1 **a** 1.09 **b** −11.47 **c** 0.33 **d** 1.89

2

a $= 1 \div \sin 30° = 1 \div \frac{1}{2} = 2$

b $= 1 \div \tan 45° = 1 \div 1 = 1$

c $= 1 \div \cos 150° = 1 \div (-\cos 30°) = 1 \div (-\frac{\sqrt{3}}{2}) = -\frac{2}{\sqrt{3}}$

d $= 1 \div \sin 300° = 1 \div (-\sin 60°) = 1 \div (-\frac{\sqrt{3}}{2}) = -\frac{2}{\sqrt{3}}$

e $= \cos 90° \div \sin 90° = 0 \div 1 = 0$

f $= 1 \div \cos 225° = 1 \div (-\cos 45°) = 1 \div (-\frac{1}{\sqrt{2}}) = -\sqrt{2}$

g $= 1 \div \sin 270° = 1 \div (-\sin 90°) = 1 \div (-1) = -1$

h $= 1 \div \tan 330° = 1 \div (-\tan 30°) = 1 \div (-\frac{1}{\sqrt{3}}) = -\sqrt{3}$

i $= 1 \div \cos 660° = 1 \div \cos 60° = 1 \div \frac{1}{2} = 2$

j $= 1 \div \sin (-45°) = 1 \div (-\sin 45°) = 1 \div (-\frac{1}{\sqrt{2}}) = -\sqrt{2}$

k $= 1 \div \tan (-240°) = 1 \div (-\tan 60°) = 1 \div (-\sqrt{3}) = -\frac{1}{\sqrt{3}}$

l $= 1 \div \cos (-315°) = 1 \div \cos 45° = 1 \div \frac{1}{\sqrt{2}} = \sqrt{2}$

3 **a** 1.60 **b** 1.01 **c** −2.09 **d** 2.54

4

a $= 1 \div \cos 0 = 1 \div 1 = 1$

b $= 1 \div \sin \frac{\pi}{4} = 1 \div \frac{1}{\sqrt{2}} = \sqrt{2}$

c $= 1 \div \tan \frac{3\pi}{4} = 1 \div (-\tan \frac{\pi}{4}) = 1 \div (-1) = -1$

d $= 1 \div \cos \frac{4\pi}{3} = 1 \div (-\cos \frac{\pi}{3}) = 1 \div (-\frac{1}{2}) = -2$

e $= 1 \div \sin \frac{2\pi}{3} = 1 \div \sin \frac{\pi}{3} = 1 \div \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$

f $= \cos \frac{7\pi}{2} \div \sin \frac{7\pi}{2} = \cos \frac{\pi}{2} \div (-\sin \frac{\pi}{2}) = 0 \div (-1) = 0$

g $= 1 \div \cos \frac{5\pi}{4} = 1 \div (-\cos \frac{\pi}{4}) = 1 \div (-\frac{1}{\sqrt{2}}) = -\sqrt{2}$

h $= 1 \div \sin (-\frac{5\pi}{6}) = 1 \div (-\sin \frac{\pi}{6}) = 1 \div (-\frac{1}{2}) = -2$

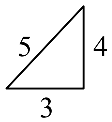
i $= 1 \div \tan \frac{11\pi}{6} = 1 \div (-\tan \frac{\pi}{6}) = 1 \div (-\frac{1}{\sqrt{3}}) = -\sqrt{3}$

j $= 1 \div \cos (-4\pi) = 1 \div \cos 0 = 1 \div 1 = 1$

k $= 1 \div \sin \frac{13\pi}{4} = 1 \div (-\sin \frac{\pi}{4}) = 1 \div (-\frac{1}{\sqrt{2}}) = -\sqrt{2}$

l $= 1 \div \tan (-\frac{7\pi}{3}) = 1 \div (-\tan \frac{\pi}{3}) = 1 \div (-\sqrt{3}) = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{1}{3}\sqrt{3}$

5

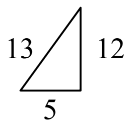


$\therefore\ \cos x = \pm\frac{3}{5},\ \tan x = \pm\frac{4}{3}$

$0 < x < 90° \Rightarrow \cos x = \frac{3}{5},\ \tan x = \frac{4}{3}$

a $= \frac{3}{5}$ **b** $= \frac{4}{3}$ **c** $= 1 \div \frac{4}{5} = \frac{5}{4}$ **d** $= 1 \div \frac{3}{5} = \frac{5}{3}$

6



$\therefore\ \sin x = \pm\frac{12}{13},\ \tan x = \pm\frac{12}{5}$

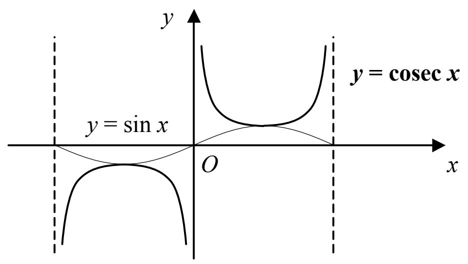
$90° < x < 180° \Rightarrow \sin x = \frac{12}{13},\ \tan x = -\frac{12}{5}$

a $= \frac{12}{13}$ **b** $= 1 \div (-\frac{5}{13}) = -\frac{13}{5}$ **c** $= 1 \div \frac{12}{13} = \frac{13}{12}$ **d** $= 1 \div -\frac{12}{5} = -\frac{5}{12}$

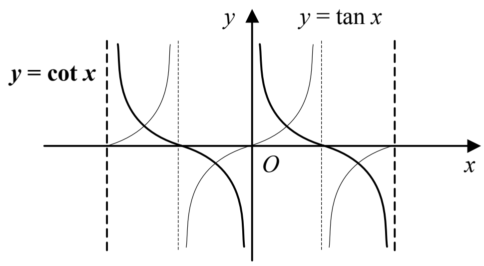
7 **a** (0, 1), (180, −1), (360, 1), (540, −1), (720, 1)

b $x = 90$, $x = 270$, $x = 450$, $x = 630$

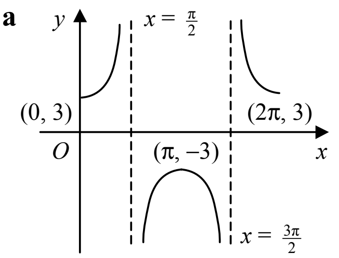
8 **a**



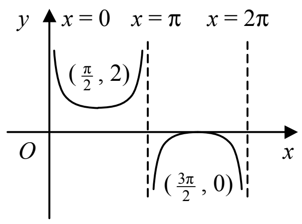
b



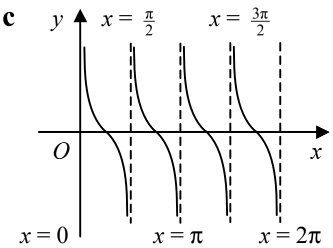
9 **a**



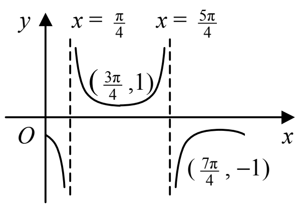
b



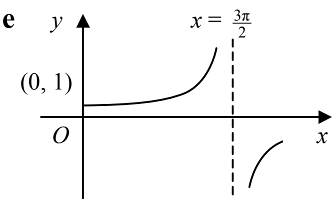
c



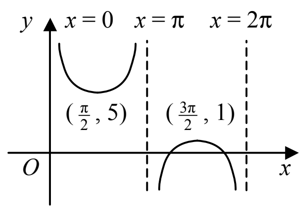
d



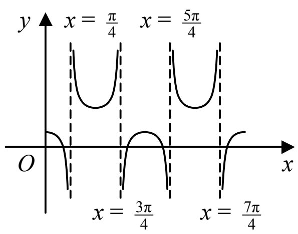
e



f

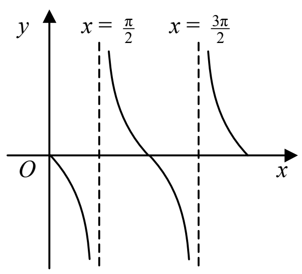


g

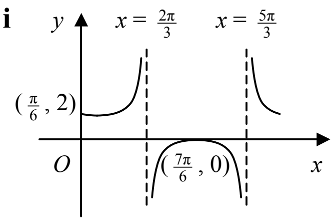


TP: (0, 0), $(\frac{\pi}{2}, 2)$, (π, 0),
$(\frac{3\pi}{2}, 2)$, (2π, 0)

h



i



10 **a** $\tan x = 1$

$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

b $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

c $\sin x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \pi - \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

d $\cos x = 0$

$x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

e $\cos x = -1$

$x = \pi$

f $\sin x = -\frac{1}{2}$

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

g $\tan x = -\frac{1}{\sqrt{3}}$

$x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

h $\cos x = -\frac{1}{\sqrt{2}}$

$x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$x = \frac{3\pi}{4}, \frac{5\pi}{4}$

11 **a** $\cos \theta = 0.5556$
$\theta = 56.3, 360 - 56.3$
$\theta = 56.3°, 303.7°$

b $\sin \theta = 0.3891$
$\theta = 22.9, 180 - 22.9$
$\theta = 22.9°, 157.1°$

c $\tan \theta = 0.9434$
$\theta = 43.3, 180 + 43.3$
$\theta = 43.3°, 223.3°$

d $\cos \theta = -0.3802$
$\theta = 180 - 67.7,$
$180 + 67.7$
$\theta = 112.3°, 247.7°$

e $\sin \theta = 0.3333$
$\theta = 19.5, 180 - 19.5$
$\theta = 19.5°, 160.5°$

f $\tan \theta = -1.0638$
$\theta = 180 - 46.8,$
$360 - 46.8$
$\theta = 133.2°, 313.2°$

g $\cos \theta = 0.5297$
$\theta = 58.0, 360 - 58.0$
$\theta = 58.0°, 302.0°$

h $\sin \theta = -0.8333$
$\theta = 180 + 56.4,$
$360 - 56.4$
$\theta = 236.4°, 303.6°$

12 **a** $\sin (x + 30) = 0.5$
$x + 30 = 30, 180 - 30$
$= 30, 150$
$x = 0, 120$

b $\tan (x - 57) = 0.625$
$x - 57 = 32.0, 32.0 - 180$
$= -148.0, 32.0$
$x = -91.0, 89.0$

c $\cos 2x = 0.4255$
$2x = 64.816, 360 - 64.816,$
$-64.816, 64.816 - 360$
$= -295.184, -64.816,$
$64.816, 295.184$
$x = -147.6, -32.4,$
$32.4, 147.6$

d $\cot x = 2.5$

$\tan x = 0.4$

$x = 21.8, 21.8 - 180$

$x = -158.2, 21.8$

e $\sec (x - 60) = \frac{2}{\sqrt{3}}$

$\cos (x - 60) = \frac{\sqrt{3}}{2}$

$x - 60 = 30, -30$

$x = 30, 90$

f $\text{cosec } \frac{1}{2}x = 3.5$

$\sin \frac{1}{2}x = 0.2857$

$\frac{1}{2}x = 16.602$

$x = 33.2$

g $\cos (2x - 18) = -0.7692$
$2x - 18 = 180 - 39.715,$
$180 + 39.715,$
$39.715 - 180,$
$-39.715 - 180$
$= -219.715, -140.285,$
$140.285, 219.715$
$2x = -201.715, -122.285,$
$158.285, 237.715$
$x = -100.9, -61.1$
$79.1, 118.9$

h $\sin 3x = -0.2941$
$3x = 180 + 17.105,$
$360 - 17.105,$
$-17.105,$
$17.105 - 180,$
$-360 - 17.105,$
$17.105 - 540$
$= -522.895, -377.105,$
$-162.895, -17.105,$
$197.105, 342.895$
$x = -174.3, -125.7, -54.3,$
$-5.7, 65.7, 114.3$

i $\tan (2x + 135) = 1$
$2x + 135 = 45, 180 + 45,$
$360 + 45,$
$45 - 180,$
$= -135, 45,$
$225, 405$
$2x = -270, -90, 90, 270$
$x = -135, -45, 45, 135$

