Sequences

1Q

One of the following sequences is arithmetic, one is geometric and one is neither.

(i) $1,\ \frac{1}{8},\ -\frac{3}{4},\ -\frac{13}{8},\ldots.$

(ii) $1,\ -\frac{1}{8},\ \frac{3}{4},\ -\frac{13}{8},\ldots..$

(iii) $1,\ -\frac{1}{8},\ \frac{1}{64},\ -\frac{1}{512},\ldots.$

(a) Identify each type of sequence. [3]

(b) Find the sum of the first 6 terms of the arithmetic sequence. [2]

(c) Find the sum to infinity of the geometric sequence. [2]

2Q

Write down the r^{th} term of the sequence

$1,\ \frac{1}{3},\ \frac{1}{9},\ \frac{1}{27}.$

State whether it is convergent, divergent or oscillatory. [3]

3Q

Write down the first four terms of the sequence

$U_n = (-1)^n + \frac{1}{n+1}.$

State whether the sequence is convergent, divergent or oscillatory. [3]

4Q

Write down the first four terms of the sequence

$U_n = 7 - \frac{1}{n}.$

State whether the sequence is convergent, divergent or oscillatory. If it is convergent, to which value does it converge? [4]

5Q

The r^{th} term of a sequence is given by $a_r = 4r + 2$.

(a) Write down the first four terms and describe the sequence. [2]

(b) Evaluate $\sum_{r=1}^{12}(4r+2)$. [3]

(c) Given that $\sum_{r=1}^{n}(4r+2) = 966$, find n. [4]

6Q

Write down the first five terms of the sequence defined by

$U_{n+1} = 3 - U_n^2 \qquad U_1 = 2.$

(a) Is this sequence convergent, divergent, oscillatory or periodic? [4]

(b) If $U_1 = \frac{\sqrt{13}-1}{2}$, show that all the terms of the sequence are the same. [3]

7Q

Find the first five terms of the sequence which is defined by

$a_{n+1} = 6 + \frac{1}{4}a_n \qquad a_1 = 12.$

(a) State whether the sequence is convergent, divergent, oscillatory or periodic. [4]

(b) Show that $a_4 = \frac{1}{64}a_1 + k$, where k is a value to be found, for any value of a_1. [3]

8Q

A sequence is defined by

$a_{n+1} = a_n + 4 \qquad a_1 = 2.$

(a) Write down the first five terms of the sequence and describe its behaviour. [4]

(b) Evaluate $\sum_{r=1}^{25}(a_r + 4)$. [2]

9Q

A sequence is defined by

$u_{n+1} = -\frac{1}{2}u_n \qquad u_1 = 4.$

(a) Write down the first five terms of the sequence and describe its behaviour. [4]

(b) Evaluate $\sum_{r=1}^{10}u_r$. [2]

10Q

Write down the first four terms of the sequences below. State whether they are convergent, divergent, oscillatory or periodic. If the sequence is convergent, state the value to which it converges.

(a) $t_n = 2n + 5$ [3]

(b) $t_n = (-1)^n + \frac{1}{n+2}$ [3]

(c) $t_n = 6 - \frac{4}{n}$ [3]

(d) $t_{n+1} = t_n^2 - 1, \quad t_1 = 0 \quad n \geq 1$ [3]