

1 **a** 20.1 **b** 0.135 **c** 13.6 **d** -0.598 **e** 1.97 **f** 0.434

2 **a** = 4 **b** = $e^{\ln 3} = 3$ **c** = $2e^{\frac{\ln 1}{6}} = \frac{1}{3}$ **d** = 7 **e** = $\ln e^{-1} = -1$ **f** = -0.5

3

a $x = 4$	b $x = 17$	c $x^2 = 25$ $x > 0 \therefore x = 5$	d $\frac{1}{x} = \frac{1}{3}$ $x = 3$
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4	a $x = e^{15}$	b $\ln t = 6$ $t = e^6$	c $x - 4 = e^7$ $x = e^7 + 4$
	d $\ln 5y = 8$ $5y = e^8$ $y = \frac{1}{5}e^8$	e $\frac{1}{2}x + 3 = e^{2.5}$ $\frac{1}{2}x = e^{2.5} - 3$ $x = 2e^{2.5} - 6$	f $4 - 3x = e^{11}$ $3x = 4 - e^{11}$ $x = \frac{1}{3}(4 - e^{11})$

5

a $x = \ln 0.7$

d $4t + 1 = \ln 12$
 $t = \frac{1}{4}(\ln 12 - 1)$

b $e^y = 2$
 $y = \ln 2$

e $e^{2x-3} = 14$
 $2x - 3 = \ln 14$
 $x = \frac{1}{2}(\ln 14 + 3)$

c $5x = \ln 3$
 $x = \frac{1}{5} \ln 3$

f $e^{4-5x} = \frac{7}{2}$
 $4 - 5x = \ln \frac{7}{2}$
 $x = \frac{1}{5}(4 - \ln \frac{7}{2})$

6 a $e^x = 12$ $x = \ln 12 = 2.48$	b $15x - 7 = e^4$ $x = \frac{1}{15}(e^4 + 7) = 4.11$	c $e^{\frac{1}{2}y+3} = \frac{11}{4}$ $\frac{1}{2}y + 3 = \ln \frac{11}{4}$ $y = 2(\ln \frac{11}{4} - 3) = -3.98$
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$$\begin{array}{lll} \text{d} \quad \ln(5-2x) = \frac{7}{3} & \text{e} \quad 10-3y = e^e & \text{f} \quad 2 \ln x + 3 \ln x = 19 \\ 5-2x = e^{\frac{7}{3}} & y = \frac{1}{3}(10 - e^e) = -1.72 & \ln x = \frac{19}{5} \\ x = \frac{1}{2}(5 - e^{\frac{7}{3}}) = -2.66 & & x = e^{\frac{19}{5}} = 44.70 \end{array}$$

g $e^{\frac{9}{4}x} = 3$	h $e^{3t-1} = 4$	i $\ln \frac{2x-5}{x} = \frac{1}{4}$
$\frac{9}{4}x = \ln 3$	$3t - 1 = \ln 4$	$2x - 5 = e^{\frac{1}{4}}x$
$x = \frac{4}{9} \ln 3 = 0.49$	$t = \frac{1}{3}(\ln 4 + 1) = 0.80$	$(2 - e^{\frac{1}{4}})x = 5$
		$x = \frac{5}{2 - e^{\frac{1}{4}}} = 6.98$

$$\begin{aligned} 7 \quad & 2e^{2x} - 11e^x + 12 = 0 \\ & (2e^x - 3)(e^x - 4) = 0 \\ & e^x = \frac{3}{2}, 4 \\ & x = \ln \frac{3}{2}, \ln 4 \end{aligned}$$

