

1 a

$$\begin{array}{r}
 x^2 + x - 2 \\
 x+1 \overline{) x^3 + 2x^2 - x - 2} \\
 \underline{x^3 + x^2} \\
 x^2 - x \\
 \underline{x^2 + x} \\
 -2x - 2 \\
 \underline{-2x - 2} \\
 0
 \end{array}$$

quotient: $x^2 + x - 2$

b

$$\begin{array}{r}
 x^2 + 4x - 1 \\
 x-2 \overline{) x^3 + 2x^2 - 9x + 2} \\
 \underline{x^3 - 2x^2} \\
 4x^2 - 9x \\
 \underline{4x^2 - 8x} \\
 -x + 2 \\
 \underline{-x + 2} \\
 0
 \end{array}$$

quotient: $x^2 + 4x - 1$

c

$$\begin{array}{r}
 x^2 - x + 5 \\
 x+4 \overline{) x^3 + 3x^2 + x + 20} \\
 \underline{x^3 + 4x^2} \\
 -x^2 + x \\
 \underline{-x^2 - 4x} \\
 5x + 20 \\
 \underline{5x + 20} \\
 0
 \end{array}$$

quotient: $x^2 - x + 5$

d

$$\begin{array}{r}
 2x^2 + x - 3 \\
 x-1 \overline{) 2x^3 - x^2 - 4x + 3} \\
 \underline{2x^3 - 2x^2} \\
 x^2 - 4x \\
 \underline{x^2 - x} \\
 -3x + 3 \\
 \underline{-3x + 3} \\
 0
 \end{array}$$

quotient: $2x^2 + x - 3$

e

$$\begin{array}{r}
 6x^2 + 11x - 18 \\
 x-5 \overline{) 6x^3 - 19x^2 - 73x + 90} \\
 \underline{6x^3 - 30x^2} \\
 11x^2 - 73x \\
 \underline{11x^2 - 55x} \\
 -18x + 90 \\
 \underline{-18x + 90} \\
 0
 \end{array}$$

quotient: $6x^2 + 11x - 18$

f

$$\begin{array}{r}
 -x^2 + 7x - 4 \\
 x+2 \overline{) -x^3 + 5x^2 + 10x - 8} \\
 \underline{-x^3 - 2x^2} \\
 7x^2 + 10x \\
 \underline{7x^2 + 14x} \\
 -4x - 8 \\
 \underline{-4x - 8} \\
 0
 \end{array}$$

quotient: $-x^2 + 7x - 4$

g

$$\begin{array}{r}
 x^2 - 3x + 7 \\
 x+3 \overline{) x^3 + 0x^2 - 2x + 21} \\
 \underline{x^3 + 3x^2} \\
 -3x^2 - 2x \\
 \underline{-3x^2 - 9x} \\
 7x + 21 \\
 \underline{7x + 21} \\
 0
 \end{array}$$

quotient: $x^2 - 3x + 7$

h

$$\begin{array}{r}
 3x^2 - 2x + 12 \\
 x+6 \overline{) 3x^3 + 16x^2 + 0x + 72} \\
 \underline{3x^3 + 18x^2} \\
 -2x^2 + 0x \\
 \underline{-2x^2 - 12x} \\
 12x + 72 \\
 \underline{12x + 72} \\
 0
 \end{array}$$

quotient: $3x^2 - 2x + 12$

2 a

$$\begin{array}{r}
 x^2 + 3x + 2 \\
 x + 5 \overline{) x^3 + 8x^2 + 17x + 16} \\
 \underline{x^3 + 5x^2} \\
 3x^2 + 17x \\
 \underline{3x^2 + 15x} \\
 2x + 16 \\
 \underline{2x + 10} \\
 6
 \end{array}$$

quotient: $x^2 + 3x + 2$ remainder: 6

b

$$\begin{array}{r}
 x^2 - 8x + 5 \\
 x - 7 \overline{) x^3 - 15x^2 + 61x - 48} \\
 \underline{x^3 - 7x^2} \\
 - 8x^2 + 61x \\
 \underline{- 8x^2 + 56x} \\
 5x - 48 \\
 \underline{5x - 35} \\
 - 13
 \end{array}$$

