

- 1 **a** $= [2x^2 - x]_1^3$
 $= (18 - 3) - (2 - 1)$
 $= 14$
- b** $= [x^3 + 2x]_0^1$
 $= (1 + 2) - (0)$
 $= 3$
- c** $= [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^3$
 $= (\frac{9}{2} - 9) - (0)$
 $= -\frac{9}{2}$
- d** $= \int_2^3 (9x^2 + 6x + 1) \, dx$ **e** $= [\frac{1}{3}x^3 - 4x^2 - 3x]_1^2$ **f** $= [8x - 2x^2 + x^3]_{-2}^4$
 $= [3x^3 + 3x^2 + x]_2^3$ $= (\frac{8}{3} - 16 - 6) - (\frac{1}{3} - 4 - 3)$ $= (32 - 32 + 64) - (-16 - 8 - 8)$
 $= (81 + 27 + 3) - (24 + 12 + 2)$ $= -12\frac{2}{3}$ $= 96$
 $= 73$
- g** $= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$ **h** $= [5x + \frac{1}{3}x^3 - x^4]_{-2}^{-1}$ **i** $= [\frac{1}{5}x^5 + 2x^3 - \frac{1}{2}x^2]_{-1}^2$
 $= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$ $= (-5 - \frac{1}{3} - 1) - (-10 - \frac{8}{3} - 16)$ $= (\frac{32}{5} + 16 - 2) - (-\frac{1}{5} - 2 - \frac{1}{2})$
 $= 27\frac{3}{4}$ $= 22\frac{1}{3}$ $= 23\frac{1}{10}$
- 2 $\int_1^4 (3x^2 + ax - 5) \, dx = [x^3 + \frac{1}{2}ax^2 - 5x]_1^4$
 $= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5) = 48 + \frac{15}{2}a$
 $\therefore 48 + \frac{15}{2}a = 18$
 $a = -4$
- 3 $\int_{-1}^k (3x^2 - 12x + 9) \, dx = [x^3 - 6x^2 + 9x]_{-1}^k$
 $= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9) = k^3 - 6k^2 + 9k + 16$
 $\therefore k^3 - 6k^2 + 9k + 16 = 16$
 $k(k^2 - 6k + 9) = 0$
 $k(k - 3)^2 = 0$
 $k \neq 0 \therefore k = 3$
- 4 **a** $= \int_1^3 (2 - x^{-2}) \, dx$ **b** $= \int_{-2}^{-1} (6x + 4x^{-3}) \, dx$ **c** $= [2x^{\frac{3}{2}} - 4x]_1^4$
 $= [2x + x^{-1}]_1^3$ $= [3x^2 - 2x^{-2}]_{-2}^{-1}$ $= (16 - 16) - (2 - 4)$
 $= (6 + \frac{1}{3}) - (2 + 1)$ $= (3 - 2) - (12 - \frac{1}{2})$ $= 2$
 $= \frac{10}{3}$ $= -10\frac{1}{2}$
- d** $= \int_{-1}^2 (2x^3 - \frac{1}{2}) \, dx$ **e** $= [\frac{1}{2}x^2 - \frac{3}{2}x^{\frac{2}{3}}]_1^8$ **f** $= \int_2^3 (\frac{1}{3}x^{-2} - 2x) \, dx$
 $= [\frac{1}{2}x^4 - \frac{1}{2}x]_{-1}^2$ $= (32 - 6) - (\frac{1}{2} - \frac{3}{2})$ $= [-\frac{1}{3}x^{-1} - x^2]_2^3$
 $= (8 - 1) - (\frac{1}{2} + \frac{1}{2})$ $= 27$ $= (-\frac{1}{9} - 9) - (-\frac{1}{6} - 4)$
 $= 6$ $= -4\frac{17}{18}$
- 5 $= \int_1^3 (3x^2 - 6x + 7) \, dx$
 $= [x^3 - 3x^2 + 7x]_1^3$
 $= (27 - 27 + 21) - (1 - 3 + 7) = 16$

$$\begin{aligned}
 6 \quad \mathbf{a} \quad &= \int_0^2 (x^2 + 2) \, dx \\
 &= \left[\frac{1}{3}x^3 + 2x \right]_0^2 \\
 &= \left(\frac{8}{3} + 4 \right) - 0 = 6\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad &= \int_{-2}^1 (3x^2 + 8x + 6) \, dx \\
 &= \left[x^3 + 4x^2 + 6x \right]_{-2}^1 \\
 &= (1 + 4 + 6) - (-8 + 16 - 12) = 15
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad &= \int_2^4 (9 + 2x - x^2) \, dx \\
 &= \left[9x + x^2 - \frac{1}{3}x^3 \right]_2^4 \\
 &= (36 + 16 - \frac{64}{3}) - (18 + 4 - \frac{8}{3}) = 11\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad &= \int_{-1}^0 (x^3 - 4x + 1) \, dx \\
 &= \left[\frac{1}{4}x^4 - 2x^2 + x \right]_{-1}^0 \\
 &= 0 - \left(\frac{1}{4} - 2 - 1 \right) = \frac{11}{4}
 \end{aligned}$$

