General Certificate of Education
Advanced Level Examination

MATHEMATICS A

Discrete 1

Paper A

Time allowed: 1 hour 20 minutes

Instructions and Information

- A graphics calculator may be used.
- Answer **all** questions.
- All necessary working should be shown or marks for method may be lost.
- The maximum mark for this paper is 60.
- You are reminded of the need for good English and clear presentation.
- Final answers to questions requiring the use of statistical tables or calculators should normally be given to three significant figures.

Written by Shaun Armstrong, Dave Hayes & Craig Hunter

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1. *A sheet is provided for use in answering this question.*

A cable TV company wishes to link 5 villages in the Scottish Highlands.
The table below shows the shortest distances, in kilometres, between these 5 villages.

	Durness	Helmsdale	Inverness	Thurso	Wick
Durness	–	68	123	81	92
Helmsdale	68	–	102	72	64
Inverness	123	102	–	148	127
Thurso	81	72	148	–	48
Wick	92	64	127	48	–

(a) Starting at Thurso, use Prim's algorithm to find a minimum spanning tree.

You should make your method clear, indicating the order in which you selected the arcs in your final tree.

(4 marks)

(b) Calculate the minimum total length of cable required. *(1 mark)*

5 marks

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2. The manager of an outdoor centre must staff each activity offered by the centre with an appropriately qualified instructor. The table below shows the sports for which each member of staff is qualified to supervise.

Name	Activities
Fatima	Windsurfing, Sailing
Gavin	Climbing, Orienteering
Hassan	Windsurfing, Climbing
Iain	Sailing, Diving
Jane	Diving, Sailing, Orienteering

(a) Draw a bipartite graph to model this situation. *(2 marks)*

Initially the manager allocates Fatima, Gavin, Iain and Jane to supervise the first sport listed against their names in the table.

(b) Starting from this matching, use the maximum matching algorithm to find a complete matching. Indicate clearly how you have applied the algorithm.

(4 marks)

6 marks

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3. *A sheet is provided for use in answering this question.*

An algorithm is described by the flow chart shown in Figure 1 below.

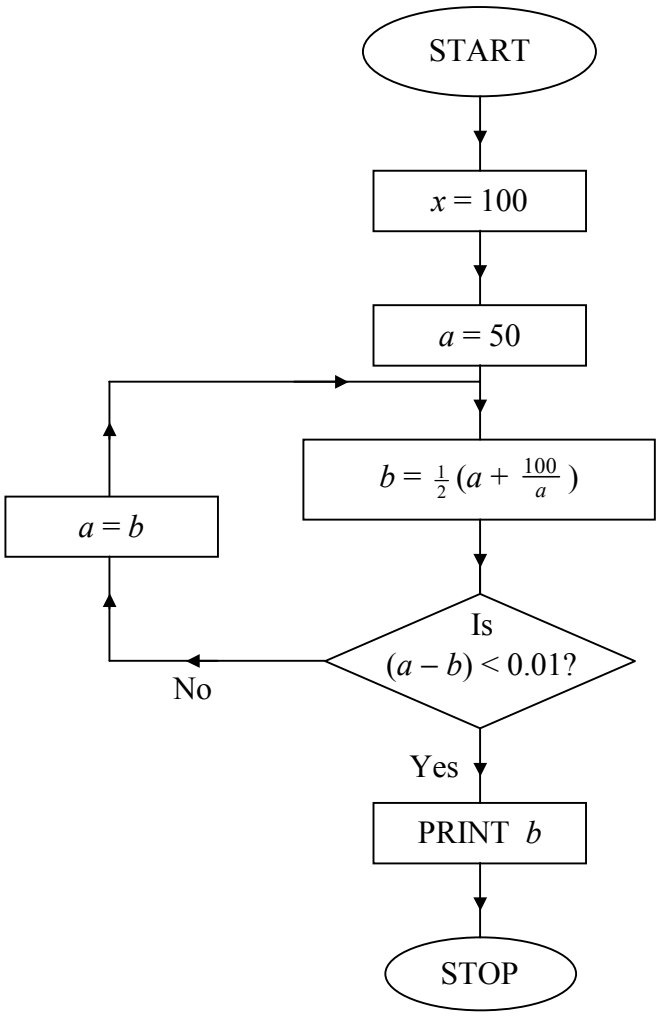

Fig. 1

(a) Complete the table on the answer sheet recording the results of each instruction as the algorithm is applied and state the final output.

