1 $\mathrm{f}: x \to 3x - 5,\ x \in \mathbb{R}$ $\quad \mathrm{g}: x \to \frac{4}{6-x},\ x \in \mathbb{R},\ x \neq 6$ $\quad \mathrm{h}: x \to x^2 + 4x - 1,\ x \in \mathbb{R}$

Find the value of

a f(3) **b** g(4) **c** h(2) **d** f(1) **e** h(–1) **f** g(8)

g g(–4) **h** $\mathrm{f}(\frac{2}{3})$ **i** $\mathrm{h}(\frac{1}{2})$ **j** f(–1) **k** h(–3) **l** $\mathrm{g}(1\frac{2}{3})$

2 $\mathrm{f}: x \to \ln(2 - 5x),\ x \in \mathbb{R},\ x < 0.4$ $\quad \mathrm{g}: x \to \sin(2x + \frac{\pi}{3}),\ x \in \mathbb{R}$ $\quad \mathrm{h}: x \to 3 + 2\mathrm{e}^{1-x},\ x \in \mathbb{R}$

Find, correct to 3 significant figures where appropriate, the value of

a $\mathrm{g}(\frac{\pi}{3})$ **b** f(0) **c** h(1) **d** $\mathrm{g}(\frac{\pi}{6})$ **e** h(2) **f** $\mathrm{f}(-\frac{1}{2})$

g h(–0.8) **h** f(0.2) **i** g(0.3) **j** $\mathrm{h}(\frac{2}{3})$ **k** g(–1) **l** $\mathrm{f}(-\frac{3}{4})$

3 Sketch each function and state its range.

a $\mathrm{f}: x \to 2x + 1,\ x \in \mathbb{R},\ 0 \le x \le 7$

b $\mathrm{f}: x \to 3x - 2,\ x \in \mathbb{R},\ x \ge 0$

c $\mathrm{f}: x \to 5 - x,\ x \in \mathbb{R},\ -5 \le x \le 5$

d $\mathrm{f}: x \to 4 - 7x,\ x \in \mathbb{R}$

e $\mathrm{f}: x \to x^2,\ x \in \mathbb{R},\ -3 < x < 3$

f $\mathrm{f}: x \to x^2 + 3,\ x \in \mathbb{R}$

g $\mathrm{f}: x \to x^2 - 6,\ x \in \mathbb{R},\ x \ge 0$

h $\mathrm{f}: x \to (x - 1)^2,\ x \in \mathbb{R},\ -2 \le x \le 4$

i $\mathrm{f}: x \to (x + 2)^2,\ x \in \mathbb{R}$

j $\mathrm{f}: x \to 4 - x^2,\ x \in \mathbb{R}$

k $\mathrm{f}: x \to x^3,\ x \in \mathbb{R},\ -10 < x \le 10$

l $\mathrm{f}: x \to -x^3,\ x \in \mathbb{R}$

4 Sketch each function and state its range.

a $\mathrm{f}: x \to x^2 + 2x - 8,\ x \in \mathbb{R}$

b $\mathrm{f}: x \to \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

c $\mathrm{f}: x \to \frac{1}{x^2},\ x \in \mathbb{R},\ x \neq 0$

d $\mathrm{f}: x \to \cos x,\ x \in \mathbb{R},\ 0 \le x \le 2\pi$

e $\mathrm{f}: x \to 5^x,\ x \in \mathbb{R}$

f $\mathrm{f}: x \to \tan x,\ x \in \mathbb{R},\ -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

5 Find the domain of each function given its range.

a $\mathrm{f}: x \to x - 1,\ \mathrm{f}(x) \in \mathbb{R},\ -1 \le \mathrm{f}(x) < 6$

b $\mathrm{f}: x \to 4 - 3x,\ \mathrm{f}(x) \in \mathbb{R},\ \mathrm{f}(x) \le 4$

c $\mathrm{f}: x \to x^3,\ \mathrm{f}(x) \in \mathbb{R},\ 0 \le \mathrm{f}(x) \le 125$

d $\mathrm{f}: x \to \frac{1}{x},\ \mathrm{f}(x) \in \mathbb{R},\ 2 < \mathrm{f}(x) < 10$

6 Given that for $x \in \mathbb{R}$, $\mathrm{f}(x) \equiv 4x + 3$, $\mathrm{g}(x) \equiv x^2 - 7$ and $\mathrm{h}(x) \equiv \frac{9}{x+2},\ x \neq -2$, solve the equations

a $\mathrm{f}(x) = 9$ **b** $\mathrm{g}(x) = 18$ **c** $\mathrm{h}(x) = 6$

d $\mathrm{f}(x) = \mathrm{h}(x)$ **e** $\mathrm{g}(x) - \frac{1}{\mathrm{h}(x)} = -6\frac{1}{3}$ **f** $\mathrm{f}(x) + \mathrm{g}(x) = 0$

7 Express each function in the form indicated and hence, state its range.

a $\mathrm{f}: x \to x^2 + 4x + 11,\ x \in \mathbb{R}$ in the form $(x + a)^2 + b$

b $\mathrm{f}: x \to x^2 - 2x - 6,\ x \in \mathbb{R}$ in the form $(x + a)^2 + b$

c $\mathrm{f}: x \to 4x^2 + 12x + 3,\ x \in \mathbb{R}$ in the form $(ax + b)^2 + c$

d $\mathrm{f}: x \to 9x^2 - 6x + 16,\ x \in \mathbb{R}$ in the form $(ax + b)^2 + c$

e $\mathrm{f}: x \to 15 - 4x - x^2,\ x \in \mathbb{R}$ in the form $a - (x + b)^2$