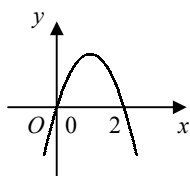
$$\begin{aligned}
 \mathbf{e} \quad &= \int_1^4 (2x + 3x^{\frac{1}{2}}) \, dx \\
 &= \left[x^2 + 2x^{\frac{3}{2}} \right]_1^4 \\
 &= (16 + 16) - (1 + 2) = 29
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f} \quad &= \int_{-5}^{-1} (3 + 5x^{-2}) \, dx \\
 &= \left[3x - 5x^{-1} \right]_{-5}^{-1} \\
 &= (-3 + 5) - (-15 + 1) = 16
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \mathbf{a} \quad &y = 0 \Rightarrow 4 - x^2 = 0 \\
 &x^2 = 4 \\
 &x = \pm 2 \\
 &\therefore (-2, 0) \text{ and } (2, 0)
 \end{aligned}$$

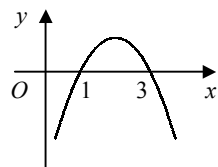
$$\begin{aligned}
 \mathbf{b} \quad &= \int_{-2}^2 (4 - x^2) \, dx \\
 &= \left[4x - \frac{1}{3}x^3 \right]_{-2}^2 \\
 &= \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) \\
 &= 10\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \mathbf{a} \quad &6x - 3x^2 = 0 \\
 &3x(2 - x) = 0 \\
 &x = 0, 2
 \end{aligned}$$



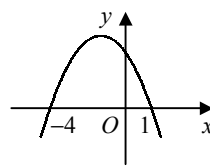
$$\begin{aligned}
 \text{area} &= \int_0^2 (6x - 3x^2) \, dx \\
 &= \left[3x^2 - x^3 \right]_0^2 \\
 &= (12 - 8) - 0 \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad &-x^2 + 4x - 3 = 0 \\
 &-(x - 1)(x - 3) = 0 \\
 &x = 1, 3
 \end{aligned}$$



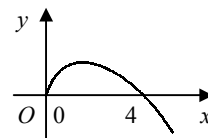
$$\begin{aligned}
 \text{area} &= \int_1^3 (-x^2 + 4x - 3) \, dx \\
 &= \left[-\frac{1}{3}x^3 + 2x^2 - 3x \right]_1^3 \\
 &= (-9 + 18 - 9) \\
 &\quad - \left(-\frac{1}{3} + 2 - 3 \right) \\
 &= \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad &4 - 3x - x^2 = 0 \\
 &-(x + 4)(x - 1) = 0 \\
 &x = -4, 1
 \end{aligned}$$



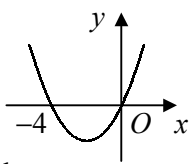
$$\begin{aligned}
 \text{area} &= \int_{-4}^1 (4 - 3x - x^2) \, dx \\
 &= \left[4x - \frac{3}{2}x^2 - \frac{1}{3}x^3 \right]_{-4}^1 \\
 &= \left(4 - \frac{3}{2} - \frac{1}{3} \right) \\
 &\quad - \left(-16 - 24 + \frac{64}{3} \right) \\
 &= 20\frac{5}{6}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad &2x^{\frac{1}{2}} - x = 0 \\
 &x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0 \\
 &x^{\frac{1}{2}} = 0, 2 \\
 &x = 0, 4
 \end{aligned}$$



$$\begin{aligned}
 \text{area} &= \int_0^4 (2x^{\frac{1}{2}} - x) \, dx \\
 &= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2 \right]_0^4 \\
 &= \left(\frac{32}{3} - 8 \right) - 0 \\
 &= \frac{8}{3}
 \end{aligned}$$