(6 marks)

(b) Explain what the algorithm achieves. *(1 mark)*

7 marks

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4. *A sheet is provided for use in answering this question.*

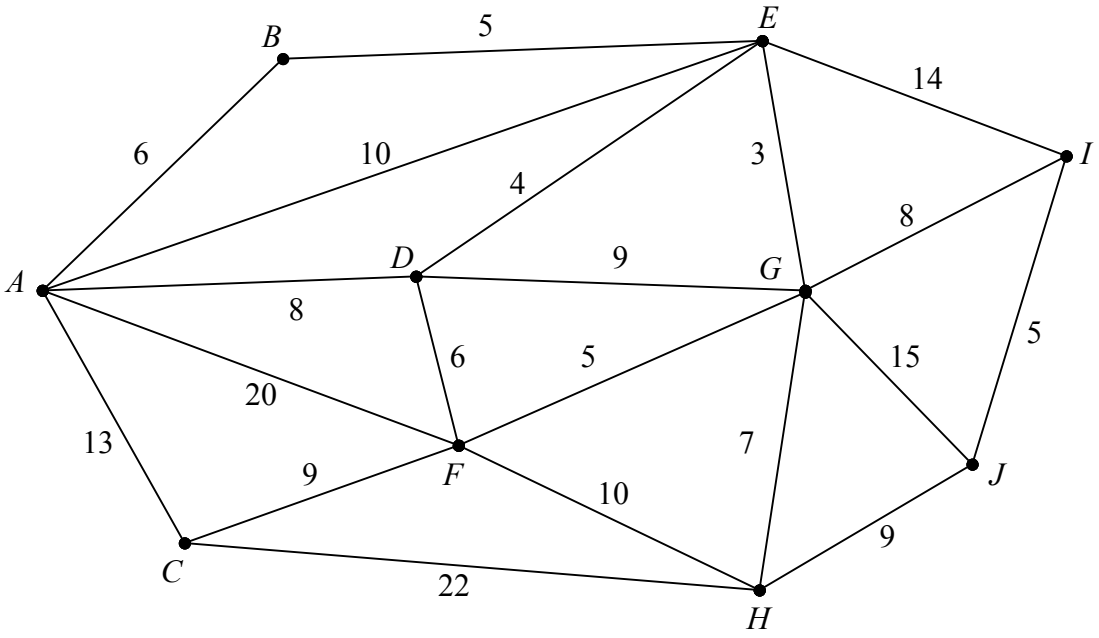

Fig. 2

Figure 2 shows a weighted network. The number on each arc indicates the weight of that arc.

Use Dijkstra's algorithm to find a path of least weight from A to J and state the weight of the path.

Your solution must show clearly how you have applied the algorithm including:

(i) the order in which the vertices were labelled,

(ii) how you determined the path of least weight.

7 marks

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5.

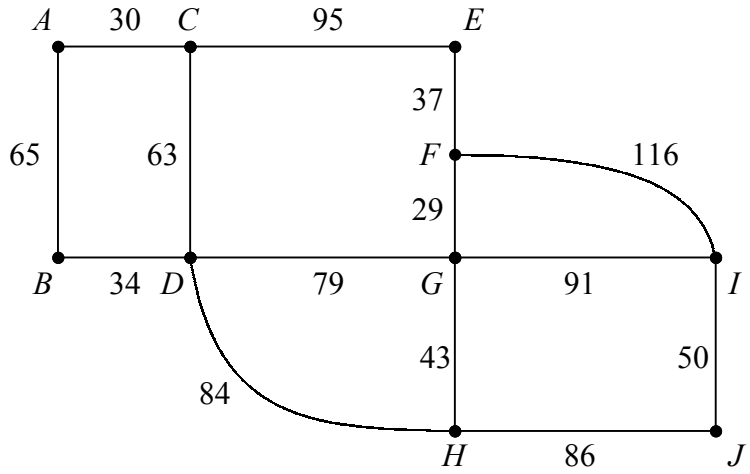

Fig. 3

The network in Figure 3 represents the roads being used in a treasure hunt and the numbers on the arcs are the length of the roads in metres. Participants in the treasure hunt start at point A and have to collect clues along all the sections of road shown before returning to A.

In order to collect the clues as quickly as possible participants need to find the shortest route that takes them along all of the roads.

