C3 EXPONENTIALS AND LOGARITHMS Worksheet A

1 Find, to 3 significant figures, the value of

a e^3 **b** e^{-2} **c** $5e$ **d** $\ln 0.55$ **e** $\frac{3}{7}\ln 100$ **f** $\log_{10} e$

2 Without using a calculator, find the value of

a $e^{\ln 4}$ **b** $e^{\frac{1}{2}\ln 9}$ **c** $2e^{-\ln 6}$ **d** $\ln e^7$ **e** $\ln \frac{1}{e}$ **f** $5\ln e^{-0.1}$

3 Find the value of x in each case.

a $e^{\ln x} = 4$ **b** $\ln e^x = 17$ **c** $e^{2\ln x} = 25$ **d** $e^{-\ln x} = \frac{1}{3}$

4 Solve each equation, giving your answers in terms of e.

a $\ln x = 15$ **b** $\frac{1}{2}\ln t - 3 = 0$ **c** $\ln (x - 4) = 7$

d $17 - \ln 5y = 9$ **e** $\ln (\frac{1}{2}x + 3) = 2.5$ **f** $\ln (4 - 3x) - 11 = 0$

5 Solve each equation, giving your answers in terms of natural logarithms.

a $e^x = 0.7$ **b** $9 - 2e^y = 5$ **c** $e^{5x} - 3 = 0$

d $e^{4t+1} = 12$ **e** $\frac{1}{2}e^{2x-3} - 7 = 0$ **f** $2e^{4-5x} + 9 = 16$

6 Solve each equation, giving your answers to 2 decimal places.

a $\frac{1}{3}e^x = 4$ **b** $\ln (15x - 7) = 4$ **c** $4e^{\frac{1}{2}y+3} = 11$

d $\frac{3}{7}\ln (5 - 2x) - 1 = 0$ **e** $\ln (10 - 3y) - e = 0$ **f** $\ln x^2 + \ln x^3 = 19$

g $e^{2x} = 3e^{-\frac{1}{4}x}$ **h** $e^{5t} = 4e^{2t+1}$ **i** $\ln (2x - 5) - \ln x = \frac{1}{4}$

7 Find, in exact form, the solutions to the equation

$$2e^{2x} + 12 = 11e^x.$$

8 **a** Simplify

$$\frac{3x^2 - 10x + 8}{x^2 - 5x + 6}.$$

b Hence, solve the equation

$$\ln (3x^2 - 10x + 8) - \ln (x^2 - 5x + 6) = \ln 2x.$$

9 Solve the following simultaneous equations, giving your answers to 2 decimal places.

$$e^{5y} - x = 0$$
$$\ln x^4 = 7 - y$$

10 Sketch each pair of curves on the same diagram, showing the coordinates of any points of intersection with the coordinate axes.

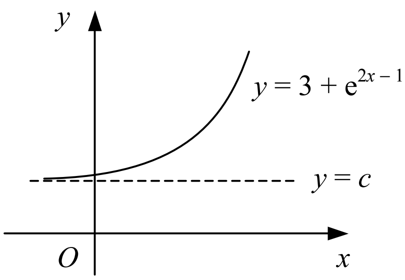
a $y = e^x$, $y = e^{-2x}$ **b** $y = 2e^x$, $y = e^{x-1}$ **c** $y = 2 + e^x$, $y = e^{2x+1}$

d $y = e^x$, $y = \ln x$ **e** $y = -\ln x$, $y = 2 + \ln x$ **f** $y = \ln (x - 2)$, $y = \ln 3x$

11 **a** Sketch on the same diagram the curves $y = \ln(x + 1)$ and $y = 1 + \ln x$.

b Show that the x-coordinate of the point where the two curves intersect is $\frac{1}{e - 1}$.

12



The diagram shows the curve with the equation $y = 3 + e^{2x-1}$ and the asymptote of the curve which has the equation $y = c$.

a State the value of the constant c.

b Find the exact coordinates of the point where the curve crosses the y-axis.

c Find the x-coordinate of the point on the curve where $y = 7$, giving your answer in the form $a + \ln b$, where a is rational and b is an integer.

13 A quantity N is decreasing such that at time t

$$N = 50e^{-0.2t}.$$

a Find the value of N when $t = 10$.

b Find the value of t when $N = 3$.

14 A radioactive substance is decaying such that its mass, m grams, at a time t years after initial observation is given by

$$m = 240e^{kt},$$

where k is a constant.

Given that when $t = 180$, $m = 160$, find

a the value of k,

b the time it takes for the mass of the substance to be halved.

15 A quantity N is increasing such that at time t

$$N = 20e^{0.04t}.$$

a Find the value of N when $t = 15$.

b Find, in terms of the constant k, expressions for the value of t when

i $N = k$,

ii $N = 2k$.

c Hence, show that the time it takes for the value of N to double is constant.