1 $f : x \to 4x - 3,\ x \in \mathbb{R}$ $g : x \to 2 - x,\ x \in \mathbb{R}$ $h : x \to x^2 + 5,\ x \in \mathbb{R}$

Evaluate

a gf(2) **b** gh(1) **c** fg(–3) **d** hf(3)

e gg(5) **f** ff($\frac{1}{2}$) **g** hg(8) **h** fh($1\frac{1}{2}$)

2 $f : x \to 5x + 2,\ x \in \mathbb{R}$ $g : x \to \cos x,\ x \in \mathbb{R}$ $h: x \to \ln x,\ x \in \mathbb{R},\ x > 0$

Evaluate, giving your answers to 3 significant figures

a fh(20) **b** gh(3) **c** fg(5) **d** gg(–4)

e gf($1\frac{3}{4}$) **f** hg(6.7) **g** hh(50) **h** hf(–0.3)

3 $f : x \to 2x + 1,\ x \in \mathbb{R}$ $g : x \to 1 - 3x,\ x \in \mathbb{R}$ $h : x \to x^2 + 4,\ x \in \mathbb{R}$

Given the functions f, g and h, express the following composite functions in a similar form.

a fg **b** ff **c** fh **d** hf

e gh **f** gg **g** hg **h** gf

4 $f : x \to 4 - x,\ x \in \mathbb{R}$ $g : x \to e^x,\ x \in \mathbb{R}$ $h : x \to 2x^2 + 7,\ x \in \mathbb{R}$

Given the functions f, g and h, express the following composite functions in a similar form.

a gf **b** hg **c** fh **d** gg

e gh **f** ff **g** fg **h** hf

5 $f : x \to 5x - 3,\ x \in \mathbb{R}$ $g : x \to 3x^2 + 1,\ x \in \mathbb{R}$ $h : x \to \frac{1}{x-2},\ x \in \mathbb{R},\ x \neq 2$

Solve

a $\text{ff}(x) = -8$ **b** $\text{hf}(x) = 2$ **c** $\text{gf}(x) = 28$ **d** $\text{hg}(x) = \frac{1}{2}$

e $\text{fh}(x) = 7$ **f** $\text{fg}(x) = 32$ **g** $\text{gh}(x) = 4$ **h** $\text{hh}(x) = -2$

6 $f : x \to \ln x,\ x \in \mathbb{R},\ x > 0$ $g : x \to 3 + 2x,\ x \in \mathbb{R}$ $h : x \to e^x,\ x \in \mathbb{R}$

Solve, giving your answers to 2 decimal places,

a $\text{gh}(x) = 9$ **b** $\text{fg}(x) = 3.6$ **c** $\text{hg}(x) = 4$ **d** $\text{gf}(x) = 10.4$

7 The functions f and g are defined by

$$f : x \to \frac{x+1}{5},\ x \in \mathbb{R} \qquad g : x \to e^x,\ x \in \mathbb{R}$$

a State the range of g.

b Solve $\text{fg}(x) = 17$.

8 The functions f and g are defined by

$$f(x) \equiv 4x - 9,\ x \in \mathbb{R} \qquad g(x) \equiv x^2,\ x \in \mathbb{R}$$

a Evaluate ff($3\frac{1}{4}$).

b Solve $\text{gf}(x) = 25$.

c Sketch the graph of $y = \text{fg}(x)$, showing the coordinates of any points of intersection with the coordinate axes.

9 $f : x \to \tan x,\ x \in \mathbb{R}$ $\quad g : x \to 4 + \ln x,\ x \in \mathbb{R}^+$ $\quad h : x \to e^{2x-1},\ x \in \mathbb{R}$

Evaluate

a $gf(\frac{\pi}{4})$ **b** $hg(e^{-2})$ **c** $gh(-1)$ **d** $ff(1)$

e $hf(0.2)$ **f** $fg(7)$ **g** $hh(\frac{1}{4})$ **h** $fg(e^e)$

10 $f : x \to 3e^x + 2,\ x \in \mathbb{R}$ $\quad g : x \to 4x + 1,\ x \in \mathbb{R}$ $\quad h : x \to \dfrac{1}{x+1},\ x \in \mathbb{R},\ x \neq -1$

Express the following composite functions in a similar form, stating the domain in each case.

a fg **b** gf **c** hf **d** gg

e hg **f** gh **g** hh **h** ggg

11 $f : x \to \sqrt{x+4},\ x \in \mathbb{R},\ x > -4$ $\quad g : x \to e^{1+2x},\ x \in \mathbb{R}$ $\quad h : x \to \dfrac{x+1}{3},\ x \in \mathbb{R}$

Solve

a $fh(x) = 3$ **b** $fg(x) = 7$ **c** $gh(x) = 11$ **d** $hh(x) = \frac{2}{3}$

e $hg(x) = 1.2$ **f** $hf(x) = \frac{1}{2}$ **g** $ff(x) = 3$ **h** $ghh(x) = \frac{1}{2}$

12 $f(x) \equiv x^3,\ x \in \mathbb{R}$ $\quad g(x) \equiv x + 2,\ x \in \mathbb{R}$

Find the composition of the functions f and g that corresponds to the function h, where

a $h(x) \equiv (x + 2)^3,\ x \in \mathbb{R}$ **b** $h(x) \equiv x^3 + 2,\ x \in \mathbb{R}$ **c** $h(x) \equiv x + 4,\ x \in \mathbb{R}$

d $h(x) \equiv x^9,\ x \in \mathbb{R}$ **e** $h(x) \equiv x^9 + 2,\ x \in \mathbb{R}$ **f** $h(x) \equiv (x + 2)^3 + 2,\ x \in \mathbb{R}$

