1 Find the quotient obtained in dividing

a $(x^3 + 2x^2 - x - 2)$ by $(x + 1)$
b $(x^3 + 2x^2 - 9x + 2)$ by $(x - 2)$
c $(20 + x + 3x^2 + x^3)$ by $(x + 4)$
d $(2x^3 - x^2 - 4x + 3)$ by $(x - 1)$
e $(6x^3 - 19x^2 - 73x + 90)$ by $(x - 5)$
f $(-x^3 + 5x^2 + 10x - 8)$ by $(x + 2)$
g $(x^3 - 2x + 21)$ by $(x + 3)$
h $(3x^3 + 16x^2 + 72)$ by $(x + 6)$

2 Find the quotient and remainder obtained in dividing

a $(x^3 + 8x^2 + 17x + 16)$ by $(x + 5)$
b $(x^3 - 15x^2 + 61x - 48)$ by $(x - 7)$
c $(3x^3 + 4x^2 + 7)$ by $(2 + x)$
d $(-x^3 - 5x^2 + 15x - 50)$ by $(x + 8)$
e $(4x^3 + 2x^2 - 16x + 3)$ by $(x - 3)$
f $(1 - 22x^2 - 6x^3)$ by $(x + 2)$

3 Use the factor theorem to determine whether or not

a $(x - 1)$ is a factor of $(x^3 + 2x^2 - 2x - 1)$
b $(x + 2)$ is a factor of $(x^3 - 5x^2 - 9x + 2)$
c $(x - 3)$ is a factor of $(x^3 - x^2 - 14x + 27)$
d $(x + 6)$ is a factor of $(2x^3 + 13x^2 + 2x - 24)$
e $(2x + 1)$ is a factor of $(2x^3 - 5x^2 + 7x - 14)$
f $(3x - 2)$ is a factor of $(2 - 17x + 25x^2 - 6x^3)$

4
$$f(x) \equiv x^3 - 2x^2 - 11x + 12.$$

a Show that $(x - 1)$ is a factor of $f(x)$.
b Hence, express $f(x)$ as the product of three linear factors.

5
$$g(x) \equiv 2x^3 + x^2 - 13x + 6.$$

Show that $(x + 3)$ is a factor of $g(x)$ and solve the equation $g(x) = 0$.

6
$$f(x) \equiv 6x^3 - 7x^2 - 71x + 12.$$

Given that $f(4) = 0$, find all solutions to the equation $f(x) = 0$.

7
$$g(x) \equiv x^3 + 7x^2 + 7x - 6.$$

Given that $x = -2$ is a solution to the equation $g(x) = 0$,

a express $g(x)$ as the product of a linear factor and a quadratic factor,
b find, to 2 decimal places, the other two solutions to the equation $g(x) = 0$.

8
$$f(x) \equiv x^3 + 2x^2 - 11x - 12.$$

a Evaluate $f(1)$, $f(2)$, $f(-1)$ and $f(-2)$.
b Hence, state a linear factor of $f(x)$ and fully factorise $f(x)$.

9 By first finding a linear factor, fully factorise

a $x^3 - 2x^2 - 5x + 6$
b $x^3 + x^2 - 5x - 2$
c $20 + 11x - 8x^2 + x^3$
d $3x^3 - 4x^2 - 35x + 12$
e $x^3 + 8$
f $12 + 29x + 8x^2 - 4x^3$

10 Solve each equation, giving your answers in exact form.

a $x^3 - x^2 - 10x - 8 = 0$
b $x^3 + 2x^2 - 9x - 18 = 0$
c $4x^3 - 12x^2 + 9x = 2$
d $x^3 - 5x^2 + 3x + 1 = 0$
e $x^2(x + 4) = 3(3x + 2)$
f $x^3 - 14x + 15 = 0$

11 $$f(x) \equiv 2x^3 - x^2 - 15x + c.$$

Given that $(x - 2)$ is a factor of $f(x)$,

a find the value of the constant c,

b fully factorise $f(x)$.

12 $$g(x) \equiv x^3 + px^2 - 13x + q.$$

Given that $(x + 1)$ and $(x - 3)$ are factors of $g(x)$,

a show that $p = 3$ and find the value of q,

b solve the equation $g(x) = 0$.

13 Use the remainder theorem to find the remainder obtained in dividing

a $(x^3 + 4x^2 - x + 6)$ by $(x - 2)$

b $(x^3 - 2x^2 + 7x + 1)$ by $(x + 1)$

c $(2x^3 + x^2 - 9x + 17)$ by $(x + 5)$

d $(8x^3 + 4x^2 - 6x - 3)$ by $(2x - 1)$

e $(2x^3 - 3x^2 - 20x - 7)$ by $(2x + 1)$

f $(3x^3 - 6x^2 + 2x - 7)$ by $(3x - 2)$

14 Given that when $(x^3 - 4x^2 + 5x + c)$ is divided by $(x - 2)$ the remainder is 5, find the value of the constant c.

15 Given that when $(2x^3 - 9x^2 + kx + 5)$ is divided by $(2x - 1)$ the remainder is -2, find the value of the constant k.

