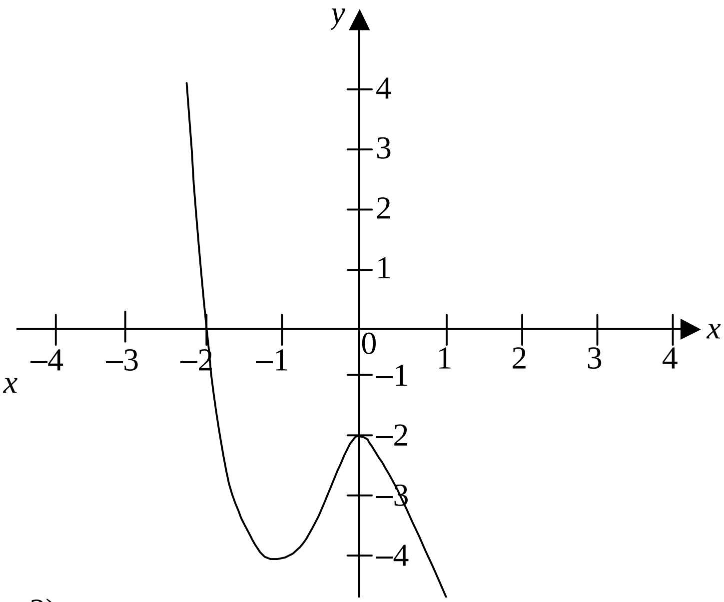
1)

(a)



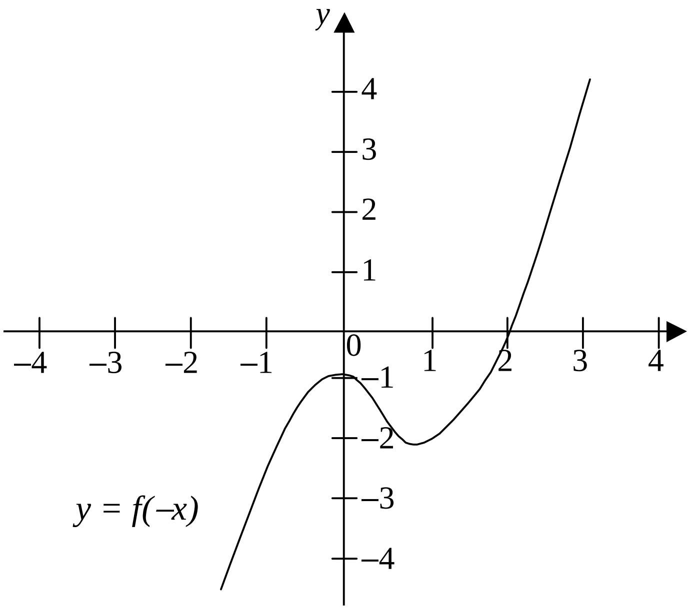
A translation of 2 units in the positive *x* direction

(b)



A stretch of 2 units along *y* axis

(c)



A reflection in the *y* axis

2)

Let $g(x) = \frac{1}{x}$

A reflection in the *y* axis is given by

$y = g(-x)$

$$g(-x) = \frac{1}{(-x)}$$

A translation of -3 units parallel to the x axis is given by
$g(-(x+3))$

$$g(-(x+3)) = -\frac{1}{(x+3)}$$

A translation of 4 units parallel to the y axis is given by
$g(-(x+3)) + 4$

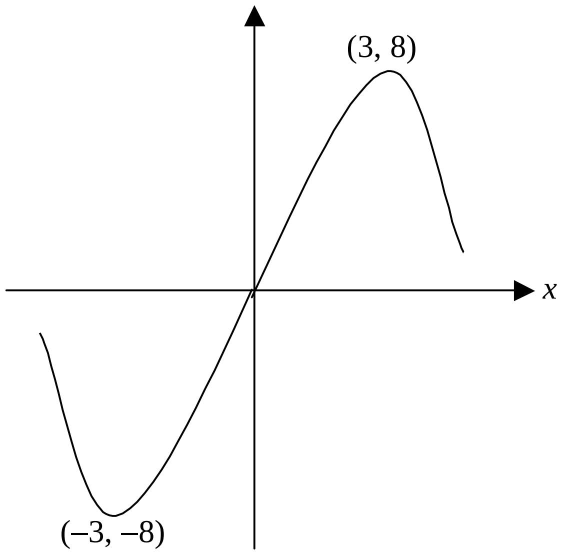
$$\begin{aligned} g(-(x+3)) + 4 &= -\frac{1}{(x+3)} + 4 \\ &= 4 - \frac{1}{x+3} \\ &= \frac{4(x+3)-1}{(x+3)} \\ &= \frac{4x+12-1}{(x+3)} \\ &= \frac{11+4x}{3+x} \end{aligned}$$