13 $f(x) \equiv x - 4,\ x \in \mathbb{R}$ $\quad g(x) \equiv 3x^2,\ x \in \mathbb{R}$ $\quad h(x) \equiv \dfrac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

Find the composition of the functions f, g and h that corresponds to the function j, where

a $j(x) \equiv 3x^2 - 4,\ x \in \mathbb{R}$ **b** $j(x) \equiv \dfrac{1}{x-4},\ x \in \mathbb{R},\ x \neq 4$

c $j(x) \equiv \dfrac{3}{x^2},\ x \in \mathbb{R},\ x \neq 0$ **d** $j(x) \equiv 27x^4,\ x \in \mathbb{R}$

e $j(x) \equiv \dfrac{1}{3x^2} - 4,\ x \in \mathbb{R},\ x \neq 0$ **f** $j(x) \equiv \dfrac{1}{3x^2 - 4},\ x \in \mathbb{R},\ x \neq \pm\dfrac{2}{\sqrt{3}}$

14 The functions f and g are defined by

$$f : x \to 5^x - 7,\ x \in \mathbb{R} \qquad g : x \to 2x + 3,\ x \in \mathbb{R}$$

a Find and simplify an expression for gf, stating its domain.

b Solve the equation $gf(x) = 10$.

15 The functions f and g are defined by

$$f : x \to 2(x + 1),\ x \in \mathbb{R} \qquad g : x \to x^2 - 9,\ x \in \mathbb{R}$$

a Express gf in terms of x and state its domain and range.

b Sketch the graph of $y = gf(x)$, showing the coordinates of any points of intersection with the coordinate axes.

The equation $gf(x) - 2f(x) = a$, where a is a constant, has no real roots.

c Show that $a < -10$.

1 The domain of each of the following functions is $x \in \mathbb{R}$. For each function, find its inverse $\mathrm{f}^{-1}(x)$.

a $\mathrm{f} : x \to 10x + 3$ **b** $\mathrm{f} : x \to 9 + 2x$ **c** $\mathrm{f} : x \to 5 - 6x$

d $\mathrm{f} : x \to \frac{x+3}{4}$ **e** $\mathrm{f} : x \to \frac{1}{3}(2x - 5)$ **f** $\mathrm{f} : x \to 8 - \frac{3}{5}x$

2 For each function, find $\mathrm{f}^{-1}(x)$ and state its domain.

a $\mathrm{f}(x) \equiv \ln x,\ x \in \mathbb{R},\ x > 0$ **b** $\mathrm{f}(x) \equiv \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

c $\mathrm{f}(x) \equiv \sqrt[4]{x},\ x \in \mathbb{R},\ x > 0$ **d** $\mathrm{f}(x) \equiv 3x - 4,\ x \in \mathbb{R},\ 0 \leq x < 3$

e $\mathrm{f}(x) \equiv \frac{1}{x-5},\ x \in \mathbb{R},\ x \neq 5$ **f** $\mathrm{f}(x) \equiv 2 + \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

3 For each of the following functions,

i find, in the form $\mathrm{f}^{-1} : x \to \ldots$, the inverse function of f and state its domain,

ii sketch $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same set of axes.

a $\mathrm{f} : x \to 2x + 1,\ x \in \mathbb{R}$ **b** $\mathrm{f} : x \to \frac{1-x}{5},\ x \in \mathbb{R}$ **c** $\mathrm{f} : x \to \frac{10}{x},\ x \in \mathbb{R},\ x \neq 0$

d $\mathrm{f} : x \to x^2,\ x \in \mathbb{R},\ x > 0$ **e** $\mathrm{f} : x \to \mathrm{e}^x,\ x \in \mathbb{R}$ **f** $\mathrm{f} : x \to x^3,\ x \in \mathbb{R}$

4 For each of the following, solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$.

a $\mathrm{f} : x \to 5x + 1,\ x \in \mathbb{R}$ $\quad$ $\mathrm{g} : x \to 2,\ x \in \mathbb{R}$

b $\mathrm{f} : x \to \frac{2x-4}{3},\ x \in \mathbb{R}$ $\quad$ $\mathrm{g} : x \to 7 - x,\ x \in \mathbb{R}$

c $\mathrm{f} : x \to \mathrm{e}^x + 2,\ x \in \mathbb{R}$ $\quad$ $\mathrm{g} : x \to \ln(3x - 8),\ x \in \mathbb{R},\ x > \frac{8}{3}$

d $\mathrm{f} : x \to \sqrt{x+2},\ x \in \mathbb{R},\ x \geq -2$ $\quad$ $\mathrm{g} : x \to 3x - 4,\ x \in \mathbb{R}$

e $\mathrm{f} : x \to \frac{4}{x+3},\ x \in \mathbb{R},\ x \neq -3$ $\quad$ $\mathrm{g} : x \to 5(x + 1),\ x \in \mathbb{R}$

5 The function f is defined by $\mathrm{f} : x \to 4 - 2x,\ x \in \mathbb{R}$.

a Sketch $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same set of axes.

b Find the coordinates of the point where the lines $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ intersect.

6 The functions f and g are defined by

$$\mathrm{f} : x \to 3 - 2x,\ x \in \mathbb{R} \qquad \mathrm{g} : x \to \frac{1}{2x+4},\ x \in \mathbb{R},\ x \neq -2$$

a Find $\mathrm{g}^{-1}(x)$ and state its domain and range.

b Express gf in terms of x and state its domain.

c Solve the equation $\mathrm{gf}(x) = \mathrm{f}^{-1}(x)$.