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13 **a** $\text{cosec}^2\,\theta = 4$

$\text{cosec}\,\theta = \pm 2$

$\sin\theta = \pm\,\frac{1}{2}$

$\theta = 30,\ 180 - 30$ or $180 + 30,\ 360 - 30$

$\theta = 30,\ 150,\ 210,\ 330$

b $(\sec\theta + 1)(\sec\theta - 3) = 0$

$\sec\theta = -1$ or 3

$\cos\theta = -1$ or $\frac{1}{3}$

$\theta = 180$ or $70.5,\ 360 - 70.5$

$\theta = 70.5,\ 180,\ 289.5$

c $\cot\theta(\text{cosec}\,\theta - 6) = 0$

$\cot\theta = 0$ or $\text{cosec}\,\theta = 6$

$\cos\theta = 0$ or $\sin\theta = \frac{1}{6}$

$\theta = 90,\ 360 - 90$ or $9.6,\ 180 - 9.6$

$\theta = 9.6,\ 90,\ 170.4,\ 270$

d $\dfrac{1}{\sin\theta} = \dfrac{4}{\cos\theta}$

$\dfrac{\sin\theta}{\cos\theta} = \frac{1}{4}$

$\tan\theta = \frac{1}{4}$

$\theta = 14.0,\ 180 + 14.0$

$\theta = 14.0,\ 194.0$

e $2\cos\theta = \dfrac{\cos\theta}{\sin\theta}$

$2\cos\theta\sin\theta = \cos\theta$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\theta = 90,\ 360 - 90$ or $30,\ 180 - 30$

$\theta = 30,\ 90,\ 150,\ 270$

f $5\sin\theta - \dfrac{2}{\sin\theta} - 3 = 0$

$5\sin^2\theta - 3\sin\theta - 2 = 0$

$(5\sin\theta + 2)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{2}{5}$ or 1

$\theta = 180 + 23.6,\ 360 - 23.6$ or 90

$\theta = 90,\ 203.6,\ 336.4$

14 **a** $(2\,\text{cosec}\,x - 3)(\text{cosec}\,x + 4) = 0$

$\text{cosec}\,x = -4$ or $\frac{3}{2}$

$\sin x = -\frac{1}{4}$ or $\frac{2}{3}$

$x = -0.2527,\ -\pi + 0.2527$ or

$0.7297,\ \pi - 0.7297$

$x = -2.89,\ -0.25,\ 0.73,\ 2.41$

b $\dfrac{1}{\cos x} = \dfrac{3\sin x}{\cos x}$

$\sin x = \frac{1}{3}$

$x = 0.3398,\ \pi - 0.3398$

$x = 0.34,\ 2.80$

c $\dfrac{3}{\cos x} = \dfrac{2\cos x}{\sin x}$

$3\sin x = 2\cos^2 x$

$3\sin x = 2(1 - \sin^2 x)$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2}$ or -2 [no solutions]

$x = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$

$x = 0.52,\ 2.62$

d $4 + \tan x - \dfrac{5}{\tan x} = 0$

$\tan^2 x + 4\tan x - 5 = 0$

$(\tan x + 5)(\tan x - 1) = 0$

$\tan x = -5$ or 1

$x = \pi - 1.3734,\ -1.3734$ or $\frac{\pi}{4},\ -\pi + \frac{\pi}{4}$

$x = -2.36,\ -1.37,\ 0.79,\ 1.77$

e $\dfrac{1}{\sin x} = -\dfrac{5\cos x}{\sin x}$

$\cos x = -\frac{1}{5}$

$x = \pi - 1.3694,\ -\pi + 1.3694$

$x = -1.77,\ 1.77$

f $\dfrac{6\sin x}{\cos x} = \dfrac{5}{\sin x}$

$6\sin^2 x = 5\cos x$

$6(1 - \cos^2 x) = 5\cos x$

$6\cos^2 x + 5\cos x - 6 = 0$

$(3\cos x - 2)(2\cos x + 3) = 0$

$\cos x = \frac{2}{3}$ or $-\frac{3}{2}$ [no solutions]

$x = -0.84,\ 0.84$

15 **a** LHS $= \frac{1}{\cos x} - \cos x$

$= \frac{1 - \cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$

$= \sin x \times \frac{\sin x}{\cos x}$

$= \sin x \tan x$

= RHS

b LHS $= \operatorname{cosec} x - \cot x + \cot x - \cos x \cot x$

$= \frac{1}{\sin x} - \cos x \times \frac{\cos x}{\sin x}$

$= \frac{1 - \cos^2 x}{\sin x}$

$= \frac{\sin^2 x}{\sin x}$

$= \sin x$

= RHS

c LHS $= \frac{\sin x(\cot x - \cos x)}{\sin x(1 - \sin x)}$

$= \frac{\cos x - \sin x \cos x}{\sin x(1 - \sin x)}$

$= \frac{\cos x(1 - \sin x)}{\sin x(1 - \sin x)}$

$= \frac{\cos x}{\sin x}$

$= \cot x$

= RHS

d LHS $= \sin x \cos x + \sin x \cot x + \tan x \cos x + 1$

$= \sin x \cos x + \cos x + \sin x + 1$

$= \sin x (\cos x + 1) + \cos x + 1$

$= (\cos x + 1)(\sin x + 1)$

= RHS

16 **a** $x = 0 \Rightarrow y = 2 - 3 - 5 = -6 \;\therefore\; (0, -6)$

b $y = 0 \Rightarrow 2 \cos x - \frac{3}{\cos x} - 5 = 0$

$2 \cos^2 x - 5 \cos x - 3 = 0$

$(2 \cos x + 1)(\cos x - 3) = 0$

$\cos x = -\frac{1}{2}$ or 3 [no solutions]

$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\therefore\; (\frac{2\pi}{3}, 0)$ and $(\frac{4\pi}{3}, 0)$

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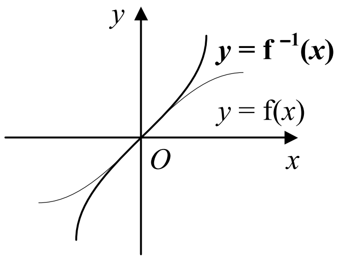
1 **a** $-1 \le \mathrm{f}(x) \le 1$

b $\mathrm{f}^{-1}(x) \equiv \arcsin x,\ x \in \mathbb{R},\ -1 \le x \le 1$

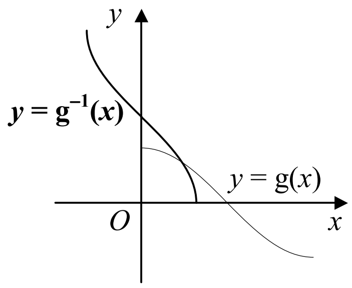
c



2 **a** 0 **b** $\frac{\pi}{4}$ **c** $-\frac{\pi}{2}$ **d** $-\frac{\pi}{3}$

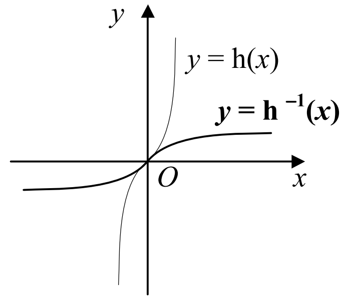
3 **a** $\mathrm{g}^{-1}(x) \equiv \arccos x,\ x \in \mathbb{R},\ -1 \le x \le 1$

b



4 **a** $\mathrm{h}^{-1}(x) \equiv \arctan x,\ x \in \mathbb{R}$

b



5 **a** 0 **b** $\frac{\pi}{3}$ **c** $\frac{\pi}{6}$ **d** $-\frac{\pi}{6}$

e $-\frac{\pi}{4}$ **f** π **g** $-\frac{\pi}{6}$ **h** $\frac{3\pi}{4}$

6 **a** 0.64 **b** 1.42 **c** 1.36 **d** -0.39

e 0.40 **f** -0.43 **g** -0.53 **h** 2.42

7 **a** $x = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ **b** $x = \cos 0 = 1$ **c** $x = \tan(-\frac{\pi}{3}) = -\sqrt{3}$

d $2x = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$x = \frac{\sqrt{3}}{4}$

e $\arctan x = \frac{\pi}{4}$

$x = \tan \frac{\pi}{4} = 1$

f $\arcsin x = -\frac{\pi}{6}$

$x = \sin(-\frac{\pi}{6}) = -\frac{1}{2}$

8 **a** $x = \cos 2 = -0.416$ **b** $x = \sin(-0.7) = -0.644$

c $3x = \tan 0.96 = 1.42836$

$x = 0.476$

d $\arcsin x = 1$

$x = \sin 1 = 0.841$

e $\arctan x = -\frac{2}{3}$

$x = \tan(-\frac{2}{3}) = -0.787$

f $\arccos 2x = 3$

$2x = \cos 3 = -0.98999$

$x = -0.495$

9 **a** $\mathrm{f}(-\frac{1}{2}) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$

b $\arccos x = \frac{\pi}{3} \Rightarrow x = \cos \frac{\pi}{3} = \frac{1}{2}$

c $y = \arccos x - \frac{\pi}{3}$ swap $x = \arccos y - \frac{\pi}{3}$

$y = \cos(x + \frac{\pi}{3})$

$\mathrm{f}^{-1}(x) \equiv \cos(x + \frac{\pi}{3}),\ x \in \mathbb{R},\ -\frac{\pi}{3} \le x \le \frac{2\pi}{3}$

1 **a** $\sin^2 x + \cos^2 x \equiv 1$

$\Rightarrow \quad \dfrac{\sin^2 x}{\cos^2 x} + 1 \equiv \dfrac{1}{\cos^2 x}$

$\Rightarrow \quad \tan^2 x + 1 \equiv \sec^2 x$

b $\sin^2 x + \cos^2 x \equiv 1$

$\Rightarrow \quad 1 + \dfrac{\cos^2 x}{\sin^2 x} \equiv \dfrac{1}{\sin^2 x}$

$\Rightarrow \quad 1 + \cot^2 x \equiv \text{cosec}^2\, x$

2 **a** $\tan^2 A = \frac{1}{9}$

$\sec^2 A = 1 + \frac{1}{9} = \frac{10}{9}$

b $\text{cosec}^2\, B = 1 + 2\sqrt{3} + 3$

$= 4 + 2\sqrt{3}$

$\cot^2 B = (4 + 2\sqrt{3}) - 1$

$= 3 + 2\sqrt{3}$

c $\sec^2 C = \frac{9}{4}$

$\tan^2 C = \frac{9}{4} - 1 = \frac{5}{4}$

$\tan C = \pm\sqrt{\frac{5}{4}} = \pm\frac{1}{2}\sqrt{5}$

3 **a** $3(1 + \tan^2 \theta) = 4\tan^2 \theta$

$\tan^2 \theta = 3$

$\tan \theta = \pm\sqrt{3}$

$\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

b $\sec^2 \theta - 1 - 2\sec \theta + 1 = 0$

$\sec^2 \theta - 2\sec \theta = 0$

$\sec \theta(\sec \theta - 2) = 0$

$\sec \theta = 2$ or 0 [no solutions]