quotient: $x^2 - 8x + 5$ remainder: -13

c

$$\begin{array}{r}
 3x^2 - 2x + 4 \\
 x + 2 \overline{) 3x^3 + 4x^2 + 0x + 7} \\
 \underline{3x^3 + 6x^2} \\
 - 2x^2 + 0x \\
 \underline{- 2x^2 - 4x} \\
 4x + 7 \\
 \underline{4x + 8} \\
 - 1
 \end{array}$$

quotient: $3x^2 - 2x + 4$ remainder: -1

d

$$\begin{array}{r}
 -x^2 + 3x - 9 \\
 x + 8 \overline{) -x^3 - 5x^2 + 15x - 50} \\
 \underline{-x^3 - 8x^2} \\
 3x^2 + 15x \\
 \underline{3x^2 + 24x} \\
 - 9x - 50 \\
 \underline{- 9x - 72} \\
 22
 \end{array}$$

quotient: $-x^2 + 3x - 9$ remainder: 22

e

$$\begin{array}{r}
 4x^2 + 14x + 26 \\
 x - 3 \overline{) 4x^3 + 2x^2 - 16x + 3} \\
 \underline{4x^3 - 12x^2} \\
 14x^2 - 16x \\
 \underline{14x^2 - 42x} \\
 26x + 3 \\
 \underline{26x - 78} \\
 81
 \end{array}$$

quotient: $4x^2 + 14x + 26$ remainder: 81

f

$$\begin{array}{r}
 -6x^2 - 10x + 20 \\
 x + 2 \overline{) -6x^3 - 22x^2 + 0x + 1} \\
 \underline{-6x^3 - 12x^2} \\
 - 10x^2 + 0x \\
 \underline{- 10x^2 - 20x} \\
 20x + 1 \\
 \underline{20x + 40} \\
 - 39
 \end{array}$$

quotient: $-6x^2 - 10x + 20$ remainder: -39

3

a let $f(x) \equiv x^3 + 2x^2 - 2x - 1$

$$f(1) = 1 + 2 - 2 - 1 = 0$$

 $\therefore (x - 1)$ is a factorc let $f(x) \equiv x^3 - x^2 - 14x + 27$

$$f(3) = 27 - 9 - 42 + 27 = 3$$

 $\therefore (x - 3)$ is not a factore let $f(x) \equiv 2x^3 - 5x^2 + 7x - 14$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{4} - \frac{5}{4} - \frac{7}{2} - 14 = -19$$

 $\therefore (2x + 1)$ is not a factorb let $f(x) \equiv x^3 - 5x^2 - 9x + 2$

$$f(-2) = -8 - 20 + 18 + 2 = -8$$

 $\therefore (x + 2)$ is not a factord let $f(x) \equiv 2x^3 + 13x^2 + 2x - 24$

$$f(-6) = -432 + 468 - 12 - 24 = 0$$

 $\therefore (x + 6)$ is a factorf let $f(x) \equiv 2 - 17x + 25x^2 - 6x^3$

$$f\left(\frac{2}{3}\right) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$$

 $\therefore (3x - 2)$ is a factor

4 a $f(1) = 1 - 2 - 11 + 12 = 0$
 $\therefore (x - 1)$ is a factor of $f(x)$

b

$$\begin{array}{r} x^2 - x - 12 \\ x-1 \overline{) x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 - x^2} \\ -x^2 - 11x \\ \underline{-x^2 + x} \\ -12x + 12 \\ \underline{-12x + 12} \\ 0 \end{array}$$

$$\therefore f(x) \equiv (x-1)(x^2 - x - 12)$$

$$\equiv (x-1)(x+3)(x-4)$$

5 $g(-3) = -54 + 9 + 39 + 6 = 0$
 $\therefore (x + 3)$ is a factor of $g(x)$

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x+3 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2} \\ -5x^2 - 13x \\ \underline{-5x^2 - 15x} \\ 2x + 6 \\ \underline{2x + 6} \\ 0 \end{array}$$

