C3 DIFFERENTIATION Worksheet A

1 Differentiate with respect to x

a e^x **b** $3e^x$ **c** $\ln x$ **d** $\frac{1}{2}\ln x$

2 Differentiate with respect to t

a $7 - 2e^t$ **b** $3t^2 + \ln t$ **c** $e^t + t^5$ **d** $t^{\frac{3}{2}} + 2e^t$

e $2\ln t + \sqrt{t}$ **f** $2.5e^t - 3.5\ln t$ **g** $\frac{1}{t} + 8\ln t$ **h** $7t^2 - 2t + 4e^t$

3 Find $\frac{d^2y}{dx^2}$ for each of the following.

a $y = 4x^3 + e^x$ **b** $y = 7e^x - 5x^2 + 3x$ **c** $y = \ln x + x^{\frac{5}{2}}$

d $y = 5e^x + 6\ln x$ **e** $y = \frac{3}{x} + 3\ln x$ **f** $y = 4\sqrt{x} + \frac{1}{4}\ln x$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 3x + e^x$, $x = 0$ **b** $f(x) = \ln x - x^2$, $x = 4$

c $f(x) = x^{\frac{1}{2}} + 2\ln x$, $x = 9$ **d** $f(x) = 5e^x + \frac{1}{x^2}$, $x = -\frac{1}{2}$

5 Find, in each case, any values of x for which $\frac{dy}{dx} = 0$.

a $y = 5\ln x - 8x$ **b** $y = 2.4e^x - 3.6x$ **c** $y = 3x^2 - 14x + 4\ln x$

6 Find the value of x for which $f'(x)$ takes the value indicated in each case.

a $f(x) = 2e^x - 3x$, $f'(x) = 7$ **b** $f(x) = 15x + \ln x$, $f'(x) = 23$

c $f(x) = \frac{x^2}{8} - 2x + \ln x$, $f'(x) = -1$ **d** $f(x) = 30\ln x - x^2$, $f'(x) = 4$

7 Find the coordinates and the nature of any stationary points on each of the following curves.

a $y = e^x - 2x$ **b** $y = \ln x - 10x$ **c** $y = 2\ln x - \sqrt{x}$

d $y = 4x - 5e^x$ **e** $y = 7 + 2x - 4\ln x$ **f** $y = x^2 - 26x + 72\ln x$

8 Given that $y = x + ke^x$, where k is a constant, show that

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

9 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = e^x$, $x = 2$ **b** $y = \ln x$, $x = 3$ **c** $y = 0.8x - 2e^x$, $x = 0$

d $y = 5\ln x + \frac{4}{x}$, $x = 1$ **e** $y = x^{\frac{1}{3}} - 3e^x$, $x = 1$ **f** $y = \ln x - \sqrt{x}$, $x = 9$

10 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate.

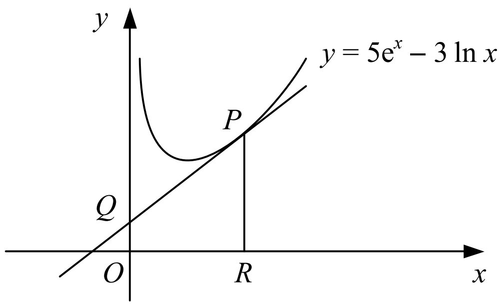
a $y = \ln x$, $x = e$ **b** $y = 4 + 3e^x$, $x = 0$ **c** $y = 10 + \ln x$, $x = 3$

d $y = 3\ln x - 2x$, $x = 1$ **e** $y = x^2 + 8\ln x$, $x = 1$ **f** $y = \frac{1}{10}x - \frac{3}{10}e^x - 1$, $x = 0$

1 **a** Find an equation for the normal to the curve $y = \frac{2}{5}x + \frac{1}{10}e^x$ at the point on the curve where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point where this normal crosses the x-axis.

2



The diagram shows the curve with equation $y = 5e^x - 3 \ln x$ and the tangent to the curve at the point P with x-coordinate 1.

a Show that the tangent at P has equation $y = (5e - 3)x + 3$.

The tangent at P meets the y-axis at Q.
The line through P parallel to the y-axis meets the x-axis at R.