$\cos \theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

c $\text{cosec}^2\, \theta - 1 - 3\,\text{cosec}\, \theta + 3 = 0$

$\text{cosec}^2\, \theta - 3\,\text{cosec}\, \theta + 2 = 0$

$(\text{cosec}\, \theta - 1)(\text{cosec}\, \theta - 2) = 0$

$\text{cosec}\, \theta = 1$ or 2

$\sin \theta = \frac{1}{2}$ or 1

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

d $1 + \cot^2 \theta + \cot^2 \theta = 3$

$\cot^2 \theta = 1$

$\cot \theta = \pm 1$

$\tan \theta = \pm 1$

$\theta = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

e $1 + \tan^2 \theta + 2\tan \theta = 0$

$(\tan \theta + 1)^2 = 0$

$\tan \theta = -1$

$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

f $1 + \cot^2 \theta - \sqrt{3}\cot \theta - 1 = 0$

$\cot^2 \theta - \sqrt{3}\cot \theta = 0$

$\cot \theta(\cot \theta - \sqrt{3}) = 0$

$\cot \theta = 0$ or $\sqrt{3}$

$\cos \theta = 0$ or $\tan \theta = \dfrac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$ or $\frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$

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4 **a** $\sec^2 x - 1 - 2\sec x - 2 = 0$
$\sec^2 x - 2\sec x - 3 = 0$
$(\sec x + 1)(\sec x - 3) = 0$
$\sec x = -1$ or 3
$\cos x = -1$ or $\frac{1}{3}$
$x = 180, -180$ or $70.5, -70.5$
$x = -180°, -70.5°, 70.5°, 180°$

b $2(1 + \cot^2 x) + 2 = 9\cot x$
$2\cot^2 x - 9\cot x + 4 = 0$
$(2\cot x - 1)(\cot x - 4) = 0$
$\cot x = \frac{1}{2}$ or 4
$\tan x = \frac{1}{4}$ or 2
$x = 14.0, 14.0 - 180$ or $63.4, 63.4 - 180$
$x = -166.0°, -116.6°, 14.0°, 63.4°$

c $\text{cosec}^2 x + 5\,\text{cosec}\, x + 2(\text{cosec}^2 x - 1) = 0$
$3\,\text{cosec}^2 x + 5\,\text{cosec}\, x - 2 = 0$
$(3\,\text{cosec}\, x - 1)(\text{cosec}\, x + 2) = 0$
$\text{cosec}\, x = -2$ or $\frac{1}{3}$ [no solutions]
$\sin x = -\frac{1}{2}$
$x = -30, 30 - 180$
$x = -150°, -30°$

d $3\tan^2 x - 3\tan x + 1 + \tan^2 x = 2$
$4\tan^2 x - 3\tan x - 1 = 0$
$(4\tan x + 1)(\tan x - 1) = 0$
$\tan x = -\frac{1}{4}$ or 1
$x = 180 - 14.0, -14.0$ or $45, 45 - 180$
$x = -135°, -14.0°, 45°, 166.0°$

e $\sec^2 x - 1 + 4\sec x - 2 = 0$
$\sec^2 x + 4\sec x - 3 = 0$
$\sec x = \dfrac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$
$\cos x = \dfrac{1}{-2 \pm \sqrt{7}}$
$\cos x = -0.2153$ or 1.5486 [no solutions]
$x = 180 - 77.6, 77.6 - 180$
$x = -102.4°, 102.4°$

f $2\cot^2 x + 3(1 + \cot^2 x) = 4\cot x + 3$
$5\cot^2 x - 4\cot x = 0$
$\cot x\,(5\cot x - 4) = 0$
$\cot x = 0$ or $\frac{4}{5}$
$\cos x = 0$ or $\tan x = \frac{5}{4}$
$x = 90, -90$ or $51.3, 51.3 - 180$
$x = -128.7°, -90°, 51.3°, 90°$

5 **a** $\text{cosec}^2 2x - 1 + \text{cosec}\, 2x - 1 = 0$
$\text{cosec}^2 2x + \text{cosec}\, 2x - 2 = 0$
$(\text{cosec}\, 2x + 2)(\text{cosec}\, 2x - 1) = 0$
$\text{cosec}\, 2x = -2$ or 1
$\sin 2x = -\frac{1}{2}$ or 1
$2x = 180 + 30, 360 - 30, 540 + 30,$
$720 - 30$ or $90, 360 + 90$
$= 90, 210, 330, 450, 570, 690$
$x = 45°, 105°, 165°, 225°, 285°, 345°$

b $8(1 - \cos^2 x) + \sec x = 8$
$8\cos^2 x = \sec x$
$\cos^3 x = \frac{1}{8}$
$\cos x = \frac{1}{2}$
$x = 60, 360 - 60$
$x = 60°, 300°$

c $\dfrac{3}{\sin^2 x} - 4\sin^2 x = 1$
$4\sin^4 x + \sin^2 x - 3 = 0$
$(4\sin^2 x - 3)(\sin^2 x + 1) = 0$
$\sin^2 x = \frac{3}{4}$ or -1 [no solutions]
$\sin x = \pm\dfrac{\sqrt{3}}{2}$
$x = 60, 180 - 60$ or $180 + 60, 360 - 60$
$x = 60°, 120°, 240°, 300°$

d $9(1 + \tan^2 x) - 8 = 1 + \cot^2 x$
$9\tan^2 x = \cot^2 x$
$\tan^4 x = \frac{1}{9}$
$\tan^2 x = \frac{1}{3}$ or $-\frac{1}{3}$ [no solutions]
$\tan x = \pm\dfrac{1}{\sqrt{3}}$
$x = 30, 180 + 30$ or $180 - 30, 360 - 30$
$x = 30°, 150°, 210°, 330°$

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6 **a** LHS $= 1 + \cot^2 x - (1 + \tan^2 x)$
$= \cot^2 x - \tan^2 x$
$=$ RHS

b LHS $= \cot^2 x - 2\cot x + 1$
$= \operatorname{cosec}^2 x - 2\cot x$
$=$ RHS

c LHS $= \cos^2 x - 4 + 4\sec^2 x$
$= \cos^2 x - 4 + 4(1 + \tan^2 x)$
$= \cos^2 x + 4\tan^2 x$
$=$ RHS

d LHS $= 1 + \tan^2 x - (1 - \cos^2 x)$
$= \tan^2 x + \cos^2 x$
$=$ RHS

e LHS $= \tan^2 x + 2 + \cot^2 x$
$= \sec^2 - 1 + 2 + \operatorname{cosec}^2 x - 1$
$= \sec^2 x + \operatorname{cosec}^2 x$
$=$ RHS

f LHS $= \sin^2 x - 2\sin x \sec x + \sec^2 x$
$= \sin^2 x - 2\tan x + 1 + \tan^2 x$
$= \sin^2 x + (\tan x - 1)^2$
$=$ RHS

g LHS $= \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$
$= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$
$= \dfrac{1}{\cos^2 x \sin^2 x}$
$= \dfrac{1}{\cos^2 x} \times \dfrac{1}{\sin^2 x}$
$= \sec^2 x \operatorname{cosec}^2 x$
$=$ RHS

h LHS $= \sec^2 x\,(1 + \tan^2 x) + \tan^2 x\,(\sec^2 x - 1)$
$= \sec^2 x + \sec^2 x \tan^2 x + \sec^2 x \tan^2 x - \tan^2 x$
$= 1 + \tan^2 x + 2\sec^2 x \tan^2 x - \tan^2 x$
$= 2\sec^2 x \tan^2 x + 1$
$=$ RHS

7 $4\sec^2 x - \sec x + 2\tan^2 x = 0 \;\Rightarrow\; 4\sec^2 x - \sec x + 2(\sec^2 x - 1) = 0$
$6\sec^2 x - \sec x - 2 = 0$
$(3\sec x - 2)(2\sec x + 1) = 0$
$\sec x = -\tfrac{1}{2},\ \tfrac{2}{3}$
for real values of x, $|\sec x| > 1 \;\therefore$ no real solutions

8 **a** LHS $= \dfrac{1}{\sin x} \times \dfrac{1}{\cos x} - \dfrac{\cos x}{\sin x}$
$= \dfrac{1 - \cos^2 x}{\sin x \cos x}$
$= \dfrac{\sin^2 x}{\sin x \cos x}$
$= \dfrac{\sin x}{\cos x}$
$= \tan x$
$=$ RHS

b $\operatorname{cosec} x \sec x - \cot x = 3$
$\tan x = 3$
$x = 71.6,\ 180 + 71.6$
$x = 71.6°,\ 251.6°$

1 **a** $\sin(A+B) \equiv \sin A\cos B + \cos A\sin B \quad (1)$
$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B \quad (2)$

b let $B = -B$ in (1) $\Rightarrow$ $\sin[A+(-B)] \equiv \sin A\cos(-B) + \cos A\sin(-B)$
$\sin(A-B) \equiv \sin A\cos B + \cos A(-\sin B)$
$\sin(A-B) \equiv \sin A\cos B - \cos A\sin B$

let $B = -B$ in (2) $\Rightarrow$ $\cos[A+(-B)] \equiv \cos A\cos(-B) - \sin A\sin(-B)$
$\cos(A-B) \equiv \cos A\cos B - \sin A(-\sin B)$
$\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$

c (1) ÷ (2) $\Rightarrow$ $\dfrac{\sin(A+B)}{\cos(A+B)} \equiv \dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$

$$\tan(A+B) \equiv \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$$

$$\tan(A+B) \equiv \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

let $B = -B$ $\Rightarrow$ $\tan[A+(-B)] \equiv \dfrac{\tan A + \tan(-B)}{1 - \tan A\tan(-B)}$

$$\tan(A-B) \equiv \frac{\tan A + (-\tan B)}{1 - \tan A(-\tan B)}$$

$$\tan(A-B) \equiv \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

2 **a** $= \sin(10+30)°$
$= \sin 40°$

b $= \sin(67-18)°$
$= \sin 49°$

c $= \sin(62+74)°$
$= \sin 136°$
$= \sin(180-136)°$
$= \sin 44°$

d $= \cos(14+39)°$
$= \cos 53°$
$= \sin(90-53)°$
$= \sin 37°$

3 **a** $= \cos(A+2A)$
$= \cos 3A$

b $= \sin(4A-B)$

c $= \tan(2A+5A)$
$= \tan 7A$

d $= \cos(A-3A)$
$= \cos(-2A)$
$= \cos 2A$

4 **a** $= \sin(45 - 30)°$

$= \sin 45°\cos 30° - \cos 45°\sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

b $= \sin 15°$

$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

c $= \frac{1}{\sin 15°}$

$= \frac{2\sqrt{2}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{2\sqrt{2}(\sqrt{3}+1)}{3-1}$

$= \sqrt{6} + \sqrt{2}$

d $= \sin(90 - 75)°$

$= \sin 15°$

$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

e $= \cos(45 - 30)°$

$= \cos 45°\cos 30° + \sin 45°\sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{1}{4}(\sqrt{6} + \sqrt{2})$

f $= \frac{1}{\cos 195°}$

$= \frac{1}{-\cos 15°}$

$= -\frac{2\sqrt{2}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= -\frac{2\sqrt{2}(\sqrt{3}-1)}{3-1}$

