

1 Simplify

a $\frac{3x-1}{18x-6}$

b $\frac{6x+15}{8x+20}$

c $\frac{3y+3}{y^2+7y+6}$

d $\frac{x^2-25}{x^2-7x+10}$

e $\frac{a^2-a-6}{a^2+3a-18}$

f $\frac{x^2+3x}{2x^2+5x-3}$

g $\frac{3t^2-11t-4}{t^2-16}$

h $\frac{6x^2-13x+6}{12x^2+x-6}$

2 Express as simply as possible

a $\frac{3x^2}{9x-9} \times \frac{4x-4}{2x}$

b $\frac{x^2-36}{x^2+7x+10} \div \frac{x-6}{x+2}$

c $\frac{n^2+2n}{n^2+6n+8} \times \frac{n+4}{n^2}$

d $\frac{4x-12}{x^2-4} \times \frac{x^2+2x}{x^2-2x-3}$

e $\frac{4y^2}{2y^2+y} \div \frac{y^2+2y-15}{2y^2+11y+5}$

f $\frac{x^2-1}{2x^2+7x-4} \times \frac{6x^2-5x+1}{3x^2-4x+1}$

g $\frac{10x-10}{5x+15} \div \frac{4-3x-x^2}{x^2+7x+12}$

h $\frac{a^3-3a^2}{8a^2-4a} \div \frac{a^2-9}{2a^2+5a-3}$

3 Express as a single fraction in its simplest form

a $\frac{2}{y} + \frac{7}{y+4}$

b $\frac{2x}{x-5} - \frac{1}{x+3}$

c $\frac{7}{x(x+2)} - \frac{3x}{x+2}$

d $\frac{x}{(x-3)(x-1)} + \frac{5}{2(x-1)}$

e $\frac{2}{q^2+3q} + \frac{5q}{4q+12}$

f $\frac{4}{3x-3} + \frac{x+2}{x^2-x}$

g $\frac{4}{x+5} + \frac{x}{x^2+8x+15}$

h $\frac{6x}{x^2-4} - \frac{3}{x+2}$

i $\frac{5t+12}{2t^2+7t+3} - \frac{4}{2t+1}$

4 Simplify

a $\frac{x^2-5x}{6x-30}$

b $\frac{16-x^2}{x^2+2x-8}$

c $\frac{2x^2-4x-6}{3x^2-12x+9}$

d $\frac{x^3-x}{2x^2-x-1}$

e $\frac{3x-x^2}{2x^2-18}$

f $\frac{x^3+x^2-2x}{3x^2+4x-4}$

g $\frac{2+5x-3x^2}{2x^2+x-10}$

h $\frac{x^4-5x^2+4}{x^2-x-2}$

5 Express as simply as possible

a $\frac{10x^2-10}{5x+10} \times \frac{x^2+6x+8}{x^2+5x+4}$

b $\frac{t^2-2t}{2t^2-t-6} \div \frac{9t^2-4}{6t^2+13t+6}$

c $\frac{2x^2+12x+10}{4x^2-7x+3} \div \frac{4x^2+20x}{4x^2-3x}$

d $\frac{8x^2+6x-9}{4x^2+12x+9} \times \frac{2x^2+3x}{6-8x}$

e $\frac{x^4+6x^2+5}{x^2-9} \times \frac{2x^2-6x}{4x^2+4}$

f $\frac{y^4-16}{5y^2+9y-2} \div \frac{y^2+4}{25y^2-10y+1}$

6 Express as a single fraction in its simplest form

a $\frac{5}{x^2-1} - \frac{1}{2x+2}$

b $\frac{3x}{x^2-4} - \frac{4}{2x^2+3x-2}$

c $\frac{4}{x^2+2x-3} + \frac{1}{x^2-3x+2}$

d $\frac{x+1}{x^2-25} + \frac{2}{x^2+5x}$

e $\frac{2x-1}{x^2+4x+4} + \frac{x}{3x+6}$

f $\frac{1}{x-3} + \frac{3}{x^2-3x} + \frac{x}{x^2-6x+9}$

g $\frac{x}{x^2-x-6} + \frac{2}{2x^2+3x-2}$

h $\frac{1}{x^2} - \frac{1}{3x^2-2x} + \frac{3}{6x-4}$

7 Solve

a $1 - \frac{2}{x} = \frac{3}{2x-5}$

b $\frac{2}{x^2-1} + \frac{3}{x+1} = 1$

c $\frac{20}{2x^2+5x+2} + 1 = \frac{10}{2x+1}$

d $\frac{y+3}{y+5} - \frac{1}{2} = \frac{2y-1}{y}$

e $5 + \frac{1}{x^2+5x+6} = \frac{11}{x+3}$

f $\frac{3}{1-4x+4x^2} - \frac{10}{1-4x^2} = \frac{5}{1+2x}$

8 $f(x) \equiv \frac{7x-15}{x^2-5x} - \frac{4}{x-5}, \quad 0 < x < 5.$

Show that $f(x) = \frac{k}{x}$, where k is an integer to be found.

9 $f(x) \equiv \frac{x-5}{3x^2+5x-2} + \frac{2}{3x-1}, \quad x > 1.$

Show that $f(x) = \frac{1}{x+2}$.