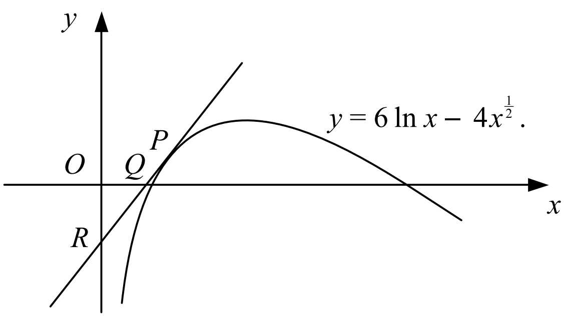
b Find the area of trapezium $ORPQ$, giving your answer in terms of e.

3 A curve has equation $y = 3x - \frac{1}{2}e^x$.

a Find the coordinates of the stationary point on the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

4



The diagram shows the curve $y = 6 \ln x - 4x^{\frac{1}{2}}$. The x-coordinate of the point P on the curve is 4.

The tangent to the curve at P meets the x-axis at Q and the y-axis at R.

a Find an equation for the tangent to the curve at P.

b Hence, show that the area of triangle OQR is $(10 - 12 \ln 2)^2$.

5 The curve with equation $y = 2x - 2 - \ln x$ passes through the point A (1, 0). The tangent to the curve at A crosses the y-axis at B and the normal to the curve at A crosses the y-axis at C.

a Find an equation for the tangent to the curve at A.

b Show that the mid-point of BC is the origin.

The curve has a minimum point at D.

c Show that the y-coordinate of D is $\ln 2 - 1$.

6 **a** Sketch the curve with equation $y = e^x + k$, where k is a positive constant.

Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

b Find an equation for the tangent to the curve at the point on the curve where $x = 2$.

Given that the tangent passes through the x-axis at the point $(-1, 0)$,

c show that $k = 2e^2$.

7 A curve has equation $y = 3x^2 - 2 \ln x$, $x > 0$.

The gradient of the curve at the point P on the curve is -1.

a Find the x-coordinate of P.

b Find an equation for the tangent to the curve at the point on the curve where $x = 1$.

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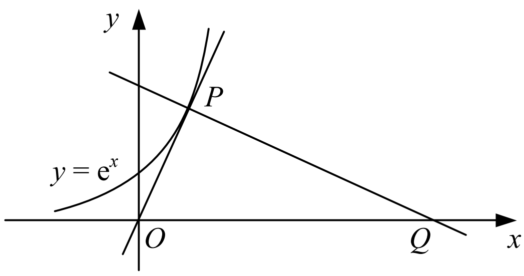


The diagram shows the curve with equation $y = e^x$ which passes through the point P (p, e^p). Given that the tangent to the curve at P passes through the origin and that the normal to the curve at P meets the x-axis at Q,

a show that $p = 1$,

b show that the area of triangle OPQ, where O is the origin, is $\frac{1}{2}e(1 + e^2)$.

9 The curve with equation $y = 4 - e^x$ meets the y-axis at the point P and the x-axis at the point Q.

a Find an equation for the normal to the curve at P.

b Find an equation for the tangent to the curve at Q.

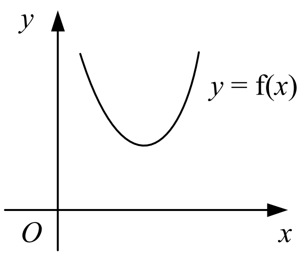
The normal to the curve at P meets the tangent to the curve at Q at the point R.

The x-coordinate of R is $a \ln 2 + b$, where a and b are rational constants.

c Show that $a = \frac{8}{5}$.

d Find the value of b.

10



The diagram shows a sketch of the curve $y = f(x)$ where

$$f : x \to 9x^4 - 16 \ln x, \ x > 0.$$

Given that the set of values of x for which $f(x)$ is a decreasing function of x is $0 < x \leq k$, find the exact value of k.