16 A quantity N is decreasing such that at time t

$$N = N_0e^{kt}.$$

Given that at time $t = 10$, $N = 300$ and that at time $t = 20$, $N = 225$, find

a the values of the constants N_0 and k,

b the value of t when $N = 150$.

1 A radioactive substance is decaying such that its mass, m grams, at a time t years after initial observation is given by

$$m = 60e^{kt},$$

where k is a constant.

Given that when $t = 100$, $m = 42$,

a find the value of k, **(3)**

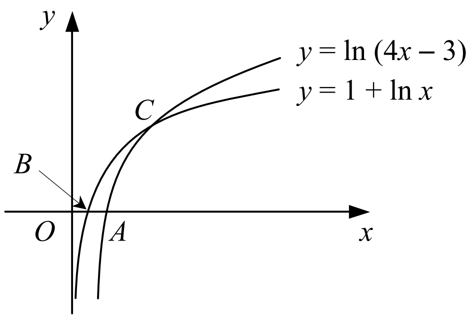
b find the value of t when $m = 30$. **(2)**

2 Solve each equation, giving your answers correct to 2 decimal places.

a $e^{2x} - 5.7e^{-x} = 0$ **(3)**

b $\ln x - \ln (x - 1) = \frac{1}{2}$ **(4)**

3



The diagram shows the curves $y = \ln (4x - 3)$ and $y = 1 + \ln x$ which cross the x-axis at the points A and B respectively.

a Find the coordinates of A and B. **(4)**

The two curves intersect at the point C.

b Find the exact x-coordinate of C, giving your answer in terms of e. **(4)**

4 Find, as natural logarithms, the roots of the equation

$$2e^x + 3e^{-x} = 7.$$ **(5)**

5 A scientist carries out an experiment to investigate the growth of a population of flies. She introduces a colony of flies into a closed environment and uses the model that after t days the number of flies in the environment, N, is given by

$$N = 800e^{0.01t}.$$

Find, according to this model,

a the number of flies introduced into the environment, **(1)**

b the size of the population after 20 days, **(2)**

c the least number of days after which the population will exceed 2000. **(3)**

6 $$f(x) = 1 + e^{2x + 1}.$$

a Solve the equation $f(x) = 10$, giving your answer in the form $a + \ln b$ where a is rational and b is an integer. **(3)**

b Find, to 3 significant figures, the x-coordinate of the point where the curve $y = f(x)$ intersects the curve $y = 3 - e^x$. **(5)**

7 Giving your answers in exact form, solve the equations

a $\ln(4x - 1) = 2$, **(3)**

b $7 - e^{1-3y} = 0$. **(3)**

8 At time $t = 0$, there are 800 bacteria present in a culture. The number of bacteria present at time t hours is modelled by the continuous variable N and the relationship

$$N = ae^{bt},$$

where a and b are constants.

a Write down the value of a. **(1)**

Given that when $t = 2$, $N = 7200$,

b find the value of b in the form $\ln k$, **(3)**

c find, to the nearest minute, how long it takes for the number of bacteria present to double. **(4)**

9 **a** Simplify

$$\frac{x^2 - 4x + 3}{x^2 + x - 2}.$$ **(3)**

b Solve the equation

$$\ln(x^2 - 4x + 3) = 1 + \ln(x^2 + x - 2),$$

giving your answer in terms of e. **(4)**

10 Giving your answers to an appropriate degree of accuracy, solve the simultaneous equations

$$e^y + 5 - 9x = 0$$

$$y - \ln(x + 4) = 2$$ **(7)**

11 **a** Describe fully the single transformation which maps the graph of $y = e^x$ onto the graph of $y = e^{-x}$. **(1)**

b Sketch the graphs of $y = e^{-x}$ and $y = e^{3x+1}$ on the same diagram, showing the coordinates of any points of intersection with the coordinate axes. **(4)**

c Find the exact coordinates of the point of intersection of the two graphs. **(3)**

12 **a** Given that $t = \ln x$, find expressions in terms of t for

i $\ln\sqrt{x}$,

ii $\ln(e^2x)$. **(4)**

b Hence, or otherwise, solve the equation

$$5 + \ln\sqrt{x} = \ln(e^2x).$$ **(3)**

13 A bead is projected vertically upwards in a jar of liquid with a velocity of 13 m s^{-1}. Its velocity, v m s^{-1}, at time t seconds after projection, is given by

$$v = ce^{-kt} - 2.$$

a Find the value of c. **(2)**

Given that the bead has a velocity of 7 m s^{-1} after 5.1 seconds, find

b the value of k correct to 4 decimal places, **(3)**

c the time taken for its velocity to decrease from 10 m s^{-1} to 4 m s^{-1}. **(5)**