1 **a** e^x **b** $3e^x$ **c** $\frac{1}{x}$ **d** $\frac{1}{2x}$

2 **a** $-2e^t$ **b** $6t + \frac{1}{t}$ **c** $e^t + 5t^4$ **d** $\frac{3}{2}t^{\frac{1}{2}} + 2e^t$

e $\frac{2}{t} + \frac{1}{2}t^{-\frac{1}{2}}$ or $\frac{2}{t} + \frac{1}{2\sqrt{t}}$ **f** $2.5e^t - \frac{7}{2t}$ **g** $-t^{-2} + \frac{8}{t}$ or $\frac{8}{t} - \frac{1}{t^2}$ **h** $14t - 2 + 4e^t$

3 **a** $\frac{dy}{dx} = 12x^2 + e^x$

$\frac{d^2y}{dx^2} = 24x + e^x$

b $\frac{dy}{dx} = 7e^x - 10x + 3$

$\frac{d^2y}{dx^2} = 7e^x - 10$

c $\frac{dy}{dx} = \frac{1}{x} + \frac{5}{2}x^{\frac{3}{2}}$

$\frac{d^2y}{dx^2} = -x^{-2} + \frac{15}{4}x^{\frac{1}{2}}$

d $\frac{dy}{dx} = 5e^x + \frac{6}{x}$

$\frac{d^2y}{dx^2} = 5e^x - 6x^{-2}$

e $\frac{dy}{dx} = -3x^{-2} + \frac{3}{x}$

$\frac{d^2y}{dx^2} = 6x^{-3} - 3x^{-2}$

f $\frac{dy}{dx} = 2x^{-\frac{1}{2}} + \frac{1}{4x}$

$\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} - \frac{1}{4}x^{-2}$

4 **a** $f'(x) = 3 + e^x$

$f'(0) = 3 + 1 = 4$

b $f'(x) = \frac{1}{x} - 2x$

$f'(4) = \frac{1}{4} - 8 = -7\frac{3}{4}$

c $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{x}$

$f'(9) = \frac{1}{6} + \frac{2}{9} = \frac{7}{18}$

d $f'(x) = 5e^x - 2x^{-3}$

$f'(-\frac{1}{2}) = 5e^{-\frac{1}{2}} + 16$ [19.0 (3sf)]

5 **a** $\frac{dy}{dx} = \frac{5}{x} - 8 = 0$

$5 = 8x$

$x = \frac{5}{8}$

b $\frac{dy}{dx} = 2.4e^x - 3.6 = 0$

$e^x = 1.5$

$x = \ln 1.5$

[0.405 (3sf)]

c $\frac{dy}{dx} = 6x - 14 + \frac{4}{x} = 0$

$3x^2 - 7x + 2 = 0$

$(3x - 1)(x - 2) = 0$

$x = \frac{1}{3}, 2$

6 **a** $f'(x) = 2e^x - 3 = 7$

$e^x = 5$

$x = \ln 5$ [1.61 (3sf)]

b $f'(x) = 15 + \frac{1}{x} = 23$

$\frac{1}{x} = 8$

$x = \frac{1}{8}$

c $f'(x) = \frac{1}{4}x - 2 + \frac{1}{x} = -1$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$x = 2$

d $f'(x) = \frac{30}{x} - 2x = 4$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$\ln x$ only real for $x > 0$ $\therefore$ $x = 3$

7 **a** $\frac{dy}{dx} = e^x - 2$

SP: $e^x - 2 = 0$

$x = \ln 2$

$\frac{d^2y}{dx^2} = e^x$

$x = \ln 2$: $\frac{d^2y}{dx^2} = 2$

$\therefore$ $(\ln 2, 2 - 2\ln 2)$, min

b $\frac{dy}{dx} = \frac{1}{x} - 10$

SP: $\frac{1}{x} - 10 = 0$

$x = \frac{1}{10}$

$\frac{d^2y}{dx^2} = -x^{-2}$

$x = \frac{1}{10}$: $\frac{d^2y}{dx^2} = -100$

$\therefore$ $(\frac{1}{10}, -1 - \ln 10)$, max

c $\frac{dy}{dx} = \frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}}$

SP: $\frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}} = 0$

$4 - x^{\frac{1}{2}} = 0$

$x^{\frac{1}{2}} = 4$, $x = 16$

$\frac{d^2y}{dx^2} = -2x^{-2} + \frac{1}{4}x^{-\frac{3}{2}}$

$x = 16$: $\frac{d^2y}{dx^2} = -\frac{1}{256}$

$\therefore$ $(16, 8\ln 2 - 4)$, max

d $\frac{dy}{dx} = 4 - 5e^x$

SP: $4 - 5e^x = 0$

$x = \ln \frac{4}{5}$

$\frac{d^2y}{dx^2} = -5e^x$

$x = \ln \frac{4}{5}$: $\frac{d^2y}{dx^2} = -4$

$\therefore$ $(\ln \frac{4}{5}, 4\ln \frac{4}{5} - 4)$, max

e $\frac{dy}{dx} = 2 - \frac{4}{x}$

SP: $2 - \frac{4}{x} = 0$

$x = 2$

$\frac{d^2y}{dx^2} = 4x^{-2}$

$x = 2$: $\frac{d^2y}{dx^2} = 1$

$\therefore$ $(2, 11 - 4\ln 2)$, min

f $\frac{dy}{dx} = 2x - 26 + \frac{72}{x}$

SP: $2x - 26 + \frac{72}{x} = 0$

$x^2 - 13x + 36 = 0$

$(x - 4)(x - 9) = 0$

$x = 4, 9$

$\frac{d^2y}{dx^2} = 2 - 72x^{-2}$

$x = 4$: $\frac{d^2y}{dx^2} = -\frac{5}{2}$

$x = 9$: $\frac{d^2y}{dx^2} = \frac{10}{9}$

$\therefore$ $(4, 144\ln 2 - 88)$, max
$(9, 144\ln 3 - 153)$, min

8 $\frac{dy}{dx} = 1 + ke^x$

$\frac{d^2y}{dx^2} = ke^x$

$$\therefore\ (1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1 - x)ke^x + x(1 + ke^x) - (x + ke^x)$$
$$= ke^x - kxe^x + x + kxe^x - x - ke^x = 0$$

9 **a** $x = 2 \;\therefore\; y = e^2$
$\frac{dy}{dx} = e^x$, grad $= e^2$
$\therefore\; y - e^2 = e^2(x - 2)$
$[\; y = e^2(x - 1) \;]$

b $x = 3 \;\therefore\; y = \ln 3$
$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{3}$
$\therefore\; y - \ln 3 = \frac{1}{3}(x - 3)$
$[\; y = \frac{1}{3}x + \ln 3 - 1 \;]$

c $x = 0 \;\therefore\; y = -2$
$\frac{dy}{dx} = 0.8 - 2e^x$, grad $= -1.2$
$\therefore\; y = -1.2x - 2$

d $x = 1 \;\therefore\; y = 4$
$\frac{dy}{dx} = \frac{5}{x} - 4x^{-2}$, grad $= 1$
$\therefore\; y - 4 = x - 1$
$[\; y = x + 3 \;]$

e $x = 1 \;\therefore\; y = 1 - 3e$
$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 3e^x$, grad $= \frac{1}{3} - 3e$
$\therefore\; y - (1 - 3e) = (\frac{1}{3} - 3e)(x - 1)$
$[\; y = (\frac{1}{3} - 3e)x + \frac{2}{3} \;]$

f $x = 9 \;\therefore\; y = \ln 9 - 3$
$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2}x^{-\frac{1}{2}}$, grad $= -\frac{1}{18}$
$\therefore\; y - (\ln 9 - 3) = -\frac{1}{18}(x - 9)$
$[\; y = \ln 9 - \frac{5}{2} - \frac{1}{18}x \;]$

10 **a** $x = e \;\therefore\; y = 1$
$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{e}$
$\therefore$ grad of normal $= -e$
$\therefore\; y - 1 = -e(x - e)$
$[\; y = e^2 + 1 - ex \;]$

b $x = 0 \;\therefore\; y = 7$
$\frac{dy}{dx} = 3e^x$, grad $= 3$
$\therefore$ grad of normal $= -\frac{1}{3}$
$\therefore\; y = 7 - \frac{1}{3}x$

c $x = 3 \;\therefore\; y = 10 + \ln 3$
$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{3}$
$\therefore$ grad of normal $= -3$
$\therefore\; y - (10 + \ln 3) = -3(x - 3)$
$[\; y = 19 + \ln 3 - 3x \;]$

d $x = 1 \;\therefore\; y = -2$
$\frac{dy}{dx} = \frac{3}{x} - 2$, grad $= 1$
$\therefore$ grad of normal $= -1$
$\therefore\; y + 2 = -(x - 1)$
$[\; y = -x - 1 \;]$

e $x = 1 \;\therefore\; y = 1$
$\frac{dy}{dx} = 2x + \frac{8}{x}$, grad $= 10$
$\therefore$ grad of normal $= -\frac{1}{10}$
$\therefore\; y - 1 = -\frac{1}{10}(x - 1)$
$[\; y = \frac{1}{10}(11 - x) \;]$

f $x = 0 \;\therefore\; y = -\frac{13}{10}$
$\frac{dy}{dx} = \frac{1}{10} - \frac{3}{10}e^x$, grad $= -\frac{1}{5}$
$\therefore$ grad of normal $= 5$
$\therefore\; y = 5x - \frac{13}{10}$