7 The functions f and g are defined by

$$\mathrm{f} : x \to 5x + 2,\ x \in \mathbb{R} \qquad \mathrm{g} : x \to \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$$

a Find the following functions, stating the domain in each case.

i f^{-1} **ii** fg **iii** $(\mathrm{fg})^{-1}$

b Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{fg}(x)$, giving your answers correct to 2 decimal places.

8 For each of the following functions, find the inverse function in the form $\mathrm{f}^{-1}: x \to \ldots$ and state its domain.

a $\mathrm{f}: x \to \frac{1}{2}\ln(4x - 9),\ x \in \mathbb{R},\ x > 2\frac{1}{4}$

b $\mathrm{f}: x \to \frac{x-2}{x+5},\ x \in \mathbb{R},\ x \neq -5$

c $\mathrm{f}: x \to \mathrm{e}^{0.4x-2},\ x \in \mathbb{R}$

d $\mathrm{f}: x \to \sqrt[3]{x^5 - 3},\ x \in \mathbb{R}$

e $\mathrm{f}: x \to \log_{10}(2 - 7x),\ x \in \mathbb{R},\ x < \frac{2}{7}$

f $\mathrm{f}: x \to \frac{4-x}{3x+2},\ x \in \mathbb{R},\ x \neq -\frac{2}{3}$

9 For each of the following functions,

i find, in the form $\mathrm{f}^{-1}: x \to \ldots,$ the inverse function of f and state its domain,

ii sketch $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same set of axes.

a $\mathrm{f}: x \to \mathrm{e}^{2x},\ x \in \mathbb{R}$

b $\mathrm{f}: x \to x^2 + 4,\ x \in \mathbb{R},\ x > 0$

c $\mathrm{f}: x \to \ln(x - 3),\ x \in \mathbb{R},\ x > 3$

d $\mathrm{f}: x \to x^2 + 6x + 9,\ x \in \mathbb{R},\ x > -3$

10 For each of the following functions,

i find the range of f,

ii find $\mathrm{f}^{-1}(x)$, stating its domain.

a $\mathrm{f}(x) \equiv x^2 + 6x + 3,\ x \in \mathbb{R},\ x < -3$

b $\mathrm{f}(x) \equiv x^2 - 4x + 5,\ x \in \mathbb{R},\ x \geq 2$

c $\mathrm{f}(x) \equiv x^2 + 5x - 2,\ x \in \mathbb{R},\ x < -2\frac{1}{2}$

d $\mathrm{f}(x) \equiv x^2 - 3x + 5,\ x \in \mathbb{R},\ 2 < x < 4$

e $\mathrm{f}(x) \equiv (2 - x)(4 + x),\ x \in \mathbb{R},\ x \geq -1$

f $\mathrm{f}(x) \equiv 20x - 5x^2,\ x \in \mathbb{R},\ x > 2$

11 For each of the following, solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$.

a $\mathrm{f}: x \to \frac{1}{3}(2x - 5),\ x \in \mathbb{R}$ $\quad$ $\mathrm{g}: x \to \frac{4}{2-x},\ x \in \mathbb{R},\ x \neq 2$

b $\mathrm{f}: x \to \ln\frac{x+3}{5},\ x \in \mathbb{R},\ x > -3$ $\quad$ $\mathrm{g}: x \to 10 - 6\mathrm{e}^{-x},\ x \in \mathbb{R}$

c $\mathrm{f}: x \to x^2 - 4,\ x \in \mathbb{R},\ x > 0$ $\quad$ $\mathrm{g}: x \to \frac{x+6}{3},\ x \in \mathbb{R}$

12 The function f is defined by

$$\mathrm{f}: x \to \frac{x+b}{x+a},\ x \in \mathbb{R},\ x \neq 2.$$

a State the value of the constant a.

Given that $\mathrm{f}(6) = 4$,

b find the value of the constant b,

c find $\mathrm{f}^{-1}(x)$ and state its domain.

13 The functions f and g are defined by

$$\mathrm{f}: x \to x^2 - 3x,\ x \in \mathbb{R},\ x \geq 1\tfrac{1}{2},$$

$$\mathrm{g}: x \to 2x + 3,\ x \in \mathbb{R}.$$

a Find, in the form $\mathrm{f}^{-1}: x \to \ldots,$ the inverse function of f and state its domain.

b On the same set of axes, sketch $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$.

Given that $\mathrm{f}^{-1}\mathrm{g}^{-1}(12) = a(1 + \sqrt{3})$,

c show that $a = 1\frac{1}{2}$.

1 $f : x \to |x - 4|,\ x \in \mathbb{R}$ $\qquad g : x \to |x| - 4,\ x \in \mathbb{R}$

Find the value of

a f(6) **b** f(3) **c** f(−2) **d** g(2) **e** g(−8) **f** g(−1)

2 $f : x \to x^2 + 2x - 3,\ x \in \mathbb{R}$ $\qquad g : x \to |2x + 1|,\ x \in \mathbb{R}$

Find the value of

a gf(0) **b** fg(0) **c** fg(4) **d** gg(−3) **e** gf(−3) **f** fg(−1)

3 Sketch each of the following graphs, showing the coordinates of any points of intersection with the axes. Where it occurs, a is a positive constant.

a $y = |x + 4|$ **b** $y = |2x - 5|$ **c** $y = |2 - 3x|$

d $y = |x^2 - 9|$ **e** $y = |x^3|$ **f** $y = |\sin x|,\ 0 \le x \le 2\pi$

g $y = |x - a|$ **h** $y = |3x + a|$ **i** $y = |a - 2x|$

j $y = |16 - x^2|$ **k** $y = |(x + 3)(2x - 1)|$ **l** $y = |\frac{1}{x}|,\ x \ne 0$

m $y = |\ln x|,\ x > 0$ **n** $y = |10 - 3x - x^2|$ **o** $y = |3x^2 + 5ax - 2a^2|$

4 For each of the following,

i sketch $y = f(x)$ and $y = g(x)$ on the same diagram,

ii solve the equation $f(x) = g(x)$.