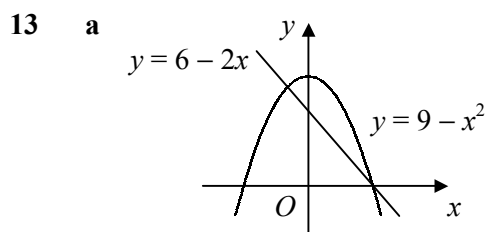
9 a $x^2 + 4x = 0$
 $x(x + 4) = 0$
 $x = -4, 0$



b $= \int_0^2 (x^2 + 4x) \, dx$
 $= \left[\frac{1}{3}x^3 + 2x^2 \right]_0^2$
 $= \left(\frac{8}{3} + 8 \right) - 0 = 10\frac{2}{3}$

11 a $x^3 - 5x^2 + 6x = 0$
 $x(x - 2)(x - 3) = 0$
 $x = 0, 2, 3$
 $\therefore (0, 0), (2, 0) \text{ and } (3, 0)$

b $\int_0^2 (x^3 - 5x^2 + 6x) \, dx$
 $= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_0^2$
 $= \left(4 - \frac{40}{3} + 12 \right) - 0 = \frac{8}{3}$
 $\int_2^3 (x^3 - 5x^2 + 6x) \, dx$
 $= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_2^3$
 $= \left(\frac{81}{4} - 45 + 27 \right) - \frac{8}{3} = -\frac{5}{12}$
 total area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$



$9 - x^2 = 6 - 2x$
 $x^2 - 2x - 3 = 0$
 $(x + 1)(x - 3) = 0$
 $x = -1, 3$

\therefore intersect at $(-1, 8)$ and $(3, 0)$
 area below curve

$= \int_{-1}^3 (9 - x^2) \, dx$
 $= \left[9x - \frac{1}{3}x^3 \right]_{-1}^3$
 $= (27 - 9) - \left(-9 + \frac{1}{3} \right)$
 $= 26\frac{2}{3}$

area below line

$= \frac{1}{2} \times 4 \times 8 = 16$

area between line and curve

$= 26\frac{2}{3} - 16 = 10\frac{2}{3}$

10 a $x^2 + 2x - 15 = 0$
 $(x + 5)(x - 3) = 0$
 $x = -5, 3$
 $\therefore (-5, 0) \text{ and } (3, 0)$

b $= \left[\frac{1}{3}x^3 + x^2 - 15x \right]_0^3$
 $= (9 + 9 - 45) - 0 = -27$

c 27

12 a $x^2 - 3x + 4 = x + 1$
 $x^2 - 4x + 3 = 0$
 $(x - 1)(x - 3) = 0$
 $x = 1, 3$

$\therefore (1, 2) \text{ and } (3, 4)$

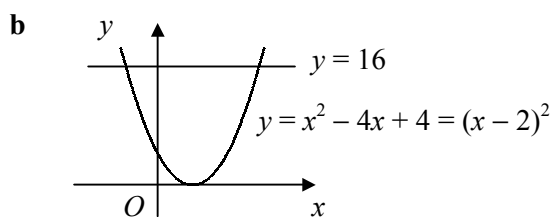
b area below curve

$= \int_1^3 (x^2 - 3x + 4) \, dx$
 $= \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x \right]_1^3$

$= \left(9 - \frac{27}{2} + 12 \right) - \left(\frac{1}{3} - \frac{3}{2} + 4 \right) = \frac{14}{3}$

area below line $= \frac{1}{2} \times 2 \times (2 + 4) = 6$

shaded area $= 6 - \frac{14}{3} = \frac{4}{3}$



$x^2 - 4x + 4 = 16$
 $x^2 - 4x - 12 = 0$
 $(x + 2)(x - 6) = 0$
 $x = -2, 6$

\therefore intersect at $(-2, 16)$ and $(6, 16)$

area below curve

$= \int_{-2}^6 (x^2 - 4x + 4) \, dx$
 $= \left[\frac{1}{3}x^3 - 2x^2 + 4x \right]_{-2}^6$
 $= (72 - 72 + 24) - \left(-\frac{8}{3} - 8 - 8 \right)$
 $= 42\frac{2}{3}$

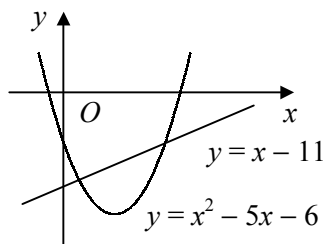
area below line

$= 8 \times 16 = 128$

area between line and curve

$= 128 - 42\frac{2}{3} = 85\frac{1}{3}$

c $y = x^2 - 5x - 6 \Rightarrow y = (x + 1)(x - 6)$



$$x^2 - 5x - 6 = x - 11$$

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

$$x = 1, 5$$

\therefore intersect at $(1, -10)$ and $(5, -6)$

area above curve

$$= -\int_1^5 (x^2 - 5x - 6) \, dx$$

$$= -\left[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x\right]_1^5$$

$$= -\left[\left(\frac{125}{3} - \frac{125}{2} - 30\right) - \left(\frac{1}{3} - \frac{5}{2} - 6\right)\right]$$