1 **a** $x = 0 \;\therefore\; y = \frac{1}{10}$

$\frac{dy}{dx} = \frac{2}{5} + \frac{1}{10}e^x$, grad $= \frac{1}{2}$

$\therefore$ grad of normal $= -2$

$\therefore\; y = -2x + \frac{1}{10}$

$20x + 10y - 1 = 0$

b $y = 0 \;\therefore\; x = \frac{1}{20}$

$(\frac{1}{20}, 0)$

2 **a** $x = 1 \;\therefore\; y = 5e$

$\frac{dy}{dx} = 5e^x - \frac{3}{x}$, grad $= 5e - 3$

$\therefore\; y - 5e = (5e - 3)(x - 1)$

$y = (5e - 3)x + 3$

b at Q, $x = 0 \;\therefore\; y = 3$

R is $(1, 0)$

area $= \frac{1}{2} \times (3 + 5e) \times 1$

$= \frac{1}{2}(5e + 3)$

3 **a** $\frac{dy}{dx} = 3 - \frac{1}{2}e^x$

SP: $3 - \frac{1}{2}e^x = 0$

$x = \ln 6$

$\therefore\; (\ln 6, 3\ln 6 - 3)$

b $\frac{d^2y}{dx^2} = -\frac{1}{2}e^x$

$x = \ln 6$: $\frac{d^2y}{dx^2} = -3$

$\therefore$ max

4 **a** at P, $x = 4 \;\therefore\; y = 6\ln 4 - 8$

$\frac{dy}{dx} = \frac{6}{x} - 2x^{-\frac{1}{2}}$, grad $= \frac{1}{2}$

$\therefore\; y - (6\ln 4 - 8) = \frac{1}{2}(x - 4)$

$[\, y = \frac{1}{2}x - 10 + 12\ln 2 \,]$

b at Q, $y = 0 \;\therefore\; x = 20 - 24\ln 2$

at R, $x = 0 \;\therefore\; y = 12\ln 2 - 10$

area $= \frac{1}{2} \times (20 - 24\ln 2) \times (10 - 12\ln 2)$

$= (10 - 12\ln 2)^2$

5 **a** $\frac{dy}{dx} = 2 - \frac{1}{x}$, grad $= 1$

$\therefore\; y = x - 1$

b grad of normal $= -1$

$\therefore\; y = -(x - 1)$ $[\, y = 1 - x \,]$

at B, $x = 0 \;\therefore\; y = -1$

at C, $x = 0 \;\therefore\; y = 1$

mid-point of $(0, -1)$ and $(0, 1)$

$= (0, \frac{-1+1}{2}) = (0, 0)$

$\therefore$ mid-point of BC is the origin

c SP: $2 - \frac{1}{x} = 0$

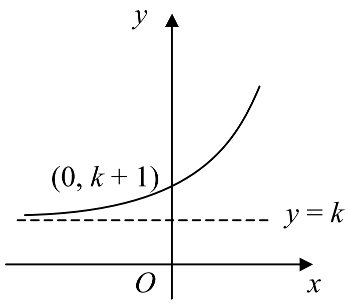
$x = \frac{1}{2}$

$\therefore\; y = 1 - 2 - \ln\frac{1}{2}$

$= -1 - \ln 2^{-1}$

$= \ln 2 - 1$

6 **a**



b $x = 2 \;\therefore\; y = e^2 + k$

$\frac{dy}{dx} = e^x$, grad $= e^2$

$\therefore\; y - (e^2 + k) = e^2(x - 2)$

$[\, y = e^2x - e^2 + k \,]$

c $(-1, 0)\;\therefore$ $0 = -e^2 - e^2 + k$

$k = 2e^2$

7 **a** $\frac{dy}{dx} = 6x - \frac{2}{x}$

at P, $6x - \frac{2}{x} = -1$

$6x^2 + x - 2 = 0$

$(3x + 2)(2x - 1) = 0$

$x > 0 \therefore x = \frac{1}{2}$

b $x = 1 \therefore y = 3$, grad $= 4$

$\therefore y - 3 = 4(x - 1)$

$[\, y = 4x - 1 \,]$

8 **a** $\frac{dy}{dx} = e^x$, grad at $P = e^p$

tangent: $y - e^p = e^p(x - p)$

$(0, 0) \therefore 0 - e^p = e^p(0 - p)$

$e^p(p - 1) = 0$

$e^p \neq 0 \therefore p = 1$

b P (1, e), grad at P = e

$\therefore$ grad of normal $= -\frac{1}{e}$

$\therefore y - e = -\frac{1}{e}(x - 1)$

at Q, $y = 0 \therefore x = e^2 + 1$

$\therefore$ area $= \frac{1}{2} \times (e^2 + 1) \times e = \frac{1}{2}e(1 + e^2)$

9 **a** at P, $x = 0 \therefore y = 3$

$\frac{dy}{dx} = -e^x$, grad $= -1$

$\therefore$ grad of normal $= 1$

$\therefore y = x + 3$

b at Q, $y = 0 \therefore x = \ln 4$

grad at $Q = -4$

$\therefore y = -4(x - \ln 4)$ $[\, y = 8 \ln 2 - 4x \,]$

c at R $x + 3 = -4(x - \ln 4)$

$5x = 4 \ln 4 - 3 = 8 \ln 2 - 3$

$x = \frac{1}{5}(8 \ln 2 - 3)$

$\therefore a = \frac{8}{5}$

d $b = -\frac{3}{5}$

10 $f'(x) = 36x^3 - \frac{16}{x}$

SP: $36x^3 - \frac{16}{x} = 0$

$x^4 = \frac{4}{9}$

$x^2 = -\frac{2}{3}$ [no solutions] or $\frac{2}{3}$

$x > 0 \therefore x = \sqrt{\frac{2}{3}}$

$\therefore$ decreasing for $0 < x \leq \sqrt{\frac{2}{3}}$

$k = \sqrt{\frac{2}{3}}$ or $\frac{1}{3}\sqrt{6}$

1 **a** $= 5(x+3)^4$
b $= 3(2x-1)^2 \times 2 = 6(2x-1)^2$
c $= 7(8-x)^6 \times (-1) = -7(8-x)^6$
d $= 12(3x+4)^5 \times 3 = 36(3x+4)^5$
e $= 4(6-5x)^3 \times (-5) = -20(6-5x)^3$
f $= -(x-2)^{-2}$
g $= -12(2x+3)^{-4} \times 2 = -24(2x+3)^{-4}$
h $= -2(7-3x)^{-3} \times (-3)$
h $= 6(7-3x)^{-3}$

2 **a** $= 6e^{3t}$
b $= \frac{1}{2}(4t-1)^{-\frac{1}{2}} \times 4 = 2(4t-1)^{-\frac{1}{2}}$
c $= \frac{5}{t}$
d $= \frac{3}{2}(8-3t)^{\frac{1}{2}} \times (-3) = -\frac{9}{2}(8-3t)^{\frac{1}{2}}$
e $= \frac{3}{6t+1} \times 6 = \frac{18}{6t+1}$
f $= \frac{1}{2}e^{5t+4} \times 5 = \frac{5}{2}e^{5t+4}$
g $= \frac{d}{dx}[6(2t-5)^{-\frac{1}{3}}] = -2(2t-5)^{-\frac{4}{3}} \times 2 = -4(2t-5)^{-\frac{4}{3}}$
h $= \frac{2}{3-\frac{1}{4}t} \times (-\frac{1}{4}) = \frac{2}{t-12}$

3 **a** $\frac{dy}{dx} = 4(3x-1)^3 \times 3 = 12(3x-1)^3$
$\frac{d^2y}{dx^2} = 36(3x-1)^2 \times 3 = 108(3x-1)^2$
b $\frac{dy}{dx} = \frac{4}{1+2x} \times 2 = 8(1+2x)^{-1}$
$\frac{d^2y}{dx^2} = -8(1+2x)^{-2} \times 2 = \frac{-16}{(1+2x)^2}$
c $\frac{dy}{dx} = \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times (-2) = -(5-2x)^{-\frac{1}{2}}$
$\frac{d^2y}{dx^2} = \frac{1}{2}(5-2x)^{-\frac{3}{2}} \times (-2) = -(5-2x)^{-\frac{3}{2}}$

4 **a** $f'(x) = 2x - \frac{6}{x}$
$f'(3) = 6 - 2 = 4$
b $f'(x) = 2 - e^{x-2}$
$f'(2) = 2 - 1 = 1$
c $f'(x) = 4(2-5x)^3 \times (-5) = -20(2-5x)^3$
$f'(\frac{1}{2}) = -20 \times (-\frac{1}{8}) = \frac{5}{2}$
d $f'(x) = -4(x+5)^{-2}$
$f'(-1) = -4 \times \frac{1}{16} = -\frac{1}{4}$

5 **a** $f'(x) = 2(3x+15)^{-\frac{1}{2}} \times 3 = 2$
$\frac{6}{\sqrt{3x+15}} = 2$
$\sqrt{3x+15} = 3$
$3x + 15 = 9$
$x = -2$
b $f'(x) = 2x - \frac{1}{x-2} = 5$
$2x(x-2) - 1 = 5(x-2)$
$2x^2 - 9x + 9 = 0$
$(2x-3)(x-3) = 0$
for real $f(x)$, $x > 2$ $\therefore x = 3$

6 **a** $= 3(x^2-4)^2 \times 2x = 6x(x^2-4)^2$
b $= 12(3x^2+1)^5 \times 6x = 72x(3x^2+1)^5$
c $= \frac{1}{3+2x^2} \times 4x = \frac{4x}{3+2x^2}$
d $= \frac{d}{dx}[(4-x^2)^3] = 3(4-x^2)^2 \times (-2x) = -6x(4-x^2)^2$
e $= \frac{d}{dx}[(\frac{1}{2}x^4+3)^8] = 8(\frac{1}{2}x^4+3)^7 \times 2x^3 = 16x^3(\frac{1}{2}x^4+3)^7$
f $= \frac{d}{dx}[(3-x^2)^{-\frac{1}{2}}] = -\frac{1}{2}(3-x^2)^{-\frac{3}{2}} \times (-2x) = x(3-x^2)^{-\frac{3}{2}}$
g $= 7e^{x^2} \times 2x = 14xe^{x^2}$
h $= 4(1-5x+x^3)^3 \times (-5+3x^2) = 4(3x^2-5)(1-5x+x^3)^3$

i $= \frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}})$
$= \frac{3}{2x - 8\sqrt{x}}$

j $= 7(e^{4x}+2)^6 \times 4e^{4x}$
$= 28e^{4x}(e^{4x}+2)^6$

k $= -(5+4\sqrt{x})^{-2} \times 2x^{-\frac{1}{2}}$
$= \frac{-2}{\sqrt{x}(5+4\sqrt{x})^2}$

l $= 5(\frac{2}{x} - x)^4 \times (-2x^{-2} - 1)$
$= -5(\frac{2}{x^2} + 1)(\frac{2}{x} - x)^4$

7 **a** $\frac{dy}{dx} = 5(2x-3)^4 \times 2$
SP: $10(2x-3)^4 = 0$
$x = \frac{3}{2}$
$\therefore (\frac{3}{2}, 0)$

b $\frac{dy}{dx} = 3(x^2-4)^2 \times 2x$
SP: $6x(x^2-4)^2 = 0$
$x = 0$ or $x^2 = 4$
$x = 0, \pm 2$
$\therefore (-2, 0), (0, -64), (2, 0)$

c $\frac{dy}{dx} = 8 - 2e^{2x}$
SP: $8 - 2e^{2x} = 0$
$e^{2x} = 4$
$x = \frac{1}{2}\ln 4 = \ln 2$
$\therefore (\ln 2, 8\ln 2 - 4)$

d $\frac{dy}{dx} = \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \times 4x$
SP: $\frac{2x}{\sqrt{1+2x^2}} = 0$
$x = 0$
$\therefore (0, 1)$

e $\frac{dy}{dx} = \frac{2}{x-x^2} \times (1-2x)$
SP: $\frac{2(1-2x)}{x-x^2} = 0$
$x = \frac{1}{2}$
$\therefore (\frac{1}{2}, -4\ln 2)$

f $\frac{dy}{dx} = 4 - (x-3)^{-2}$
SP: $4 - \frac{1}{(x-3)^2} = 0$
$(x-3)^2 = \frac{1}{4},\ x - 3 = \pm\frac{1}{2}$
$x = \frac{5}{2}, \frac{7}{2}$
$\therefore (\frac{5}{2}, 8), (\frac{7}{2}, 16)$

8 **a** $x = 2 \ \therefore y = 1$
$\frac{dy}{dx} = 4(3x-7)^3 \times 3 = 12(3x-7)^3$
grad $= -12$
$\therefore y - 1 = -12(x-2)$
$[\, y = 25 - 12x \,]$

b $x = 0 \ \therefore y = 2$
$\frac{dy}{dx} = \frac{1}{1+4x} \times 4 = \frac{4}{1+4x}$
grad $= 4$
$\therefore y = 4x + 2$

c $x = 1 \ \therefore y = 3$
$\frac{dy}{dx} = -9(x^2+2)^{-2} \times 2x = -18x(x^2+2)^{-2}$
grad $= -2$
$\therefore y - 3 = -2(x-1)$
$[\, y = 5 - 2x \,]$

d $x = \frac{1}{4} \ \therefore y = \frac{1}{2}$
$\frac{dy}{dx} = \frac{1}{2}(5x-1)^{-\frac{1}{2}} \times 5 = \frac{5}{2}(5x-1)^{-\frac{1}{2}}$
grad $= 5$
$\therefore y - \frac{1}{2} = 5(x - \frac{1}{4})$
$[\, y = 5x - \frac{3}{4} \,]$