1 Differentiate with respect to x

a $(x+3)^5$ **b** $(2x-1)^3$ **c** $(8-x)^7$ **d** $2(3x+4)^6$

e $(6-5x)^4$ **f** $\frac{1}{x-2}$ **g** $\frac{4}{(2x+3)^3}$ **h** $\frac{1}{(7-3x)^2}$

2 Differentiate with respect to t

a $2e^{3t}$ **b** $\sqrt{4t-1}$ **c** $5 \ln 2t$ **d** $(8-3t)^{\frac{3}{2}}$

e $3 \ln (6t+1)$ **f** $\frac{1}{2}e^{5t+4}$ **g** $\frac{6}{\sqrt[3]{2t-5}}$ **h** $2 \ln (3 - \frac{1}{4}t)$

3 Find $\frac{d^2y}{dx^2}$ for each of the following.

a $y = (3x-1)^4$ **b** $y = 4 \ln (1+2x)$ **c** $y = \sqrt{5-2x}$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = x^2 - 6 \ln 2x$, $x = 3$ **b** $f(x) = 3 + 2x - e^{x-2}$, $x = 2$

c $f(x) = (2-5x)^4$, $x = \frac{1}{2}$ **d** $f(x) = \frac{4}{x+5}$, $x = -1$

5 Find the value of x for which $f'(x)$ takes the value indicated in each case.

a $f(x) = 4\sqrt{3x+15}$, $f'(x) = 2$ **b** $f(x) = x^2 - \ln (x-2)$, $f'(x) = 5$

6 Differentiate with respect to x

a $(x^2-4)^3$ **b** $2(3x^2+1)^6$ **c** $\ln (3+2x^2)$ **d** $(2+x)^3(2-x)^3$

e $\left(\frac{x^4+6}{2}\right)^8$ **f** $\frac{1}{\sqrt{3-x^2}}$ **g** $4 + 7e^{x^2}$ **h** $(1-5x+x^3)^4$

i $3 \ln (4 - \sqrt{x})$ **j** $(e^{4x}+2)^7$ **k** $\frac{1}{5+4\sqrt{x}}$ **l** $(\frac{2}{x} - x)^5$

7 Find the coordinates of any stationary points on each curve.

a $y = (2x-3)^5$ **b** $y = (x^2-4)^3$ **c** $y = 8x - e^{2x}$

d $y = \sqrt{1+2x^2}$ **e** $y = 2 \ln (x - x^2)$ **f** $y = 4x + \frac{1}{x-3}$

8 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = (3x-7)^4$, $x = 2$ **b** $y = 2 + \ln (1+4x)$, $x = 0$

c $y = \frac{9}{x^2+2}$, $x = 1$ **d** $y = \sqrt{5x-1}$, $x = \frac{1}{4}$

9 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate.

a $y = e^{4-x^2} - 10$, $x = -2$ **b** $y = (1-2x^2)^3$, $x = \frac{1}{2}$

c $y = \frac{1}{2 - \ln x}$, $x = 1$ **d** $y = 6e^{\frac{x}{3}}$, $x = 3$

1 Find an equation for the tangent to the curve with equation $y = x^2 + \ln(4x - 1)$ at the point on the curve where $x = \frac{1}{2}$.

2 A curve has the equation $y = \sqrt{8 - e^{2x}}$.

The point P on the curve has y-coordinate 2.

a Find the x-coordinate of P.

b Show that the tangent to the curve at P has equation

$$2x + y = 2 + \ln 4.$$

3 A curve has the equation $y = 2x + 1 + \ln(4 - 2x)$, $x < 2$.

a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the coordinates of the stationary point of the curve.

c Determine the nature of this stationary point.

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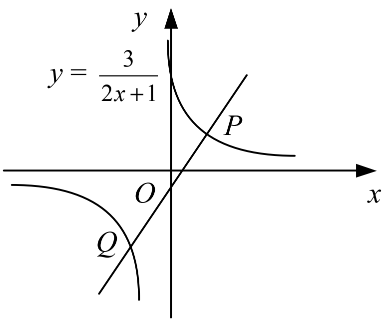

The diagram shows the curve with equation $y = \frac{3}{2x+1}$.

a Find an equation for the normal to the curve at the point P (1, 1).

The normal to the curve at P intersects the curve again at the point Q.

b Find the exact coordinates of Q.

5 A quantity N is increasing such that at time t seconds,

$$N = ae^{kt}.$$

Given that at time $t = 0$, $N = 20$ and that at time $t = 8$, $N = 60$, find

a the values of the constants a and k,

b the value of N when $t = 12$,

c the rate at which N is increasing when $t = 12$.