$$8 \quad a \quad = \frac{(3x-4)(x-2)}{(x-2)(x-3)} = \frac{3x-4}{x-3}$$

$$b \quad \ln \frac{3x^2-10x+8}{x^2-5x+6} = \ln 2x$$

$$\frac{3x^2-10x+8}{x^2-5x+6} = 2x$$

$$\frac{3x-4}{x-3} = 2x$$

$$3x-4 = 2x(x-3)$$

$$2x^2 - 9x + 4 = 0$$

$$(2x-1)(x-4) = 0$$

$$x = \frac{1}{2}, 4$$

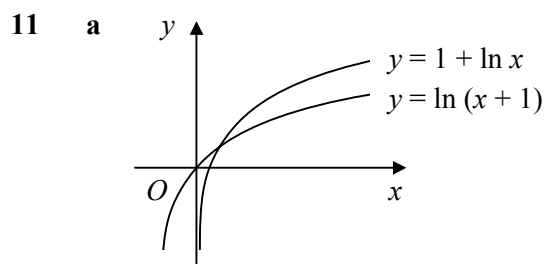
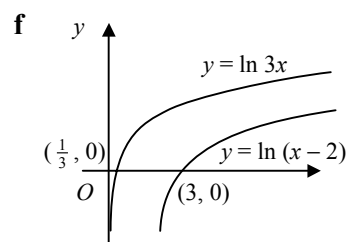
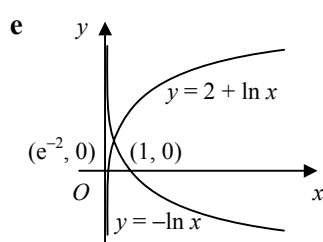
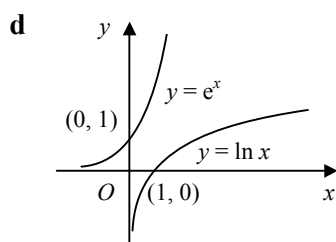
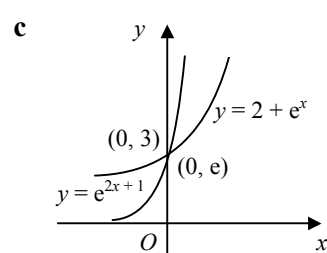
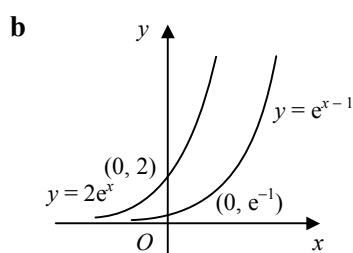
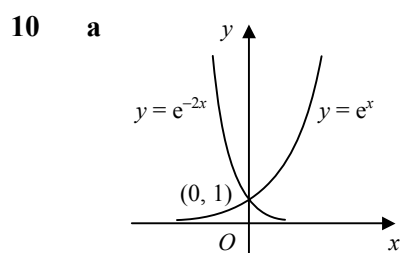
$$9 \quad e^{5y} - x = 0 \Rightarrow 5y = \ln x$$

$$\ln x^4 = 7 - y \Rightarrow 4 \ln x = 7 - y$$

$$\text{sub.} \quad 20y = 7 - y$$

$$y = \frac{1}{3}$$

$$\therefore x = e^{\frac{5}{3}} = 5.29, y = 0.33$$



$$b \quad \ln(x+1) = 1 + \ln x$$

$$\ln(x+1) - \ln x = 1$$

$$\ln \frac{x+1}{x} = 1$$

$$\frac{x+1}{x} = e$$

$$x+1 = ex$$

$$1 = x(e-1)$$

$$x = \frac{1}{e-1}$$

12 a 3

$$\text{b } x = 0 \therefore y = 3 + e^{-1} \\ \therefore (0, 3 + e^{-1})$$

$$\text{c } 3 + e^{2x-1} = 7 \\ e^{2x-1} = 4$$

$$2x - 1 = \ln 4$$

$$x = \frac{1}{2}(1 + \ln 4)$$

$$x = \frac{1}{2} + \ln 2$$

13 a $t = 10, N = 50e^{-2} = 6.77$ (3sf)

$$\text{b } 3 = 50e^{-0.2t}$$

$$t = -5 \ln \frac{3}{50} = 14.1$$
 (3sf)