9 **a** $x = -2 \ \therefore y = -9$
$\frac{dy}{dx} = e^{4-x^2} \times (-2x) = -2x\,e^{4-x^2}$
grad $= 4 \ \therefore$ grad of normal $= -\frac{1}{4}$
$\therefore y + 9 = -\frac{1}{4}(x+2)$
$[\, y = -\frac{1}{4}x - \frac{19}{2} \,]$

b $x = \frac{1}{2} \ \therefore y = \frac{1}{8}$
$\frac{dy}{dx} = 3(1-2x^2)^2 \times (-4x) = -12x(1-2x^2)^2$
grad $= -\frac{3}{2} \ \therefore$ grad of normal $= \frac{2}{3}$
$\therefore y - \frac{1}{8} = \frac{2}{3}(x - \frac{1}{2})$
$[\, 16x - 24y - 5 = 0 \,]$

c $x = 1 \ \therefore y = \frac{1}{2}$
$\frac{dy}{dx} = -(2 - \ln x)^{-2} \times (-\frac{1}{x}) = \frac{1}{x(2-\ln x)^2}$
grad $= \frac{1}{4} \ \therefore$ grad of normal $= -4$
$\therefore y - \frac{1}{2} = -4(x-1)$
$[\, y = \frac{9}{2} - 4x \,]$

d $x = 3 \ \therefore y = 6e$
$\frac{dy}{dx} = 2e^{\frac{x}{3}}$
grad $= 2e \ \therefore$ grad of normal $= -\frac{1}{2e}$
$\therefore y - 6e = -\frac{1}{2e}(x-3)$
$[\, x + 2ey - 12e^2 - 3 = 0 \,]$

1 $x = \frac{1}{2}$ $\therefore y = \frac{1}{4}$

$\frac{dy}{dx} = 2x + \frac{1}{4x-1} \times 4 = 2x + \frac{4}{4x-1}$

grad = 1 + 4 = 5

$\therefore y - \frac{1}{4} = 5(x - \frac{1}{2})$

$[\, y = 5x - \frac{9}{4} \,]$

2 **a** $\sqrt{8 - e^{2x}} = 2$

$8 - e^{2x} = 4$

$x = \frac{1}{2}\ln 4 = \ln 2$

b $\frac{dy}{dx} = \frac{1}{2}(8 - e^{2x})^{-\frac{1}{2}} \times (-2e^{2x})$

$= \frac{-e^{2x}}{\sqrt{8 - e^{2x}}}$

grad = –2

$\therefore y - 2 = -2(x - \ln 2)$

$2x + y = 2 + 2\ln 2$

$2x + y = 2 + \ln 2^2$

$2x + y = 2 + \ln 4$

3 **a** $\frac{dy}{dx} = 2 + \frac{1}{4-2x} \times (-2) = 2 - \frac{1}{2-x}$

$\frac{d^2y}{dx^2} = (2 - x)^{-2} \times (-1) = \frac{-1}{(2-x)^2}$

b SP: $2 - \frac{1}{2-x} = 0$

$2 - x = \frac{1}{2}$

$x = \frac{3}{2}$ $\therefore (\frac{3}{2}, 4)$

c $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = -4$ $\therefore$ maximum

4 **a** $\frac{dy}{dx} = -3(2x + 1)^{-2} \times 2 = \frac{-6}{(2x+1)^2}$

$x = 1$, grad $= -\frac{2}{3}$, $\therefore$ grad of normal $= \frac{3}{2}$

$\therefore y - 1 = \frac{3}{2}(x - 1)$

$[\, y = \frac{3}{2}x - \frac{1}{2} \,]$

b at Q $\frac{3x-1}{2} = \frac{3}{2x+1}$

$(3x - 1)(2x + 1) = 6$

$6x^2 + x - 7 = 0$

$(6x + 7)(x - 1) = 0$

$x = 1$ (at P) or $-\frac{7}{6}$

$\therefore Q\,(-\frac{7}{6}, -\frac{9}{4})$

5 **a** $t = 0, N = 20$ $\therefore a = 20$

$t = 8, N = 60$ $\therefore 60 = 20e^{8k}$

$k = \frac{1}{8}\ln 3 = 0.137$ (3sf)

b $N = 20e^{0.1373t}$

$t = 12, N = 104$ (3sf)

c $\frac{dN}{dt} = 20 \times 0.1373e^{0.1373t} = 2.747e^{0.1373t}$

$t = 12$, $\frac{dN}{dt} = 14.3$

$\therefore$ N increasing at 14.3 per second (3sf)

6 **a** $= 3(5 - 2x^2)^2 \times (-4x)$

$= -12x(5 - 2x^2)^2$

b SP: $-12x(5 - 2x^2)^2 = 0$

$x = 0$ or $x^2 = \frac{5}{2}$

$x = 0, \pm\frac{1}{2}\sqrt{10}$

$\therefore (-\frac{1}{2}\sqrt{10}, 0), (0, 125), (\frac{1}{2}\sqrt{10}, 0)$

c $x = \frac{3}{2}$, $y = \frac{1}{8}$

grad $= -18 \times \frac{1}{4} = -\frac{9}{2}$

$\therefore y - \frac{1}{8} = -\frac{9}{2}(x - \frac{3}{2})$

$8y - 1 = -36x + 54$

$36x + 8y - 55 = 0$

7 **a** $\frac{dy}{dx} = 4 - e^{2x}$

SP: $4 - e^{2x} = 0$

$x = \frac{1}{2}\ln 4 = \ln 2$

$\therefore$ (ln 2, 4 ln 2 – 2)

b $\frac{d^2y}{dx^2} = -2e^{2x}$

$x = \ln 2$: $\frac{d^2y}{dx^2} = -8$ $\therefore$ maximum

8 **a** $f'(x) = \frac{3}{x} - 2$

b grad of curve = 4

$\therefore$ $\frac{3}{x} - 2 = 4$

$x = \frac{1}{2}$

c SP: $\frac{3}{x} - 2 = 0$

$x = \frac{3}{2}$ $\therefore$ $(\frac{3}{2}, 3\ln\frac{15}{2} - 3)$

d $x \geq \frac{3}{2}$

9 **a** $\frac{dy}{dx} = \frac{1}{2}(x^2+3)^{-\frac{1}{2}} \times 2x = \frac{x}{\sqrt{x^2+3}}$

at A, grad = $-\frac{1}{2}$

$\therefore$ $y - 2 = -\frac{1}{2}(x + 1)$

[$y = \frac{3}{2} - \frac{1}{2}x$]

b at B, grad = $\frac{1}{2}$

$\therefore$ grad of normal = –2

$\therefore$ $y - 2 = -2(x - 1)$

[$y = 4 - 2x$]

c $\frac{3}{2} - \frac{1}{2}x = 4 - 2x$

$x = \frac{5}{3}$

10 **a** 80°C

b 20°C, as $t \to \infty$, $T \to 20$

c $30 = 20 + 60e^{-25k}$

$e^{-25k} = \frac{30-20}{60} = \frac{1}{6}$

$k = \frac{-1}{25}\ln\frac{1}{6} = 0.0717$ (3sf)

d $T = 20 + 60e^{-0.07167t}$

$\frac{dT}{dt} = 60 \times (-0.07167)e^{-0.07167t}$

$= -4.300e^{-0.07167t}$

$t = 40$, $\frac{dT}{dt} = -0.245$

$\therefore$ temp. decreasing at 0.245°C min^{-1} (3sf)

11 **a** $f'(x) = 2x - 7 + \frac{4}{x} = 0$

$2x^2 - 7x + 4 = 0$

$x = \frac{7 \pm \sqrt{49-32}}{4} = \frac{7 \pm \sqrt{17}}{4}$

$x = 0.72, 2.78$

b $x = 2$ $\therefore$ $y = -10$, grad = –1

$\therefore$ $y + 10 = -(x - 2)$

[$y = -x - 8$]

12 **a** $\frac{dy}{dx} = 2x + 8(x - 1)^{-2}$

SP: $2x + \frac{8}{(x-1)^2} = 0$

$2x(x - 1)^2 + 8 = 0$

$2x(x^2 - 2x + 1) + 8 = 0$

$2x^3 - 4x^2 + 2x + 8 = 0$

$x^3 - 2x^2 + x + 4 = 0$

b let $f(x) = x^3 - 2x^2 + x + 4$

$f(1) = 4$, $f(2) = 6$, $f(-1) = 0$

$\therefore$ $(x + 1)$ is a factor

$\therefore$ $(x + 1)(x^2 - 3x + 4) = 0$

$x = -1$ or $x^2 - 3x + 4 = 0$

$b^2 - 4ac = 9 - 16 = -7$

$b^2 - 4ac < 0$ $\therefore$ no real roots

$\therefore$ exactly one SP

(–1, 5)

c $\frac{d^2y}{dx^2} = 2 - 16(x - 1)^{-3}$

when $x = -1$, $\frac{d^2y}{dx^2} = 4$

$\frac{d^2y}{dx^2} > 0$ $\therefore$ minimum

1 **a** $f(x) = x[x^3 + 3x^2(2) + 3x(2)^2 + 2^3]$
$= x^4 + 6x^3 + 12x^2 + 8x$
$f'(x) = 4x^3 + 18x^2 + 24x + 8$

b $f'(x) = 1 \times (x+2)^3 + x \times 3(x+2)^2$
$= (x+2)^2[(x+2) + 3x]$
$= 2(2x+1)(x+2)^2$

2 **a** $= 1 \times e^x + x \times e^x$
$= e^x(1 + x)$

b $= 1 \times (x+1)^5 + x \times 5(x+1)^4$
$= (x+1)^4[(x+1) + 5x]$
$= (6x+1)(x+1)^4$

c $= 1 \times \ln x + x \times \frac{1}{x}$
$= \ln x + 1$

d $= 2x \times (x-1)^3 + x^2 \times 3(x-1)^2$
$= x(x-1)^2[2(x-1) + 3x]$
$= x(5x-2)(x-1)^2$

e $= 3x^2 \times \ln 2x + x^3 \times \frac{1}{x}$
$= x^2(3 \ln 2x + 1)$

f $= 2x \times e^{-x} + x^2 \times (-e^{-x})$
$= xe^{-x}(2 - x)$

g $= 8x^3 \times (5+x)^3 + 2x^4 \times 3(5+x)^2$
$= 2x^3(5+x)^2[4(5+x) + 3x]$
$= 2x^3(20 + 7x)(5+x)^2$

h $= 2x \times (x-3)^4 + x^2 \times 4(x-3)^3$
$= 2x(x-3)^3[(x-3) + 2x]$
$= 6x(x-1)(x-3)^3$