$= \sqrt{2} - \sqrt{6}$

g $= \tan(30 + 45)°$

$= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$

$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \times 1}$

$= \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{1 + 2\sqrt{3} + 3}{3-1} = 2 + \sqrt{3}$

h $= \frac{1}{\sin 105°}$

$= \frac{1}{\sin 75°}$

$= \frac{1}{\cos 15°}$

$= \sqrt{6} - \sqrt{2}$

5 **a** $= \cos(x - 30°)$

$\therefore$ max. $= 1$ when $x = 30°$

b $= 3\sin(x + 45°)$

$\therefore$ max. $= 3$ when $x = 45°$

c $= \sin(x - 67°)$

$\therefore$ max. $= 1$ when $x = 157°$

d $= -4(\cos x \cos 108° - \sin x \sin 108°)$

$= -4\cos(x + 108°)$

$\therefore$ max. $= 4$ when $x = 72°$

6 **a** $= \sin(x - \frac{\pi}{3})$

$\therefore$ min. $= -1$ when $x = \frac{11\pi}{6}$

b $= 2\cos(x + \frac{\pi}{6})$

$\therefore$ min. $= -2$ when $x = \frac{5\pi}{6}$

c $= \cos(4x - x)$

$= \cos 3x$

$\therefore$ min. $= -1$ when $x = \frac{\pi}{3}$

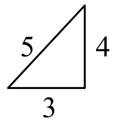
d $= 6\sin(2x - 3x)$

$= 6\sin(-x)$

$= -6\sin x$

$\therefore$ min. $= -6$ when $x = \frac{\pi}{2}$

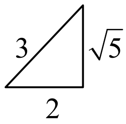
7 **a**



$\therefore\ \tan A = \pm\frac{4}{3}$

$0 < A < 90° \Rightarrow \tan A = \frac{4}{3}$

b



$\therefore\ \sin B = \pm\frac{\sqrt{5}}{3}$

$0 < B < 90° \Rightarrow \sin B = \frac{\sqrt{5}}{3}$

c $= \cos A \cos B - \sin A \sin B$

$= \frac{3}{5}\times\frac{2}{3} - \frac{4}{5}\times\frac{\sqrt{5}}{3}$

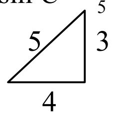
$= \frac{2}{15}(3 - 2\sqrt{5})$

d $= \sin A \cos B + \cos A \sin B$

$= \frac{4}{5}\times\frac{2}{3} + \frac{3}{5}\times\frac{\sqrt{5}}{3}$

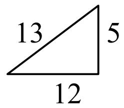
$= \frac{1}{15}(8 + 3\sqrt{5})$

8 **a** $\sin C = \frac{3}{5}$ $\quad\therefore\ \cos C = \pm\frac{4}{5}$



$0 < C < 90° \Rightarrow \cos C = \frac{4}{5}$

b



$\therefore\ \cos D = \pm\frac{12}{13}$

$90° < D < 180° \Rightarrow \cos D = -\frac{12}{13}$

c $= \sin C \cos D - \cos C \sin D$

$= \frac{3}{5}\times(-\frac{12}{13}) - \frac{4}{5}\times\frac{5}{13}$

$= -\frac{56}{65}$

d $\cos(C - D) = \cos C \cos D + \sin C \sin D$

$= \frac{4}{5}\times(-\frac{12}{13}) + \frac{3}{5}\times\frac{5}{13}$

$= -\frac{33}{65}$

$\therefore\ \sec(C - D) = -\frac{65}{33}$

9 **a** $\sin(\theta + 15) = 0.4$

$\theta + 15 = 23.6,\ 180 - 23.6$

$= 23.6,\ 156.4$

$\theta = 8.6,\ 141.4$

b $\tan(2\theta - 60) = 1$

$2\theta - 60 = 45,\ 180 + 45,\ 360 + 45,\ 540 + 45$

$= 45,\ 225,\ 405,\ 585$

$2\theta = 105,\ 285,\ 465,\ 645$

$\theta = 52.5,\ 142.5,\ 232.5,\ 322.5$

c $\cos\theta\cos 60 + \sin\theta\sin 60 = \sin\theta$

$\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = \sin\theta$

$(1 - \frac{\sqrt{3}}{2})\sin\theta = \frac{1}{2}\cos\theta$

$\tan\theta = \frac{1}{2} \div (1 - \frac{\sqrt{3}}{2}) = 3.7321$

$\theta = 75,\ 180 + 75$

$\theta = 75,\ 255$

d $2\sin\theta + \sin\theta\cos 45 + \cos\theta\sin 45 = 0$

$2\sin\theta + \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 0$

$(2 + \frac{1}{\sqrt{2}})\sin\theta = -\frac{1}{\sqrt{2}}\cos\theta$

$\tan\theta = -\frac{1}{\sqrt{2}} \div (2 + \frac{1}{\sqrt{2}}) = -0.2612$

$\theta = 180 - 14.6,\ 360 - 14.6$

$\theta = 165.4,\ 345.4$

e $\sin\theta\cos 30 + \cos\theta\sin 30$

$= \cos\theta\cos 45 + \sin\theta\sin 45$

$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta$

$(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}})\sin\theta = (\frac{1}{\sqrt{2}} - \frac{1}{2})\cos\theta$

$\tan\theta = (\frac{1}{\sqrt{2}} - \frac{1}{2}) \div (\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}) = 1.3032$

$\theta = 52.2,\ 180 + 52.5$

$\theta = 52.5,\ 232.5$

f $3(\cos 2\theta\cos 60 - \sin 2\theta\sin 60)$

$- (\sin 2\theta\cos 30 - \cos 2\theta\sin 30) = 0$

$\frac{3}{2}\cos 2\theta - \frac{3\sqrt{3}}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta + \frac{1}{2}\cos 2\theta = 0$

$2\sqrt{3}\sin 2\theta = 2\cos 2\theta$

$\tan 2\theta = \frac{1}{\sqrt{3}}$

$2\theta = 30,\ 180 + 30,\ 360 + 30,\ 540 + 30$

$= 30,\ 210,\ 390,\ 570$

$\theta = 15,\ 105,\ 195,\ 285$

10 LHS $= \cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3} - (\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3})$

$= -2\sin x\sin\frac{\pi}{3}$

$= -\sqrt{3}\sin x \quad\therefore\ k = -\sqrt{3}$

11 **a** LHS = $\cos x - (\cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3})$

$= \cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$= \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$= \cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}$

$= \cos (x + \frac{\pi}{3})$ = RHS

b LHS = $\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6} + \cos x$

$= \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x + \cos x$

$= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$

$= \sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6}$

$= \sin (x + \frac{\pi}{6})$ = RHS

12 **a** $\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$
let $B = A \Rightarrow \sin (A + A) \equiv \sin A \cos A + \cos A \sin A$
$\sin 2A \equiv 2 \sin A \cos A$

b $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$
let $B = A \Rightarrow \cos (A + A) \equiv \cos A \cos A - \sin A \sin A$
$\cos 2A \equiv \cos^2 A - \sin^2 A$

c **i** $\cos 2A \equiv \cos^2 A - (1 - \cos^2 A)$
$\cos 2A \equiv 2 \cos^2 A - 1$

ii $\cos 2A \equiv 1 - \sin^2 A - \sin^2 A$
$\cos 2A \equiv 1 - 2 \sin^2 A$

d $\tan (A + B) \equiv \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

let $B = A \Rightarrow \tan (A + A) \equiv \dfrac{\tan A + \tan A}{1 - \tan A \tan A}$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

13 **a** $2 \cos^2 x - 1 + \cos x = 0$
$(2 \cos x - 1)(\cos x + 1) = 0$
$\cos x = -1$ or $\frac{1}{2}$
$x = 180$ or $60, 360 - 60$
$x = 60°, 180°, 300°$

b $2 \sin x \cos x + \cos x = 0$
$\cos x(2 \sin x + 1) = 0$
$\cos x = 0$ or $\sin x = -\frac{1}{2}$
$x = 90, 360 - 90$ or $180 + 30, 360 - 30$
$x = 90°, 210°, 270°, 330°$

c $2(1 - 2 \sin^2 x) = 7 \sin x$
$4 \sin^2 x + 7 \sin x - 2 = 0$
$(4 \sin x - 1)(\sin x + 2) = 0$
$\sin x = \frac{1}{4}$ or -2 [no solutions]
$x = 14.5, 180 - 14.5$
$x = 14.5°, 165.5°$

d $11 \cos x = 4 + 3(2 \cos^2 x - 1)$
$6 \cos^2 x - 11 \cos x + 1 = 0$
$\cos x = \dfrac{11 \pm \sqrt{121 - 24}}{12} = \dfrac{11 \pm \sqrt{97}}{12}$
$\cos x = 0.09593$ or 1.7374 [no solutions]
$x = 84.5, 360 - 84.5$
$x = 84.5°, 275.5°$

e $\dfrac{2\tan x}{1-\tan^2 x} - \tan x = 0$
$2 \tan x = \tan x(1 - \tan^2 x)$
$\tan^3 x + \tan x = 0$
$\tan x(\tan^2 x + 1) = 0$
$\tan x = 0$ or $\tan^2 x = -1$ [no solutions]
$x = 0, 180°, 360°$

f $\dfrac{1}{\cos x} = 4 \sin x$
$1 = 4 \sin x \cos x$
$1 = 2 \sin 2x$
$\sin 2x = \frac{1}{2}$
$2x = 30, 180 - 30, 360 + 30, 540 - 30$
$= 30, 150, 390, 510$
$x = 15°, 75°, 195°, 255°$

g $10 \sin 2x \cos 2x = 2 \sin 2x$
$2 \sin 2x(5 \cos 2x - 1) = 0$
$\sin 2x = 0$ or $\cos 2x = \frac{1}{5}$
$2x = 0, 180, 360, 540, 720$
or $78.463, 360 - 78.463,$
$360 + 78.463, 720 - 78.463$
$= 0, 78.463, 180, 281.537, 360$
$438.463, 540, 641.537, 720$
$x = 0, 39.2°, 90°, 140.8°, 180°,$
$219.2°, 270°, 320.8°, 360°$

h $2(1 - \cos^2 x) - (2 \cos^2 x - 1) - \cos x = 0$
$4 \cos^2 x + \cos x - 3 = 0$
$(4 \cos x - 3)(\cos x + 1) = 0$
$\cos x = -1$ or $\frac{3}{4}$
$x = 180$ or $41.4, 360 - 41.4$
$x = 41.4°, 180, 318.6°$

14 **a** LHS $= \cos^2 x + 2 \sin x \cos x + \sin^2 x$
$= \cos^2 x + \sin^2 x + \sin 2x$
$= 1 + \sin 2x$
$=$ RHS

b LHS $= \tan x(1 + 2 \cos^2 x - 1)$
$= \dfrac{\sin x}{\cos x} \times 2 \cos^2 x$
$= 2 \sin x \cos x$
$= \sin 2x$
$=$ RHS

c LHS $= \dfrac{2\sin x\cos x}{\cos x(2\cos x - \sec x)}$
$= \dfrac{2\sin x\cos x}{2\cos^2 x - 1}$
$= \dfrac{\sin 2x}{\cos 2x}$
$= \tan 2x$
$=$ RHS

d LHS $= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$
$= \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}$
$= \dfrac{1}{\frac{1}{2}\sin 2x}$
$= 2 \operatorname{cosec} 2x$
$=$ RHS

e LHS $= \dfrac{1}{\sin 2x} - \dfrac{\cos 2x}{\sin 2x}$
$= \dfrac{1 - \cos 2x}{\sin 2x}$
$= \dfrac{1 - (1 - 2\sin^2 x)}{\sin 2x}$
$= \dfrac{2\sin^2 x}{2\sin x\cos x}$
$= \dfrac{\sin x}{\cos x}$
$= \tan x$
$=$ RHS

f LHS $= \cos x \operatorname{cosec} x - 1 + 1 - \sin x \sec x$
$= \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$
$= \dfrac{\cos^2 x - \sin^2 x}{\sin x\cos x}$
$= \dfrac{\cos 2x}{\frac{1}{2}\sin 2x}$
$= 2 \cot 2x$
$=$ RHS

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g LHS = $\dfrac{\sin x(1-\sin 2x)}{\sin x(\operatorname{cosec} x - 2\cos x)}$

$= \dfrac{\sin x(1-\sin 2x)}{1-2\sin x\cos x}$

$= \dfrac{\sin x(1-\sin 2x)}{1-\sin 2x}$

$= \sin x$

= RHS

h LHS = $\cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= \cos x(2\cos^2 x - 1) - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$