$$= 42\frac{2}{3}$$

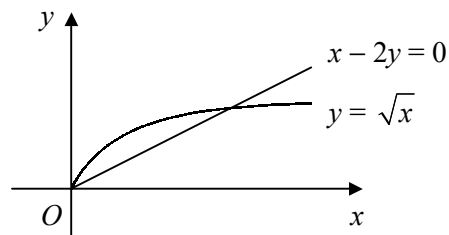
area above line

$$= \frac{1}{2} \times 4 \times (10 + 6) = 32$$

area between line and curve

$$= 42\frac{2}{3} - 32 = 10\frac{2}{3}$$

d $x - 2y = 0 \Rightarrow y = \frac{1}{2}x$



$$x^{\frac{1}{2}} = \frac{1}{2}x$$

$$\frac{1}{2}x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$$

$$x^{\frac{1}{2}} = 0, 2$$

$$x = 0, 4$$

\therefore intersect at $(0, 0)$ and $(4, 2)$

area below curve

$$= \int_0^4 x^{\frac{1}{2}} \, dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4$$

$$= \frac{16}{3} - 0 = \frac{16}{3}$$

area below line

$$= \frac{1}{2} \times 4 \times 2 = 4$$

area between line and curve

$$= \frac{16}{3} - 4 = \frac{4}{3}$$

1 a $f(x) = -[x^2 - 4x] + 3$
 $= -[(x-2)^2 - 4] + 3$
 $= -(x-2)^2 + 7$
 $\therefore a = -1, b = -2, c = 7$

b (2, 7)

c intersect when

$$3 + 4x - x^2 = 3$$

$$x(4-x) = 0$$

$$x = 0, 4$$

area below curve

$$= \int_0^4 (3 + 4x - x^2) dx$$

$$= [3x + 2x^2 - \frac{1}{3}x^3]_0^4$$

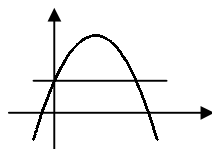
$$= (12 + 32 - \frac{64}{3}) - 0 = \frac{68}{3}$$

area below line

$$= 4 \times 3 = 12$$

area between line and curve

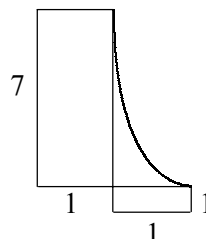
$$= \frac{68}{3} - 12 = 10\frac{2}{3}$$



2 a $= [-4x^{-2}]_1^2$
 $= -1 - (-4)$
 $= 3$

b $y = 1 \Rightarrow x = 2$

$$y = 8 \Rightarrow x = 1$$



shaded area

$$= 3 - (1 \times 1) + (7 \times 1)$$

$$= 9$$

3 a $\frac{dy}{dx} = 5 - 4x$

grad = 1

$$\therefore \text{grad of normal} = -1$$

$$\therefore y - 3 = -(x - 1)$$

$$[y = 4 - x]$$

b area below curve

$$= \int_0^1 (5x - 2x^2) dx$$

$$= [\frac{5}{2}x^2 - \frac{2}{3}x^3]_0^1$$

$$= (\frac{5}{2} - \frac{2}{3}) - 0 = \frac{11}{6}$$

normal meets y-axis at (0, 4)

area below line

$$= \frac{1}{2} \times 1 \times (4 + 3) = \frac{7}{2}$$

shaded area

$$= \frac{7}{2} - \frac{11}{6} = \frac{5}{3}$$

4 a $\frac{4-x^2}{x^2} = 0$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x > 0 \therefore x = 2, P(2, 0)$$

b l: $y - 0 = -3(x - 2)$

$$y = 6 - 3x$$

intersect when $\frac{4-x^2}{x^2} = 6 - 3x$

$$4 - x^2 = 6x^2 - 3x^3$$

$$3x^3 - 7x^2 + 4 = 0$$

$x = 2$ is a solution $\therefore (x - 2)$ is a factor

$$(x - 2)(3x^2 - x - 2) = 0$$

$$(x - 2)(3x + 2)(x - 1) = 0$$

$$x = 2 \text{ (at } P), -\frac{2}{3}, 1$$

$x > 0 \therefore Q(1, 3)$

c area below curve

$$= \int_1^2 (4x^{-2} - 1) dx$$

$$= [-4x^{-1} - x]_1^2$$

$$= (-2 - 2) - (-4 - 1) = 1$$

area below line

$$= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$$

area between line and curve

$$= \frac{3}{2} - 1 = \frac{1}{2}$$

1

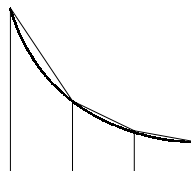
a

x	1	2	3	4
y	3	$\frac{3}{2}$	1	$\frac{3}{4}$

$$b = \frac{1}{2} \times 1 \times [3 + \frac{3}{4} + 2(\frac{3}{2} + 1)] = 4\frac{3}{8}$$

