

### Quadratic functions and their graphs

1Q

- (a) Show that  $x^2 + 4x + 7 = (x + 2)^2 + a$  where  $a$  is to be determined. [2]  
(b) Sketch the graph of  $y = x^2 + 4x + 7$ , giving the equation of the axis of symmetry and the co-ordinates of its vertex [4]

2Q

A quadratic function is defined by  $f(x) = x^2 + 8x - 3$

- (a) Find values of  $p$  and  $q$  such that  $f(x) = (x - p)^2 + q$  [3]  
(b) Hence find the value of  $x$  for which  $f(x)$  has a minimum value. State this minimum value. [2]

3Q

- (a) Write  $x^2 + 4x + 14$  in the form  $(x + p)^2 + q$ , where  $p$  and  $q$  are integers to be found [2]  
(b) Find the minimum value of  $x^2 + 4x + 14$ . State the value of  $x$  for which this minimum occurs. [2]  
(c) Find the value of the constant  $k$  for which  $x^2 + 4x + k = 0$  has equal roots [2]

4Q

Given a quadratic function  $f(x) = 12 - 8x - x^2$

- (a) Find values of  $p$  and  $q$  such that  $f(x) = q - (x + p)^2$  [2]  
(b) Hence find the value of  $x$  for which  $f(x)$  has a maximum value. State this maximum value [2]  
(c) Find the minimum value of  $\frac{1}{12 - 8x - x^2}$  [1]

5Q

- (a) Determine the values of  $a$  and  $b$  such that  $x^2 - 8x - 1 \equiv (x + a)^2 + b$ . Hence find the minimum value of  $x^2 - 8x - 1$  and state the value of  $x$  for which this occurs. [4]  
(b) Write down the maximum value of  $\frac{1}{x^2 - 8x - 1}$  [1]  
(c) Sketch the graph of  $y = x^2 - 8x - 1$ . On your graph mark the co-ordinates of the points where the curve cuts the co-ordinate axes. What is the equation of the line of symmetry of the curve? [3]

6Q

Ashana and Bill cycle 28 miles along a cycle track. On average Ashana cycles 2mph faster than Bill, and completes the journey 20 minutes before Bill. What was the average speed of each cyclist? [5]

7Q

$(k - 1)x^2 - 2(2k + 1)x + 4k + 9 = 0$  is a quadratic equation in  $x$ . If this equation has equal roots find the value of  $k$ . [4]

8Q

A quadratic function is defined by  $f(x) = 5 - 8x - x^2$

(a) Find values of  $\alpha$  and  $\beta$  such that  $f(x) = \alpha - (x + \beta)^2$  [3]

(b) Hence find the value of  $x$  for which  $f(x)$  has a maximum value. State this maximum value. [2]

(c) Sketch the graph of  $y = 5 - 8x - x^2$  giving the equation of the axis of symmetry, the co-ordinates of the points where the curve crosses the co-ordinate axes, and the co-ordinates of the vertex. [5]

9Q

(a) Find the values of  $a$  for which the quadratic equation  $(3a - 2)x^2 + 2ax + 1 = 0$  has equal roots. [4]

(b) Show that the equation  $25x^2 - 10x + 1 = 0$  has equal roots. Solve the equation and sketch the curve  $y = 25x^2 - 10x + 1$  [3]

10Q

(a) By completing the square, find the maximum value of the quadratic function  $5 + 8x - 2x^2$ . State the value of  $x$  at which this occurs. [4]

(b) Sketch the function  $y = 5 + 8x - 2x^2$ , giving the equation of the axis of symmetry of the curve. Show on your sketch the co-ordinates of the points where the curve cuts the co-ordinate axes. [3]

(c) Write down the minimum value of  $\frac{1}{5 + 8x - 2x^2}$  [1]