The domain of all the functions is $x \in \mathbb{R}$ and a is a positive constant where it occurs.

a $f(x) \equiv |2x - 3|,\ g(x) \equiv 2$ **b** $f(x) \equiv |7 - 3x|,\ g(x) \equiv 7$

c $f(x) \equiv |4x + 3a|,\ g(x) \equiv 5a$ **d** $f(x) \equiv |x^2 - 4|,\ g(x) \equiv 9$

e $f(x) \equiv |x^2 - 4x - 12|,\ g(x) \equiv 20$ **f** $f(x) \equiv |2a - 5x|,\ g(x) \equiv x$

5 Solve each equation.

a $|x - 5| = 3$ **b** $|x + 1| = 15$ **c** $|2x - 7| = 4$

d $|x - 2| = |x + 4|$ **e** $|x - 5| = |7 - x|$ **f** $|2x + 1| = |9 - 2x|$

g $|x + 3| = |2x|$ **h** $|4x - 1| = |2 - x|$ **i** $|3x - 4| = |2x + 3|$

6 Find the set of values of x for which

a $|x - 20| < 2$ **b** $|2x - 11| \le 5$ **c** $|x - 17| > 12$

d $|5x - 22| < 40$ **e** $|x + 4| \le |x + 1|$ **f** $|x + 2| > |2x - 5|$

7 For each of the following, sketch $y = |f(x)|$ and $y = f(|x|)$ on separate diagrams showing the coordinates of any points of intersection with the axes.

a $f : x \to 3x - 1,\ x \in \mathbb{R}$ **b** $f : x \to 3 - 4x,\ x \in \mathbb{R}$

c $f : x \to 4x^2 - 25,\ x \in \mathbb{R}$ **d** $f : x \to (1 + x)(5 - x),\ x \in \mathbb{R}$

e $f : x \to \tan x,\ x \in \mathbb{R},\ -\frac{\pi}{2} < x < \frac{\pi}{2}$ **f** $f : x \to e^x,\ x \in \mathbb{R}$

1 Describe how the graph of $y = \mathrm{f}(x)$ is transformed to give the graph of

a $y = 2 + \mathrm{f}(x + 3)$ **b** $y = 2\mathrm{f}(-x)$ **c** $y = 3\mathrm{f}(x - 1)$ **d** $y = 4 - \mathrm{f}(x)$

2 **a** Express $x^2 + 6x + 2$ in the form $a(x + b)^2 + c$.

b Hence, describe two transformations that would map the graph of $y = x^2$ onto the graph of $y = x^2 + 6x + 2$.

3 Each of the following graphs is translated by 3 units in the positive x-direction and then stretched by a factor of 2 in the y-direction, about the x-axis.

Find and simplify an equation of the graph obtained in each case.

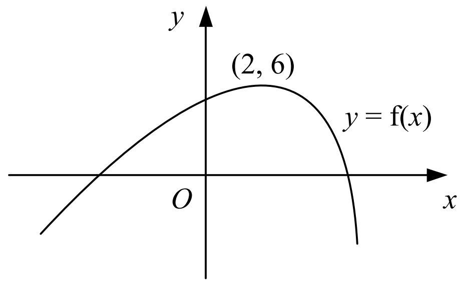
a $y = 2x + 7$ **b** $y = 3\mathrm{e}^x$ **c** $y = x^2 - 3x + 1$ **d** $y = \frac{1}{x}$

4 Describe in order two transformations that would map the graph of

a $y = |x|$ onto the graph of $y = -|3x|$ **b** $y = \mathrm{e}^x$ onto the graph of $y = 5 + \mathrm{e}^{-x}$

c $y = \frac{1}{x}$ onto the graph of $y = \frac{3}{x+4}$ **d** $y = \ln x$ onto the graph of $y = 2 + 3 \ln x$

5



The diagram shows the curve with equation $y = \mathrm{f}(x)$ which is stationary at the point (2, 6).

Showing the coordinates of the stationary point in each case, sketch on separate diagrams the graphs of

a $y = 1 + \mathrm{f}(x - 4)$ **b** $y = 3 - \mathrm{f}(x)$ **c** $y = 2\mathrm{f}(x + 1)$ **d** $y = \frac{1}{2}\mathrm{f}(2x)$

6 The graph of $y = x^2 + 4x - 2$ undergoes the following three transformations:

first: translation by -2 units in the positive x-direction,
second: stretch by a factor of 3 in the y-direction, about the x-axis,
third: reflection in the y-axis.

Find and simplify an equation of the graph obtained.

7 **a** Express $2x^2 - 4x + 7$ in the form $a(x + b)^2 + c$.

b Hence, describe in order a sequence of transformations that would map the graph of $y = 2x^2 - 4x + 7$ onto the graph of $y = x^2$.

8 $$\mathrm{f}(x) \equiv x^3 - 3x^2 + 4, \ x \in \mathbb{R}.$$

a Find the coordinates of the stationary points on the graph of $y = \mathrm{f}(x)$.