14 a $160 = 240e^{180k}$

$$k = \frac{1}{180} \ln \frac{2}{3} = -0.00225$$
 (3sf)

$$\text{b } m = 240e^{-0.002253t}$$

$$120 = 240e^{-0.002253t}$$

$$t = \frac{-1}{0.002253} \ln \frac{1}{2} = 308 \text{ years}$$
 (3sf)

15 a $t = 15, N = 20e^{0.6} = 36.4$ (3sf)

$$\text{b i } k = 20e^{0.04t}$$

$$t = \frac{\ln(\frac{k}{20})}{0.04} = 25 \ln \frac{k}{20}$$

$$\text{ii } 2k = 20e^{0.04t}$$

$$t = \frac{\ln(\frac{k}{10})}{0.04} = 25 \ln \frac{k}{10}$$

c time for N to increase from k to $2k$

$$= 25 \ln \frac{k}{10} - 25 \ln \frac{k}{20}$$

$$= 25 \ln \frac{(\frac{k}{10})}{(\frac{k}{20})}$$

$$= 25 \ln 2$$

 \therefore time for N to double is constant16 a $300 = N_0 e^{10k} \Rightarrow N_0 = \frac{300}{e^{10k}}$

$$225 = N_0 e^{20k}$$

$$\therefore 225 = \frac{300}{e^{10k}} \times e^{20k}$$

$$e^{10k} = \frac{3}{4}$$

$$k = \frac{1}{10} \ln \frac{3}{4} = -0.0288$$
 (3sf)

$$N_0 = \frac{300}{\frac{3}{4}} = 400$$

$$\text{b } N = 400e^{-0.02877t}$$

$$150 = 400e^{-0.02877t}$$

$$t = \frac{-1}{0.02877} \ln \frac{3}{8} = 34.1$$
 (3sf)

1 a $42 = 60e^{100k}$
 $100k = \ln 0.7$
 $k = \frac{1}{100} \ln 0.7 = -0.00357$ (3sf)

b $30 = 60e^{kt}$
 $kt = \ln 0.5$
 $t = \frac{100 \ln 0.5}{\ln 0.7} = 194$ (3sf)

2 a $e^{3x} = 5.7$
 $x = \frac{1}{3} \ln 5.7 = 0.58$ (2dp)

b $\ln \frac{x}{x-1} = \frac{1}{2}$
 $\frac{x}{x-1} = e^{\frac{1}{2}}$
 $x = e^{\frac{1}{2}}(x-1)$
 $x(e^{\frac{1}{2}} - 1) = e^{\frac{1}{2}}$
 $x = \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}} - 1} = 2.54$ (2dp)

3 a $\ln(4x-3) = 0$
 $4x-3 = 1$
 $x = 1 \quad \therefore A(1, 0)$
 $1 + \ln x = 0$
 $\ln x = -1$
 $x = e^{-1} \quad \therefore B(e^{-1}, 0)$

b $\ln(4x-3) = 1 + \ln x$
 $\ln(4x-3) - \ln x = 1$
 $\ln \frac{4x-3}{x} = 1$
 $\frac{4x-3}{x} = e$
 $4x-3 = ex$
 $x(4-e) = 3$
 $x = \frac{3}{4-e}$

4 $2e^{2x} - 7e^x + 3 = 0$
 $(2e^x - 1)(e^x - 3) = 0$
 $e^x = \frac{1}{2}, 3$
 $x = \ln \frac{1}{2}, \ln 3$

5 a $t = 0 \Rightarrow N = 800$
b $t = 20 \Rightarrow N = 800e^{0.2}$
 $= 977$ (nearest unit)

c $800e^{0.01t} > 2000$
 $e^{0.01t} > 2.5$
 $0.01t > \ln 2.5$
 $t > 91.6 \quad \therefore 92$ days