$$\therefore g(x) \equiv (x+3)(2x^2 - 5x + 2)$$

$$\equiv (x+3)(2x-1)(x-2)$$

$$g(x) = 0 \Rightarrow (x+3)(2x-1)(x-2) = 0$$

$$x = -3, \frac{1}{2} \text{ or } 2$$

6 $f(4) = 0 \therefore (x - 4)$ is a factor of $f(x)$

$$\begin{array}{r} 6x^2 + 17x - 3 \\ x-4 \overline{) 6x^3 - 7x^2 - 71x + 12} \\ \underline{6x^3 - 24x^2} \\ 17x^2 - 71x \\ \underline{17x^2 - 68x} \\ -3x + 12 \\ \underline{-3x + 12} \\ 0 \end{array}$$

$$\therefore f(x) \equiv (x-4)(6x^2 + 17x - 3)$$

$$\equiv (x-4)(6x-1)(x+3)$$

$$f(x) = 0 \Rightarrow (x-4)(6x-1)(x+3) = 0$$

$$x = -3, \frac{1}{6} \text{ or } 4$$

7 a $g(-2) = 0 \therefore (x + 2)$ is a factor of $g(x)$

$$\begin{array}{r} x^2 + 5x - 3 \\ x+2 \overline{) x^3 + 7x^2 + 7x - 6} \\ \underline{x^3 + 2x^2} \\ 5x^2 + 7x \\ \underline{5x^2 + 10x} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

$$\therefore g(x) \equiv (x+2)(x^2 + 5x - 3)$$

b other solutions given by $x^2 + 5x - 3 = 0$

$$x = \frac{-5 \pm \sqrt{25+12}}{2} = \frac{-5 \pm \sqrt{37}}{2}$$

$$x = -5.54 \text{ or } 0.54$$

8 a $f(1) = 1 + 2 - 11 - 12 = -20$
 $f(2) = 8 + 8 - 22 - 12 = -18$
 $f(-1) = -1 + 2 + 11 - 12 = 0$
 $f(-2) = -8 + 8 + 22 - 12 = 10$

b $(x + 1)$ is a factor of $f(x)$

$$\begin{array}{r} x^2 + x - 12 \\ x+1 \overline{) x^3 + 2x^2 - 11x - 12} \\ \underline{x^3 + x^2} \\ x^2 - 11x \\ \underline{x^2 + x} \\ -12x - 12 \\ \underline{-12x - 12} \\ 0 \end{array}$$

$$\therefore f(x) = (x+1)(x^2 + x - 12)$$

$$= (x+1)(x+4)(x-3)$$

- 9 a let $f(x) = x^3 - 2x^2 - 5x + 6$ b let $f(x) = x^3 + x^2 - 5x - 2$ c let $f(x) = 20 + 11x - 8x^2 + x^3$
 $f(1) = 0$ $f(1) = -5, f(2) = 0$ $f(1) = 24, f(2) = 18, f(-1) = 0$
 $\therefore (x - 1)$ is a factor $\therefore (x - 2)$ is a factor $\therefore (x + 1)$ is a factor

$$\begin{array}{r} x^2 - x - 6 \\ x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 - x^2} \\ -x^2 - 5x \\ \underline{-x^2 + x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x - 1)(x^2 - x - 6) \\ &= (x - 1)(x + 2)(x - 3) \end{aligned}$$

$$\begin{array}{r} x^2 + 3x + 1 \\ x - 2 \overline{) x^3 + x^2 - 5x - 2} \\ \underline{x^3 + 2x^2} \\ -x^2 - 5x - 2 \\ \underline{-x^2 - 2x} \\ -3x - 2 \\ \underline{-3x + 6} \\ x - 2 \\ \underline{x - 2} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x - 2)(x^2 + 3x + 1) \end{aligned}$$

$$\begin{array}{r} x^2 - 9x + 20 \\ x + 1 \overline{) x^3 - 8x^2 + 11x + 20} \\ \underline{x^3 + x^2} \\ -9x^2 + 11x + 20 \\ \underline{-9x^2 - 9x} \\ 20x + 20 \\ \underline{20x + 20} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x + 1)(x^2 - 9x + 20) \\ &= (x + 1)(x - 4)(x - 5) \end{aligned}$$