(a) Explain why participants have to traverse some roads more than once. *(2 marks)*

(b) Find those sections of road which participants should repeat in order to minimise the total distance they have to cover and state the minimum total distance.

(6 marks)

8 marks

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6. *A sheet is provided for use in answering this question.*

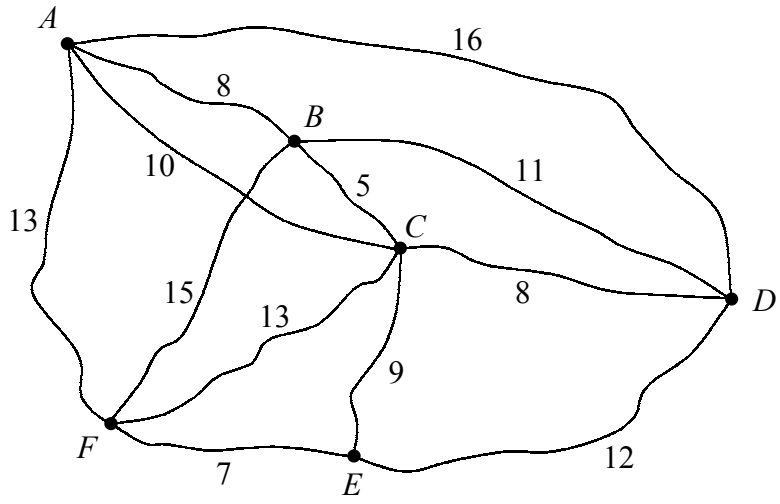

Fig. 4

The network in Figure 4 shows the distances, in miles, between a newspaper distributor based at area A, and five areas, B, C, D, E, and F, to which the distributor must deliver newspapers. Each morning a delivery van has to set out from A and visit each of these areas before again returning to A, and the driver wishes to keep the total mileage to a minimum.

(a) Draw a complete network showing the shortest distances between the six areas.

(2 marks)

(b) Obtain a minimum spanning tree for the complete network and use shortcuts to find an upper bound for the length of the driver's route of less than 55 miles.

(6 marks)

(c) By deleting A, find a lower bound for the total length of the route. *(3 marks)*

11 marks

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7. A couple decide to divide their new garden into three regions. They plan to have x m^2 of lawn, y m^2 of paving and a flower bed of area z m^2.

Given that the total area of their garden is 240 m^2,

(a) express z in terms of x and y. *(1 mark)*

The couple decide that the lawn should cover at least 40% of the area of the garden but that the lawn must not be more than 3 times as big as the area of paving. They also want the area of paving to be at least as big as the flower bed.

The couple plan to spend £4 per m^2 of lawn, £12 per m^2 of paving and £20 per m^2 on the flower bed and wish to minimise the total amount they spend on their garden.

(b) Formulate the above situation as a linear programming problem expressing the objective function and constraints in terms of x and y.

(6 marks)

(c) Draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line.

(6 marks)

(d) Use your diagram to find the area of the garden that should be allocated to each region and state the cost of this solution.

(3 marks)

16 marks

Total 60 marks

END

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Sheet for answering question 1 *NAME* …….

Please hand this sheet in for marking

	Durness	Helmsdale	Inverness	Thurso	Wick
Durness	–	68	123	81	92
Helmsdale	68	–	102	72	64
Inverness	123	102	–	148	127
Thurso	81	72	148	–	48
Wick	92	64	127	48	–

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Sheet for answering question 3 *NAME* …….

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x	a	b	$(a - b) < 0.01$?
100	50	26	No
–	26	14.923	No

Final output …….

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Sheet for answering question 4 *NAME* …….