3 **a** $= 1 \times (2x-1)^3 + x \times 3(2x-1)^2 \times 2$
$= (2x-1)^2[(2x-1) + 6x]$
$= (8x-1)(2x-1)^2$

b $= 12x^3 \times e^{2x+3} + 3x^4 \times e^{2x+3} \times 2$
$= 6x^3e^{2x+3}(2 + x)$

c $= 1 \times \sqrt{x-1} + x \times \frac{1}{2}(x-1)^{-\frac{1}{2}}$
$= \frac{1}{2}(x-1)^{-\frac{1}{2}}[2(x-1) + x]$
$= \dfrac{3x-2}{2\sqrt{x-1}}$

d $= 2x \times \ln(x+6) + x^2 \times \dfrac{1}{x+6}$
$= 2x \ln(x+6) + \dfrac{x^2}{x+6}$

e $= 1 \times (1-5x)^4 + x \times 4(1-5x)^3 \times (-5)$
$= (1-5x)^3[(1-5x) - 20x]$
$= (1-25x)(1-5x)^3$

f $= 1 \times (x-3)^3 + (x+2) \times 3(x-3)^2$
$= (x-3)^2[(x-3) + 3(x+2)]$
$= (4x+3)(x-3)^2$

g $= \frac{4}{3}x^{\frac{1}{3}} \times e^{3x} + x^{\frac{4}{3}} \times 3e^{3x}$
$= \frac{1}{3}x^{\frac{1}{3}}e^{3x}(4 + 9x)$

h $= 1 \times \ln(x^2-1) + (x+1) \times \dfrac{1}{x^2-1} \times 2x$
$= \ln(x^2-1) + \dfrac{2x(x+1)}{(x+1)(x-1)}$
$= \ln(x^2-1) + \dfrac{2x}{x-1}$

i $= 2x \times \sqrt{3x+1} + x^2 \times \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3$
$= \frac{1}{2}x(3x+1)^{-\frac{1}{2}}[4(3x+1) + 3x]$
$= \dfrac{x(15x+4)}{2\sqrt{3x+1}}$

4 **a** $f'(x) = 4 \times e^{3x} + 4x \times 3e^{3x}$
$= 4e^{3x}(1 + 3x)$
$f'(0) = 4 \times 1 \times 1 = 4$

b $f'(x) = 2 \times (x^2 + 2)^3 + 2x \times 3(x^2 + 2)^2 \times 2x$
$= 2(x^2 + 2)^2[(x^2 + 2) + 6x^2]$
$= 2(7x^2 + 2)(x^2 + 2)^2$
$f'(-1) = 2 \times 9 \times 9 = 162$

c $f'(x) = 5 \times \ln 3x + (5x - 4) \times \frac{1}{x}$
$= 5 \ln 3x + 5 - \frac{4}{x}$
$f'(\frac{1}{3}) = 0 + 5 - 12 = -7$

d $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \times (1 - 2x)^3 + x^{\frac{1}{2}} \times 3(1 - 2x)^2 \times (-2)$
$= \frac{1}{2}x^{-\frac{1}{2}}(1 - 2x)^2[(1 - 2x) - 12x]$
$= \frac{1}{2}x^{-\frac{1}{2}}(1 - 14x)(1 - 2x)^2$
$f'(\frac{1}{4}) = \frac{1}{2} \times 2 \times (-\frac{5}{2}) \times \frac{1}{4} = -\frac{5}{8}$

5 **a** $\frac{dy}{dx} = 1 \times e^{2x} + x \times 2e^{2x}$
SP: $e^{2x}(1 + 2x) = 0$
$x = -\frac{1}{2}$
$\therefore (-\frac{1}{2}, -\frac{1}{2}e^{-1})$

b $\frac{dy}{dx} = 1 \times (x - 4)^3 + x \times 3(x - 4)^2$
$= (x - 4)^2[(x - 4) + 3x]$
SP: $4(x - 1)(x - 4)^2 = 0$
$x = 1, 4$
$\therefore (1, -27), (4, 0)$

c $\frac{dy}{dx} = 2x \times (2x - 3)^4 + x^2 \times 4(2x - 3)^3 \times 2$
$= 2x(2x - 3)^3[(2x - 3) + 4x]$
SP: $6x(2x - 1)(2x - 3)^3 = 0$
$x = 0, \frac{1}{2}, \frac{3}{2}$
$\therefore (0, 0), (\frac{1}{2}, 4), (\frac{3}{2}, 0)$

d $\frac{dy}{dx} = 1 \times \sqrt{x+12} + x \times \frac{1}{2}(x+12)^{-\frac{1}{2}}$
$= \frac{1}{2}(x+12)^{-\frac{1}{2}}[2(x + 12) + x]$
SP: $\frac{3(x+8)}{2\sqrt{x+12}} = 0$
$x = -8$
$\therefore (-8, -16)$

e $\frac{dy}{dx} = 2x \times e^{-4x} + x^2 \times (-4e^{-4x})$
SP: $2xe^{-4x}(1 - 2x) = 0$
$x = \frac{1}{2}$
$\therefore (\frac{1}{2}, 2 + \frac{1}{4}e^{-2})$

f $\frac{dy}{dx} = -3 \times (3 - x)^3 + (1 - 3x) \times 3(3 - x)^2 \times (-1)$
$= -3(3 - x)^2[(3 - x) + (1 - 3x)]$
SP: $-12(1 - x)(3 - x)^2 = 0$
$x = 1, 3$
$\therefore (1, -16), (3, 0)$

6 **a** $x = 1 \quad \therefore y = 1$
$\frac{dy}{dx} = 1 \times (x - 2)^4 + x \times 4(x - 2)^3$
$= (x - 2)^3[(x - 2) + 4x]$
$= (5x - 2)(x - 2)^3$
grad $= -3$
$\therefore y - 1 = -3(x - 1)$
$[\, y = 4 - 3x \,]$

b $x = 1 \quad \therefore y = 3e$
$\frac{dy}{dx} = 6x \times e^x + 3x^2 \times e^x$
$= 3xe^x(2 + x)$
grad $= 9e$
$\therefore y - 3e = 9e(x - 1)$
$[\, y = 3e(3x - 2) \,]$

c $x = \frac{1}{2} \quad \therefore y = 0$
$\frac{dy}{dx} = 4 \times \ln 2x + (4x - 1) \times \frac{1}{x}$
$= 4 \ln 2x + 4 - \frac{1}{x}$
grad $= 2$
$\therefore y - 0 = 2(x - \frac{1}{2})$
$[\, y = 2x - 1 \,]$

d $x = -2 \quad \therefore y = 8$
$\frac{dy}{dx} = 2x \times \sqrt{x+6} + x^2 \times \frac{1}{2}(x+6)^{-\frac{1}{2}}$
$= \frac{1}{2}x(x+6)^{-\frac{1}{2}}[4(x + 6) + x] = \frac{x(5x+24)}{2\sqrt{x+6}}$
grad $= -7$
$\therefore y - 8 = -7(x + 2)$
$[\, y = -7x - 6 \,]$

7 **a** $x = 1 \;\therefore\; y = 1$

$\frac{dy}{dx} = 2x \times (2 - x)^3 + x^2 \times 3(2 - x)^2 \times (-1)$

$= x(2 - x)^2[2(2 - x) - 3x]$

$= x(4 - 5x)(2 - x)^2$

grad $= -1$

$\therefore$ grad of normal $= 1$

$\therefore\; y - 1 = 1(x - 1)$

$x - y = 0$

b $x = 2 \;\therefore\; y = 0$

$\frac{dy}{dx} = 1 \times \ln(3x - 5) + x \times \frac{1}{3x-5} \times 3$

$= \ln(3x - 5) + \frac{3x}{3x-5}$

grad $= 6$

$\therefore$ grad of normal $= -\frac{1}{6}$

$\therefore\; y - 0 = -\frac{1}{6}(x - 2)$

$x + 6y - 2 = 0$

c $x = 0 \;\therefore\; y = -1$

$\frac{dy}{dx} = 2x \times e^{3x} + (x^2 - 1) \times 3e^{3x}$

$= e^{3x}[2x + 3(x^2 - 1)]$

$= e^{3x}(3x^2 + 2x - 3)$

grad $= -3$

$\therefore$ grad of normal $= \frac{1}{3}$

$\therefore\; y + 1 = \frac{1}{3}(x - 0)$

$x - 3y - 3 = 0$

d $x = 8 \;\therefore\; y = 16$

$\frac{dy}{dx} = 1 \times \sqrt{x-4} + x \times \frac{1}{2}(x-4)^{-\frac{1}{2}}$

$= \frac{1}{2}(x-4)^{-\frac{1}{2}}[2(x - 4) + x]$

$= \frac{3x-8}{2\sqrt{x-4}}$

grad $= 4$

$\therefore$ grad of normal $= -\frac{1}{4}$

$\therefore\; y - 16 = -\frac{1}{4}(x - 8)$

$x + 4y - 72 = 0$

8 **a** $x = 1 \;\therefore\; y = e$

$\frac{dy}{dx} = 1 \times e^{x^2} + x \times e^{x^2} \times 2x$

$= e^{x^2}(1 + 2x^2)$

grad $= 3e$

$\therefore\; y - e = 3e(x - 1)$

$[\, y = e(3x - 2) \,]$

b $x = 0 \;\Rightarrow\; y = -2e$

$y = 0 \;\Rightarrow\; x = \frac{2}{3}$

$\therefore$ area $= \frac{1}{2} \times 2e \times \frac{2}{3} = \frac{2}{3}e$

1 **a** $f(x) = x(x + 2)^{-1}$

$f'(x) = 1 \times (x + 2)^{-1} + x \times [-(x + 2)^{-2}]$

$= (x + 2)^{-2}[(x + 2) - x]$

$= \dfrac{2}{(x+2)^2}$

b $f'(x) = \dfrac{1\times(x+2) - x\times 1}{(x+2)^2}$

$= \dfrac{2}{(x+2)^2}$

2 **a** $= \dfrac{4\times(1-3x) - 4x\times(-3)}{(1-3x)^2}$

$= \dfrac{4}{(1-3x)^2}$

b $= \dfrac{e^x\times(x-4) - e^x\times 1}{(x-4)^2}$

$= \dfrac{e^x(x-5)}{(x-4)^2}$

c $= \dfrac{1\times(2x+3) - (x+1)\times 2}{(2x+3)^2}$

$= \dfrac{1}{(2x+3)^2}$

d $= \dfrac{\frac{1}{x}\times 2x - \ln x\times 2}{(2x)^2}$

$= \dfrac{1-\ln x}{2x^2}$

e $= \dfrac{1\times(2-x^2) - x\times(-2x)}{(2-x^2)^2}$

$= \dfrac{2+x^2}{(2-x^2)^2}$

f $= \dfrac{\frac{1}{2}x^{-\frac{1}{2}}\times(3x+2) - \sqrt{x}\times 3}{(3x+2)^2}$

$= \dfrac{(3x+2)-6x}{2\sqrt{x}(3x+2)^2} = \dfrac{2-3x}{2\sqrt{x}(3x+2)^2}$

g $= \dfrac{2e^{2x}\times(1-e^{2x}) - e^{2x}\times(-2e^{2x})}{(1-e^{2x})^2}$

$= \dfrac{2e^{2x}}{(1-e^{2x})^2}$

h $= \dfrac{2\times\sqrt{x-3} - (2x+1)\times\frac{1}{2}(x-3)^{-\frac{1}{2}}}{x-3}$

$= \dfrac{4(x-3)-(2x+1)}{2(x-3)^{\frac{3}{2}}} = \dfrac{2x-13}{2(x-3)^{\frac{3}{2}}}$

3 **a** $= \dfrac{2x\times(x+4) - x^2\times 1}{(x+4)^2}$

$= \dfrac{x^2+8x}{(x+4)^2}$

b $= \dfrac{\frac{1}{2}(x-4)^{-\frac{1}{2}}\times 2x^2 - \sqrt{x-4}\times 4x}{(2x^2)^2}$