10 Given that $f(x) \equiv \frac{x+2}{x-2}$, $x \neq \pm 2$, show that $f(x) - \frac{1}{f(x)} = \frac{8x}{x^2-4}$.

11 a Express $\frac{2}{x+5} + \frac{3}{(x+2)(x+5)}$ as a single fraction in its simplest form.

b Hence solve the equation

$$\frac{2}{x+5} + \frac{3}{(x+2)(x+5)} = \frac{1}{3},$$

giving your answers to 2 decimal places.

12 Show that the equation $\frac{3}{4x+2} - \frac{5}{4x^2+4x+1} = 2$ has no real roots.

13 Express $\left(\frac{6}{x+5} - \frac{1}{x}\right) \div \frac{x-1}{x^2-25}$ as a single fraction in its simplest form.

1 Find the quotient and remainder obtained in dividing

a $(3x^3 - 10x^2 - 9x + 15)$ by $(x - 4)$

b $(2x^3 - 11x^2 - x + 3)$ by $(2x - 1)$

c $(4x^3 + 8x^2 + 7x + 32)$ by $(2x + 5)$

d $(1 - 22x^2 - 6x^3)$ by $(3x + 2)$

2 **a** Show that $(x + 2)$ is a factor of $(x^3 + 4x^2 + x - 6)$.

b Fully factorise $x^3 + 4x^2 + x - 6$.

c Simplify $\frac{x^3 + 4x^2 + x - 6}{x^2 - 9}$.

3 **a** Show that $(2x - 3)$ is a factor of $(2x^3 - 5x^2 + 13x - 15)$.

b Simplify $\frac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6}$.

4 **a** State a linear factor of $x^3 - 1$.

b Simplify $\frac{x^3 - 1}{x^2 + x - 2}$.

5 Find the integers A and B such that

$$\frac{2x+5}{x+3} \equiv A + \frac{B}{x+3}.$$

6 Express each of the following in the form $A + \frac{B}{f(x)}$, where $f(x)$ is linear.

a $\frac{x+2}{x+1}$

b $\frac{x+3}{x-2}$

c $\frac{x}{1-x}$

d $\frac{2x+1}{x+2}$

e $\frac{x-1}{2x-1}$

f $\frac{1-4x}{3+2x}$

7 Find the quotient and remainder obtained in dividing

a $(x^2 + 3x + 5)$ by $(x^2 + x + 2)$

b $(2x^2 + 3x - 8)$ by $(x^2 - x - 4)$

c $(x^2 + 7)$ by $(x^2 + 3x - 1)$

d $(3x^2 - x - 4)$ by $(x^2 + 2)$

e $(x^3 - 2x^2 - 5x + 8)$ by $(x^2 + x - 2)$

f $(2x^3 - 7x^2 + 1)$ by $(x^2 - 5x + 1)$

g $(3x^3 + 6x^2 - 2x + 5)$ by $(3x^2 + 4)$

h $(6x^3 - x^2 - 44x - 6)$ by $(2x^2 - 5x - 2)$

8 **a** Divide $(x^3 + 5x^2 + 7x - 13)$ by $(x^2 + 3x - 4)$.

b Hence show that

$$\frac{x^3 + 5x^2 + 7x - 13}{x^2 + 3x - 4} \equiv x + 2 + \frac{5}{x + 4}.$$

9 $f(x) = \frac{x^3 - 2x^2 - 21x + 70}{x^2 + 2x - 15}, \quad x \neq 3.$

a Express $f(x)$ in the form $Ax + B + \frac{C}{g(x)}$, where $g(x)$ is linear.

b Hence, or otherwise, solve the equation $f(x) = \frac{3x-7}{x-3}$.

1 Express $\frac{6}{x^2-9} - \frac{7}{2x^2-5x-3}$ as a single fraction in its simplest form. (6)

2
$$f(x) \equiv \frac{3}{2x+3} - \frac{x+9}{2x^2+11x+12}, \quad x > 0.$$

Show that $f(x) = \frac{1}{x+4}$. (5)

3 a Express $\frac{1}{x-6} - \frac{2}{x^2-36}$ as a single fraction in its simplest form. (3)

b Hence solve the equation

$$\frac{1}{x-6} - \frac{2}{x^2-36} = \frac{1}{2},$$

giving your answers in the form $a + b\sqrt{5}$, where $a, b \in \mathbb{Z}$. (4)

4
$$f(x) \equiv 2x^3 - 5x^2 - 23x - 10.$$

a Show that $(x-5)$ is a factor of $f(x)$. (2)

b Express $\frac{f(x)}{2x^2-9x-5}$ in its simplest form. (5)

5 Given that the equation

$$\frac{x+6}{x^2+9x+18} + \frac{x-p}{x+7} = 0$$

has real, equal roots, find the possible values of the constant p . (7)

6 Express $\frac{1}{3x-1} - \frac{3x}{9x^2-6x+1} - \frac{1}{3x^2-x}$ as a single fraction in its simplest form. (5)

7 a Simplify

i $\frac{7x+14}{4-x^2},$

ii $\frac{2x^2+x-28}{3x^2+12x}.$ (4)

b Hence show that the equation $\frac{7x+14}{4-x^2} = \frac{2x^2+x-28}{3x^2+12x}$ has no real roots. (4)

8 The first three terms of an arithmetic series are $\frac{1}{t-2}$, $\frac{1}{2}$ and $\frac{4}{t^2-2t}$ respectively.

a Show that $\frac{4}{t^2-2t} + \frac{1}{t-2} = 1.$ (2)

b Given that the common difference of the series is not zero, find the value of t and the first term of the series. (5)