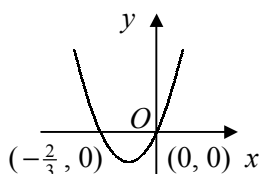
c the true area is less

the curve passes below the top of each trapezium as shown:



2

a



x	0	0.5	1	1.5	2
$x(3x+2)$	0	1.75	5	9.75	16

$$\text{area} \approx \frac{1}{2} \times 0.5 \times [0 + 16 + 2(1.75 + 5 + 9.75)] = 12.25$$

$$c = \int_0^2 (3x^2 + 2x) dx = [x^3 + x^2]_0^2 = (8 + 4) - 0 = 12$$

$$d \text{ \% error} = \frac{12.25 - 12}{12} \times 100\% = 2.08\% \text{ (3sf)}$$

3

a

x	4	5	6
$\frac{3}{2x+1}$	$\frac{1}{3}$	$\frac{3}{11}$	$\frac{3}{13}$

$$\therefore \text{area} \approx \frac{1}{2} \times 1 \times [\frac{1}{3} + \frac{3}{13} + 2(\frac{3}{11})] = 0.555 \text{ (3sf)}$$

x	0	1	2	3
$\lg(x^2 + 9)$	$\lg 9$	$\lg 10$	$\lg 13$	$\lg 18$

$$\therefore \text{area} \approx \frac{1}{2} \times 1 \times [\lg 9 + \lg 18 + 2(\lg 10 + \lg 13)] = 3.22 \text{ (3sf)}$$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$x^2 \sin x$	0	0.436	2.467	3.926	0

$$\therefore \text{area} \approx \frac{1}{2} \times \frac{\pi}{4} \times [0 + 0 + 2(0.436 + 2.467 + 3.926)] = 5.36 \text{ (3sf)}$$

x	-2	-1	0	1	2
$\sqrt[3]{2x+5}$	1	$\sqrt[3]{3}$	$\sqrt[3]{5}$	$\sqrt[3]{7}$	$\sqrt[3]{9}$

$$\therefore \text{area} \approx \frac{1}{2} \times 1 \times [1 + \sqrt[3]{9} + 2(\sqrt[3]{3} + \sqrt[3]{5} + \sqrt[3]{7})] = 6.61 \text{ (3sf)}$$

4

a

x	0	1	2	3
3^x	1	3	9	27

$$\therefore \text{area} \approx \frac{1}{2} \times 1 \times [1 + 27 + 2(3 + 9)] = 26$$

x	2	2.2	2.4
$\sin(\lg x)$	0.2965	0.3358	0.3711

$$\therefore \text{area} \approx \frac{1}{2} \times 0.2 \times [0.2965 + 0.3711 + 2(0.3358)] = 0.134 \text{ (3sf)}$$

c	x	0	0.1	0.2	0.3	0.4	0.5
	$\frac{x}{x^3+2}$	0	0.04998	0.09960	0.14800	0.19380	0.23529

$$\therefore \text{area} \approx \frac{1}{2} \times 0.1 \times [0 + 0.23529 + 2(0.04998 + 0.09960 + 0.14800 + 0.19380)] = 0.0609 \text{ (3sf)}$$

d	x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
	$\sqrt{\cos(\frac{1}{2}x)}$	1	0.9828	0.9306	0.8409	0.7071

$$\therefore \text{area} \approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.7071 + 2(0.9828 + 0.9306 + 0.8409)] = 1.89 \text{ (3sf)}$$

5 a	x	3	3.8	4.6	5.4	6.2	7
	$2 - 3x^{-\frac{1}{2}}$	0.2679	0.4610	0.6012	0.7090	0.7952	0.8661

$$\text{area} \approx \frac{1}{2} \times 0.8 \times [0.2679 + 0.8661 + 2(0.4610 + 0.6012 + 0.7090 + 0.7952)] = 2.51 \text{ (3sf)}$$

$$\begin{aligned} \text{b } &= \int_3^7 (2 - 3x^{-\frac{1}{2}}) \, dx \\ &= [2x - 6x^{\frac{1}{2}}]_3^7 = (14 - 6\sqrt{7}) - (6 - 6\sqrt{3}) = 8 + 6(\sqrt{3} - \sqrt{7}) \end{aligned}$$