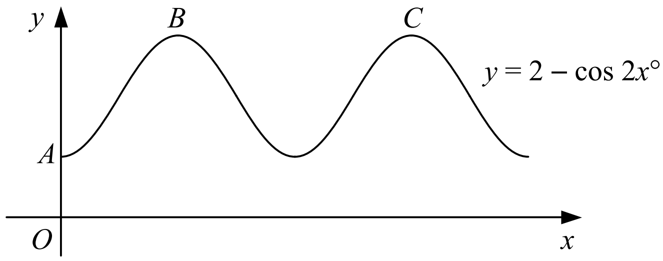
b Hence, find the coordinates of the stationary points on each of the following graphs.

i $y = -2\mathrm{f}(x)$ **ii** $y = 3 + \mathrm{f}(\frac{1}{2}x)$ **iii** $y = \frac{1}{4}\mathrm{f}(x - 2)$

9 **a** Describe clearly, in order, the sequence of transformations that would map the graph of $y = \sqrt{x}$ onto the graph of $y = 2 - 3\sqrt{x}$.

b Sketch the graph of $y = 2 - 3\sqrt{x}$ showing the coordinates of any points where the graph meets the coordinate axes.

10



The diagram shows part of the curve with equation $y = 2 - \cos 2x°$, $x > 0$.

a State the period of the curve.

b Write down the coordinates of the point A where the curve meets the y-axis.

c Write down the coordinates of B and C, the first two maximum points on the curve.

11 Sketch each of the following curves for x in the interval $0 \leq x \leq 360$. Show the coordinates of any turning points and the equations of any asymptotes.

a $y = 3 \cos 2x°$ **b** $y = \tan(-2x°)$ **c** $y = 1 + 2 \sin x°$

d $y = -\sin(x + 60)°$ **e** $y = 2 \cos(x - 45)°$ **f** $y = 3 - \tan x°$

g $y = 2 + \cos \frac{1}{2}x°$ **h** $y = 4 \sin \frac{3}{2}x°$ **i** $y = 1 - 2 \cos x°$

12 State the period of the curves with the equations

a $y = 2 \tan 3x°$,

b $y = 1 + \sin kx°$, giving your answer in terms of k.

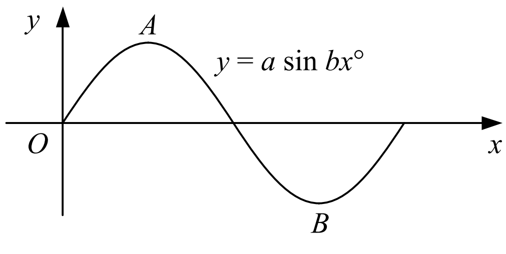
13 $$\mathrm{f}(x) \equiv 2 \sin \tfrac{1}{2}x, \quad 0 \leq x \leq 2\pi.$$

a Sketch the graph $y = \mathrm{f}(x)$.

b State the coordinates of the maximum point of the curve.

c Solve the equation $\mathrm{f}(x) = \sqrt{2}$, giving your answers in terms of π.

14



The graph shows the curve $y = a \sin bx°$, $0 \leq x \leq 180$.

The curve has a maximum at the point A with coordinates (45, 4).

a Find the values of the constants a and b.

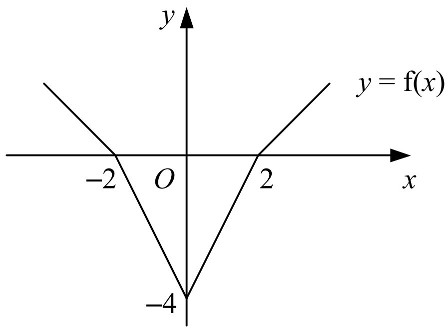
b Write down the coordinates of the minimum point of the curve, B.

1 **a** Express $x^2 - 8x + 18$ in the form $(x + a)^2 + b$.

b Find the distance of the vertex of the curve $y = x^2 - 8x + 18$ from the origin, giving your answer in the form $k\sqrt{5}$.

c Describe two transformations that would map the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 18$.

2



The diagram shows the graph of $y = \text{f}(x)$ which meets the coordinate axes at the points $(-2, 0)$, $(0, -4)$ and $(2, 0)$.

Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the graphs of

a $y = \frac{1}{2}\,|\,\text{f}(x)\,|$,

b $y = 4 + \text{f}(x + 2)$.

3 Sketch the curve with equation $y = 2 - 2 \sin x$ for x in the interval $0 \le x \le 2\pi$.

Label on your sketch the coordinates of any maximum or minimum points and any points where the curve meets the coordinate axes.

4 $$\text{f}(x) \equiv |\,2x + 5\,|, \ x \in \mathbb{R}.$$

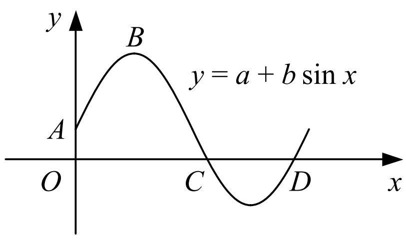
a Sketch the graph $y = \text{f}(x)$, showing the coordinates of any points where the graph meets the coordinate axes.

b Evaluate $\text{ff}(-4)$.

$$\text{g}(x) \equiv \text{f}(x + k), \ x \in \mathbb{R}.$$

c State the value of the constant k for which $\text{g}(x)$ is symmetrical about the y-axis.

5



The diagram shows the curve $y = a + b \sin x$, $0 \le x \le 360°$.

The curve meets the y-axis at the point A $(0, 2)$ and has a maximum at the point B $(90°, 7)$.

a Find the values of the constants a and b.