= RHS

15 **a** $\cos 2A \equiv 2\cos^2 A - 1$

let $A = \frac{x}{2}$

$\cos x \equiv 2\cos^2 \frac{x}{2} - 1$

b $\cos 2A \equiv 1 - 2\sin^2 A$

let $A = \frac{x}{2}$

$\cos x \equiv 1 - 2\sin^2 \frac{x}{2}$

$\sin^2 \frac{x}{2} \equiv \frac{1}{2}(1 - \cos x)$

16 **a** $\sin^2 \frac{A}{2} = \frac{1}{2}(1 - \frac{7}{9}) = \frac{1}{9}$

$\sin \frac{A}{2} = \pm\frac{1}{3}$

$0 < \frac{A}{2} < 45° \ \therefore \sin \frac{A}{2} = \frac{1}{3}$

b $-\frac{3}{8} = 2\cos^2 \frac{B}{2} - 1$

$\cos^2 \frac{B}{2} = \frac{1}{2}(-\frac{3}{8} + 1) = \frac{5}{16}$

$\cos \frac{B}{2} = \pm\frac{1}{4}\sqrt{5}$

$45° < \frac{B}{2} < 90° \ \therefore \cos \frac{B}{2} = \frac{1}{4}\sqrt{5}$

17 **a** LHS = $\dfrac{2}{1+(2\cos^2 \frac{x}{2} - 1)}$

$= \dfrac{2}{2\cos^2 \frac{x}{2}}$

$= \sec^2 \frac{x}{2}$

= RHS

b LHS = $\dfrac{1+(2\cos^2 \frac{x}{2} - 1)}{1-(1-2\sin^2 \frac{x}{2})}$

$= \dfrac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}}$

$= \cot^2 \frac{x}{2}$

= RHS

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1 **a** $\sin(A+B) \equiv \sin A\cos B + \cos A\sin B$ (1)
$\sin(A-B) \equiv \sin A\cos B - \cos A\sin B$ (2)

b (1) + (2) $\sin(A+B)+\sin(A-B) \equiv \sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$
$\Rightarrow$ $2\sin A\cos B \equiv \sin(A+B)+\sin(A-B)$

c $\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$ (3)
$\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$ (4)
(3) + (4) $2\cos A\cos B \equiv \cos(A+B)+\cos(A-B)$
(4) − (3) $2\sin A\sin B \equiv \cos(A-B)-\cos(A+B)$

2 **a** $= \sin(30+10)° + \sin(30-10)°$
$= \sin 40° + \sin 20°$

b $= \cos(36+18)° + \cos(36-18)°$
$= \cos 54° + \cos 18°$

c $= \frac{1}{2}[\sin(49+25)° - \sin(49-25)°]$
$= \frac{1}{2}\sin 74° - \frac{1}{2}\sin 24°$

d $= \cos(3A-A) - \cos(3A+A)$
$= \cos 2A - \cos 4A$

e $= \sin(5A+2A) - \sin(5A-2A)$
$= \sin 7A - \sin 3A$

f $= 2[\cos(3A+B) + \cos(3A-B)]$
$= 2\cos(3A+B) + 2\cos(3A-B)$

g $= \frac{1}{2}[\sin(A+6B) + \sin(A-6B)]$
$= \frac{1}{2}\sin(A+6B) + \frac{1}{2}\sin(A-6B)$

h $= \sin[A+(A+40°)] - \sin[A-(A+40°)]$
$= \sin(2A+40°) - \sin(-40°)$
$= \sin(2A+40°) + \sin 40°$

3 **a** $2\sin A\cos B \equiv \sin(A+B)+\sin(A-B)$
let $P = A+B$ (1) and $Q = A-B$ (2)
(1) + (2) $\Rightarrow 2A = P+Q \Rightarrow A = \frac{P+Q}{2}$, (1) − (2) $\Rightarrow 2B = P-Q \Rightarrow B = \frac{P-Q}{2}$
$\therefore\ \sin P + \sin Q \equiv 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

b let $P = A+B$ and $Q = A-B$ in each part

i $2\cos A\sin B \equiv \sin(A+B)-\sin(A-B) \Rightarrow \sin P - \sin Q \equiv 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$

ii $2\cos A\cos B \equiv \cos(A+B)+\cos(A-B) \Rightarrow \cos P + \cos Q \equiv 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

iii $2\sin A\sin B \equiv \cos(A-B)-\cos(A+B) \Rightarrow \cos Q - \cos P \equiv 2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$
$\Rightarrow \cos P - \cos Q \equiv -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$

4 **a** $= 2\cos\frac{25+15}{2}\cos\frac{25-15}{2}$
$= 2\cos 20°\cos 5°$

b $= 2\cos\frac{84+30}{2}\sin\frac{84-30}{2}$
$= 2\cos 57°\sin 27°$

c $= 2\sin\frac{5A+A}{2}\cos\frac{5A-A}{2}$
$= 2\sin 3A\cos 2A$

d $= -2\sin\frac{A+2A}{2}\sin\frac{A-2A}{2}$
$= -2\sin\frac{3A}{2}\sin(-\frac{A}{2}) = 2\sin\frac{3A}{2}\sin\frac{A}{2}$

e $= -2\sin\frac{2A+4B}{2}\sin\frac{2A-4B}{2}$
$= -2\sin(A+2B)\sin(A-2B)$

f $= 2\sin\frac{2A+90}{2}\cos(\frac{-30}{2})$
$= 2\sin(A+45)\cos(-15) = 2\sin(A+45°)\cos 15°$

g $= 4\cos\frac{A+3A}{2}\cos\frac{A-3A}{2}$
$= 4\cos 2A\cos(-A) = 4\cos 2A\cos A$

h $= 2\cos\frac{4A+B}{2}\sin\frac{3B-2A}{2}$
$= 2\cos(2A+\frac{1}{2}B)\sin(\frac{3}{2}B - A)$

5 **a** $2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}=0$

$\cos 2x\sin x=0$

$\cos 2x=0$ or $\sin x=0$

$2x=\frac{\pi}{2},\ 2\pi-\frac{\pi}{2}$ or $x=0,\ \pi$

$2x=\frac{\pi}{2},\ \frac{3\pi}{2}$ or $x=0,\ \pi$

$x=0,\ \frac{\pi}{4},\ \frac{3\pi}{4},\ \pi$

b $\cos 4x-\cos x=0$

$-2\sin\frac{4x+x}{2}\sin\frac{4x-x}{2}=0$

$\sin\frac{5}{2}x\sin\frac{3}{2}x=0$

$\sin\frac{5}{2}x=0$ or $\sin\frac{3}{2}x=0$

$\frac{5}{2}x=0,\ \pi,\ 2\pi$ or $\sin\frac{3}{2}x=0,\ \pi$

$x=0,\ \frac{2\pi}{5},\ \frac{2\pi}{3},\ \frac{4\pi}{5}$

c $\cos(x-5x)-\cos(x+5x)=\cos 4x$

$\cos(-4x)-\cos 6x=\cos 4x$

$\cos 4x-\cos 6x=\cos 4x$

$\cos 6x=0$

$6x=\frac{\pi}{2},\ 2\pi-\frac{\pi}{2},\ 2\pi+\frac{\pi}{2},$

$4\pi-\frac{\pi}{2},\ 4\pi+\frac{\pi}{2},\ 6\pi-\frac{\pi}{2}$

$=\frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2},\ \frac{7\pi}{2},\ \frac{9\pi}{2},\ \frac{11\pi}{2}$

$x=\frac{\pi}{12},\ \frac{\pi}{4},\ \frac{5\pi}{12},\ \frac{7\pi}{12},\ \frac{3\pi}{4},\ \frac{11\pi}{12}$

d $4[\sin(2x+\frac{\pi}{2})-\sin\frac{\pi}{6}]=1$

$\sin(2x+\frac{\pi}{2})-\frac{1}{2}=\frac{1}{4}$

$\sin(2x+\frac{\pi}{2})=\frac{3}{4}$

$2x+\frac{\pi}{2}=\pi-0.8481,\ 2\pi+0.8481$

$=2.2935,\ 7.1312$

$2x=0.7227,\ 5.5605$

$x=0.36,\ 2.78$

e $2\sin\frac{x+\frac{x}{2}}{2}\cos\frac{x-\frac{x}{2}}{2}=0$

$\sin\frac{3}{4}x\cos\frac{1}{4}x=0$

$\sin\frac{3}{4}x=0$ or $\cos\frac{1}{4}x=0$

$\frac{3}{4}x=0$ or (none in interval)

$x=0$

f $2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}=\cos 2x$

$2\cos 2x\cos x=\cos 2x$

$\cos 2x(2\cos x-1)=0$

$\cos 2x=0$ or $\cos x=\frac{1}{2}$

$2x=\frac{\pi}{2},\ 2\pi-\frac{\pi}{2}$ or $x=\frac{\pi}{3}$

$2x=\frac{\pi}{2},\ \frac{3\pi}{2}$ or $x=\frac{\pi}{3}$

$x=\frac{\pi}{4},\ \frac{\pi}{3},\ \frac{3\pi}{4}$

6 **a** $\cos(2x+3x)+\cos(2x-3x)-\cos x=0$

$\cos 5x+\cos(-x)-\cos x=0$

$\cos 5x+\cos x-\cos x=0$

$\cos 5x=0$

$5x=90,\ 360-90,\ 360+90,$

$720-90,\ 720+90$

$=90,\ 270,\ 450,\ 630,\ 810$

$x=18°,\ 54°,\ 90°,\ 126°,\ 162°$

b $2\cos\frac{3x+2x}{2}\sin\frac{3x-2x}{2}=0$

$\cos\frac{5}{2}x\sin\frac{1}{2}x=0$

$\cos\frac{5}{2}x=0$ or $\sin\frac{1}{2}x=0$

$\frac{5}{2}x=90,\ 360-90,\ 360+90$ or $\frac{1}{2}x=0$

$\frac{5}{2}x=90,\ 270,\ 450$ or $\frac{1}{2}x=0$

$x=0,\ 36°,\ 108°,\ 180°$

c $2\sin\frac{4x+2x}{2}\cos\frac{4x-2x}{2} = \sin 3x$

$2\sin 3x\cos x = \sin 3x$

$\sin 3x(2\cos x - 1) = 0$

$\sin 3x = 0$ or $\cos x = \frac{1}{2}$

$3x = 0, 180, 360, 540$ or $x = 60$

$x = 0, 60°, 120°, 180°$

d $\cos 2x - \cos(x - 60) = 0$

$-2\sin\frac{3x-60}{2}\sin\frac{x+60}{2} = 0$

$\sin(\frac{3}{2}x - 30)\sin(\frac{1}{2}x + 30) = 0$

$\sin(\frac{3}{2}x - 30) = 0$ or $\sin(\frac{1}{2}x + 30) = 0$

$\frac{3}{2}x - 30 = 0, 180$ or (none in interval)

$\frac{3}{2}x = 30, 210$

$x = 20°, 140°$

e $\frac{1}{2}[\sin(5x + x) - \sin(5x - x)] + \sin 4x = 0$

$\frac{1}{2}\sin 6x - \frac{1}{2}\sin 4x + \sin 4x = 0$

$\frac{1}{2}\sin 6x + \frac{1}{2}\sin 4x = 0$

$\sin\frac{6x+4x}{2}\cos\frac{6x-4x}{2} = 0$

$\sin 5x\cos x = 0$

$\sin 5x = 0$ or $\cos x = 0$

$5x = 0, 180, 360, 540, 720, 900$ or $x = 90$

$x = 0, 36°, 72°, 90°, 108°, 144°, 180°$

f $2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2} = 2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2}$

$\sin 2x\cos(-x) = \cos 2x\cos(-x)$

$\sin 2x\cos x = \cos 2x\cos x$

$\cos x(\sin 2x - \cos 2x) = 0$

$\cos x = 0$ or $\sin 2x = \cos 2x$

$\cos x = 0$ or $\tan 2x = 1$

$x = 90$ or $2x = 45, 180 + 45 = 45, 225$

$x = 22.5°, 90°, 112.5°$

7 **a** LHS $= 2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2} + \sin 2x$