6 a	x	0	0.2	0.4	0.6	0.8	1
	$\sin x^2$	0	0.03999	0.15932	0.35227	0.59720	0.84147

$$\text{area} \approx \frac{1}{2} \times 0.2 \times [0 + 0.84147 + 2(0.03999 + 0.15932 + 0.35227 + 0.59720)] = 0.3139$$

$$\begin{aligned} \text{b } &\text{area of rectangle} = 1 \times 0.8415 = 0.8415 \\ &\therefore \text{area of flower bed} \approx 0.8415 - 0.3139 = 0.53 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{7 a } &= 1 + 6\left(\frac{x}{2}\right) + \frac{6 \times 5}{2} \left(\frac{x}{2}\right)^2 + \frac{6 \times 5 \times 4}{3 \times 2} \left(\frac{x}{2}\right)^3 + \dots \\ &= 1 + 3x + \frac{15}{4}x^2 + \frac{5}{2}x^3 + \dots \end{aligned}$$

$$\begin{aligned} \text{b } \text{area} &\approx \int_0^{0.5} \left(1 + 3x + \frac{15}{4}x^2 + \frac{5}{2}x^3\right) \, dx \\ &= \left[x + \frac{3}{2}x^2 + \frac{5}{4}x^3 + \frac{5}{8}x^4\right]_0^{0.5} \\ &= \left(\frac{1}{2} + \frac{3}{8} + \frac{5}{32} + \frac{5}{128}\right) - 0 = 1.07 \text{ (3sf)} \end{aligned}$$

c	x	0	0.1	0.2	0.3	0.4	0.5
	$(1 + \frac{x}{2})^6$	1	1.3401	1.7716	2.3131	2.9860	3.8147

$$\text{area} \approx \frac{1}{2} \times 0.1 \times [1 + 3.8147 + 2(1.3401 + 1.7716 + 2.3131 + 2.9860)] = 1.08 \text{ (3sf)}$$

$$\text{8 a } \frac{dy}{dx} = 2x - 16x^{-2}$$

$$\begin{aligned} \text{SP: } 2x - 16x^{-2} &= 0 \\ x^3 &= 8 \\ x &= 2 \end{aligned}$$

$$\text{when } x = 2, y = 4 + 8 = 12 \therefore \text{SP } (2, 12)$$

b	x	2	2.5	3	3.5	4
	$x^2 + \frac{16}{x}$	12	12.65	14.333	16.821	20

$$\text{area} \approx \frac{1}{2} \times 0.5 \times [12 + 20 + 2(12.65 + 14.333 + 16.821)] = 29.9 \text{ (3sf)}$$

c over-estimate

1 a $= [-2x^{-1}]_1^4$

$$= -\frac{1}{2} - (-2)$$

$$= \frac{3}{2}$$

b $= \int_0^2 (x^2 - 6x + 9) \, dx$

$$= [\frac{1}{3}x^3 - 3x^2 + 9x]_0^2$$

$$= (\frac{8}{3} - 12 + 18) - 0$$

$$= 8\frac{2}{3}$$

3 a $= 3\sqrt{2} - \frac{1}{\sqrt{2}}$

$$= 3\sqrt{2} - \frac{1}{2}\sqrt{2}$$

$$= \frac{5}{2}\sqrt{2}$$

b $\int_3^4 (3x^{\frac{1}{2}} - x^{-\frac{1}{2}}) \, dx$

$$= [2x^{\frac{3}{2}} - 2x^{\frac{1}{2}}]_3^4$$

$$= [16 - 4] - [(2 \times 3\sqrt{3}) - 2\sqrt{3}]$$

$$= 12 - 4\sqrt{3}$$

2 a

x	0	2	4	6
$\sqrt{x^2+4}$	2	$\sqrt{8}$	$\sqrt{20}$	$\sqrt{40}$

$$\text{area} \approx \frac{1}{2} \times 2 \times [2 + \sqrt{40} + 2(\sqrt{8} + \sqrt{20})]$$

$$= 22.9 \text{ (3sf)}$$

b over-estimate

curve passes below top of each trapezium

4 a $4x^{\frac{1}{2}} - x^{\frac{3}{2}} = 0$

$$x^{\frac{1}{2}}(4 - x) = 0$$

$$x^{\frac{1}{2}} = 0 \Rightarrow x = 0, \text{ at } O \text{ or } x = 4$$

$$\therefore A(4, 0)$$

b $\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$$\text{SP: } 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$$