- d let $f(x) = 3x^3 - 4x^2 - 35x + 12$ e let $f(x) = x^3 + 8$ f let $f(x) = 12 + 29x + 8x^2 - 4x^3$
 $f(1) = -24, f(2) = -50,$ $f(1) = 9, f(2) = 16$ $f(1) = 45, f(2) = 70,$
 $f(-1) = 40, f(-2) = 42$ $f(-1) = 7, f(-2) = 0$ $f(-1) = -5, f(-2) = 18$
 $f(3) = -48, f(-3) = 0$ $\therefore (x + 2)$ is a factor $f(3) = 63, f(-3) = 105$
 $\therefore (x + 3)$ is a factor $f(4) = 0$
 $\therefore (x - 4)$ is a factor

$$\begin{array}{r} 3x^2 - 13x + 4 \\ x + 3 \overline{) 3x^3 - 4x^2 - 35x + 12} \\ \underline{3x^3 + 9x^2} \\ -13x^2 - 35x + 12 \\ \underline{-13x^2 - 39x} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x + 3)(3x^2 - 13x + 4) \\ &= (x + 3)(3x - 1)(x - 4) \end{aligned}$$

$$\begin{array}{r} x^2 - 2x + 4 \\ x + 2 \overline{) x^3 + 0x^2 + 0x + 8} \\ \underline{x^3 + 2x^2} \\ -2x^2 + 0x + 8 \\ \underline{-2x^2 - 4x} \\ 4x + 8 \\ \underline{4x + 8} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x + 2)(x^2 - 2x + 4) \end{aligned}$$

$$\begin{array}{r} -4x^2 - 8x - 3 \\ x - 4 \overline{) -4x^3 + 8x^2 + 29x + 12} \\ \underline{-4x^3 + 16x^2} \\ -8x^2 + 29x + 12 \\ \underline{-8x^2 + 32x} \\ -3x + 12 \\ \underline{-3x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x - 4)(-4x^2 - 8x - 3) \\ &= -(x - 4)(4x^2 + 8x + 3) \\ &= (4 - x)(2x + 1)(2x + 3) \end{aligned}$$

- 10 a** let $f(x) = x^3 - x^2 - 10x - 8$
 $f(1) = -18, f(2) = -24,$
 $f(-1) = 0$
 $\therefore (x + 1)$ is a factor
- b** let $f(x) = x^3 + 2x^2 - 9x - 18$
 $f(1) = -24, f(2) = -20$
 $f(-1) = -8, f(-2) = 0$
 $\therefore (x + 2)$ is a factor
- c** let $f(x) = 4x^3 - 12x^2 + 9x - 2$
 $f(1) = -1, f(2) = 0$
 $\therefore (x - 2)$ is a factor

$$\begin{array}{r} x^2 - 2x - 8 \\ x+1 \overline{) x^3 - x^2 - 10x - 8} \\ \underline{x^3 + x^2} \\ -2x^2 - 10x \\ \underline{-2x^2 - 2x} \\ -8x - 8 \\ \underline{-8x - 8} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ (x+1)(x^2 - 2x - 8) &= 0 \\ (x+1)(x+2)(x-4) &= 0 \\ x &= -2, -1, 4 \end{aligned}$$

$$\begin{array}{r} x^2 + 0x - 9 \\ x+2 \overline{) x^3 + 2x^2 - 9x - 18} \\ \underline{x^3 + 2x^2} \\ 0x^2 - 9x - 18 \\ \underline{0x^2 + 0x} \\ -9x - 18 \\ \underline{-9x - 18} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ (x+2)(x^2 - 9) &= 0 \\ (x+2)(x+3)(x-3) &= 0 \\ x &= -3, -2, 3 \end{aligned}$$