$= 2\sin 2x\cos(-x) + \sin 2x$

$= 2\sin 2x\cos x + \sin 2x$

$= \sin 2x(2\cos x + 1)$

$=$ RHS

b LHS $= \dfrac{-2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}}{2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2}}$

$= \dfrac{-\sin 2x\sin(-x)}{\cos 2x\cos(-x)}$

$= \dfrac{\sin 2x\sin x}{\cos 2x\cos x}$

$= \tan x\tan 2x$

$=$ RHS

1 **a** $\cos x + \sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = 1$
$\therefore\ R = \sqrt{1+1} = \sqrt{2} = 1.4$
$\tan\alpha = 1,\ \alpha = 45$
$\therefore\ \cos x° + \sin x° = 1.4\cos(x - 45)°$

b $3\cos x + 4\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 3,\ R\sin\alpha = 4$
$\therefore\ R = \sqrt{9+16} = 5$
$\tan\alpha = \frac{4}{3},\ \alpha = 53.1$
$\therefore\ 3\cos x° + 4\sin x° = 5\cos(x - 53.1)°$

c $2\sin x + \cos x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = 2$
$\therefore\ R = \sqrt{1+4} = \sqrt{5} = 2.2$
$\tan\alpha = 2,\ \alpha = 63.4$
$\therefore\ 2\sin x° + \cos x° = 2.2\cos(x - 63.4)°$

d $\cos x + \sqrt{3}\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$
$\therefore\ R = \sqrt{1+3} = 2$
$\tan\alpha = \sqrt{3},\ \alpha = 60$
$\therefore\ \cos x° + \sqrt{3}\sin x° = 2\cos(x - 60)°$

2 **a** $5\cos x - 12\sin x$
$= R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 5,\ R\sin\alpha = 12$
$\therefore\ R = \sqrt{25+144} = 13$
$\tan\alpha = \frac{12}{5},\ \alpha = 67.4$
$\therefore\ 5\cos x° - 12\sin x° = 13\cos(x + 67.4)°$

b $4\sin x + 2\cos x$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 4,\ R\sin\alpha = 2$
$\therefore\ R = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$
$\tan\alpha = \frac{1}{2},\ \alpha = 26.6$
$\therefore\ 4\sin x° + 2\cos x° = 2\sqrt{5}\sin(x + 26.6)°$

c $\sin x - 7\cos x$
$= R\sin x\cos\alpha - R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = 7$
$\therefore\ R = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$
$\tan\alpha = 7,\ \alpha = 81.9$
$\therefore\ \sin x° - 7\cos x° = 5\sqrt{2}\sin(x - 81.9)°$

d $8\cos 2x - 15\sin 2x$
$= R\cos 2x\cos\alpha - R\sin 2x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 8,\ R\sin\alpha = 15$
$\therefore\ R = \sqrt{64+225} = 17$
$\tan\alpha = \frac{15}{8},\ \alpha = 61.9$
$\therefore\ 8\cos 2x° - 15\sin 2x° = 17\cos(2x + 61.9)°$

3 **a** $3\sin x - 2\cos x$
$= R\sin x\cos\alpha - R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 3,\ R\sin\alpha = 2$
$\therefore\ R = \sqrt{9+4} = \sqrt{13}$
$\tan\alpha = \frac{2}{3},\ \alpha = 0.59$
$\therefore\ 3\sin x - 2\cos x = \sqrt{13}\sin(x - 0.59)$

b $3\cos x + \sqrt{3}\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 3,\ R\sin\alpha = \sqrt{3}$
$\therefore\ R = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$
$\tan\alpha = \frac{1}{\sqrt{3}},\ \alpha = \frac{\pi}{6}$
$\therefore\ 3\cos x + \sqrt{3}\sin x = 2\sqrt{3}\cos(x - \frac{\pi}{6})$

c $8\sin 3x + 6\cos 3x$
$= R\sin 3x\cos\alpha + R\cos 3x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 8,\ R\sin\alpha = 6$
$\therefore\ R = \sqrt{64+36} = 10$
$\tan\alpha = \frac{3}{4},\ \alpha = 0.64$
$\therefore\ 8\sin 3x + 6\cos 3x = 10\sin(3x + 0.64)$

d $\cos x + \frac{1}{2}\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = \frac{1}{2}$
$\therefore\ R = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$
$\tan\alpha = \frac{1}{2},\ \alpha = 0.46$
$\therefore\ \cos x + \frac{1}{2}\sin x = \frac{1}{2}\sqrt{5}\cos(x - 0.46)$

4 **a** $24 \sin x - 7 \cos x = R \sin (x - \alpha)$
$= R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 24,\ R \sin \alpha = 7$
$\therefore R = \sqrt{576 + 49} = 25$
$\tan \alpha = \frac{7}{24},\ \alpha = 16.3°$
$\therefore 24 \sin x - 7 \cos x = 25 \sin (x - 16.3°)$
$\therefore$ max. $= 25$ when $x - 16.3 = 90$
$x = 106.3°$ (1dp)

b $4 \cos 2x + 4 \sin 2x = R \cos (2x - \alpha)$
$= R \cos 2x \cos \alpha + R \sin 2x \sin \alpha$
$\Rightarrow R \cos \alpha = 4,\ R \sin \alpha = 4$
$\therefore R = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$
$\tan \alpha = 1,\ \alpha = 45°$
$\therefore 4 \cos 2x + 4 \sin 2x = 4\sqrt{2} \cos (2x - 45°)$
$\therefore$ max. $= 4\sqrt{2}$ when $2x - 45 = 0$
$x = 22.5°$

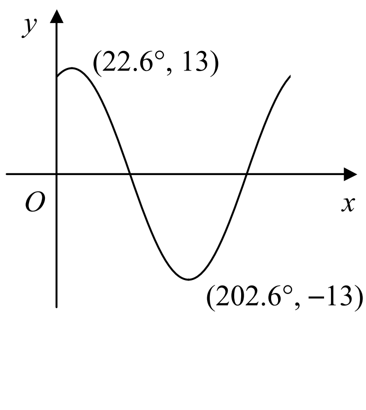
c $3 \cos x - 5 \sin x = R \cos (x + \alpha)$
$= R \cos x \cos \alpha - R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 3,\ R \sin \alpha = 5$
$\therefore R = \sqrt{9 + 25} = \sqrt{34}$
$\tan \alpha = \frac{5}{3},\ \alpha = 59.0°$
$\therefore 3 \cos x - 5 \sin x = \sqrt{34} \cos (x + 59.0°)$
$\therefore$ max. $= \sqrt{34}$ when $x + 59.0 = 360$
$x = 301.0°$ (1dp)

d $5 \sin 3x + \cos 3x = R \sin (3x + \alpha)$
$= R \sin 3x \cos \alpha + R \cos 3x \sin \alpha$
$\Rightarrow R \cos \alpha = 5,\ R \sin \alpha = 1$
$\therefore R = \sqrt{25 + 1} = \sqrt{26}$
$\tan \alpha = \frac{1}{5},\ \alpha = 11.3°$
$\therefore 5 \sin 3x + \cos 3x = \sqrt{26} \sin (3x + 11.3°)$
$\therefore$ max. $= \sqrt{26}$ when $3x + 11.3 = 90$
$x = 26.2°$ (1dp)

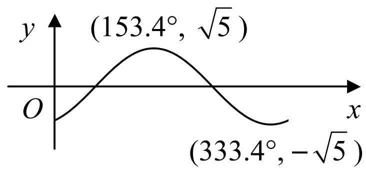
5 **a** $3 \sin x - 3 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 3,\ R \sin \alpha = 3$
$\therefore R = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$
$\tan \alpha = 1,\ \alpha = 45°$
$\therefore 3 \sin x° - 3 \cos x° = 3\sqrt{2} \sin (x - 45)°$

b translation by 45 units in positive x-direction and stretch by a factor of $3\sqrt{2}$ in y-direction

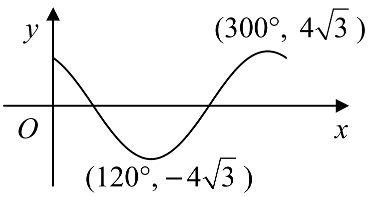
6 **a** $12 \cos x + 5 \sin x = R \cos (x - \alpha)$
$= R \cos x \cos \alpha + R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 12,\ R \sin \alpha = 5$
$\therefore R = \sqrt{144 + 25} = 13$
$\tan \alpha = \frac{5}{12},\ \alpha = 22.6°$ (1dp)
$\therefore y = 13 \cos (x - 22.6°)$



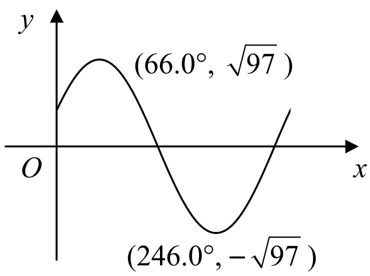
b $\sin x - 2 \cos x = R \sin (x - \alpha)$
$= R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 1,\ R \sin \alpha = 2$
$\therefore R = \sqrt{1 + 4} = \sqrt{5}$
$\tan \alpha = 2,\ \alpha = 63.4°$ (1dp)
$\therefore y = \sqrt{5} \sin (x - 63.4°)$



c $2\sqrt{3}\cos x - 6\sin x = R\cos(x + \alpha)$
$= R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 2\sqrt{3},\ R\sin\alpha = 6$
$\therefore\ R = \sqrt{12+36} = \sqrt{48} = 4\sqrt{3}$
$\tan\alpha = \sqrt{3},\ \alpha = 60°$
$\therefore\ y = 4\sqrt{3}\cos(x + 60°)$



d $9\sin x + 4\cos x = R\sin(x + \alpha)$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 9,\ R\sin\alpha = 4$
$\therefore\ R = \sqrt{81+16} = \sqrt{97}$
$\tan\alpha = \frac{4}{9},\ \alpha = 24.0°$ (1dp)
$\therefore\ y = \sqrt{97}\sin(x + 24.0°)$



7 **a** $\sqrt{3}\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = \sqrt{3},\ R\sin\alpha = 1$
$\therefore\ R = \sqrt{3+1} = 2$
$\tan\alpha = \frac{1}{\sqrt{3}},\ \alpha = \frac{\pi}{6}$
$\therefore\ \sqrt{3}\cos x - \sin x = 2\cos(x + \frac{\pi}{6})$

b $2\cos(x + \frac{\pi}{6}) = 1$
$\cos(x + \frac{\pi}{6}) = \frac{1}{2}$
$x + \frac{\pi}{6} = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3} = \frac{\pi}{3},\ \frac{5\pi}{3}$
$x = \frac{\pi}{6},\ \frac{3\pi}{2}$