6 $$f(x) \equiv (5 - 2x^2)^3.$$

a Find $f'(x)$.

b Find the coordinates of the stationary points of the curve $y = f(x)$.

c Find the equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate $\frac{3}{2}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

7 A curve has the equation $y = 4x - \frac{1}{2}e^{2x}$.

a Find the coordinates of the stationary point of the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

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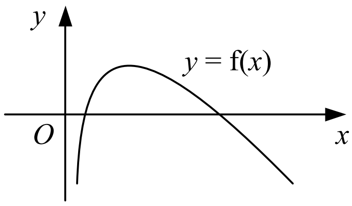

The diagram shows the curve $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 3 \ln 5x - 2x$, $x > 0$.

a Find $\mathrm{f}'(x)$.

b Find the x-coordinate of the point on the curve at which the gradient of the normal to the curve is $-\frac{1}{4}$.

c Find the coordinates of the maximum turning point of the curve.

d Write down the set of values of x for which $\mathrm{f}(x)$ is a decreasing function.

9 The curve C has the equation $y = \sqrt{x^2 + 3}$.

a Find an equation for the tangent to C at the point A $(-1, 2)$.

b Find an equation for the normal to C at the point B $(1, 2)$.

c Find the x-coordinate of the point where the tangent to C at A meets the normal to C at B.

10 A bucket of hot water is placed outside and allowed to cool. The surface temperature of the water, T °C, after t minutes is given by

$$T = 20 + 60e^{-kt},$$

where k is a positive constant.

a State the initial surface temperature of the water.

b State, with a reason, the air temperature around the bucket.

Given that $T = 30$ when $t = 25$,

c find the value of k,

d find the rate at which the surface temperature of the water is decreasing when $t = 40$.

11
$$\mathrm{f}(x) \equiv x^2 - 7x + 4 \ln \left(\frac{x}{2}\right), \quad x > 0.$$

a Solve the equation $\mathrm{f}'(x) = 0$, giving your answers correct to 2 decimal places.

b Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point on the curve where $x = 2$.

12 A curve has the equation $y = x^2 - \dfrac{8}{x-1}$.

a Show that the x-coordinate of any stationary point of the curve satisfies the equation

$$x^3 - 2x^2 + x + 4 = 0.$$

b Hence, show that the curve has exactly one stationary point and find its coordinates.

c Determine the nature of this stationary point.

1 Given that $f(x) = x(x + 2)^3$, find $f'(x)$

a by first expanding $f(x)$, **b** using the product rule.

2 Differentiate each of the following with respect to x and simplify your answers.

a xe^x **b** $x(x + 1)^5$ **c** $x \ln x$ **d** $x^2(x - 1)^3$

e $x^3 \ln 2x$ **f** x^2e^{-x} **g** $2x^4(5 + x)^3$ **h** $x^2(x - 3)^4$

3 Find $\frac{dy}{dx}$, simplifying your answer in each case.

a $y = x(2x - 1)^3$ **b** $y = 3x^4e^{2x+3}$ **c** $y = x\sqrt{x-1}$

d $y = x^2 \ln (x + 6)$ **e** $y = x(1 - 5x)^4$ **f** $y = (x + 2)(x - 3)^3$

g $y = x^{\frac{4}{3}}e^{3x}$ **h** $y = (x + 1) \ln (x^2 - 1)$ **i** $y = x^2\sqrt{3x+1}$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 4xe^{3x}$, $x = 0$ **b** $f(x) = 2x(x^2 + 2)^3$, $x = -1$

c $f(x) = (5x - 4) \ln 3x$, $x = \frac{1}{3}$ **d** $f(x) = x^{\frac{1}{2}}(1 - 2x)^3$, $x = \frac{1}{4}$