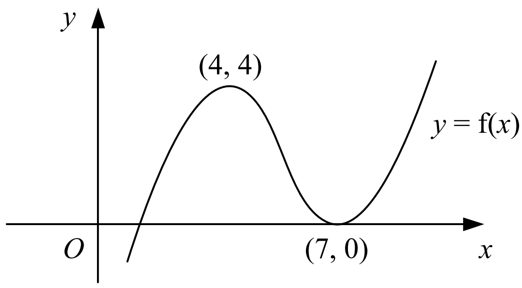
b Find the x-coordinates of the points C and D, where the curve crosses the x-axis.

6 **a** Sketch the curve $y = 3\cos 2x°$ for x in the interval $0 \le x \le 360$.

b Write down the coordinates of the points where the curve intersects the x-axis.

c Write down the coordinates of the turning points of the curve.

7



The diagram shows the curve with equation $y = f(x)$ which has two stationary points with coordinates (4, 4) and (7, 0).

Showing the coordinates of any stationary points, sketch on separate diagrams the curves

a $y = 1 + 2f(x)$,

b $y = f(-3x)$.

8 **a** Sketch the curve $y = \frac{1}{2} + \sin 3x$ for x in the interval $0 \le x \le 180°$.

b Write down the coordinates of the turning points of the curve.

c Find the x-coordinates of the points where the curve crosses the x-axis.

9 The function f is defined by

$$f : x \to x^{\frac{1}{2}} - 2, \ x \in \mathbb{R}, \ x \ge 0.$$

Showing the coordinates of any points where each graph meets the coordinate axes, sketch on separate diagrams the graphs of

a $y = f(x)$,

b $y = 2 + |f(x)|$,

c $y = 3f(x + 1)$.

10 Sketch the curve $y = 4\sin(x + \frac{\pi}{3})$ for x in the interval $0 \le x \le 2\pi$.

Label on your sketch

i the value of x at each point where the curve intersects the x-axis,

ii the coordinates of the maximum and minimum points of the curve.

11 $$f(x) \equiv \frac{3x - 5}{x - 2}, \ x \in \mathbb{R}, \ x \ne 2.$$

a Find $f^{-1}(x)$ and state its domain.

b Hence, or otherwise, solve the equation $f(x) = 4$.

c Find the values of a and b such that

$$f(x) = a + \frac{b}{x - 2}.$$

d Hence, describe two transformations that map the graph of $y = \frac{1}{x}$ onto the graph of $y = f(x)$.

1 $\quad \mathrm{f} : x \to 2 + \log_4 x, \; x \in \mathbb{R}, \; x > 0.$

a Evaluate ff(1). **(3)**

b Solve the equation $\mathrm{f}(x) = 0$. **(2)**

c Find the inverse function $\mathrm{f}^{-1}(x)$. **(3)**

2 The function f is defined by

$$\mathrm{f} : x \to |3x - a|, \; x \in \mathbb{R}.$$

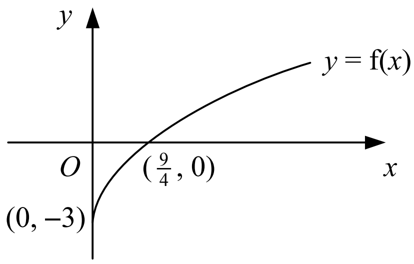
where a is a positive constant.

a Find $\mathrm{ff}(-2a)$. **(2)**

b Sketch the graph $y = \mathrm{f}(x)$, showing the coordinates of any points where the graph meets the coordinate axes. **(3)**

c Solve the equation $\mathrm{f}(x) = x$, giving your answers in terms of a. **(3)**

3



The diagram shows the graph of $y = \mathrm{f}(x)$ which meets the x-axis at the point $(\frac{9}{4}, 0)$ and the y-axis at the point $(0, -3)$.

a Sketch on separate diagrams the graphs of

i $y = |\mathrm{f}(x)|$,

ii $y = \mathrm{f}^{-1}(x)$. **(4)**

Given that f(x) is of the form $\mathrm{f}(x) \equiv ax^{\frac{1}{2}} + b, \; x \in \mathbb{R}, \; x \geq 0$,

b find the values of the constants a and b, **(3)**

c find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

4 The function f is defined by

$$\mathrm{f} : x \to \frac{x+2}{x-1}, \; x \in \mathbb{R}, \; x \neq 1.$$

a Show that $\mathrm{ff}(x) = x$ for all $x \in \mathbb{R}, \; x \neq 1$. **(3)**

b Hence, write down an expression for $\mathrm{f}^{-1}(x)$. **(1)**

The function g is defined by

$$\mathrm{g} : x \to 2x - 3, \; x \in \mathbb{R}.$$

c Solve the equation $\mathrm{gf}(x) = 0$. **(4)**

5 **a** Sketch on the same set of axes the graphs of $y = |x|$ and $y = |2x - 3|$. **(3)**

b Hence, or otherwise, solve the equation

$$|x| = |2x - 3|.$$ **(4)**

6 The function f(x) is defined for all real values of x by

$$f(x) = x + 2, \quad x < 1,$$
$$f(x) = 4 - x^2, \quad x \geq 1.$$

a Sketch the graph of f(x) showing the coordinates of any points of intersection with the coordinate axes. **(4)**

b Evaluate ff(3). **(2)**

c Solve the equation f(x) = 1. **(4)**

7 The functions f and g are defined by

$$f : x \to kx + 2, \ x \in \mathbb{R},$$
$$g : x \to x - 3k, \ x \in \mathbb{R},$$

where k is a constant.