8 **a** $6\sin x + 8\cos x = R\sin(x + \alpha)$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 6,\ R\sin\alpha = 8$
$\therefore\ R = \sqrt{36+64} = 10$
$\tan\alpha = \frac{4}{3},\ \alpha = 0.9273$
$\therefore\ 10\sin(x + 0.9273) = 5$
$\sin(x + 0.9273) = \frac{1}{2}$
$x + 0.9273 = \pi - \frac{\pi}{6},\ 2\pi + \frac{\pi}{6}$
$= \frac{5\pi}{6},\ \frac{13\pi}{6}$
$x = 1.69,\ 5.88$

b $2\cos x - 2\sin x = R\cos(x + \alpha)$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 2,\ R\sin\alpha = 2$
$\therefore\ R = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
$\tan\alpha = 1,\ \alpha = \frac{\pi}{4}$
$\therefore\ 2\sqrt{2}\cos(x + \frac{\pi}{4}) = 1$
$\cos(x + \frac{\pi}{4}) = \frac{1}{2\sqrt{2}}$
$x + \frac{\pi}{4} = 1.2094,\ 2\pi - 1.2094$
$= 1.2094,\ 5.0738$
$x = 0.42,\ 4.29$

c $7\sin x - 24\cos x = R\sin(x - \alpha)$
$= R\sin x\cos\alpha - R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 7,\ R\sin\alpha = 24$
$\therefore\ R = \sqrt{49+576} = 25$
$\tan\alpha = \frac{24}{7},\ \alpha = 1.2870$
$\therefore\ 25\sin(x - 1.2870) - 10 = 0$
$\sin(x - 1.2870) = \frac{2}{5}$
$x - 1.2870 = 0.4115,\ \pi - 0.4115$
$= 0.4115,\ 2.7301$
$x = 1.70,\ 4.02$

d $3\cos x + \sin x = R\cos(x - \alpha)$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 3,\ R\sin\alpha = 1$
$\therefore\ R = \sqrt{9+1} = \sqrt{10}$
$\tan\alpha = \frac{1}{3},\ \alpha = 0.3218$
$\therefore\ \sqrt{10}\cos(x - 0.3218) + 1 = 0$
$\cos(x - 0.3218) = -\frac{1}{\sqrt{10}}$
$x - 0.3218 = \pi - 1.2490,\ \pi + 1.2490$
$= 1.8925,\ 4.3906$
$x = 2.21,\ 4.71$

e $\cos 2x + 4\sin 2x = R\cos(2x - \alpha)$
$= R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = 4$
$\therefore\ R = \sqrt{1+16} = \sqrt{17}$
$\tan\alpha = 4,\ \alpha = 1.3258$
$\therefore\ \sqrt{17}\cos(2x - 1.3258) = 3$
$\cos(2x - 1.3258) = \frac{3}{\sqrt{17}}$
$2x - 1.3258 = 0.7560,\ 2\pi - 0.7560,$
$2\pi + 0.7560,\ -0.7560$
$= -0.7560,\ 0.7560,\ 5.5272,\ 7.0392$
$2x = 0.5698,\ 2.0818,\ 6.8530,\ 8.3650$
$x = 0.28,\ 1.04,\ 3.43,\ 4.18$

f $5\sin x - 8\cos x = R\sin(x - \alpha)$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 5,\ R\sin\alpha = 8$
$\therefore\ R = \sqrt{25+64} = \sqrt{89}$
$\tan\alpha = \frac{8}{5},\ \alpha = 1.0122$
$\therefore\ \sqrt{89}\sin(x - 1.0122) + 7 = 0$
$\sin(x - 1.0122) = -\frac{7}{\sqrt{89}}$
$x - 1.0122 = -0.8360,\ \pi + 0.8360$
$= -0.8360,\ 3.9776$
$x = 0.18,\ 4.99$

9 **a** $\sin x + \cos x = R\sin(x + \alpha)$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = 1$
$\therefore\ R = \sqrt{1+1} = \sqrt{2}$
$\tan\alpha = 1,\ \alpha = 45°$
$\therefore\ \sqrt{2}\sin(x + 45°) = 1$
$\sin(x + 45°) = \frac{1}{\sqrt{2}}$
$x + 45 = 45,\ 180 - 45$
$= 45,\ 135$
$x = 0,\ 90°$

b $4\cos x - \sin x = R\cos(x + \alpha)$
$= R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 4,\ R\sin\alpha = 1$
$\therefore\ R = \sqrt{16+1} = \sqrt{17}$
$\tan\alpha = \frac{1}{4},\ \alpha = 14.04°$
$\therefore\ \sqrt{17}\cos(x + 14.04°) + 2 = 0$
$\cos(x + 14.04°) = -\frac{2}{\sqrt{17}}$
$x + 14.04 = 180 - 60.98,\ 60.98 - 180$
$= -119.02,\ 119.02$
$x = -133.1°,\ 105.0°$

c $\cos\frac{x}{2} + 5\sin\frac{x}{2} = R\cos(\frac{x}{2} - \alpha)$
$= R\cos\frac{x}{2}\cos\alpha + R\sin\frac{x}{2}\sin\alpha$
$\Rightarrow\ R\cos\alpha = 1,\ R\sin\alpha = 5$
$\therefore\ R = \sqrt{1+25} = \sqrt{26}$
$\tan\alpha = 5,\ \alpha = 78.69°$
$\therefore\ \sqrt{26}\cos(\frac{x}{2} - 78.69°) - 4 = 0$
$\cos(\frac{x}{2} - 78.69°) = \frac{4}{\sqrt{26}}$
$\frac{x}{2} - 78.69 = -38.33$
$\frac{x}{2} = 40.36$
$x = 80.7°$

d $6\sin x + 3\cos x = R\sin(x + \alpha)$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow\ R\cos\alpha = 6,\ R\sin\alpha = 3$
$\therefore\ R = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$
$\tan\alpha = \frac{1}{2},\ \alpha = 26.57°$
$\therefore\ 3\sqrt{5}\sin(x + 26.57°) = 5$
$\sin(x + 26.57°) = \frac{\sqrt{5}}{3}$
$x + 26.57 = 48.19,\ 180 - 48.19$
$= 48.19,\ 131.81$
$x = 21.6°,\ 105.2°$

1

$\sec^2 x - 1 - \sec x = 1$
$\sec^2 x - \sec x - 2 = 0$
$(\sec x + 1)(\sec x - 2) = 0$
$\sec x = -1$ or 2
$\cos x = -1$ or $\frac{1}{2}$
$x = 180$ or $60, 360 - 60$
$x = 60°, 180°, 300°$

2

a $2\cos x + 5\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = 5$
$\therefore R = \sqrt{4+25} = \sqrt{29} = 5.39$
$\tan\alpha = \frac{5}{2},\ \alpha = 68.2$
$\therefore 2\cos x + 5\sin x = 5.39\cos(x - 68.2)°$

b $\sqrt{29}\cos(x - 68.199) = 3$
$\cos(x - 68.199) = \frac{3}{\sqrt{29}} = 0.5571$
$x - 68.199 = 56.145, -56.145$
$x = 12.1, 124.3$

3

a $\arctan 2x = \frac{\pi}{6}$
$2x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$
$x = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

b $4\sin x\cos x = 3\cos x$
$\cos x(4\sin x - 3) = 0$
$\cos x = 0$ or $\sin x = \frac{3}{4}$
$x = 90, 360 - 90$ or $48.6, 180 - 48.6$
$x = 48.6°$ (1dp), $90°$, $131.4°$ (1dp), $270°$

4

a $\sin(A + B) \equiv \sin A\cos B + \cos A\sin B$
$\sin(A - B) \equiv \sin A\cos B - \cos A\sin B$

subtracting

$\sin(A + B) - \sin(A + B) \equiv 2\cos A\sin B$
let $P = A + B$ (1) and $Q = A - B$ (2)
(1) + (2) $\Rightarrow 2A = P + Q \Rightarrow A = \frac{P+Q}{2}$
(1) − (2) $\Rightarrow 2B = P - Q \Rightarrow B = \frac{P-Q}{2}$
$\therefore \sin P - \sin Q \equiv 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$

b $\sin 4x - \sin 2x = 0$
$2\cos\frac{4x+2x}{2}\sin\frac{4x-2x}{2} = 0$
$\cos 3x\sin x = 0$
$\cos 3x = 0$ or $\sin x = 0$
$3x = 90, 360 - 90, 360 + 90$ or $x = 0, 180$
$3x = 90, 270, 450$ or $x = 0, 180$
$x = 0, 30°, 90°, 150°, 180°$

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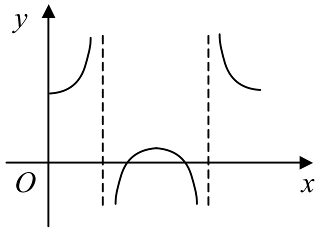
5 **a** LHS = $4\sin^2\theta - 4 + \operatorname{cosec}^2\theta$
$= 4(1 - \cos^2\theta) - 4 + \operatorname{cosec}^2\theta$
$= \operatorname{cosec}^2\theta - 4\cos^2\theta$ = RHS

b **i**



ii (0, 5)

iii $3 + 2\sec x = 0$

$\sec x = -\frac{3}{2}$, $\cos x = -\frac{2}{3}$

$x = \pi - 0.841, \pi + 0.841 = 2.30, 3.98$

$\therefore$ (2.30, 0) and (3.98, 0) [x to 2dp]

6 **a** $\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 1$
$\therefore R = \sqrt{1+1} = \sqrt{2}$
$\tan\alpha = 1,\ \alpha = \frac{\pi}{4}$

b $\cos x - \sin x + \sqrt{2}\cos(3x - \frac{\pi}{4}) = 0$

$\sqrt{2}\cos(x + \frac{\pi}{4}) + \sqrt{2}\cos(3x - \frac{\pi}{4}) = 0$

$2\sqrt{2}\cos\frac{4x}{2}\cos\frac{-2x+\frac{\pi}{2}}{2} = 0$

$\cos 2x\cos(\frac{\pi}{4} - x) = 0$

$\cos 2x\cos(x - \frac{\pi}{4}) = 0$

$\cos 2x = 0$ or $\cos(x - \frac{\pi}{4}) = 0$

$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$

or $x - \frac{\pi}{4} = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $x - \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

7 **a** LHS = $\frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$
$= \frac{\cos 2x + 1}{\sin 2x}$
$= \frac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$
$= \frac{2\cos^2 x}{2\sin x\cos x}$
$= \frac{\cos x}{\sin x}$
$= \cot x$ = RHS

b $\cot x = 6 - \cot^2 x$
$\cot^2 x + \cot x - 6 = 0$

$(\cot x + 3)(\cot x - 2) = 0$

$\cot x = -3$ or 2
$\tan x = -\frac{1}{3}$ or $\frac{1}{2}$
$x = \pi - 0.3218, 2\pi - 0.3218$
or $0.4636, \pi + 0.4636$
$x = 0.46, 2.82, 3.61, 5.96$

8 **a** LHS = $\cos x\cos 30 - \sin x\sin 30 + \sin x$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + \sin x$

$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$= \cos x\cos 30 + \sin x\sin 30$

$= \cos(x - 30)°$ = RHS

b let $x = 45$
$\cos 75° + \sin 45° = \cos 15°$
$\therefore \cos 75° - \cos 15° = -\sin 45°$
$= -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$

c $3\cos(x + 30) + 3\sin x - 2\sin x$
$= 3\cos(x - 30) + 1$

$-2\sin x = 1$
$\sin x = -\frac{1}{2}$
$x = -30, 30 - 180$
$x = -150, -30$

9 **a** $a = 3$

$b \sin x° + c \cos x°$ can be expressed in the form $k \sin (x + \alpha)°$ which will vary between $-k$ and $+k$

$\therefore\ a + k = 5$ and $a - k = 1$, hence $a = 3$

b $3 + k = 5 \ \therefore\ k = 2$

$60 + \alpha = 90 \ \therefore\ \alpha = 30$

c $f(x) = 3 + 2 \sin (x + 30)$

$= 3 + 2 \sin x \cos 30 + 2 \cos x \sin 30$

$= 3 + \sqrt{3} \sin x + \cos x$

$\therefore\ b = \sqrt{3},\ c = 1$

10 **a** LHS $= \dfrac{1-(1-2\sin^2 \frac{x}{2})}{1+(2\cos^2 \frac{x}{2}-1)}$

$= \dfrac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$

$= \tan^2 \dfrac{x}{2} =$ RHS

b **i** let $x = \frac{\pi}{6}$, $\dfrac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}} = \tan^2 \frac{\pi}{12}$

