GCE Examinations

Pure Mathematics Module P5

Advanced Subsidiary / Advanced Level

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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1. A curve has the equation

$$y = x + 2x^2 + 5x^3.$$

Show that the radius of curvature of the curve at the origin is $\frac{1}{\sqrt{2}}$. **(5 marks)**

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2. Show that

$$\int_0^{\ln 2} x \operatorname{sech}^2 x \ \mathrm{d}x = \frac{3}{5}\ln 2 - \ln\left(\frac{5}{4}\right).$$ **(8 marks)**

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3. *(a)* Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin 2x) = \frac{2}{\sqrt{1-4x^2}}.$$ **(3 marks)**

Given that

$$\mathrm{f}(x) = 2x \arcsin 2x + \sqrt{1-4x^2},$$

(b) show that

$$\mathrm{f}''(x)\left[\mathrm{f}(x) - x\mathrm{f}'(x)\right] = 4.$$ **(6 marks)**

4.

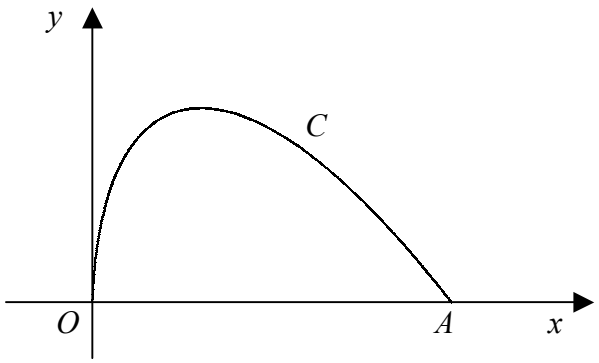

Fig. 1

The parametric coordinates of the curve C shown in Figure 1 are

$$x = t^2, \qquad y = t - \tfrac{1}{3}t^3, \qquad 0 \leq t \leq a.$$

The curve C meets the x-axis at the point A where $t = a$.

(a) Find the value of a. **(2 marks)**

The curve C is rotated through 2π about Ox.

(b) Find the surface area of the solid generated. **(8 marks)**

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5. *(a)* Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, prove that

$$\cosh 2x = 2\cosh^2 x - 1.$$ **(3 marks)**

(b) Solve the equation

$$2\cosh 2x = 13\cosh x - 12,$$

giving your answers in terms of natural logarithms. **(7 marks)**

Turn over

6. $x^2 - 10x + 41 \equiv (x + a)^2 + b$.

(a) Find the values of the constants a and b. **(2 marks)**

(b) Show that

$$\int_5^9 \frac{x}{\sqrt{x^2 - 10x + 41}}\, \mathrm{d}x = p\left(\sqrt{2} - 1\right) + q \ln r,$$

stating your values of p, q and r. **(8 marks)**

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7. $$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \ \mathrm{d}x, \quad n \geq 0.$$

(a) Prove that

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2} \quad n \geq 2.$$ **(5 marks)**

(b) Hence find the value of I_4, giving your answer in terms of π. **(6 marks)**

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8. The rectangular hyperbola C has equation $xy = c^2$, where c is a positive constant.

(a) Show that an equation of the tangent to C at the point $P\left(cp, \frac{c}{p}\right)$ is

$$x + yp^2 = 2cp.$$ **(4 marks)**

The tangent to C at P meets the x-axis at the point X.

The point Q on C has coordinates $\left(cq, \frac{c}{q}\right)$, $q \neq p$ such that QX is parallel to the y-axis.

(b) Show that $q = 2p$. **(3 marks)**

M is the mid-point of PQ.

(c) Find, in Cartesian form, an equation of the locus of M as p varies. **(5 marks)**

END