$\Rightarrow$ $$f(x) = \frac{(11+4x)}{(3+x)}$$

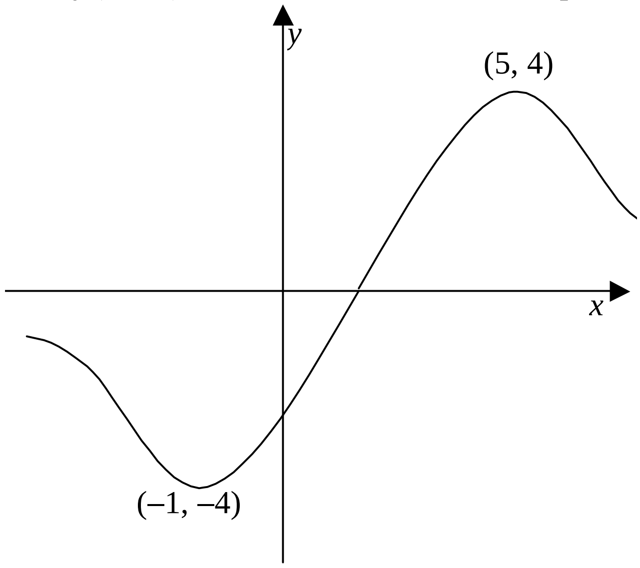
$\Rightarrow$ $$\frac{11+4x}{3+x} \equiv \frac{a-bx}{c-x}$$

$\Rightarrow$ $a = 11, \quad b = -4, \quad c = -3$

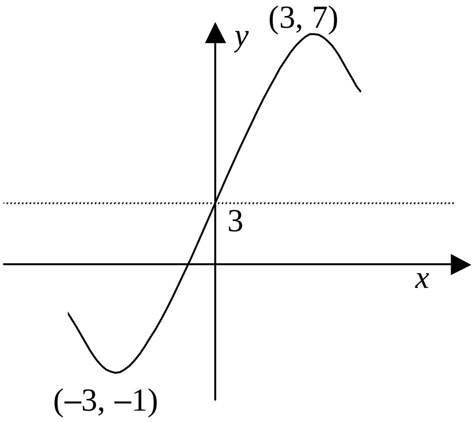
3)
(a) $y = 2f(x)$ is a stretch of 2 units along the y axis



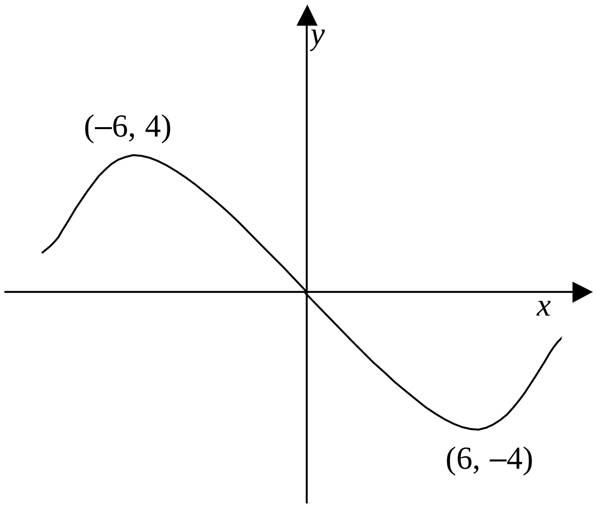
(b) $y = f(x-2)$ is a translation of 2 units parallel to the x axis



(c) $y = 3 + f(x)$ is a translation of 3 units parallel to the y axis



(d) $y = -f\left(\frac{x}{2}\right)$ is a stretch of 2 units parallel to the x axis followed by a reflection in the x axis



4)

