Arithmetic progression solutions

1) The n^{th} term of an arithmetic progression is

$$U_n = a + (n-1)d$$

$$\therefore \quad a + 11d = 32.5 \quad - \quad \{1\}$$

$$a + 19d = 52.5 \quad - \quad \{2\}$$

Now solve the equations {1} and {2} simultaneously

$$\{2\} - \{1\} \quad 8d = 20$$

$$d = 2.5$$

Substitute in {1}

$$a + 11 \times 2.5 = 32.5$$

$$\therefore \Rightarrow \quad a = 5$$

(a) First term, $a = 5$

(b) Common difference $d = 2.5$

(c) The sum of n terms $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore$ The sum of 18 terms $S_{18} = \frac{18}{2}\left[2 \times 5 + (18-1) \times 2.5\right]$

$$S_{18} = 472.5$$

2) (a) Firstly, find the first term and the common difference.
The n^{th} term of an arithmetic progression is

$$U_n = a + (n-1)d$$

$$\therefore \quad a + 4d = 6 \quad - \quad \{1\}$$

$$a + 14d = 36 \quad - \quad \{2\}$$

Solve equations {1} and {2} simultaneously

$$\{2\} - \{1\} \quad 10d = 30$$

$$d = 3$$

Substitute in {1}

$$a + 4 \times 3 = 6$$

$$a + 12 = 6$$

$$\therefore \quad a = -6$$

The 20^{th} term $U_{20} = -6 + (20-1) \times 3$

$$U_{20} = 51$$

(b) The sum of n terms $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

The sum of 20 terms $S_{20} = \frac{20}{2}\left[2 \times (-6) + 19 \times 3\right]$

$$S_{20} = 450$$

3) (a) The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$\therefore \quad S_{20} = \frac{20}{2}\left[2a+19d\right] = 325 \quad - \quad \{1\}$$

$$S_{30} = \frac{30}{2}\left[2a+29d\right] = 712.5 \quad - \quad \{2\}$$

$\{1\} \div 10$ $\quad 2a + 19d = 32.5 \quad - \quad \{3\}$

$\{2\} \div 15$ $\quad 2a + 29d = 47.5 \quad - \quad \{4\}$

$\{4\} - \{3\}$ $\quad 10d = 15$

$\div 10$ $\quad d = 1.5$

(b) Now find the first term, a

Substitute in $\{3\}$

$$2a + 19 \times 1.5 = 32.5$$
$$2a + 28.5 = 32.5$$
$$a = 2$$

$\therefore$ Sum of the first 50 terms,

$$\therefore \quad S_{50} = \frac{50}{2}\left[2\times 2+(50-1)\times 1.5\right]$$

$$S_{50} = 1937.5$$

4) (a) The n^{th} term of an arithmetic series is

$$U_n = a + (n-1)d$$

$$\therefore \quad a + 9d = -2 \quad - \quad \{1\}$$

The Sum of n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$\frac{30}{2}(2a+29d) = 105$$

$\div 15$ $\quad 2a + 29d = 7 \quad - \quad \{2\}$

$\{1\} \times 2$ $\quad 2a + 18d = -4 \quad - \quad \{3\}$

$\{2\} - \{3\}$ $\quad 11d = 11$

$$d = 1$$

(b) Firstly, find the first term

Substitute $d = 1$ in $\{1\}$

$$a + 9 \times 1 = -2$$

$$\Rightarrow \quad a = -11$$

The sum of the first 60 terms

$$S_{60} = \frac{60}{2}\left[2\times(-11)+59\times 1\right]$$

$$S_{60} = 1110$$

5) (a) The sum of the first 40 odd, positive integers is

$$S_{40} = 1 + 3 + 5 + 7 + \ldots. + 79$$

This is an arithmetic series, where $a = 1$, $d = 2$, $n = 40$

The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore \quad S_{40} = \frac{40}{2}\left[2\times 1 + (40-1)\times 2\right]$$

$$S_{40} = 1600$$

(b) The sum of the first n natural numbers is

$$S_n = 1 + 2 + 3 + \ldots + n$$

This is an arithmetic series where $a = 1$, $d = 1$

The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore \quad \frac{n}{2}\left(2\times 1 + (n-1)\times 1\right) = 15{,}400$$

