C4 VECTORS Worksheet E

1 Relative to a fixed origin, the points A and B have position vectors $(\mathbf{i} + 6\mathbf{j} + 4\mathbf{k})$ and $(5\mathbf{i} - 6\mathbf{k})$ respectively.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line m has equation

$$\mathbf{r} = 5\mathbf{i} - 5\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}),$$

where μ is a scalar parameter.

Given that lines l and m intersect at the point C,

b find the position vector of C, **(4)**

c show that C is the mid-point of AB. **(2)**

2 Relative to a fixed origin, the points P and Q have position vectors $(5\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and $(3\mathbf{i} + \mathbf{j})$ respectively.

a Find, in vector form, an equation of the line L_1 which passes through P and Q. **(2)**

The line L_2 has equation

$$\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} - \mathbf{k} + \mu(5\mathbf{i} - \mathbf{j} + 3\mathbf{k}),$$

where μ is a scalar parameter.

b Show that lines L_1 and L_2 intersect and find the position vector of their point of intersection. **(5)**

c Find, in degrees to 1 decimal place, the acute angle between lines L_1 and L_2. **(4)**

3 Relative to a fixed origin, the lines l_1 and l_2 have vector equations as follows:

$$l_1 : \quad \mathbf{r} = 5\mathbf{i} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),$$

$$l_2 : \quad \mathbf{r} = 7\mathbf{i} - 3\mathbf{j} + 7\mathbf{k} + \mu(-\mathbf{i} + \mathbf{j} - 2\mathbf{k}),$$

where λ and μ are scalar parameters.

a Show that lines l_1 and l_2 intersect and find the position vector of their point of intersection. **(5)**

The points A and C lie on l_1 and the points B and D lie on l_2.

Given that $ABCD$ is a parallelogram and that A has position vector $(9\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$,

b find the position vector of C. **(3)**

Given also that the area of parallelogram $ABCD$ is 54,

c find the distance of the point B from the line l_1. **(4)**

4 Relative to a fixed origin, the points A and B have position vectors $(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$ and $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ respectively.

a Find, in vector form, an equation of the line l_1 which passes through A and B. **(2)**

The line l_2 passes through the point C with position vector $(4\mathbf{i} - 7\mathbf{j} - \mathbf{k})$ and is parallel to the vector $(6\mathbf{j} - 2\mathbf{k})$.

b Write down, in vector form, an equation of the line l_2. **(1)**

c Show that A lies on l_2. **(2)**

d Find, in degrees, the acute angle between lines l_1 and l_2. **(4)**

5 Relative to a fixed origin O, the points A and B have position vectors $(5\mathbf{i} - \mathbf{j} - 10\mathbf{k})$ and $(4\mathbf{i} + \mathbf{j} - 8\mathbf{k})$ respectively.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line l intersects the y-axis at the point C.

b Find the coordinates of C. **(2)**

The point D on the line l is such that OD is perpendicular to l.

c Find the coordinates of D. **(5)**

d Find the area of triangle OCD, giving your answer in the form $k\sqrt{5}$. **(3)**

6 Relative to a fixed origin, the line l_1 has the equation

$$\mathbf{r} = \mathbf{i} - 6\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{j} - \mathbf{k}),$$

where λ is a scalar parameter.

a Show that the point P with position vector $(\mathbf{i} + 6\mathbf{j} - 5\mathbf{k})$ lies on l_1. **(1)**

The line l_2 has the equation

$$\mathbf{r} = 4\mathbf{i} - 4\mathbf{j} - \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}),$$

where μ is a scalar parameter, and intersects l_1 at the point Q.

b Find the position vector of Q. **(3)**

The point R lies on l_2 such that $PQ = QR$.

c Find the two possible position vectors of the point R. **(5)**

7 Relative to a fixed origin, the points A and B have position vectors $(4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k})$ and $(4\mathbf{i} + 6\mathbf{j} + 2\mathbf{k})$ respectively.

a Find, in vector form, an equation of the line l_1 which passes through A and B. **(2)**

The line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where μ is a scalar parameter.

b Show that l_1 and l_2 intersect and find the position vector of their point of intersection. **(4)**

c Find the acute angle between lines l_1 and l_2. **(3)**

d Show that the point on l_2 closest to A has position vector $(-\mathbf{i} + 3\mathbf{j} - \mathbf{k})$. **(5)**

8 A tunnel is to be made under a hill in order to link two major roads.

Relative to a fixed origin, digging at one end of the tunnel begins at the point with position vector $(\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$, in the direction $(4\mathbf{i} - 7\mathbf{j} - \mathbf{k})$.

Digging at the other end of the tunnel begins at the point with position vector $(18\mathbf{i} - 20\mathbf{j} + 4\mathbf{k})$, in the direction $(-3\mathbf{i} + 6\mathbf{j} - \mathbf{k})$.

Assuming that each section of the tunnel is straight,

a write down vector equations for the lines along which each section of tunnel lies, **(2)**

b show that the two sections of tunnel will meet and find the position vector of the point where this occurs. **(5)**

Given that one unit on each coordinate axis represents 20 metres,

c find how much shorter the tunnel would be if it was constructed along a straight line between the two ends. **(5)**