(a) At the vertex $x = -1 \Rightarrow y = 2 - 6(-1) - 3(-1)^2$

$= 5$

$\Rightarrow$ Co-ordinates of vertex are $(-1, 5)$

When $x = 0$ $\quad y = 2 - 6(0) - 3(0)^2$

$= 2$

Curve cuts y axis at $(0, 2)$

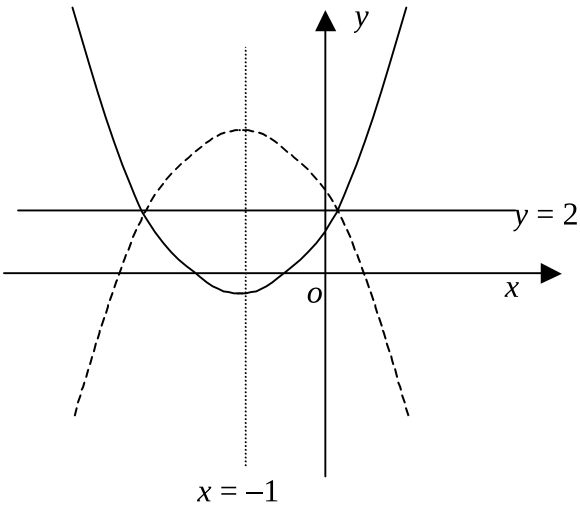
(b) By "completing the square"

$2 - 6x - 3x^2 \equiv a(x + 1)^2 + c = ax^2 + 2ax + a + c$

$ax^2 = -3x^2 \qquad a = -3$

$a + c = 2 \qquad c = 5 \qquad a = -3 \quad c = +5$

(c)



(d) To find the equation of the new curve

$y = 2 - 6x - 3x^2$

(i) Translate the curve -2 units parallel to the y axis

$y = 2 - 6x - 3x^2 - 2$

$y = -6x - 3x^2$

(ii) Reflect in x axis

$y = 6x + 3x^2$

(iii) Translate 2 units parallel to y axis

$y = 6x + 3x^2 + 2$

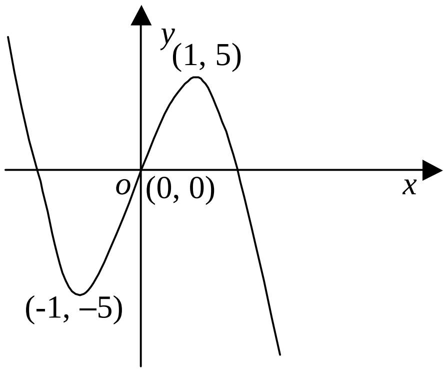
$\Rightarrow$ The new curve has equation $y = 3x^2 + 6x + 2$

When $x = -1$, $y = 3(-1)^2 + 6(-1) + 2 = -1$

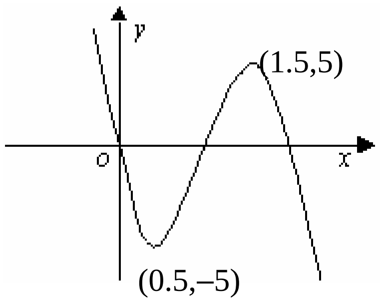
Co-ordinates of the vertex are $(-1, -1)$

5)

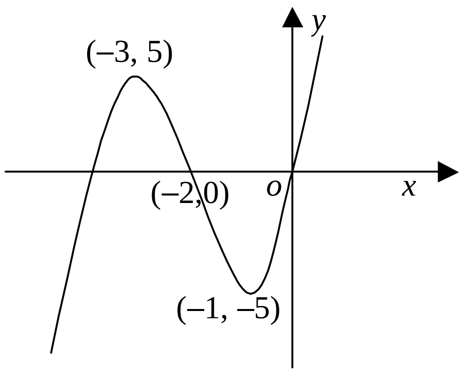
(a) $y = f(x + 2)$ is a translation of $y = f(x)$ by -2 units parallel to the x axis



(b) $y = f(2x)$ is a stretch of $\frac{1}{2}$ unit parallel to the x axis



(c) $y = f(-x)$ is a reflection in the y axis



6)

(a) The dashed line is a reflection of $y = f(x)$ in the x axis

$\Rightarrow$ $y = -f(x)$

(b) The dashed line is a stretch of scale factor 3 parallel to the y axis

$\Rightarrow$ $y = 3f(x)$

(c) The dashed line is a translation of -7 units parallel to the x axis

$\Rightarrow$ $y = f(x + 7)$

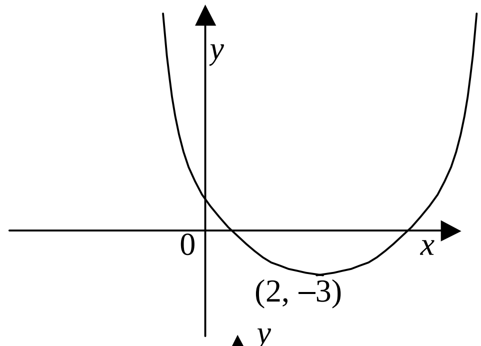
(d) The dashed line is a translation of -3 units parallel to the y axis