6 a $1 + e^{2x+1} = 10$
 $e^{2x+1} = 9$
 $2x+1 = \ln 9$
 $x = \frac{1}{2}(-1 + \ln 9)$
 $x = -\frac{1}{2} + \ln 3$

b $1 + e^{2x+1} = 3 - e^x$
 $e(e^{2x}) + e^x - 2 = 0$
 $e^x = \frac{-1 \pm \sqrt{1+8e}}{2e}$
 $x = \ln \frac{-1 - \sqrt{1+8e}}{2e}$ (not real)
or $\ln \frac{-1 + \sqrt{1+8e}}{2e}$
 $\therefore x = -0.366$ (3sf)

7 a $4x - 1 = e^2$
 $x = \frac{1}{4}(e^2 + 1)$

b $7 = e^{1-3y}$
 $1 - 3y = \ln 7$
 $y = \frac{1}{3}(1 - \ln 7)$

9 a $= \frac{(x-1)(x-3)}{(x+2)(x-1)}$
 $= \frac{x-3}{x+2}$

b $\ln(x^2 - 4x + 3) - \ln(x^2 + x - 2) = 1$

$$\ln \frac{x^2 - 4x + 3}{x^2 + x - 2} = \ln \frac{x-3}{x+2} = 1$$

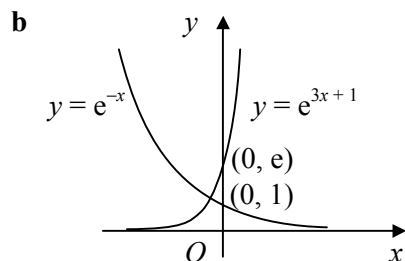
$$\frac{x-3}{x+2} = e$$

$$x - 3 = e(x + 2)$$

$$x(1 - e) = 2e + 3$$

$$x = \frac{2e+3}{1-e}$$

11 a reflection in y-axis



c $e^{-x} = e^{3x+1}$
 $1 = e^{4x+1}$
 $4x + 1 = 0$
 $x = -\frac{1}{4}$
 $\therefore (-\frac{1}{4}, e^{\frac{1}{4}})$

13 a when $t = 0$, $v = 13$

$$\therefore 13 = c - 2$$

$$c = 15$$

b $7 = 15e^{-5.1k} - 2$

$$e^{-5.1k} = \frac{3}{5}$$

$$k = \frac{\ln \frac{3}{5}}{-5.1} = 0.1002$$

c $10 = 15e^{-0.1002t} - 2$, $4 = 15e^{-0.1002T} - 2$

$$t = \frac{\ln \frac{4}{5}}{-0.1002} = 2.2278, \quad T = \frac{\ln \frac{2}{5}}{-0.1002} = 9.1481$$

$$T - t = 6.92 \text{ seconds (3sf)}$$

8 a $a = 800$

b $7200 = 800e^{2b}$

$$b = \frac{1}{2} \ln 9 = \ln 3$$

c $1600 = 800e^{t \ln 3}$

$$t = \frac{\ln 2}{\ln 3} = 0.631 \text{ hours}$$

$$\therefore 60 \times 0.631 = 38 \text{ minutes}$$

10 $e^y + 5 - 9x = 0 \Rightarrow y = \ln(9x - 5)$

sub. $\ln(9x - 5) - \ln(x + 4) = 2$

$$\frac{9x-5}{x+4} = e^2$$

$$9x - 5 = e^2(x + 4)$$

$$x(9 - e^2) = 4e^2 + 5$$

$$x = \frac{4e^2 + 5}{9 - e^2} = 21.4509$$

$$\therefore x = 21.5, y = 5.24 \text{ (3sf)}$$

12 a i $= \ln x^{\frac{1}{2}} = \frac{1}{2} \ln x = \frac{1}{2} t$

ii $= \ln e^2 + \ln x = 2 + t$

b $5 + \frac{1}{2} t = 2 + t$

$$t = \ln x = 6$$

$$x = e^6$$