$$\frac{1}{2}x^{-\frac{1}{2}}(4 - 3x) = 0$$

$$x^{-\frac{1}{2}} = 0 \Rightarrow \text{no solutions}$$

$$\therefore x = \frac{4}{3} \text{ at } B$$

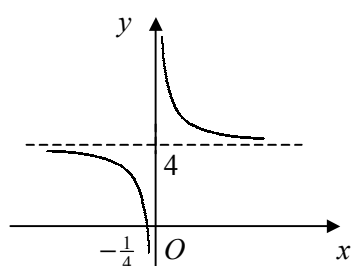
c $= \int_0^4 (4x^{\frac{1}{2}} - x^{\frac{3}{2}}) \, dx$

$$= [\frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}]_0^4$$

$$= (\frac{64}{3} - \frac{64}{5}) - 0 = 8\frac{8}{15}$$

5 a $p = -\frac{1}{4}, q = 4$

b



c

x	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$4 + \frac{1}{x}$	5	$4\frac{2}{3}$	$4\frac{1}{2}$	$4\frac{2}{5}$	$4\frac{1}{3}$

$$\text{area} \approx \frac{1}{2} \times \frac{1}{2} \times [5 + 4\frac{1}{3} + 2(4\frac{2}{3} + 4\frac{1}{2} + 4\frac{2}{5})]$$

$$= 9\frac{7}{60} \text{ or } 9.12 \text{ (3sf)}$$

6 a $4x - y + 11 = 0 \Rightarrow y = 4x + 11$

$$\text{intersect when } 2x^2 + 6x + 7 = 4x + 11$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1$$

$$\therefore (-2, 3) \text{ and } (1, 15)$$

b area below curve

$$= \int_{-2}^1 (2x^2 + 6x + 7) \, dx$$

$$= [\frac{2}{3}x^3 + 3x^2 + 7x]_{-2}^1$$

$$= (\frac{2}{3} + 3 + 7) - (-\frac{16}{3} + 12 - 14) = 18$$

area below line

$$= \frac{1}{2} \times 3 \times (3 + 15) = 27$$

area between line and curve

$$= 27 - 18 = 9$$

- 7 a minimum when $\sin x = 1$

$$\therefore x = \frac{\pi}{2}$$

$$\therefore \left(\frac{\pi}{2}, \frac{1}{2}\right)$$

b	x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
	$\frac{1}{1+\sin x}$	1	0.6667	0.5359

$$\therefore \text{area} \approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.5359 + 2(0.6667)] = 0.751 \text{ (3sf)}$$

8 a $= 1 + 12\left(\frac{x}{10}\right) + \frac{12 \times 11}{2} \left(\frac{x}{10}\right)^2 + \frac{12 \times 11 \times 10}{3 \times 2} \left(\frac{x}{10}\right)^3 + \dots$

$$= 1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3 + \dots$$

b $\approx \int_0^1 \left(1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3\right) dx$

$$= \left[x + \frac{3}{5}x^2 + \frac{11}{50}x^3 + \frac{11}{200}x^4\right]_0^1$$

$$= \left(1 + \frac{3}{5} + \frac{11}{50} + \frac{11}{200}\right) - 0 = 1\frac{7}{8}$$

- 9 a at A, $x = 0 \Rightarrow (0, 2)$

$$\frac{dy}{dx} = -1 - 2x$$

$$\text{grad at A} = -1$$

$$\therefore y = 2 - x$$

- b curve cuts x-axis when $y = 0$

$$2 - x - x^2 = 0$$

$$(2 + x)(1 - x) = 0$$

$$x = -2, 1$$

area below curve

$$= \int_0^1 (2 - x - x^2) dx$$

$$= \left[2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - 0 = \frac{7}{6}$$

tangent cuts x-axis when $y = 0$

$$x = 2$$

area below line

$$= \frac{1}{2} \times 2 \times 2 = 2$$

shaded area

$$= 2 - \frac{7}{6}$$

$$= \frac{5}{6}$$

1 a at A , $x = 0 \therefore A(0, 4)$

at B , $y = 0$

$$(x^{\frac{1}{2}} - 2)^2 = 0$$

$$x^{\frac{1}{2}} = 2$$

$$x = 4 \therefore B(4, 0)$$

b $= \int_0^4 (x - 4x^{\frac{1}{2}} + 4) \, dx$

$$= \left[\frac{1}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 4x \right]_0^4$$