16 Given that when $(2x^3 + ax^2 + 13)$ is divided by $(x + 3)$ the remainder is 22,

a find the value of the constant a,

b find the remainder when $(2x^3 + ax^2 + 13)$ is divided by $(x - 4)$.

17 $$f(x) \equiv px^3 + qx^2 + qx + 3.$$

Given that $(x + 1)$ is a factor of $f(x)$,

a find the value of the constant p.

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is 15,

b find the value of the constant q.

18 $$p(x) \equiv x^3 + ax^2 + 9x + b.$$

Given that $(x - 3)$ is a factor of $p(x)$,

a find a linear relationship between the constants a and b.

Given also that when $p(x)$ is divided by $(x + 2)$ the remainder is -30,

b find the values of the constants a and b.

19 $$f(x) \equiv 4x^3 - 6x^2 + mx + n.$$

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 3 and that when $f(x)$ is divided by $(2x - 1)$ the remainder is 15, find the values of the constants m and n.

20 $$g(x) \equiv x^3 + cx + 3.$$

Given that when $g(x)$ is divided by $(x - 4)$ the remainder is 39,

a find the value of the constant c,

b find the quotient and remainder when $g(x)$ is divided by $(x + 2)$.

1 $$\mathrm{f}(x) \equiv x^3 - 5x^2 + ax + b.$$

Given that $(x + 2)$ and $(x - 3)$ are factors of $\mathrm{f}(x)$,

a show that $a = -2$ and find the value of b.

b Hence, express $\mathrm{f}(x)$ as the product of three linear factors.

2 $$\mathrm{f}(x) \equiv 8x^3 - x^2 + 7.$$

The remainder when $\mathrm{f}(x)$ is divided by $(x - k)$ is eight times the remainder when $\mathrm{f}(x)$ is divided by $(2x - k)$.

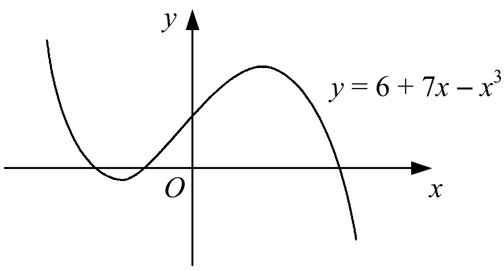
Find the two possible values of the constant k.

3 $$\mathrm{f}(x) \equiv 3x^3 - x^2 - 12x + 4.$$

a Show that $(x - 2)$ is a factor of $\mathrm{f}(x)$.

b Solve the equation $\mathrm{f}(x) = 0$.

4



The diagram shows the curve with the equation $y = 6 + 7x - x^3$.

Find the coordinates of the points where the curve crosses the x-axis.

5 $$\mathrm{f}(x) \equiv 3x^3 + px^2 + 8x + q.$$

When $\mathrm{f}(x)$ is divided by $(x + 1)$ there is a remainder of -4.

When $\mathrm{f}(x)$ is divided by $(x - 2)$ there is a remainder of 80.

a Find the values of the constants p and q.

b Show that $(x + 2)$ is a factor of $\mathrm{f}(x)$.

c Solve the equation $\mathrm{f}(x) = 0$.

6 **a** Solve the equation

$$x^3 - 4x^2 - 7x + 10 = 0.$$

b Hence, solve the equation

$$y^6 - 4y^4 - 7y^2 + 10 = 0.$$

7 $$\mathrm{f}(n) \equiv n^3 + 7n^2 + 14n + 3.$$

a Find the remainder when $\mathrm{f}(n)$ is divided by $(n + 1)$.

b Express $\mathrm{f}(n)$ in the form

$$\mathrm{f}(n) \equiv (n + 1)(n + a)(n + b) + c,$$

where a, b and c are integers.

c Hence, show that $\mathrm{f}(n)$ is odd for all positive integer values of n.

