

1 a  $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$

$$a \times \left(\frac{3}{4}\right)^2 = 27$$

$$a = \frac{16}{9} \times 27 = 48$$

b  $S_{\infty} = \frac{48}{1 - \frac{3}{4}} = 192$

2 a  $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$

$$(k+4)^2 = (3k+2)(k-8)$$

$$k^2 - 15k - 16 = 0$$

$$(k+1)(k-16) = 0$$

$$k > 0 \therefore k = 16$$

b  $u_1 = 8, u_2 = 20 \therefore a = 8, r = \frac{5}{2}$

$$u_6 = 8 \times \left(\frac{5}{2}\right)^5 = 781\frac{1}{4}$$

c  $S_{10} = \frac{8\left[\left(\frac{5}{2}\right)^{10} - 1\right]}{\frac{5}{2} - 1} = 50\,857.3$

3 a  $ar = 75, ar^4 = 129.6$   
 $r^3 = 129.6 \div 75 = 1.728$

$$r = \sqrt[3]{1.728} = 1.2$$

$$a = 75 \div 1.2 = 62.5$$

b  $u_{10} = 62.5 \times (1.2)^9 = 322.5$

c  $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 a  $S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$   
 $rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$

subtracting,

$$S_n - rS_n = a - ar^n$$

$$(1 - r)S_n = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

b  $\frac{2[1 - (\sqrt{2})^n]}{1 - \sqrt{2}} = 126(\sqrt{2} + 1)$

$$1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$$

$$1 - (\sqrt{2})^n = 63(1 - 2)$$

$$(\sqrt{2})^n = 64$$

$$2^{\frac{1}{2}n} = 2^6$$

$$n = 12$$

5 a  $\frac{18}{1-r} = 15$

$$\therefore 1 - r = \frac{18}{15} = 1.2$$

$$r = -0.2$$

b  $u_3 = 18 \times (-0.2)^2 = 0.72$

c  $S_8 = \frac{18[1 - (-0.2)^8]}{1 - (-0.2)} = 14.9999616$

$$S_{\infty} - S_8 = 0.000\,0384$$

6 a  $S_3 = 5(3^3 - 1) = 130$

$$S_2 = 5(3^2 - 1) = 40$$

$$u_3 = S_3 - S_2 = 90$$

b  $S_{n-1} = 5(3^{n-1} - 1)$

$$u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$$

$$= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$$

7 a  $4 \times (1.25)^7 = 19.1 \text{ mm (3sf)}$

b GP:  $a = 4, r = 1.25$

$$S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8 \text{ mm}$$

$$\therefore \text{length} = 1.37 \text{ m (3sf)}$$

8 a  $ar = 30, ar^3 = 2.7 \therefore r^2 = 2.7 \div 30 = 0.09$   
 $r > 0 \therefore r = \sqrt{0.09} = 0.3$

$$a = 30 \div 0.3 = 100$$

b  $S_{\infty} = \frac{100}{1 - 0.3} = 142.9 \text{ (1dp)}$

9 a GP:  $a = 27, r = 3$

$$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$$

b  $\sum_{r=1}^{15} 2^r$ : GP,  $a = 2, r = 2$

$$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$$

$\sum_{r=1}^{15} 12r$ : AP,  $a = 12, d = 12$

$$S_{15} = \frac{15}{2} [24 + (14 \times 12)] = 1440$$

$$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$$

11 a  $u_8 = 4 \times \left(\frac{1}{2}\right)^7 = \frac{1}{32}$

b  $u_n = 4 \times \left(\frac{1}{2}\right)^{n-1}$

$$= 2^2 \times 2^{1-n}$$

$$= 2^{3-n}$$

c  $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$$= 8(1 - 2^{-n})$$

$$= 8 - (2^3 \times 2^{-n})$$

$$= 8 - 2^{3-n}$$

13 a  $a + ar^2 = a(1 + r^2) = 150$   
 $ar + ar^3 = ar(1 + r^2) = -75$   
 $\therefore r = -75 \div 150 = -\frac{1}{2}$

$$a = 150 \div \frac{5}{4} = 120$$

b  $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

10 a  $a = 64, ar^2 - ar = 20$

$$\therefore 64r^2 - 64r = 20$$

$$16r^2 - 16r - 5 = 0$$

b  $(4r + 1)(4r - 5) = 0$

$$r = -\frac{1}{4} \text{ or } \frac{5}{4}$$

c  $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times \left(-\frac{1}{4}\right)^3 = -1$

$$r = \frac{5}{4} \Rightarrow u_4 = 64 \times \left(\frac{5}{4}\right)^3 = 125$$

d  $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

12 a  $u_6 = 4 \times 3^6 = 2916$

b GP:  $a = 12, r = 3$

$$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$$

$$\therefore 6(3^t - 1) > 10^{25}$$

$$3^t > \frac{10^{25}}{6} + 1$$

$$t \lg 3 > \lg \left( \frac{10^{25}}{6} + 1 \right)$$

$$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$$

$$t > 50.8 \therefore \text{smallest } t = 51$$

14 a  $b - a = (3a + 4) - b$   
 $2b = 4a + 4$

$$b = 2a + 2$$

b  $\frac{2a+2}{a} = \frac{6a+1}{2a+2}$

$$(2a + 2)^2 = a(6a + 1)$$

$$2a^2 - 7a - 4 = 0$$

$$(2a + 1)(a - 4) = 0$$

$$a \text{ integer } \therefore a = 4$$

$$\text{sub. } b = 10$$

15 a after 4<sup>th</sup> bounce,  
reaches  $3 \times (0.6)^4 = 0.3888$  m

b total distance

$$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \dots]$$

$$= h + 2 \times S_\infty \text{ of GP, } a = 0.6h, r = 0.6$$

$$= h + \frac{2 \times 0.6h}{1 - 0.6}$$

$$= h + 3h = 4h \text{ metres}$$