$\tan^2 \frac{\pi}{12} = \dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}} = \dfrac{2-\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{4-4\sqrt{3}+3}{4-3} = 7 - 4\sqrt{3}$

ii $\tan^2 \dfrac{x}{2} = 1 - \sec \dfrac{x}{2}$

$\sec^2 \dfrac{x}{2} - 1 = 1 - \sec \dfrac{x}{2}$

$\sec^2 \dfrac{x}{2} + \sec \dfrac{x}{2} - 2 = 0$

$(\sec \dfrac{x}{2} + 2)(\sec \dfrac{x}{2} - 1) = 0$

$\sec \dfrac{x}{2} = -2$ or 1

$\cos \dfrac{x}{2} = -\frac{1}{2}$ or 1

$\dfrac{x}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or 0

$x = 0,\ \frac{4\pi}{3}$

11 **a** $6 \cot^2 x - \operatorname{cosec} x + 5 = 0$

$\Rightarrow\ 6(\operatorname{cosec}^2 x - 1) - \operatorname{cosec} x + 5 = 0$

$6 \operatorname{cosec}^2 x - \operatorname{cosec} x - 1 = 0$

$(3 \operatorname{cosec} x + 1)(2 \operatorname{cosec} x - 1) = 0$

$\operatorname{cosec} x = -\frac{1}{3},\ \frac{1}{2}$

for real x, $|\operatorname{cosec} x| \geq 1$

$\therefore$ no real solutions

b $\cos 5y - \cos y = 0$

$-2 \sin \frac{5y+y}{2} \sin \frac{5y-y}{2} = 0$

$\sin 3y \sin 2y = 0$

$\sin 3y = 0$ or $\sin 2y = 0$

$3y = 0, 180, 360, 540$ or $2y = 0, 180, 360$

$y = 0, 60°, 90°, 120°, 180°$

12 **a** $\cos (A - B) \equiv \cos A \cos B + \sin A \sin B$

$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$

subtracting

$\cos (A - B) - \cos (A + B) \equiv 2 \sin A \sin B$

$\therefore\ \sin A \sin B \equiv \frac{1}{2}[\cos (A - B) - \cos (A + B)]$

b $4 \sin (x + \frac{\pi}{3}) = \dfrac{1}{\sin(x - \frac{\pi}{6})}$

$4 \sin (x + \frac{\pi}{3}) \sin (x - \frac{\pi}{6}) = 1$

$2[\cos \frac{\pi}{2} - \cos (2x + \frac{\pi}{6})] = 1$

$2[0 - \cos (2x + \frac{\pi}{6})] = 1$

$\cos (2x + \frac{\pi}{6}) = -\frac{1}{2}$

$2x + \frac{\pi}{6} = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3}$

$= \frac{2\pi}{3},\ \frac{4\pi}{3}$

$2x = \frac{\pi}{2},\ \frac{7\pi}{6}$

$x = \frac{\pi}{4},\ \frac{7\pi}{12}$

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1 **a** $\dfrac{2}{\cos x} = \dfrac{3}{\sin x}$

$\dfrac{\sin x}{\cos x} = \frac{3}{2}$

$\tan x = \frac{3}{2}$

$x = 56.3,\ 56.3 - 180$

$x = -123.7°,\ 56.3°$

b $\cot^2\theta - \cot\theta + 1 + \cot^2\theta = 4$

$2\cot^2\theta - \cot\theta - 3 = 0$

$(2\cot\theta - 3)(\cot\theta + 1) = 0$

$\cot\theta = -1$ or $\frac{3}{2}$

$\tan\theta = -1$ or $\frac{2}{3}$

$\theta = \pi - \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$ or $0.5880,\ \pi + 0.5880$

$\theta = 0.59$ (2dp), $\frac{3\pi}{4}$, 3.73 (2dp), $\frac{7\pi}{4}$

2 $2\sin\theta\cos 30 + 2\cos\theta\sin 30$

$= \sin\theta\cos 30 - \cos\theta\sin 30$

$\sqrt{3}\sin\theta + \cos\theta = \dfrac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta$

$\dfrac{\sqrt{3}}{2}\sin\theta = -\frac{3}{2}\cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = -\sqrt{3}$

$\tan\theta = -\sqrt{3}$

$\theta = 180 - 60,\ 360 - 60$

$\theta = 120°,\ 300°$

3 **a** **i** $\operatorname{cosec} A = \dfrac{1}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \dfrac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$

ii $\operatorname{cosec}^2 A = (2 + \sqrt{3})^2$

$= 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$

$\cot^2 A = \operatorname{cosec}^2 A - 1 = 6 + 4\sqrt{3}$

b $3(1 - 2\sin^2 x) - 8\sin x + 5 = 0$

$3\sin^2 x + 4\sin x - 4 = 0$

$(3\sin x - 2)(\sin x + 2) = 0$

$\sin x = \frac{2}{3}$ or -2 [no solutions]

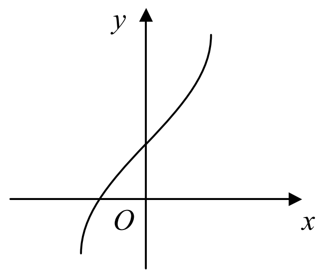
$x = 41.8,\ 180 - 41.8$

$x = 41.8°,\ 138.2°$

4 **a** $= \frac{\pi}{2} + 2 \times \frac{\pi}{6} = \frac{5\pi}{6}$

b $-\frac{\pi}{2} \le f(x) \le \frac{3\pi}{2}$

c



d $\frac{\pi}{2} + 2\arcsin x = 0$

$\arcsin x = -\frac{\pi}{4}$

$x = \sin(-\frac{\pi}{4}) = -\dfrac{1}{\sqrt{2}}$

5 **a** $2 \sin x - 3 \cos x$

$= R \sin x \cos \alpha - R \cos x \sin \alpha$

$\Rightarrow R \cos \alpha = 2, \; R \sin \alpha = 3$

$\therefore R = \sqrt{4+9} = \sqrt{13} = 3.61$

$\tan \alpha = \frac{3}{2}, \; \alpha = 0.983$

$\therefore 2 \sin x - 3 \cos x = 3.61 \sin (x - 0.983)$

b min. value = –3.61 (3sf)

when $x - 0.9828 = \frac{3\pi}{2}, \; x = 5.70$ (3sf)

c $\sqrt{13} \sin (2x - 0.9828) + 1 = 0$

$\sin (2x - 0.9828) = -\frac{1}{\sqrt{13}}$

$2x - 0.983 = \pi + 0.2810, -0.2810$

$= -0.2810, 3.4226$

$2x = 0.7018, 4.4054$

$x = 0.35, 2.20$

6 **a** $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$

let $A = B = \frac{x}{2}$

$\cos x \equiv \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$

$\cos x \equiv \cos^2 \frac{x}{2} - (1 - \cos^2 \frac{x}{2})$

$\cos x \equiv 2 \cos^2 \frac{x}{2} - 1$

b $\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1+(2\cos^2\frac{x}{2}-1)} = 3 \cot \frac{x}{2}$

$\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = 3 \cot \frac{x}{2}$

$\tan \frac{x}{2} = \frac{3}{\tan\frac{x}{2}}$

$\tan^2 \frac{x}{2} = 3$

$\tan \frac{x}{2} = \pm\sqrt{3}$

$\frac{x}{2} = 60$ or $180 - 60$

$\frac{x}{2} = 60, 120$

$x = 120°, 240°$

7 **a** LHS $= \frac{1}{\sin\theta} - \sin \theta$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$

$= \cos \theta \times \frac{\cos\theta}{\sin\theta}$

$= \cos \theta \cot \theta$

= RHS

b $\frac{2}{\cos x} + \frac{\sin x}{\cos x} = 2 \cos x$

$2 + \sin x = 2 \cos^2 x$

$2 + \sin x = 2(1 - \sin^2 x)$

$2 \sin^2 x + \sin x = 0$

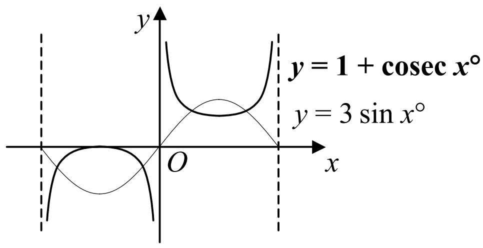
$\sin x(2 \sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or 0

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ or $0, \pi, 2\pi$

$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

8 **a**



b $3 \sin x = 1 + \frac{1}{\sin x}$

$3 \sin^2 x - \sin x - 1 = 0$

$\sin x = \frac{1\pm\sqrt{1+12}}{6} = \frac{1\pm\sqrt{13}}{6}$

$\sin x = -0.4343$ or 0.7676

$x = -25.7, 25.7 - 180$ or $50.1, 180 - 50.1$

$x = -154.3, -25.7, 50.1, 129.9$

9 **a** LHS $= \sec x + \tan x - \tan x - \sin x \tan x$

$= \frac{1}{\cos x} - \sin x \times \frac{\sin x}{\cos x}$

$= \frac{1-\sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x}$

$= \cos x$

= RHS

b $2(1 + \tan^2 2y) + \tan^2 2y = 3$

$\tan^2 2y = \frac{1}{3}$

$\tan 2y = \pm\frac{1}{\sqrt{3}}$

$2y = \frac{\pi}{6}, \pi + \frac{\pi}{6}$ or $\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$y = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

10 **a** $4\sin x - \cos x$

$= R\sin x\cos\alpha - R\cos x\sin\alpha$

$\Rightarrow R\cos\alpha = 4,\ R\sin\alpha = 1$

$\therefore\ R = \sqrt{16+1} = \sqrt{17} = 4.12$

$\tan\alpha = \frac{1}{4},\ \alpha = 14.0$

$\therefore\ 4\sin x° - \cos x° = 4.12\sin(x - 14.0)°$

b $\frac{2}{\sin x} - \frac{\cos x}{\sin x} + 4 = 0$

$2 - \cos x + 4\sin x = 0$

$\therefore\ 4\sin x° - \cos x° + 2 = 0$

c $\sqrt{17}\sin(x - 14.04) + 2 = 0$

$\sin(x - 14.04) = -\frac{2}{\sqrt{17}}$

$x - 14.04 = 180 + 29.02,\ 360 - 29.02$

$= 209.02,\ 330.98$

$x = 223.1,\ 345.0$ (1dp)

11 **a** adding

$\cos(A+B) + \cos(A-B) \equiv 2\cos A\cos B$

let $P = A + B$ (1) and $Q = A - B$ (2)

(1) + (2) $\Rightarrow 2A = P + Q \Rightarrow A = \frac{P+Q}{2}$

(1) − (2) $\Rightarrow 2B = P - Q \Rightarrow B = \frac{P-Q}{2}$

$\therefore\ \cos P + \cos Q \equiv 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

b $2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2} + \cos 2x = 0$

$2\cos 2x\cos(-x) + \cos 2x = 0$

$\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$ or $\cos x = -\frac{1}{2}$

$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$

or $x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}$

12 **a** $3\cos\theta + 4\sin\theta$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$

$\therefore\ R = \sqrt{9+16} = 5$

$\tan\alpha = \frac{4}{3},\ \alpha = 0.927$ (3sf)

$\therefore\ 3\cos\theta + 4\sin\theta = 5\cos(\theta - 0.927)$

b **i** $-4 \le f(\theta) \le 6$

ii $1 - 5\cos(2\theta - 0.9273) = 0$

$\cos(2\theta - 0.9273) = \frac{1}{5}$

$2\theta - 0.9273 = 1.3694,\ 2\pi - 1.3694$

$= 1.3694,\ 4.9137$

$2\theta = 2.2967,\ 5.8410$

$\theta = 1.15,\ 2.92$ (2dp)

c $y = \frac{2}{5\cos(x - 0.9273)}$

TP: $y = \frac{2}{5}$ when $x - 0.9273 = 0$

$y = -\frac{2}{5}$ when $x - 0.9273 = \pi$

$\therefore\ (0.93, \frac{2}{5})$ and $(4.07, -\frac{2}{5})$