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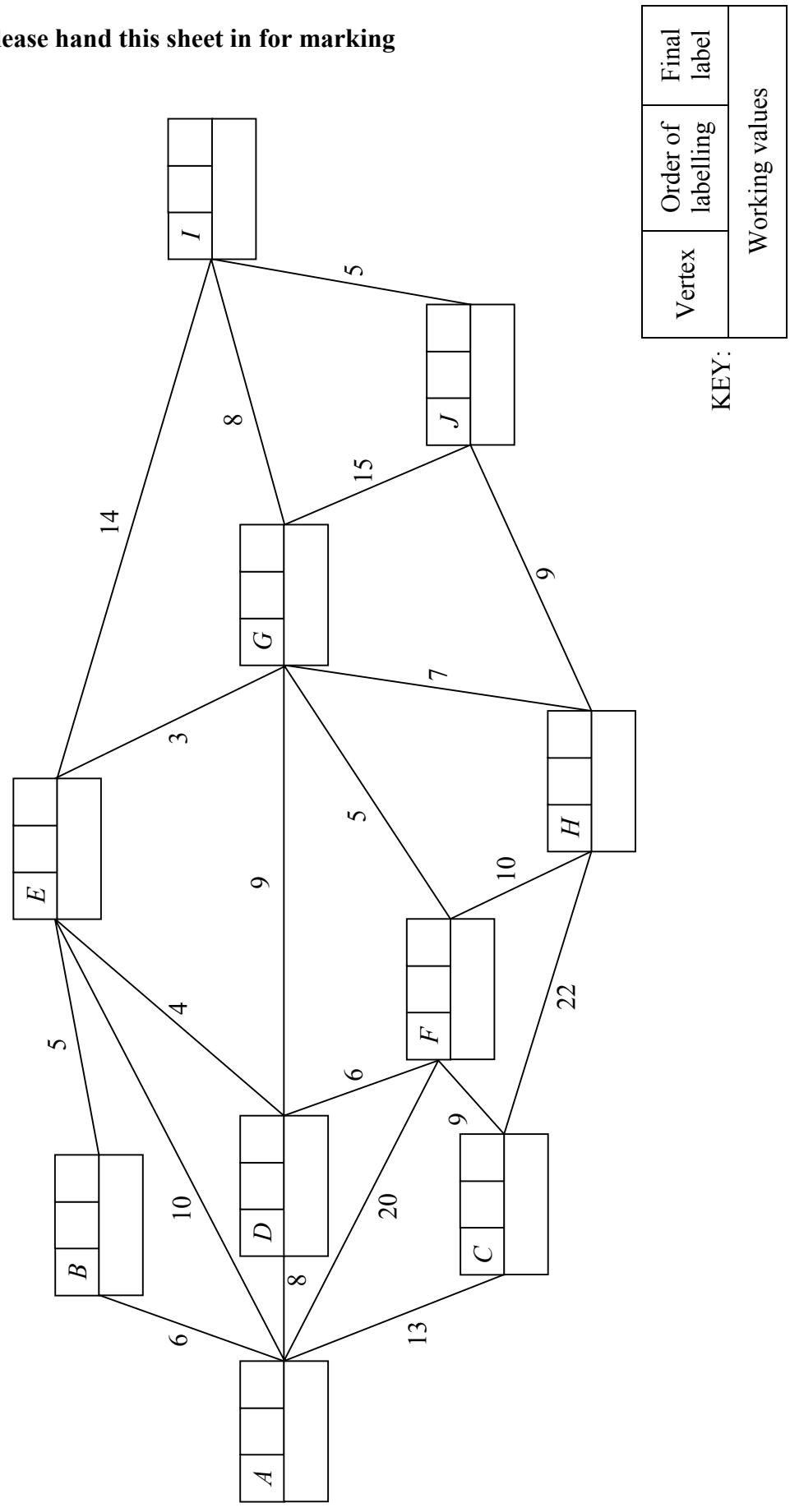

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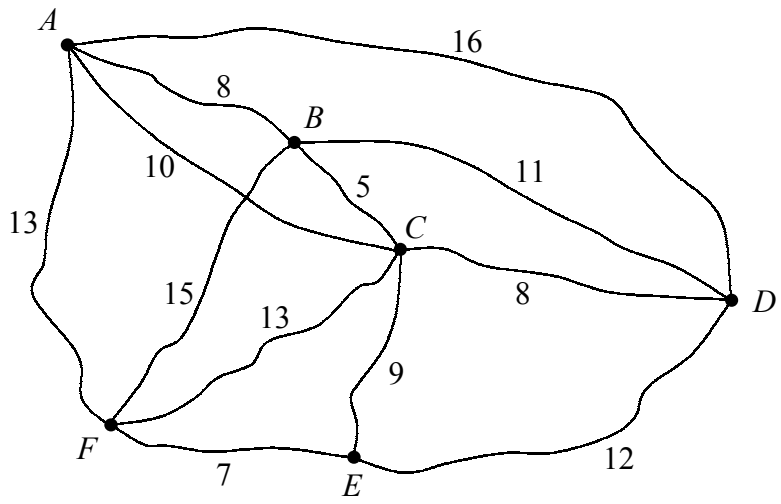

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