$$\frac{n}{2}(n+1) = 15{,}400$$

$$\times 2 \quad n(n + 1) = 30{,}800$$

$$\therefore \quad n^2 + n - 30{,}800 = 0$$

$$(n - 175)(n + 176) = 0$$

$$\therefore \quad n - 175 = 0 \quad \text{or} \quad n + 176 = 0$$

$n = 175$ as n must be a positive integer

6) (a) The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$a = 1$

$$\therefore \quad \frac{20}{2}\left(2\times 1 + (20-1)d\right) = 1{,}540$$

$$10(2 + 19d) = 1{,}540$$

$$\div 10 \quad 2 + 19d = 154$$

$$19d = 152$$

$$\div 19 \quad d = 8$$

(b) The n^{th} term of an arithmetic series is

$$U_n = a + (n - 1)d$$

$$\therefore \ 30^{th} \text{ term } = \quad U_{30} = 1 + (30 - 1)8$$

$$U_{30} = 233$$

7) $a = -5 \quad d = -2 - (-5)$

$d = 3$

The sum of n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore \quad \frac{n}{2}\left(2\times(-5)+(n-1)\times 3\right) = 918$$

$$\frac{n}{2}\left(-10+3n-3\right) = 918$$

$$\frac{n}{2}\left(3n-13\right) = 918$$

$$\times 2 \quad n(3n-13) = 1836$$

$$3n^2 - 13n - 1836 = 0$$

$$\Rightarrow \quad (3n+68)(n-27) = 0$$

$$\Rightarrow \quad 3n + 68 = 0 \quad \text{or} \quad n - 27 = 0$$

$$n = -\frac{68}{3} \quad \text{or} \quad n = 27$$

As n must be a positive integer

27 terms are required so that the sum is 918

8) The sum of the first p terms of an arithmetic series is

$$S_n = \frac{p}{2}\left[2a + (p-1)d\right]$$

$a = 6$, $d = 3$

$$\therefore \quad \frac{p}{2}\left(2\times 6+(p-1)\times 3\right) = 5{,}130$$

$$\times 2 \quad p(3p+9) = 10{,}260$$

$$3p^2 + 9p - 10{,}260 = 0$$

$$\div 3 \quad p^2 + 3p - 3{,}420 = 0$$

$$(p-57)(p+60) = 0$$

$$p - 57 = 0 \quad \text{or} \quad p + 60 = 0$$

$$p = 57 \quad \text{or} \quad p = -60$$

But p is a positive whole number

$$\therefore \quad p = 57$$

9) (a) $22 + 18 + 14 + 10 + \ldots.$

is an arithmetic series because

$$18 - 22 = -4$$

$$14 - 18 = -4$$

$$10 - 14 = -4$$

This means that the common difference $d = -4$, and $a = 22$

The n^{th} term of the arithmetic series is

$$U_n = a + (n-1)d$$

the 30^{th} term $=$ $U_{30} = 22 + (30-1)\times(-4)$

$$= 22 - 29\times 4$$

$$= 22 - 116$$

$$U_{30} = -94$$

(b) The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

The sum of the first 50 terms

$$S_{50} = \frac{50}{2}\left[2\times 22+(50-1)\times(-4)\right]$$

$$= 25(44 - 49 \times 4)$$

$$S_{50} = -3{,}800$$

10) (a) $S_n = a + (a + d) + (a + 2d) + \ldots + (a + (n-1)d)$

Now reverse the order and add

$S_n = a + (n-1)d + (a + (n-2)d) + (a + (n-3)d) + \ldots + a$

$2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \ldots + (2a + (n-1)d)$

By reversing, each pair added has the same sum.

$\therefore \quad 2S_n = n(2a + (n-1)d)$

$\div 2 \quad S_n = \frac{n}{2}\left(2a+(n-1)d\right)$

(b) $a = -12$, $d = 2$

$\therefore$ For $S_n \geq 0$

$$\frac{n}{2}\left(2\times(-12)+(n-1)2\right) \geq 0$$

$$\frac{n}{2}(2n-26) \geq 0$$

$$\frac{2n}{n}(n-13) \geq 0$$

$$n(n-13) \geq 0$$

$\therefore$ As n is positive

$$n - 13 \geq 0$$

$$n \geq 13$$

$\therefore$ At least 13 terms must be added in order that the sum is positive.