a Find expressions in terms of k for

i $f^{-1}(x)$,

ii fg(x). **(4)**

Given that fg(7) = 4,

b find the two possible values of k. **(3)**

8
$$f(x) \equiv x^2 - 4x + 5, \ x \in \mathbb{R}, \ x \geq 2.$$

a Express f(x) in the form $a(x + b)^2 + c$. **(2)**

b State the range of f. **(1)**

c Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

d Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram and state the relationship between the graphs. **(4)**

9 The functions f and g are defined by

$$f : x \to x^2 + 4, \ x \in \mathbb{R},$$
$$g : x \to 2x - \frac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0.$$

a Evaluate gf(−2). **(2)**

b Find and simplify an expression for fg(x). **(3)**

c Find the values of x for which fg(x) = 5. **(4)**

10 The function f is given by

$$f : x \to e^{\frac{1}{2}x} - 3, \ x \in \mathbb{R}.$$

a Find $f^{-1}(x)$ and state its domain. **(4)**

b Sketch the curve $y = f^{-1}(x)$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

The function g is given by

$$g : x \to \ln (x + 5), \ x \in \mathbb{R}, \ x > -5.$$

c Evaluate fg(4). **(2)**

d Solve the equation $f^{-1}(x) = g(x)$. **(4)**

1 The function f is defined by

$$f : x \rightarrow 3 + \ln (x + 2), \; x \in \mathbb{R}, \; x \geq k,$$

where k is a constant.

Given that the range of f is $f(x) \geq 3$,

a find the value of k, **(3)**

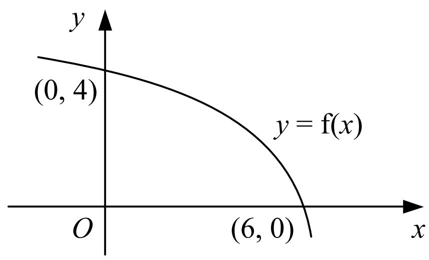
b find $f^{-1}(x)$, stating its domain clearly. **(4)**

The function g is defined by

$$g : x \rightarrow 4 + \ln (x - 1), \; x \in \mathbb{R}, \; x > 1.$$

c Find, in terms of e, the value of x such that $f(x) = g(x)$. **(4)**

2



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points (0, 4) and (6, 0).

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

a $y = f(|x|)$, **(2)**

b $y = 4 - f(x)$, **(2)**

c $y = 2f(3x)$. **(3)**

3 The functions f and g are given by

$$f(x) \equiv \frac{x}{x+2}, \; x \in \mathbb{R}, \; x \neq -2,$$

$$g(x) \equiv \frac{3}{x}, \; x \in \mathbb{R}, \; x \neq 0$$

a Solve the equation $fg(x) = 4$. **(4)**

b Find $f^{-1}(x)$, stating its domain clearly. **(4)**

c Solve the equation $f(x) = f^{-1}(x)$. **(3)**

4 The function f is defined by

$$f(x) \equiv x^2 - 2x - 9, \; x \in \mathbb{R}, \; x \geq k.$$

a Find the minimum value of the constant k for which $f^{-1}(x)$ exists. **(3)**

Given that k takes the value found in part **a**,

b solve the equation $f^{-1}(x) = 4$, **(2)**

c sketch the curve $y = |f(x)|$, **(3)**

d find the values of x for which $|f(x)| = 6$. **(5)**

5 The function f is defined by

$$\mathrm{f} : x \to 2 - \frac{3}{x}, \; x \in \mathbb{R}, \; x \neq 0.$$

a Find the value of ff(1). **(2)**

b Find $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$\mathrm{g} : x \to x^2, \; x \in \mathbb{R}.$$

c Solve the equation $\mathrm{gf}(x) = 1$. **(4)**

6 The function f is defined by

$$\mathrm{f} : x \to \mathrm{e}^{\frac{1}{2}x} - 2, \; x \in \mathbb{R}.$$

a Evaluate f(ln 9). **(2)**

b State the range of f. **(1)**

c Find $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

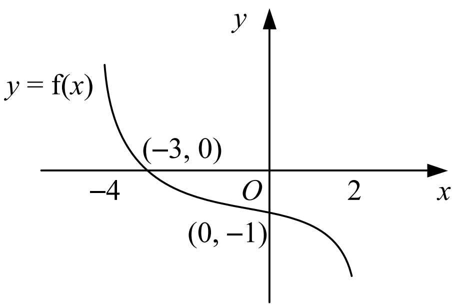
The function g is defined by

$$\mathrm{g} : x \to x^2 + 4x, \; x \in \mathbb{R}.$$

d Find and simplify an expression for $\mathrm{gf}(x)$. **(3)**

e Solve the equation $\mathrm{gf}(x) + 1 = 0$. **(2)**

7



The diagram shows the curve $y = \mathrm{f}(x)$. The domain of f is $-4 \leq x \leq 2$ and the curve intersects the coordinate axes at the points $(-3, 0)$ and $(0, -1)$.

a Explain how the graph shows that f is one-one. **(1)**

b Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

i $y = |\mathrm{f}(x)|$,

ii $y = \mathrm{f}^{-1}(x)$. **(5)**

8

$$\mathrm{f}(x) \equiv \frac{5}{(x+1)(2x-3)} + \frac{1}{x+1}, \; x \in \mathbb{R}, \; x \geq 2.$$

a Show that $\mathrm{f}(x) = \frac{2}{2x-3}$. **(4)**

b Find the range of f. **(2)**

c Find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

$$\mathrm{g}(x) \equiv \frac{1}{x-2}, \; x \in \mathbb{R}, \; x \neq 2.$$

d Solve the equation $\mathrm{fg}(x) = \frac{2}{3}$. **(4)**