1 $$f(x) \equiv x^3 + x^2 - 22x - 40.$$

a Show that $(x + 2)$ is a factor of $f(x)$. **(2)**

b Express $f(x)$ as the product of three linear factors. **(4)**

c Solve the equation $f(x) = 0$. **(1)**

2 $$f(x) \equiv x^3 - 2x^2 + kx + 1.$$

Given that the remainder when $f(x)$ is divided by $(x - 2)$ and the remainder when $f(x)$ is divided by $(x + 3)$ are equal,

a find the value of the constant k, **(4)**

b find the remainder when $f(x)$ is divided by $(x + 2)$. **(2)**

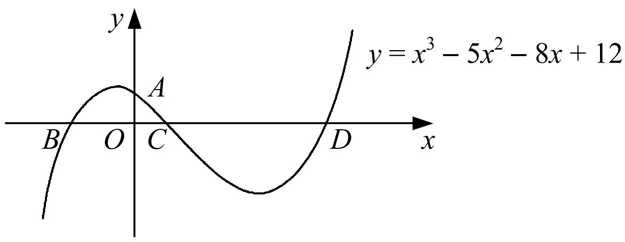
3 The polynomial $p(x)$ is defined by

$$p(x) \equiv 2x^3 - 9x^2 - 2x + 11.$$

a Find the remainder when $p(x)$ is divided by $(x + 2)$. **(2)**

b Find the quotient and remainder when $p(x)$ is divided by $(x - 4)$. **(3)**

4



The diagram shows the curve with the equation $y = x^3 - 5x^2 - 8x + 12$.

a State the coordinates of the point A where the curve crosses the y-axis. **(1)**

The curve crosses the x-axis at the points B, C and D. Given that C has coordinates $(1, 0)$,

b find the coordinates of the points B and D. **(6)**

5 $$f(x) \equiv x^3 - 3x^2 + kx + 8.$$

Given that $(x - 1)$ is a factor of $f(x)$,

a find the value of k, **(2)**

b solve the equation $f(x) = 0$. **(5)**

6 Solve the equation

$$2x^3 + x^2 - 13x + 6 = 0.$$ **(7)**

7 The polynomial $p(x)$ is defined by

$$p(x) \equiv bx^3 + ax^2 - 10x + b,$$

where a and b are constants.

Given that when $p(x)$ is divided by $(x + 1)$ the remainder is 3,

a find the value of a. **(2)**

Given also that when $p(x)$ is divided by $(3x - 1)$ the remainder is -1,

b find the value of b. **(3)**

8 $\mathrm{f}(x) \equiv x^3 - 7x^2 + x + 10.$

a Find the remainder when $\mathrm{f}(x)$ is divided by $(x + 1)$. **(2)**

b Hence, or otherwise, solve the equation $\mathrm{f}(x) = 1$, giving your answers in exact form. **(6)**

9 $\mathrm{f}(x) \equiv 3x^3 + kx^2 - 7x + 2k.$

When $\mathrm{f}(x)$ is divided by $(3x - 2)$ the remainder is 6.

Find the value of the constant k. **(3)**

10 $\mathrm{f}(x) \equiv 2x^3 - 7x^2 + 4x - 3.$

a Show that $(x - 3)$ is a factor of $\mathrm{f}(x)$. **(2)**

b Hence, express $\mathrm{f}(x)$ as the product of a linear factor and a quadratic factor. **(3)**

c Show that there is only one real solution to the equation $\mathrm{f}(x) = 0$. **(3)**

11 The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) \equiv x^3 + px + q,$$

where p and q are constants.

Given that $(x - 2)$ is a factor of $\mathrm{f}(x)$,

a find an expression for q in terms of p. **(2)**

Given also that when $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is -15,

b find the values of p and q. **(4)**

12 $\mathrm{f}(x) \equiv x^3 + 4x^2 - 9.$

Given that $x = -3$ is a solution to the equation $\mathrm{f}(x) = 0$, find the other two solutions correct to 2 decimal places. **(6)**

13 $\mathrm{f}(x) \equiv (x + k)^3 - 8.$

Given that when $\mathrm{f}(x)$ is divided by $(x + 2)$ the remainder is -7,

a find the value of the constant k, **(3)**

b show that $(x + 1)$ is a factor of $\mathrm{f}(x)$. **(2)**

14 $\mathrm{f}(x) \equiv x^3 - 4x^2 - 7x + 8.$

a Find the remainder when $\mathrm{f}(x)$ is divided by $(x + 2)$. **(2)**

Given that

$$\mathrm{g}(x) \equiv \mathrm{f}(x) + c,$$

and that $(x + 2)$ is a factor of $\mathrm{g}(x)$,

b state the value of the constant c, **(1)**

c solve the equation $\mathrm{g}(x) = 0$. **(4)**

15 $\mathrm{f}(x) \equiv x^3 - 4x + 1.$

Given that when $\mathrm{f}(x)$ is divded by $(2x - k)$, where k is a constant, the remainder is 4,

a show that $k^3 - 16k - 24 = 0$. **(3)**

Given also that when $\mathrm{f}(x)$ is divided by $(x + k)$ the remainder is 1,

b find the value of k. **(3)**