$= \dfrac{x-4(x-4)}{4x^3\sqrt{x-4}} = \dfrac{16-3x}{4x^3\sqrt{x-4}}$

c $= \dfrac{2e^x\times(1-3e^x) - (2e^x+1)\times(-3e^x)}{(1-3e^x)^2}$

$= \dfrac{5e^x}{(1-3e^x)^2}$

d $= \dfrac{-1\times(x^3+2) - (1-x)\times 3x^2}{(x^3+2)^2}$

$= \dfrac{2x^3-3x^2-2}{(x^3+2)^2}$

e $= \dfrac{\frac{3}{3x-1}\times(x+2) - \ln(3x-1)\times 1}{(x+2)^2}$

$= \dfrac{3}{(3x-1)(x+2)} - \dfrac{\ln(3x-1)}{(x+2)^2}$

f $y = \dfrac{\sqrt{x+1}}{\sqrt{x+3}}$

$\dfrac{dy}{dx} = \dfrac{\frac{1}{2}(x+1)^{-\frac{1}{2}}\times\sqrt{x+3} - \sqrt{x+1}\times\frac{1}{2}(x+3)^{-\frac{1}{2}}}{x+3}$

$= \dfrac{(x+3)-(x+1)}{2(x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}} = \dfrac{1}{(x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}}$

4 **a** $\frac{dy}{dx} = \frac{2x\times(3-x) - x^2\times(-1)}{(3-x)^2}$

$= \frac{6x - x^2}{(3-x)^2}$

SP: $\frac{x(6-x)}{(3-x)^2} = 0$

$x = 0, 6$

$\therefore$ (0, 0), (6, −12)

b $\frac{dy}{dx} = \frac{4e^{4x}\times(2x-1) - e^{4x}\times 2}{(2x-1)^2}$

$= \frac{2e^{4x}(4x-3)}{(2x-1)^2}$

SP: $\frac{2e^{4x}(4x-3)}{(2x-1)^2} = 0$

$x = \frac{3}{4}$

$\therefore$ $(\frac{3}{4}, 2e^3)$

c $\frac{dy}{dx} = \frac{1\times\sqrt{2x+1} - (x+5)\times\frac{1}{2}(2x+1)^{-\frac{1}{2}}\times 2}{2x+1}$

$= \frac{(2x+1)-(x+5)}{(2x+1)^{\frac{3}{2}}} = \frac{x-4}{(2x+1)^{\frac{3}{2}}}$

SP: $\frac{x-4}{(2x+1)^{\frac{3}{2}}} = 0$

$x = 4$

$\therefore$ (4, 3)

d $\frac{dy}{dx} = \frac{\frac{1}{x}\times 2x - \ln 3x\times 2}{(2x)^2}$

$= \frac{1-\ln 3x}{2x^2}$

SP: $\frac{1-\ln 3x}{2x^2} = 0$

$x = \frac{1}{3}e$

$\therefore$ $(\frac{1}{3}e, \frac{3}{2}e^{-1})$

e $y = \frac{(x+1)^2}{(x-2)^2}$

$\frac{dy}{dx} = \frac{2(x+1)\times(x-2)^2 - (x+1)^2\times 2(x-2)}{(x-2)^4}$

$= \frac{2(x+1)[(x-2)-(x+1)]}{(x-2)^3} = \frac{-6(x+1)}{(x-2)^3}$

SP: $\frac{-6(x+1)}{(x-2)^3} = 0$

$x = -1$

$\therefore$ (−1, 0)

f $\frac{dy}{dx} = \frac{2x\times(x+2) - (x^2-3)\times 1}{(x+2)^2}$

$= \frac{x^2+4x+3}{(x+2)^2}$

SP: $\frac{(x+1)(x+3)}{(x+2)^2} = 0$

$x = -3, -1$

$\therefore$ (−3, −6), (−1, −2)

5 **a** $x = 2 \therefore y = 4$

$\frac{dy}{dx} = \frac{2\times(3-x) - 2x\times(-1)}{(3-x)^2}$

$= \frac{6}{(3-x)^2}$

grad = 6

$\therefore$ $y - 4 = 6(x - 2)$

[$y = 6x - 8$]

b $x = 0 \therefore y = 2$

$\frac{dy}{dx} = \frac{e^x\times(e^x+1) - (e^x+3)\times e^x}{(e^x+1)^2}$

$= \frac{-2e^x}{(e^x+1)^2}$

grad = $-\frac{1}{2}$

$\therefore$ $y - 2 = -\frac{1}{2}(x - 0)$

[$y = 2 - \frac{1}{2}x$]

c $x = 4 \therefore y = 2$

$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}\times(5-x) - \sqrt{x}\times(-1)}{(5-x)^2}$

$= \frac{(5-x)+2x}{2\sqrt{x}(5-x)^2} = \frac{5+x}{2\sqrt{x}(5-x)^2}$

grad = $\frac{9}{4}$

$\therefore$ $y - 2 = \frac{9}{4}(x - 4)$

[$y = \frac{9}{4}x - 7$]

d $x = -1 \therefore y = \frac{1}{2}$

$\frac{dy}{dx} = \frac{3\times(x^2+1) - (3x+4)\times 2x}{(x^2+1)^2}$

$= \frac{3-8x-3x^2}{(x^2+1)^2}$

grad = 2

$\therefore$ $y - \frac{1}{2} = 2(x + 1)$

[$y = 2x + \frac{5}{2}$]

6 **a** $x = 1 \;\therefore\; y = 0$

$$\frac{dy}{dx} = \frac{-1\times(3x+1)-(1-x)\times 3}{(3x+1)^2}$$

$$= \frac{-4}{(3x+1)^2}$$

grad $= -\frac{1}{4}$

$\therefore$ grad of normal = 4

$\therefore\; y - 0 = 4(x - 1)$

$4x - y - 4 = 0$

b $x = -2 \;\therefore\; y = -4$

$$\frac{dy}{dx} = \frac{4\times\sqrt{2-x} - 4x\times\frac{1}{2}(2-x)^{-\frac{1}{2}}\times(-1)}{2-x}$$

$$= \frac{4(2-x)+2x}{(2-x)^{\frac{3}{2}}} = \frac{8-2x}{(2-x)^{\frac{3}{2}}}$$

grad $= \frac{3}{2}$

$\therefore$ grad of normal $= -\frac{2}{3}$

$\therefore\; y + 4 = -\frac{2}{3}(x + 2)$

$2x + 3y + 16 = 0$

c $x = 3 \;\therefore\; y = 0$

$$\frac{dy}{dx} = \frac{\frac{2}{2x-5}\times(3x-5) - \ln(2x-5)\times 3}{(3x-5)^2}$$

$$= \frac{2}{(2x-5)(3x-5)} - \frac{3\ln(2x-5)}{(3x-5)^2}$$

grad $= \frac{1}{2}$

$\therefore$ grad of normal = –2

$\therefore\; y - 0 = -2(x - 3)$

$2x + y - 6 = 0$

d $x = 2 \;\therefore\; y = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1\times(x^3-4) - x\times 3x^2}{(x^3-4)^2}$$

$$= \frac{-4-2x^3}{(x^3-4)^2}$$

grad $= -\frac{5}{4}$

$\therefore$ grad of normal $= \frac{4}{5}$

$\therefore\; y - \frac{1}{2} = \frac{4}{5}(x - 2)$

$8x - 10y - 11 = 0$

7 **a**

$$\frac{dy}{dx} = \frac{x^{-\frac{1}{2}}\times(x-2) - (2\sqrt{x}-3)\times 1}{(x-2)^2}$$

$$= \frac{(x-2) - \sqrt{x}(2\sqrt{x}-3)}{\sqrt{x}(x-2)^2}$$

$$= \frac{-(x-3\sqrt{x}+2)}{\sqrt{x}(x-2)^2}$$

at A and B, $\frac{dy}{dx} = 0 \;\therefore\; x - 3\sqrt{x} + 2 = 0$

b $(\sqrt{x} - 1)(\sqrt{x} - 2) = 0$

$\sqrt{x} = 1, 2$

$x = 1, 4$

$\therefore\; A\,(1, 1),\; B\,(4, \frac{1}{2})$

1 **a** $\frac{dy}{dx} = 2x \times (2-x)^3 + x^2 \times 3(2-x)^2 \times (-1)$

$= x(2-x)^2(4-5x)$

grad $= -1$

$\therefore\ y - 1 = -(x-1)$ $[\, y = 2 - x \,]$

b grad of normal $= 1$

$\therefore\ y - 1 = x - 1$

$y = x$

when $x = 0$, $y = 0$ $\therefore$ passes through origin

2 **a** $\frac{dy}{dx} = \frac{1 \times (2x+3) - x \times 2}{(2x+3)^2} = \frac{3}{(2x+3)^2}$

grad $= 3$

$\therefore\ y + 1 = 3(x+1)$ $[\, y = 3x + 2 \,]$

b at $(0, 0)$, grad $= \frac{1}{3}$

$\therefore$ grad of normal $= -3$

$\therefore\ y = -3x$

c $3x + 2 = -3x$

$x = -\frac{1}{3}$ $\therefore\ (-\frac{1}{3}, 1)$

3 **a** $P(-3, 0)$, $Q(1, 0)$

b $\frac{dy}{dx} = 1 \times (x-1)^3 + (x+3) \times 3(x-1)^2$

$= (x-1)^2[(x-1) + 3(x+3)]$

$= 4(x+2)(x-1)^2$

SP: $4(x+2)(x-1)^2 = 0$

$x = 1$ (at Q) or -2

$\therefore\ R(-2, -27)$

4 **a** $\frac{dy}{dx} = 1 \times \sqrt{4x+1} + x \times \frac{1}{2}(4x+1)^{-\frac{1}{2}} \times 4$

$= (4x+1)^{-\frac{1}{2}}[(4x+1) + 2x] = \frac{6x+1}{\sqrt{4x+1}}$

b $\frac{6x+1}{\sqrt{4x+1}} - 5x\sqrt{4x+1} = 0$

$6x + 1 = 5x(4x+1)$

$20x^2 - x - 1 = 0$

$(5x+1)(4x-1) = 0$

$x = -\frac{1}{5}, \frac{1}{4}$

5 **a** at A, $y = 0$ $\therefore\ x = 1$

$\frac{dy}{dx} = \frac{2 \times (x^2+3) - 2(x-1) \times 2x}{(x^2+3)^2}$

$= \frac{6 + 4x - 2x^2}{(x^2+3)^2}$

$\therefore$ grad $= \frac{1}{2}$ $\therefore$ grad of normal $= -2$

$\therefore\ y - 0 = -2(x-1)$

$y = 2 - 2x$

b SP: $\frac{6 + 4x - 2x^2}{(x^2+3)^2} = 0$

$2(1+x)(3-x) = 0$

$x = -1, 3$

$\therefore\ (-1, -1), (3, \frac{1}{3})$

6 **a** $f'(x) = \frac{3}{2}x^{\frac{1}{2}} \times (x-3)^3 + x^{\frac{3}{2}} \times 3(x-3)^2$

$= \frac{3}{2}x^{\frac{1}{2}}(x-3)^2[(x-3) + 2x]$

$= \frac{3}{2}x^{\frac{1}{2}}(3x-3)(x-3)^2$

$= \frac{9}{2}x^{\frac{1}{2}}(x-1)(x-3)^2$ $[k = \frac{9}{2}]$

b SP: $\frac{9}{2}x^{\frac{1}{2}}(x-1)(x-3)^2 = 0$

$x > 0$ $\therefore\ x = 1, 3$

$\therefore\ (1, -8), (3, 0)$

7 **a** $f'(x) = 1 \times \sqrt{2x+12} + x \times \frac{1}{2}(2x+12)^{-\frac{1}{2}} \times 2$

$= (2x+12)^{-\frac{1}{2}}[(2x+12) + x]$

$= \frac{3x+12}{\sqrt{2x+12}}$

$f''(x) = \frac{3 \times \sqrt{2x+12} - (3x+12) \times \frac{1}{2}(2x+12)^{-\frac{1}{2}} \times 2}{2x+12}$

