1 **a** $= [-2x^{-1}]_1^4$

$= -\frac{1}{2} - (-2)$

$= \frac{3}{2}$

b $= \int_0^2 (x^2 - 6x + 9)\ \mathrm{d}x$

$= [\frac{1}{3}x^3 - 3x^2 + 9x]_0^2$

$= (\frac{8}{3} - 12 + 18) - 0$

$= 8\frac{2}{3}$

2 **a**

x	0	2	4	6
$\sqrt{x^2+4}$	2	$\sqrt{8}$	$\sqrt{20}$	$\sqrt{40}$

area $\approx \frac{1}{2} \times 2 \times [2 + \sqrt{40} + 2(\sqrt{8} + \sqrt{20})]$

$= 22.9$ (3sf)

b over-estimate

curve passes below top of each trapezium

3 **a** $= 3\sqrt{2} - \frac{1}{\sqrt{2}}$

$= 3\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \frac{5}{2}\sqrt{2}$

b $\int_3^4 (3x^{\frac{1}{2}} - x^{-\frac{1}{2}})\ \mathrm{d}x$

$= [2x^{\frac{3}{2}} - 2x^{\frac{1}{2}}]_3^4$

$= [16 - 4] - [(2 \times 3\sqrt{3}) - 2\sqrt{3}]$

$= 12 - 4\sqrt{3}$

4 **a** $4x^{\frac{1}{2}} - x^{\frac{3}{2}} = 0$

$x^{\frac{1}{2}}(4 - x) = 0$

$x^{\frac{1}{2}} = 0$ $[\Rightarrow x = 0$, at $O]$ or $x = 4$

$\therefore A(4, 0)$

b $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

SP: $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

$\frac{1}{2}x^{-\frac{1}{2}}(4 - 3x) = 0$

$x^{-\frac{1}{2}} = 0 \Rightarrow$ no solutions

$\therefore x = \frac{4}{3}$ at B

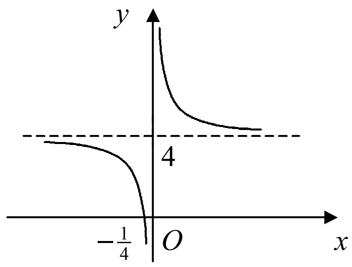
c $= \int_0^4 (4x^{\frac{1}{2}} - x^{\frac{3}{2}})\ \mathrm{d}x$

$= [\frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}]_0^4$

$= (\frac{64}{3} - \frac{64}{5}) - 0 = 8\frac{8}{15}$

5 **a** $p = -\frac{1}{4}$, $q = 4$

b



c

x	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$4 + \frac{1}{x}$	5	$4\frac{2}{3}$	$4\frac{1}{2}$	$4\frac{2}{5}$	$4\frac{1}{3}$

area $\approx \frac{1}{2} \times \frac{1}{2} \times [5 + 4\frac{1}{3} + 2(4\frac{2}{3} + 4\frac{1}{2} + 4\frac{2}{5})]$

$= 9\frac{7}{60}$ or 9.12 (3sf)

6 **a** $4x - y + 11 = 0 \Rightarrow y = 4x + 11$

intersect when $2x^2 + 6x + 7 = 4x + 11$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, 1$

$\therefore (-2, 3)$ and $(1, 15)$

b area below curve

$= \int_{-2}^1 (2x^2 + 6x + 7)\ \mathrm{d}x$

$= [\frac{2}{3}x^3 + 3x^2 + 7x]_{-2}^1$

$= (\frac{2}{3} + 3 + 7) - (-\frac{16}{3} + 12 - 14) = 18$

area below line

$= \frac{1}{2} \times 3 \times (3 + 15) = 27$

area between line and curve

$= 27 - 18 = 9$

7 **a** minimum when $\sin x = 1$

$\therefore\ x = \frac{\pi}{2}$

$\therefore\ (\frac{\pi}{2}, \frac{1}{2})$

b

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$\frac{1}{1+\sin x}$	1	0.6667	0.5359

$\therefore$ area $\approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.5359 + 2(0.6667)] = 0.751$ (3sf)

8 **a** $= 1 + 12(\frac{x}{10}) + \frac{12\times11}{2}(\frac{x}{10})^2 + \frac{12\times11\times10}{3\times2}(\frac{x}{10})^3 + \ldots$

$= 1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3 + \ldots$

b $\approx \int_0^1 (1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3)\ dx$

$= [x + \frac{3}{5}x^2 + \frac{11}{50}x^3 + \frac{11}{200}x^4]_0^1$

$= (1 + \frac{3}{5} + \frac{11}{50} + \frac{11}{200}) - 0 = 1\frac{7}{8}$

9 **a** at A, $x = 0 \Rightarrow (0, 2)$

$\frac{dy}{dx} = -1 - 2x$

grad at $A = -1$

$\therefore\ y = 2 - x$

b curve cuts x-axis when $y = 0$

$2 - x - x^2 = 0$

$(2 + x)(1 - x) = 0$

$x = -2, 1$

area below curve

$= \int_0^1 (2 - x - x^2)\ dx$

$= [2x - \frac{1}{2}x^2 - \frac{1}{3}x^3]_0^1$

$= (2 - \frac{1}{2} - \frac{1}{3}) - 0 = \frac{7}{6}$

tangent cuts x-axis when $y = 0$

$x = 2$

area below line

$= \frac{1}{2} \times 2 \times 2 = 2$

shaded area

$= 2 - \frac{7}{6}$

$= \frac{5}{6}$