$$= (8 - \frac{64}{3} + 16) - 0$$

$$= \frac{8}{3}$$

3 a $4^{x+1} = 32$

$$(2^2)^{x+1} = 2^5$$

$$2x + 2 = 5$$

$$x = \frac{3}{2}$$

b

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$
4^{x+1}	4	8	16	32

$$\therefore \text{area} \approx \frac{1}{2} \times \frac{1}{2} \times [4 + 32 + 2(8 + 16)]$$

$$= 21$$

2 $= \int_1^2 (\frac{3}{2}x + \frac{1}{2}x^{-2}) \, dx$

$$= \left[\frac{3}{4}x^2 - \frac{1}{2}x^{-1} \right]_1^2$$

$$= (3 - \frac{1}{4}) - (\frac{3}{4} - \frac{1}{2})$$

$$= \frac{5}{2}$$

4 a at A , $x^2 - 2x = 0$

$$x(x - 2) = 0$$

$$x = 0 \text{ (at } O) \text{ or } 2 \therefore A(2, 0)$$

at B , $x^2 - 2x = x$

$$x(x - 3) = 0$$

$$x = 0 \text{ (at } O) \text{ or } 3 \therefore B(3, 3)$$

b $\int_0^2 (x^2 - 2x) \, dx$

$$= \left[\frac{1}{3}x^3 - x^2 \right]_0^2$$

$$= (\frac{8}{3} - 4) - 0 = -\frac{4}{3}$$

$$\therefore \text{area} = \frac{4}{3}$$

c area below curve between A and B

$$= \int_2^3 (x^2 - 2x) \, dx$$

$$= \left[\frac{1}{3}x^3 - x^2 \right]_2^3$$

$$= (9 - 9) - (-\frac{4}{3}) = \frac{4}{3}$$

area below straight line OB

$$= \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$$

area between curve and line

$$= \frac{9}{2} - \frac{4}{3} + \frac{4}{3}$$

$$= \frac{9}{2}$$

5 a

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.319	1.024	0

$$\mathbf{b} \approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0 + 2(1.319 + 1.024)]$$

$$= 1.49 \text{ (3sf)}$$

c under-estimate
curve passes above top of each trapezium

$$7 \quad \mathbf{a} \quad \frac{dy}{dx} = 3x^2 - 6x$$

$$\text{SP: } 3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ (at } P) \text{ or } 2$$

$$\therefore Q(2, 1)$$

$$\mathbf{b} \quad x^3 - 3x^2 + 5 = 5$$

$$x^2(x - 3) = 0$$

$$x = 0 \text{ (at } P) \text{ or } 3$$

$$\therefore R(3, 5)$$

c area below curve

$$= \int_0^3 (x^3 - 3x^2 + 5) \, dx$$

$$= \left[\frac{1}{4}x^4 - x^3 + 5x \right]_0^3$$

$$= \left(\frac{81}{4} - 27 + 15 \right) - 0 = \frac{33}{4}$$

area below line

$$= 3 \times 5 = 15$$

shaded area

$$= 15 - \frac{33}{4}$$

$$= 6\frac{3}{4}$$

$$6 \quad \int_1^k (3 - 4x^{-2}) \, dx$$

$$= [3x + 4x^{-1}]_1^k$$

$$= \left(3k + \frac{4}{k}\right) - (3 + 4)$$

$$\therefore 3k + \frac{4}{k} - 7 = 6$$

$$3k^2 - 13k + 4 = 0$$

$$(3k - 1)(k - 4) = 0$$

$$k > 1 \quad \therefore k = 4$$

$$8 \quad \mathbf{a} \quad (2, 0)$$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$(2-x)^3$	8	$\frac{27}{8}$	1	$\frac{1}{8}$	0

$$\text{area} \approx \frac{1}{2} \times \frac{1}{2} \times [8 + 0 + 2(\frac{27}{8} + 1 + \frac{1}{8})]$$

$$= 4\frac{1}{4}$$

$$\mathbf{c} = 2^3 + 3(2^2)(-x) + 3(2)(-x)^2 + (-x)^3$$

$$= 8 - 12x + 6x^2 - x^3$$

$$\mathbf{d} \quad \text{area} = \int_0^2 (8 - 12x + 6x^2 - x^3) \, dx$$

$$= [8x - 6x^2 + 2x^3 - \frac{1}{4}x^4]_0^2$$

$$= (16 - 24 + 16 - 4) - 0$$

$$= 4$$

$$\therefore \% \text{ error} = \frac{4\frac{1}{4} - 4}{4} \times 100\% = 6.25\%$$