$= \frac{3(2x+12) - (3x+12)}{(2x+12)^{\frac{3}{2}}} = \frac{3x+24}{(2x+12)^{\frac{3}{2}}}$

$= \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$

b SP: $\frac{3x+12}{\sqrt{2x+12}} = 0$

$x = -4$

$\therefore\ (-4, -8)$

$f''(-4) = \frac{3}{2}$

$f''(-4) > 0$ $\therefore$ minimum

1

a $= -\sin x$

b $= 5\cos x$

c $= -3\sin 3x$

d $= \frac{1}{4}\cos \frac{1}{4}x$

e $= \cos(x + 1)$

f $= -3\sin(3x - 2)$

g $= -4\cos(\frac{\pi}{3} - x)$

h $= -\frac{1}{2}\sin(\frac{1}{2}x + \frac{\pi}{6})$

i $= 2\sin x\cos x$

j $= 6\cos^2 x \times (-\sin x)$
$= -6\cos^2 x\sin x$

k $= 2\cos(x - 1) \times [-\sin(x - 1)]$
$= -2\cos(x - 1)\sin(x - 1)$

l $= 4\sin^3 2x \times 2\cos 2x$
$= 8\sin^3 2x\cos 2x$

2

a $= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin x}{\cos x}\right)$
$= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$
$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$
$= \frac{1}{\cos^2 x}$
$= \sec^2 x$

b $= \frac{\mathrm{d}}{\mathrm{d}x}[(\cos x)^{-1}]$
$= -(\cos x)^{-2} \times (-\sin x)$
$= \frac{\sin x}{\cos^2 x}$
$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$
$= \sec x\tan x$

c $= \frac{\mathrm{d}}{\mathrm{d}x}[(\sin x)^{-1}]$
$= -(\sin x)^{-2} \times \cos x$
$= -\frac{\cos x}{\sin^2 x}$
$= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$
$= -\operatorname{cosec} x\cot x$

d $= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right)$
$= \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$
$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$
$= -\frac{1}{\sin^2 x}$
$= -\operatorname{cosec}^2 x$

3

a $= -2\operatorname{cosec}^2 2t$

b $= \sec(t + 2)\tan(t + 2)$

c $= 4\sec^2(4t - 3)$

d $= -3\operatorname{cosec} 3t\cot 3t$

e $= 2\tan t \times \sec^2 t$
$= 2\tan t\sec^2 t$

f $= -3\operatorname{cosec}(t + \frac{\pi}{6})\cot(t + \frac{\pi}{6})$

g $= 3\cot^2 t \times (-\operatorname{cosec}^2 t)$
$= -3\cot^2 t\operatorname{cosec}^2 t$

h $= 2\sec\frac{1}{2}t\tan\frac{1}{2}t$

i $= -2\operatorname{cosec}^2(2t - 3)$

j $= 2\sec 2t \times 2\sec 2t\tan 2t$
$= 4\sec^2 2t\tan 2t$

k $= -2\sec^2(\pi - 4t)$

l $= 2\operatorname{cosec}(3t + 1) \times -3\operatorname{cosec}(3t + 1)\cot(3t + 1)$
$= -6\operatorname{cosec}^2(3t + 1)\cot(3t + 1)$

4 **a** $= \frac{1}{\sin x} \times \cos x$
$= \cot x$

b $= 6e^{\tan x} \times \sec^2 x$
$= 6e^{\tan x} \sec^2 x$

c $= \frac{1}{2}(\cos 2x)^{-\frac{1}{2}} \times (-2 \sin 2x)$
$= -\frac{\sin 2x}{\sqrt{\cos 2x}}$

d $= e^{\sin 3x} \times 3 \cos 3x$
$= 3e^{\sin 3x} \cos 3x$

e $= -2 \operatorname{cosec}^2 x^2 \times 2x$
$= -4x \operatorname{cosec}^2 x^2$

f $= \frac{1}{2}(\sec x)^{-\frac{1}{2}} \times \sec x \tan x$
$= \frac{1}{2}\sqrt{\sec x} \tan x$

g $= 3e^{-\operatorname{cosec} 2x} \times 2 \operatorname{cosec} 2x \cot 2x$
$= 6 \operatorname{cosec} 2x \cot 2x \, e^{-\operatorname{cosec} 2x}$

h $= \frac{1}{\tan 4x} \times 4 \sec^2 4x$
$= \frac{\cos 4x}{\sin 4x} \times \frac{4}{\cos^2 4x}$
$= 4 \sec 4x \operatorname{cosec} 4x$

5 **a** $\frac{dy}{dx} = 1 + 2 \cos x$
SP: $1 + 2 \cos x = 0$
$\cos x = -\frac{1}{2}$
$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$
$\therefore (\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3})$,
$(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3})$

b $\frac{dy}{dx} = 2 \sec x \tan x - \sec^2 x$
$= \sec x(2 \tan x - \sec x)$
SP: $\sec x(2 \tan x - \sec x) = 0$
$2 \tan x - \sec x = 0$
$\frac{2 \sin x}{\cos x} = \frac{1}{\cos x}$
$\sin x = \frac{1}{2}$
$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$
$\therefore (\frac{\pi}{6}, \sqrt{3}), (\frac{5\pi}{6}, -\sqrt{3})$

c $\frac{dy}{dx} = \cos x - 2 \sin 2x$
SP: $\cos x - 2 \sin 2x = 0$
$\cos x - 4 \sin x \cos x = 0$
$\cos x(1 - 4 \sin x) = 0$
$\cos x = 0$ or $\sin x = \frac{1}{4}$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $0.253, \pi - 0.25268$
$x = 0.253$ (3sf), $\frac{\pi}{2}$, 2.89 (3sf), $\frac{3\pi}{2}$
$\therefore (0.253, \frac{9}{8}), (\frac{\pi}{2}, 0)$,
$(2.89, \frac{9}{8}), (\frac{3\pi}{2}, -2)$

6 **a** $x = 0 \therefore y = 1$
$\frac{dy}{dx} = 2 \cos 2x$
grad $= 2$
$\therefore y - 1 = 2(x - 0)$
$[\, y = 2x + 1 \,]$

b $x = \frac{\pi}{3} \therefore y = \frac{1}{2}$
$\frac{dy}{dx} = -\sin x$
grad $= -\frac{\sqrt{3}}{2}$
$\therefore y - \frac{1}{2} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$
$[\, 3\sqrt{3}\, x + 6y - 3 - \sqrt{3}\,\pi = 0 \,]$

c $x = \frac{\pi}{4} \therefore y = -1$
$\frac{dy}{dx} = 3 \sec^2 3x$
grad $= 6$
$\therefore y + 1 = 6(x - \frac{\pi}{4})$
$[\, 12x - 2y - 2 - 3\pi = 0 \,]$

d $x = \frac{\pi}{6} \therefore y = 1$
$\frac{dy}{dx} = -\operatorname{cosec} x \cot x - 2 \cos x$
grad $= -3\sqrt{3}$
$\therefore y - 1 = -3\sqrt{3}\,(x - \frac{\pi}{6})$
$[\, 6\sqrt{3}\, x + 2y - 2 - \sqrt{3}\,\pi = 0 \,]$

7 **a** $= 1 \times \sin x + x \times \cos x$

$= \sin x + x \cos x$

b $= \dfrac{-2\sin 2x \times x - \cos 2x \times 1}{x^2}$

$= -\dfrac{2x\sin 2x + \cos 2x}{x^2}$

c $= e^x \times \cos x + e^x \times (-\sin x)$
$= e^x(\cos x - \sin x)$

d $= \cos x \times \cos x + \sin x \times (-\sin x)$
$= \cos^2 x - \sin^2 x = \cos 2x$

e $= 2x \times \operatorname{cosec} x + x^2 \times (-\operatorname{cosec} x \cot x)$
$= x \operatorname{cosec} x(2 - x \cot x)$

f $= \sec x \tan x \times \tan x + \sec x \times \sec^2 x$
$= \sec x(\tan^2 x + \sec^2 x)$

g $= \dfrac{1 \times \tan x - x \times \sec^2 x}{\tan^2 x}$

$= \cot x - x \operatorname{cosec}^2 x$

h $= \dfrac{2\cos 2x \times e^{3x} - \sin 2x \times 3e^{3x}}{(e^{3x})^2}$

$= \dfrac{2\cos 2x - 3\sin 2x}{e^{3x}}$

i $= 2\cos x(-\sin x)\times\cot x + \cos^2 x\times(-\operatorname{cosec}^2 x)$

$= -2\cos^2 x - \cot^2 x$

j $= \dfrac{2\sec 2x \tan 2x \times x^2 - \sec 2x \times 2x}{x^4}$

$= \dfrac{2\sec 2x(x\tan 2x - 1)}{x^3}$

k $= 1 \times \tan^2 4x + x \times 2\tan 4x \times 4\sec^2 4x$

$= \tan 4x(\tan 4x + 8x\sec^2 4x)$

l $= \dfrac{\cos x \times \cos 2x - \sin x \times (-2\sin 2x)}{\cos^2 2x}$

$= \dfrac{\cos x(1 - 2\sin^2 x) + 4\sin^2 x\cos x}{\cos^2 2x} = \dfrac{\cos x(1 + 2\sin^2 x)}{\cos^2 2x}$

8 **a** $f'(x) = 3\cos 3x\times\cos 5x + \sin 3x\times(-5\sin 5x)$
$= 3\cos 3x\cos 5x - 5\sin 3x\sin 5x$

$f'(\frac{\pi}{4}) = 3(-\frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}) - 5\times\frac{1}{\sqrt{2}}(-\frac{1}{\sqrt{2}})$

$= \frac{3}{2} + \frac{5}{2} = 4$

b $f'(x) = 2\sec^2 2x \times \sin x + \tan 2x \times \cos x$
$= 2\sec^2 2x\sin x + \tan 2x\cos x$

$f'(\frac{\pi}{3}) = 2\times4\times\frac{\sqrt{3}}{2} + (-\sqrt{3})\times\frac{1}{2}$

$= 4\sqrt{3} - \frac{1}{2}\sqrt{3} = \frac{7}{2}\sqrt{3}$

c $f'(x) = \dfrac{\frac{1}{2\cos x}\times(-2\sin x)\times\sin x - \ln(2\cos x)\times\cos x}{\sin^2 x}$

$= -\sec x - \dfrac{\cos x\ln(2\cos x)}{\sin^2 x}$

$f'(\frac{\pi}{3}) = -2 - 0 = -2$

d $f'(x) =$

$2\sin x\cos x \times \cos^3 x + \sin^2 x \times 3\cos^2 x \times (-\sin x)$

$= \sin x\cos^2 x(2\cos^2 x - 3\sin^2 x)$

$f'(\frac{\pi}{6}) = \frac{1}{2}\times\frac{3}{4}(2\times\frac{3}{4} - 3\times\frac{1}{4}) = \frac{9}{32}$

9 $x = 0 \;\therefore\; y = 3$

$\dfrac{dy}{dx} = 1 \times \cos 2x + x \times (-2\sin 2x)$

$= \cos 2x - 2x\sin 2x$

grad = 1
$\therefore$ grad of normal = -1

$\therefore\; y - 3 = -(x - 0)$

[$y = 3 - x$]

10 **a** $= \dfrac{\cos x\times(1 - \sin x) - (2 + \sin x)\times(-\cos x)}{(1 - \sin x)^2}$