$$\begin{array}{r} 4x^2 - 4x + 1 \\ x-2 \overline{) 4x^3 - 12x^2 + 9x - 2} \\ \underline{4x^3 - 8x^2} \\ -4x^2 + 9x \\ \underline{-4x^2 + 8x} \\ x - 2 \\ \underline{x - 2} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ (x-2)(4x^2 - 4x + 1) &= 0 \\ (x-2)(2x-1)^2 &= 0 \\ x &= \frac{1}{2}, 2 \end{aligned}$$

- d** let $f(x) = x^3 - 5x^2 + 3x + 1$
 $f(1) = 0$
 $\therefore (x - 1)$ is a factor
- e** let $f(x) = x^3 + 4x^2 - 9x - 6$
 $f(1) = -10, f(2) = 0$
 $\therefore (x - 2)$ is a factor
- f** let $f(x) = x^3 - 14x + 15$
 $f(1) = 2, f(2) = -5, f(-1) = 28,$
 $f(-2) = 35, f(3) = 0$
 $\therefore (x - 3)$ is a factor

$$\begin{array}{r} x^2 - 4x - 1 \\ x-1 \overline{) x^3 - 5x^2 + 3x + 1} \\ \underline{x^3 - x^2} \\ -4x^2 + 3x \\ \underline{-4x^2 + 4x} \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ (x-1)(x^2 - 4x - 1) &= 0 \\ x = 1 \text{ or } \frac{4 \pm \sqrt{16+4}}{2} \\ x &= 1, 2 \pm \sqrt{5} \end{aligned}$$

$$\begin{array}{r} x^2 + 6x + 3 \\ x-2 \overline{) x^3 + 4x^2 - 9x - 6} \\ \underline{x^3 - 2x^2} \\ 6x^2 - 9x - 6 \\ \underline{6x^2 - 12x} \\ 3x - 6 \\ \underline{3x - 6} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ (x-2)(x^2 + 6x + 3) &= 0 \\ x = 2 \text{ or } \frac{-6 \pm \sqrt{36-12}}{2} \\ x &= 2, -3 \pm \sqrt{6} \end{aligned}$$

$$\begin{array}{r} x^2 + 3x - 5 \\ x-3 \overline{) x^3 + 0x^2 - 14x + 15} \\ \underline{x^3 - 3x^2} \\ 3x^2 - 14x \\ \underline{3x^2 - 9x} \\ -5x + 15 \\ \underline{-5x + 15} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ (x-3)(x^2 + 3x - 5) &= 0 \\ x = 3 \text{ or } \frac{-3 \pm \sqrt{9+20}}{2} \\ x &= 3, \frac{1}{2}(-3 \pm \sqrt{29}) \end{aligned}$$

- 11 a** $f(2) = 0$
 $\therefore 16 - 4 - 30 + c = 0$
 $c = 18$

$$\begin{array}{r} 2x^2 + 3x - 9 \\ x-2 \overline{) 2x^3 - x^2 - 15x + 18} \\ \underline{2x^3 - 4x^2} \\ 3x^2 - 15x \\ \underline{3x^2 - 6x} \\ -9x + 18 \\ \underline{-9x + 18} \\ 0 \end{array}$$

$$\begin{aligned} \therefore f(x) &\equiv (x-2)(2x^2 + 3x - 9) \\ &\equiv (x-2)(2x-3)(x+3) \end{aligned}$$

- 12 a** $g(-1) = 0$
 $\therefore -1 + p + 13 + q = 0$
 $p + q + 12 = 0 \quad (1)$

$$\begin{aligned} g(3) &= 0 \\ \therefore 27 + 9p - 39 + q &= 0 \\ 9p + q - 12 &= 0 \quad (2) \\ (2) - (1) &\Rightarrow 8p - 24 = 0 \Rightarrow p = 3 \\ \text{sub (1)} &\Rightarrow 3 + q + 12 = 0 \Rightarrow q = -15 \end{aligned}$$