$\Rightarrow$ $y = f(x) - 3$

7)
(a)

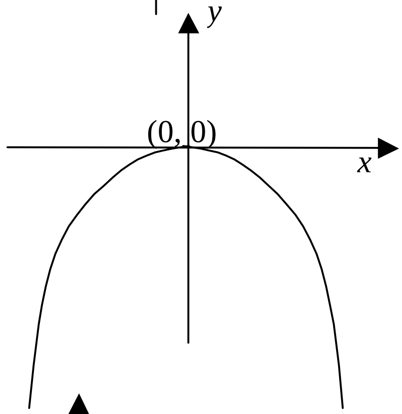
Equation of curve $\quad y = (x-2)^2 - 3$

$$y = x^2 - 4x + 1$$



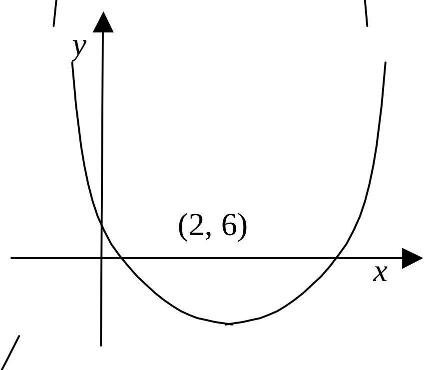
(b)

$$y = -(2x)^2$$
$$y = -4x^2$$

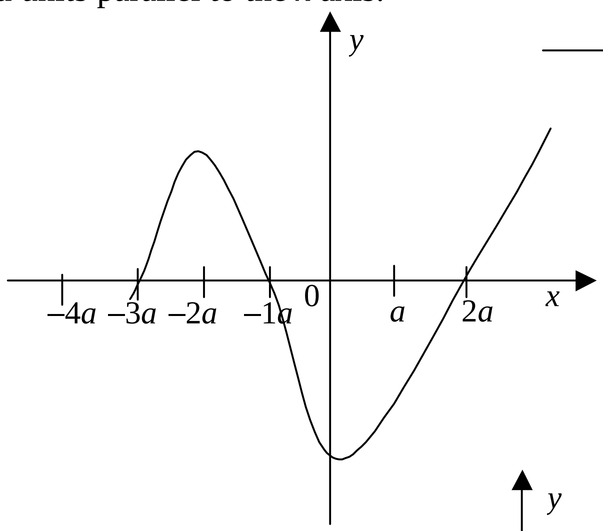


(c)

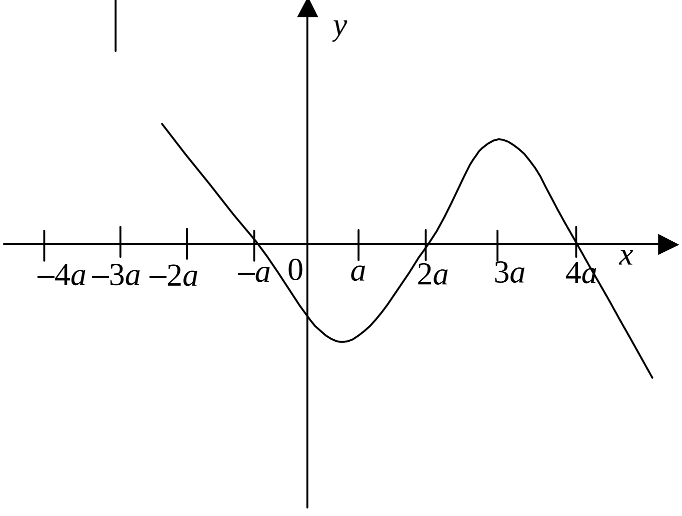
$$y = 2[(x-2)^2 + 3]$$
$$y = 2(x^2 - 4x + 7)$$
$$y = 2x^2 - 8x + 14$$