$= \dfrac{3\cos x}{(1 - \sin x)^2}$

b SP: $\dfrac{3\cos x}{(1 - \sin x)^2} = 0$

$\cos x = 0$

$x \neq \frac{\pi}{2} \;\therefore\; x = \frac{3\pi}{2}$

$\therefore\; (\frac{3\pi}{2}, \frac{1}{2})$

c $x = \frac{\pi}{6} \;\therefore\; y = 5$

grad = $6\sqrt{3}$

$y - 5 = 6\sqrt{3}(x - \frac{\pi}{6})$

$y = 6\sqrt{3}x + 5 - \sqrt{3}\pi$

11 **a** $\frac{dy}{dx} = -e^{-x} \times \sin x + e^{-x} \times \cos x$

$= e^{-x}(\cos x - \sin x)$

$\frac{d^2y}{dx^2} = -e^{-x} \times (\cos x - \sin x)$

$+ e^{-x} \times (-\sin x - \cos x)$

$= -2e^{-x} \cos x$

b SP: $e^{-x}(\cos x - \sin x) = 0$

$\cos x - \sin x = 0$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{\pi}{4} - \pi = -\frac{3\pi}{4}, \frac{\pi}{4}$

$x = -\frac{3\pi}{4}$: $\frac{d^2y}{dx^2} = \sqrt{2}\,e^{\frac{3\pi}{4}}$ (> 0)

$x = \frac{\pi}{4}$: $\frac{d^2y}{dx^2} = -\sqrt{2}\,e^{-\frac{\pi}{4}}$ (< 0)

$\therefore$ $(-\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}})$, minimum

$(\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}})$, maximum

12 **a** $\frac{dy}{dx} = 1 \times \sec x + x \times \sec x \tan x$

$= \sec x(1 + x \tan x)$

SP: $\sec x(1 + x \tan x) = 0$

no real values of x for which $\sec x = 0$

$\therefore$ $1 + x \tan x = 0$

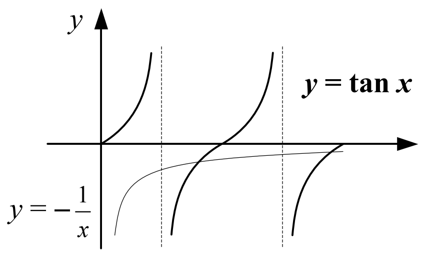
b $1 + x \tan x = 0$

$\Rightarrow$ $x \tan x = -1$

$\tan x = -\frac{1}{x}$

$\therefore$ x-coord of SP where curves

$y = \tan x$ and $y = -\frac{1}{x}$ intersect



intersect at 2 points $\therefore$ 2 SP in interval

13 **a** $f'(x) = -\sin x \times \sin 2x + \cos x \times 2\cos 2x$

$= 2\cos x(1 - 2\sin^2 x) - 2\sin^2 x \cos x$

$= 2\cos x(1 - 3\sin^2 x)$

b SP: $2\cos x(1 - 3\sin^2 x) = 0$

$\cos x = 0$ or $\sin x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $0.615, \pi - 0.61548,$

$\pi + 0.61548, 2\pi - 0.61548$

$x = 0.615, \frac{\pi}{2}, 2.53, 3.76, \frac{3\pi}{2}, 5.67$

c using graph, max. f(x) when $\sin x = \frac{1}{\sqrt{3}}$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$f(x) = \cos x \times 2\sin x \cos x = 2\sin x \cos^2 x$

$\therefore$ max. $f(x) = 2 \times \frac{1}{\sqrt{3}} \times \frac{2}{3}$

$= \frac{4}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4}{9}\sqrt{3}$

d period of $\cos x = 2\pi$, period of $\sin 2x = \pi$

$\therefore$ period of $f(x) = 2\pi$

$\therefore$ values of $f(x)$ in this interval are repeated

14 **a** $x = 0$ $\therefore$ $y = -2$

$\frac{dy}{dx} = -\text{cosec}\,(x - \frac{\pi}{6})\cot(x - \frac{\pi}{6})$

grad $= -2\sqrt{3}$

$\therefore$ grad of normal $= \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

$\therefore$ $y + 2 = \frac{1}{6}\sqrt{3}\,(x - 0)$

[$y = \frac{1}{6}\sqrt{3}\,x - 2$]

b $x = \frac{\pi}{3}$ $\therefore$ $y = 2$

grad $= -2\sqrt{3}$

$\therefore$ $y - 2 = -2\sqrt{3}\,(x - \frac{\pi}{3})$

[$6\sqrt{3}\,x + 3y - 6 - 2\sqrt{3}\,\pi = 0$]

c $\frac{1}{6}\sqrt{3}\,x - 2 = 2 - 2\sqrt{3}\,(x - \frac{\pi}{3})$

$x - 4\sqrt{3} = 4\sqrt{3} - 12x + 4\pi$

$13x = 8\sqrt{3} + 4\pi$

$x = \frac{8\sqrt{3} + 4\pi}{13}$

1 **a** $\frac{1}{2}y^{-\frac{1}{2}}$

b $y = x^2$

c $2x$

d $\frac{dx}{dy} = \frac{1}{2}y^{-\frac{1}{2}} = \frac{1}{2\sqrt{y}} = \frac{1}{2x}$

$\frac{1}{(\frac{dx}{dy})} = \frac{1}{(\frac{1}{2x})} = 2x \quad \therefore \quad \frac{dy}{dx} = \frac{1}{(\frac{dx}{dy})}$

2 **a** $\frac{dy}{dx} = 2e^{2x-1}$

$x = \frac{1}{2}(\ln y + 1)$

$\frac{dx}{dy} = \frac{1}{2y} = \frac{1}{2e^{2x-1}}$

$\frac{dy}{dx} \times \frac{dx}{dy} = 2e^{2x-1} \times \frac{1}{2e^{2x-1}} = 1$

b $\frac{dy}{dx} = 3x^2$

$x = (y-2)^{\frac{1}{3}}$

$\frac{dx}{dy} = \frac{1}{3}(y-2)^{-\frac{2}{3}} = \frac{1}{3x^2}$

$\frac{dy}{dx} \times \frac{dx}{dy} = 3x^2 \times \frac{1}{3x^2} = 1$

c $\frac{dx}{dy} = \frac{1}{2}(\ln y)^{-\frac{1}{2}} \times \frac{1}{y} = \frac{1}{2y\sqrt{\ln y}}$

$y = e^{x^2}$

$\frac{dy}{dx} = 2xe^{x^2} = 2y\sqrt{\ln y}$

$\frac{dy}{dx} \times \frac{dx}{dy} = 2y\sqrt{\ln y} \times \frac{1}{2y\sqrt{\ln y}} = 1$

3 **a** $\frac{dx}{dy} = 2y$

$\therefore \frac{dy}{dx} = \frac{1}{2y}$

b $\frac{dx}{dy} = 3(y-1)^2 \times 1$

$\therefore \frac{dy}{dx} = \frac{1}{3(y-1)^2}$

c $\frac{dx}{dy} = \sec^2 y$

$\therefore \frac{dy}{dx} = \cos^2 y$

d $\frac{dx}{dy} = \frac{1}{3y+2} \times 3$

$\therefore \frac{dy}{dx} = \frac{3y+2}{3}$

e $\frac{dx}{dy} = 2\sin y \cos y = \sin 2y$

$\therefore \frac{dy}{dx} = \text{cosec } 2y$

f $\frac{dx}{dy} = \frac{1 \times e^y - (y-2) \times e^y}{(e^y)^2} = \frac{3-y}{e^y}$

$\therefore \frac{dy}{dx} = \frac{e^y}{3-y}$

4 **a** $\frac{dx}{dy} = 3y^2 - 8y$

b $y = 3 \quad \therefore x = -9$

$\frac{dx}{dy} = 3$

$\therefore$ grad $= \frac{dy}{dx} = \frac{1}{3}$

$\therefore y - 3 = \frac{1}{3}(x + 9)$

$[\, y = \frac{1}{3}x + 6 \,]$

5 **a** $e^y = ax + b$

$x = \frac{1}{a}(e^y - b)$

b $\frac{dx}{dy} = \frac{1}{a}e^y$

c $\frac{d}{dx}[\ln(ax + b)] = \frac{dy}{dx} = 1 \div \frac{dx}{dy}$

$= \frac{a}{e^y} = \frac{a}{ax+b}$

6 **a** $\ln y = \ln 3^x = x \ln 3$

$\therefore x = \frac{\ln y}{\ln 3}$

b $\frac{dx}{dy} = \frac{1}{\ln 3} \times \frac{1}{y} = \frac{1}{y \ln 3}$

c $\frac{dy}{dx} = 1 \div \frac{dx}{dy} = y \ln 3$

$= 3^x \ln 3$

d grad $= 9 \ln 3$

$\therefore y - 9 = (9 \ln 3)(x - 2)$

$[\, y = 9x \ln 3 + 9 - 18 \ln 3 \,]$

1 $x = -1,\ y = 8$

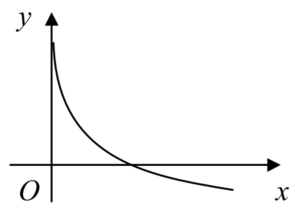
$\frac{dy}{dx} = \frac{3}{2}(3-x)^{\frac{1}{2}} \times (-1) = -\frac{3}{2}(3-x)^{\frac{1}{2}}$

grad $= -3$

$\therefore\ y - 8 = -3(x + 1)$

$[\ y = 5 - 3x\ ]$

2 **a**



b $y = 0\ \therefore\ x = \frac{1}{2}e^3$

$(\frac{1}{2}e^3, 0)$

c $x = 5\ \therefore\ y = 3 - \ln 10$

$\frac{dy}{dx} = -\frac{1}{x}$, grad $= -\frac{1}{5}$

$\therefore\ y - (3 - \ln 10) = -\frac{1}{5}(x - 5)$

$[\ y = -\frac{1}{5}x + 4 - \ln 10\ ]$

d at A, $y = 0\ \therefore\ x = 5(4 - \ln 10)$

at B, $x = 0\ \therefore\ y = 4 - \ln 10$

area $= \frac{1}{2} \times 5(4 - \ln 10) \times (4 - \ln 10)$

$= 7.20$ (3sf)

3 **a** $= 4(3x - 1)^3 \times 3$

$= 12(3x - 1)^3$

b $= \dfrac{2x \times \sin 2x - x^2 \times 2\cos 2x}{\sin^2 2x}$

$= \dfrac{2x(\sin 2x - x\cos 2x)}{\sin^2 2x}$

4 **a** $t = 3\ \therefore$ area $= 2e^{1.5} = 8.96\text{ cm}^2$ (3sf)

b $\frac{dA}{dt} = 2 \times 0.5e^{0.5t} = e^{0.5t}$

$t = 3$, $\frac{dA}{dt} = e^{1.5} = 4.4817\text{ cm}^2\text{ yr}^{-1}$

$\therefore$ rate per day $= 4.4817 \div 365 = 0.0123$

area increasing at 0.0123 cm^2 per day (3sf)

c $65 = 2e^{0.5t}$

$t = 2\ln 32.5 = 6.96$

$\therefore$ 7 years

d A increases exponentially and would become larger than the surface area of the boulder