- b** $(x+1)(x-3)(ax+b) \equiv x^3 + 3x^2 - 13x - 15$
by inspection
 $g(x) \equiv (x+1)(x-3)(x+5)$
 $g(x) = 0 \Rightarrow (x+1)(x-3)(x+5) = 0$
 $x = -5, -1 \text{ or } 3$

$$13 \quad \mathbf{a} \quad f(2) = 8 + 16 - 2 + 6 = 28$$

$$\mathbf{c} \quad f(-5) = -250 + 25 - 45 + 17 = -163$$

$$\mathbf{e} \quad f\left(-\frac{1}{2}\right) = -\frac{1}{4} - \frac{3}{4} + 10 - 7 = 2$$

$$\mathbf{b} \quad f(-1) = -1 - 2 - 7 + 1 = -9$$

$$\mathbf{d} \quad f\left(\frac{1}{2}\right) = 1 + 1 - 3 - 3 = -4$$

$$\mathbf{f} \quad f\left(\frac{2}{3}\right) = \frac{8}{9} - \frac{8}{3} + \frac{4}{3} - 7 = -7\frac{4}{9}$$

$$14 \quad f(2) = 5$$

$$\therefore 8 - 16 + 10 + c = 5$$

$$c = 3$$

$$15 \quad f\left(\frac{1}{2}\right) = -2$$

$$\therefore \frac{1}{4} - \frac{9}{4} + \frac{1}{2}k + 5 = -2$$

$$k = -10$$

$$16 \quad \mathbf{a} \quad f(-3) = 22$$

$$\therefore -54 + 9a + 13 = 22$$

$$a = 7$$

$$\mathbf{b} \quad f(x) = 2x^3 + 7x^2 + 13$$

$$\begin{aligned} \text{remainder} &= f(4) \\ &= 128 + 112 + 13 \\ &= 253 \end{aligned}$$

$$17 \quad \mathbf{a} \quad f(-1) = 0$$

$$\therefore -p + q - q + 3 = 0$$

$$p = 3$$

$$\mathbf{b} \quad f(x) = 3x^3 + qx^2 + qx + 3$$

$$f(2) = 15$$

$$\therefore 24 + 4q + 2q + 3 = 15$$

$$q = -2$$

$$18 \quad \mathbf{a} \quad p(3) = 0$$

$$\therefore 27 + 9a + 27 + b = 0$$

$$9a + b = -54 \quad (1)$$

$$\mathbf{b} \quad p(-2) = -30$$

$$\therefore -8 + 4a - 18 + b = -30$$

$$4a + b = -4 \quad (2)$$

$$(1) - (2) \Rightarrow 5a = -50$$

$$\therefore a = -10, b = 36$$

$$19 \quad f(-1) = 3$$

$$\therefore -4 - 6 - m + n = 3$$

$$n - m = 13 \quad (1)$$

$$f\left(\frac{1}{2}\right) = 15$$

$$\therefore \frac{1}{2} - \frac{3}{2} + \frac{1}{2}m + n = 15$$

$$n + \frac{1}{2}m = 16 \quad (2)$$

$$(2) - (1) \Rightarrow \frac{3}{2}m = 3$$

$$\therefore m = 2, n = 15$$

$$20 \quad \mathbf{a} \quad g(4) = 39$$

$$\therefore 64 + 4c + 3 = 39$$

$$c = -7$$

$$\mathbf{b} \quad g(x) = x^3 - 7x + 3$$

$$\begin{array}{r} x^2 - 2x - 3 \\ x+2 \overline{) x^3 + 0x^2 - 7x + 3} \\ \underline{x^3 + 2x^2} \\ -2x^2 - 7x \\ \underline{-2x^2 - 4x} \\ -3x + 3 \\ \underline{-3x - 6} \\ 9 \end{array}$$

$$\text{quotient} = x^2 - 2x - 3$$

$$\text{remainder} = 9$$