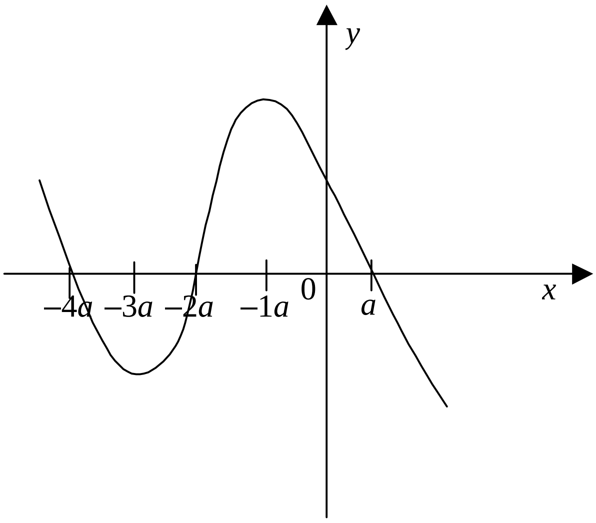
8)
(a) A translation of **a** units parallel to the x axis.



(b) A reflection in the y axis



(c) A reflection in the x axis



(d) A stretch of $\frac{1}{2}$ parallel to the x axis

9)

(a) $f(x)$ is given by a stretch of 2 units along the y axis

$\Rightarrow \quad f(x) = 2\sin x$

$y(x)$ is given by a translation of $\frac{\pi}{2}$ along the x axis

$\Rightarrow \quad y(x) = \sin\left(x - \frac{\pi}{2}\right)$

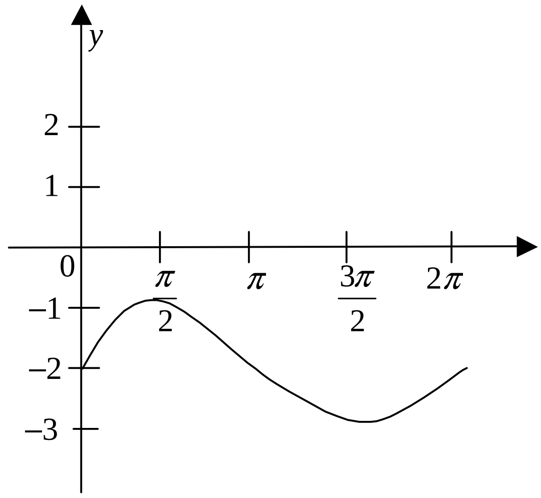
$h(x)$ is given by a stretch of 2 units along the x axis and a stretch of 3 units along the y axis

$\Rightarrow \quad h(x) = 3\sin\left(\frac{x}{2}\right)$

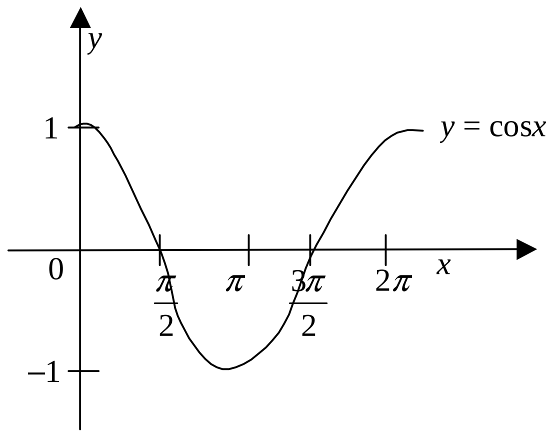
(b) (i) A translation of π in the positive x direction

$\Rightarrow \quad y = \sin(x - \pi)$

(ii) A translation of 2 in the negative y direction $\Rightarrow y = \sin x - 2$



(c)



To obtain the graph of $y = \cos x$ by moving the graph of $y = \sin x$ in the positive x direction, a translation of $\frac{\pi}{2}$ in the negative x direction is required

$\Rightarrow \quad \sin\left(x + \frac{\pi}{2}\right) = \cos x$

$\Rightarrow \quad m = \frac{\pi}{2}$

10)

(i) Reflect $y = e^x$ in the y axis

$\Rightarrow \quad y = e^{-x}$

(ii) Translate –1 unit parallel to the x axis

$\Rightarrow \quad y = e^{(-x+1)}$

then $\quad y = e^{(1-x)}$

(iii) Stretch, scale factor $\frac{1}{3}$ parallel to the x axis

$\Rightarrow \quad y = e^{3(1-x)}$

then

(iv) Translate 4 units parallel to the y axis

$\Rightarrow \quad y = e^{3(1-x)} + 4$