5 **a** $\frac{dy}{dx} = \frac{a}{x} - 4$

SP: $\frac{a}{x} - 4 = 0$

$x = \frac{1}{4}a$

$\therefore\ (\frac{1}{4}a, a\ln\frac{a}{4} - a)$

b $x = 1\ \therefore\ y = -4$, grad $= a - 4$

$\therefore\ y + 4 = (a - 4)(x - 1)$

$[\ y = (a - 4)x - a\ ]$

c $(3, 0)\ \therefore\ 0 = 3(a - 4) - a$

$a = 6$

6 **a** $\frac{dy}{dx} = 2e^{2x}\sin x + e^{2x}\cos x$

$= e^{2x}(2\sin x + \cos x)$

b $\frac{d^2y}{dx^2} = 2e^{2x}(2\sin x + \cos x) + e^{2x}(2\cos x - \sin x)$

$= e^{2x}(3\sin x + 4\cos x)$

$\therefore\ \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y$

$= e^{2x}(3\sin x + 4\cos x)$

$\quad - 4e^{2x}(2\sin x + \cos x) + 5e^{2x}\sin x$

$= e^{2x}(3\sin x + 4\cos x - 8\sin x - 4\cos x + 5\sin x)$

$= 0$

7 **a** $\frac{dx}{dy} = 2\tan y \sec^2 y$

$= 2\tan y\,(\tan^2 y + 1)$

$= 2\sqrt{x}\,(x + 1)$

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{2\sqrt{x}(x+1)}$

b $y = \frac{\pi}{4} \;\therefore\; x = 1$

grad $= \frac{1}{4}$

$\therefore$ grad of normal $= -4$

$\therefore\; y - \frac{\pi}{4} = -4(x - 1)$

$[\; 16x + 4y - \pi - 16 = 0 \;]$

8 **a** $\frac{dy}{dx} = \frac{1\times\sqrt{x-2} - (x+2)\times\frac{1}{2}(x-2)^{-\frac{1}{2}}}{x-2}$

$= \frac{2(x-2)-(x+2)}{2(x-2)^{\frac{3}{2}}}$

$= \frac{x-6}{2(x-2)^{\frac{3}{2}}}$

b SP: $\frac{x-6}{2(x-2)^{\frac{3}{2}}} = 0$

$x = 6 \quad\therefore\; (6, 4)$

c $x = 3 \;\therefore\; y = 5$, grad $= -\frac{3}{2}$

$\therefore$ grad of normal $= \frac{2}{3}$

$\therefore\; y - 5 = \frac{2}{3}(x - 3)$

$3y - 15 = 2x - 6$

$2x - 3y + 9 = 0$

9 **a** $= e^x \times (x - 1)^2 + e^x \times 2(x - 1) \times 1$

$= e^x(x^2 - 2x + 1 + 2x - 2)$

$= e^x(x^2 - 1)$

b $= e^x \times (x^2 - 1) + e^x \times 2x$

$= e^x(x^2 + 2x - 1)$

c SP: $e^x(x^2 - 1) = 0$

$x = \pm 1$

$\therefore\; (-1, 4e^{-1}), (1, 0)$

$(-1, 4e^{-1})$, $\frac{d^2y}{dx^2} = -2e^{-1}$ $\therefore$ maximum

$(1, 0)$, $\frac{d^2y}{dx^2} = 2e$ $\therefore$ minimum

d $x = 2 \;\therefore\; y = e^2$

grad $= 3e^2$

$\therefore\; y - e^2 = 3e^2(x - 2)$

$y = 3e^2x - 5e^2$

$y = e^2(3x - 5)$

10 **a** $\frac{dy}{dx} = x - \frac{3}{x}$

SP: $x - \frac{3}{x} = 0$

$x^2 = 3$

$x > 0 \;\therefore\; x = \sqrt{3}$

b $\frac{d^2y}{dx^2} = 1 + 3x^{-2}$

$x = \sqrt{3}$, $\frac{d^2y}{dx^2} = 2$

$\therefore$ minimum

c $y = \frac{1}{2}(\sqrt{3})^2 - 3\ln\sqrt{3}$

$= \frac{3}{2} - 3\ln 3^{\frac{1}{2}}$

$= \frac{3}{2} - \frac{3}{2}\ln 3$

$= \frac{3}{2}(1 - \ln 3)$

d $x = 1 \;\therefore\; y = \frac{1}{2}$

grad $= -2$

$\therefore\; y - \frac{1}{2} = -2(x - 1)$

$4x + 2y - 5 = 0$

11 **a** $f(x) = \frac{6x - 2(x+2)}{(x-1)(x+2)} = \frac{4x-4}{(x-1)(x+2)}$

$= \frac{4(x-1)}{(x-1)(x+2)} = \frac{4}{x+2}$

b $x = 2 \;\therefore\; y = 1$

$f'(x) = -4(x + 2)^{-2}$

grad $= -\frac{1}{4}$

$\therefore\; y - 1 = -\frac{1}{4}(x - 2)$

$4y - 4 = -x + 2$

$x + 4y = 6$

1 **a** $\frac{dy}{dx} = -\frac{1}{4}x^{-2} - \frac{1}{x}$

$x = 1 \;\therefore$ grad $= -\frac{5}{4}$

b grad of normal $= \frac{4}{5}$

$\therefore\; y - \frac{1}{4} = \frac{4}{5}(x - 1)$

$16x - 20y - 11 = 0$

2 **a** $\frac{dy}{dx} = 1 \times e^{-2x} + x \times (-2e^{-2x})$

$= e^{-2x}(1 - 2x)$

$\frac{d^2y}{dx^2} = -2e^{-2x} \times (1 - 2x) + e^{-2x} \times (-2)$

$= 4e^{-2x}(x - 1)$

b SP: $e^{-2x}(1 - 2x) = 0$

$x = \frac{1}{2}$

$\therefore\; (\frac{1}{2}, \frac{1}{2}e^{-1})$

when $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = -2e^{-1}$

$\frac{d^2y}{dx^2} < 0 \;\therefore$ maximum

3 **a** $y = 0 \Rightarrow x = \sqrt{3}$

$\therefore\; (\sqrt{3}, 0)$

b $= \frac{1}{2}(e^y + 2)^{-\frac{1}{2}} \times e^y$

$= \frac{e^y}{2\sqrt{e^y + 2}}$

c $\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{2\sqrt{e^y + 2}}{e^y}$

grad $= 2\sqrt{3}$

$\therefore\; y - 0 = 2\sqrt{3}\,(x - \sqrt{3})$

at Q, $x = 0 \;\therefore y = -6$

area $= \frac{1}{2} \times \sqrt{3} \times 6 = 3\sqrt{3}$

4 **a** $t = 0$, $m = 680$

$t = 100$, $m = 653.63$

% red'n $= \frac{680 - 653.63}{680} \times 100\% = 3.88\%$ (3sf)

b $640 = 600 + 80e^{-0.004t}$

$t = \frac{-1}{0.004}\ln\frac{1}{2} = 173$ (3sf)

c $\frac{dm}{dt} = 80 \times (-0.004)e^{-0.004t} = -0.32e^{-0.004t}$

$t = 150$, $\frac{dm}{dt} = -0.176$

$\therefore$ mass decreasing at 0.176 g yr^{-1} (3sf)

5 **a** $= \frac{1}{2}(\sin x + \cos x)^{-\frac{1}{2}} \times (\cos x - \sin x)$

$= \frac{\cos x - \sin x}{2\sqrt{\sin x + \cos x}}$

b $= \frac{d}{dx}[\ln(x - 1) - \ln(2x + 1)]$

$= \frac{1}{x-1} - \frac{1}{2x+1} \times 2$

$= \frac{1}{x-1} - \frac{2}{2x+1}$

6 **a** $\frac{dy}{dx} = 5(2x - 3)^4 \times 2 = 10(2x - 3)^4$

$x = 1 \;\therefore$ grad $= 10$

$\therefore\; y + 1 = 10(x - 1)$

$[\, y = 10x - 11 \,]$

b at Q $\quad 10(2x - 3)^4 = 10$

$2x - 3 = \pm 1$

$x = 1$ (at P) or 2

$\therefore\; Q\,(2, 1)$

7 **a** $\frac{dy}{dx} = -2(x^2 - 5)^{-2} \times 2x = \frac{-4x}{(x^2-5)^2}$

SP: $\frac{-4x}{(x^2-5)^2} = 0$

$x = 0$

$\therefore\ (0, -\frac{2}{5})$

b $x = 3, y = \frac{1}{2}$

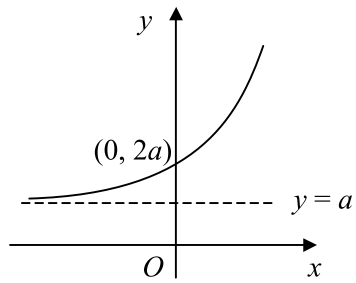
grad $= -\frac{3}{4}$

$\therefore\ y - \frac{1}{2} = -\frac{3}{4}(x - 3)$

$4y - 2 = -3x + 9$

$3x + 4y - 11 = 0$

8 **a**



b $y = ae^x + a$

swap $x = ae^y + a$

$y = \ln \frac{x-a}{a}$

$f^{-1} : x \to \ln \frac{x-a}{a},\ x \in \mathbb{R},\ x > a$

c $x = 1\ \therefore\ y = ae + a$

$f'(x) = ae^x$, grad $= ae$

$\therefore\ y - (ae + a) = ae(x - 1)$

$[\ y = aex + a\ ]$

9 **a** $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

$= \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$

$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$

$= -\frac{1}{\sin^2 x} = -\text{cosec}^2\, x$

b $\frac{dy}{dx} = e^x \times \cot x + e^x \times (-\text{cosec}^2\, x)$

$= e^x(\cot x - \text{cosec}^2\, x)$

SP: $e^x(\cot x - \text{cosec}^2\, x) = 0$

$e^x \neq 0\ \therefore\ \cot x = \text{cosec}^2\, x$

$\frac{\cos x}{\sin x} = \frac{1}{\sin^2 x}$

$\sin x \cos x = 1$

$\sin 2x = 2$

$|\sin 2x| \leq 1\ \therefore$ no solutions

$\therefore$ no turning points

10 **a** $\frac{dy}{dx} = 3(2 + \ln x)^2 \times \frac{1}{x}$

$= \frac{3}{x}(2 + \ln x)^2$

b SP: $\frac{3}{x}(2 + \ln x)^2 = 0$

$\ln x = -2$

$x = e^{-2}$

$\therefore\ (e^{-2}, 0)$

c $x = e,\ y = 27$

grad $= \frac{27}{e}$

$\therefore\ y - 27 = \frac{27}{e}(x - e)$

$y = \frac{27}{e}x$

when $x = 0,\ y = 0$

$\therefore$ passes through origin

11 **a** $= \dfrac{1}{9-x^2} \times (-2x)$

$= \dfrac{-2x}{9-x^2}$

b SP: $\dfrac{-2x}{9-x^2} = 0$

$x = 0$

$\therefore$ (0, ln 9)

c $x = 1,\ y = \ln 8 = \ln 2^3 = 3 \ln 2$

grad $= -\frac{1}{4}$

$\therefore$ grad of normal = 4

$\therefore\ y - 3 \ln 2 = 4(x - 1)$

$y = 4x - 4 + 3 \ln 2$

12 **a** model A: $t = 3$, $M = 764$ (3sf)

model B: $t = 3$, $M = 732$ (3sf)

b model A:

$\dfrac{dM}{dt} = 1500(3t + 2)^{-2} \times 3 = \dfrac{4500}{(3t+2)^2}$

$t = 3,\ \dfrac{dM}{dt} = 37.2$

$\therefore$ increasing at 37.2 tonnes yr^{-1} (3sf)

model B:

$\dfrac{dM}{dt} = 1500[2 + 5 \ln (t + 1)]^{-2} \times \dfrac{5}{t+1}$

$= \dfrac{7500}{(t+1)[2+5\ln(t+1)]^2}$

$t = 3,\ \dfrac{dM}{dt} = 23.5$

$\therefore$ increasing at 23.5 tonnes yr^{-1} (3sf)