5 Find the coordinates of any stationary points on each curve.

a $y = xe^{2x}$ **b** $y = x(x - 4)^3$ **c** $y = x^2(2x - 3)^4$

d $y = x\sqrt{x+12}$ **e** $y = 2 + x^2e^{-4x}$ **f** $y = (1 - 3x)(3 - x)^3$

6 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = x(x - 2)^4$, $x = 1$ **b** $y = 3x^2e^x$, $x = 1$

c $y = (4x - 1) \ln 2x$, $x = \frac{1}{2}$ **d** $y = x^2\sqrt{x+6}$, $x = -2$

7 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = x^2(2 - x)^3$, $x = 1$ **b** $y = x \ln (3x - 5)$, $x = 2$

c $y = (x^2 - 1)e^{3x}$, $x = 0$ **d** $y = x\sqrt{x-4}$, $x = 8$

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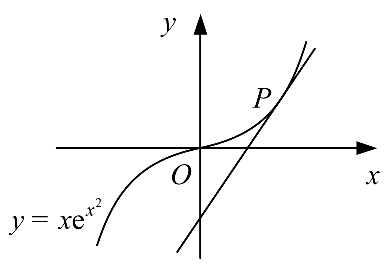


The diagram shows part of the curve with equation $y = xe^{x^2}$ and the tangent to the curve at the point P with x-coordinate 1.

a Find an equation for the tangent to the curve at P.

b Show that the area of the triangle bounded by this tangent and the coordinate axes is $\frac{2}{3}$e.

1 Given that $f(x) = \dfrac{x}{x+2}$, find $f'(x)$

a using the product rule, **b** using the quotient rule.

2 Differentiate each of the following with respect to x and simplify your answers.

a $\dfrac{4x}{1-3x}$ **b** $\dfrac{e^x}{x-4}$ **c** $\dfrac{x+1}{2x+3}$ **d** $\dfrac{\ln x}{2x}$

e $\dfrac{x}{2-x^2}$ **f** $\dfrac{\sqrt{x}}{3x+2}$ **g** $\dfrac{e^{2x}}{1-e^{2x}}$ **h** $\dfrac{2x+1}{\sqrt{x-3}}$

3 Find $\dfrac{dy}{dx}$, simplifying your answer in each case.

a $y = \dfrac{x^2}{x+4}$ **b** $y = \dfrac{\sqrt{x-4}}{2x^2}$ **c** $y = \dfrac{2e^x+1}{1-3e^x}$

d $y = \dfrac{1-x}{x^3+2}$ **e** $y = \dfrac{\ln(3x-1)}{x+2}$ **f** $y = \sqrt{\dfrac{x+1}{x+3}}$

4 Find the coordinates of any stationary points on each curve.

a $y = \dfrac{x^2}{3-x}$ **b** $y = \dfrac{e^{4x}}{2x-1}$ **c** $y = \dfrac{x+5}{\sqrt{2x+1}}$

d $y = \dfrac{\ln 3x}{2x}$ **e** $y = \left(\dfrac{x+1}{x-2}\right)^2$ **f** $y = \dfrac{x^2-3}{x+2}$

5 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = \dfrac{2x}{3-x}$, $x = 2$ **b** $y = \dfrac{e^x+3}{e^x+1}$, $x = 0$

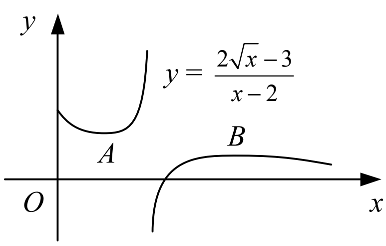
c $y = \dfrac{\sqrt{x}}{5-x}$, $x = 4$ **d** $y = \dfrac{3x+4}{x^2+1}$, $x = -1$

6 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = \dfrac{1-x}{3x+1}$, $x = 1$ **b** $y = \dfrac{4x}{\sqrt{2-x}}$, $x = -2$

c $y = \dfrac{\ln(2x-5)}{3x-5}$, $x = 3$ **d** $y = \dfrac{x}{x^3-4}$, $x = 2$

7



The diagram shows part of the curve $y = \dfrac{2\sqrt{x}-3}{x-2}$ which is stationary at the points A and B.

a Show that the x-coordinates of A and B satisfy the equation $x - 3\sqrt{x} + 2 = 0$.

b Hence, find the coordinates of A and B.

1 A curve has the equation $y = x^2(2 - x)^3$ and passes through the point A (1, 1).

a Find an equation for the tangent to the curve at A.

b Show that the normal to the curve at A passes through the origin.

2 A curve has the equation $y = \frac{x}{2x+3}$.

a Find an equation for the tangent to the curve at the point P (−1, −1).

b Find an equation for the normal to the curve at the origin, O.

c Find the coordinates of the point where the tangent to the curve at P meets the normal to the curve at O.

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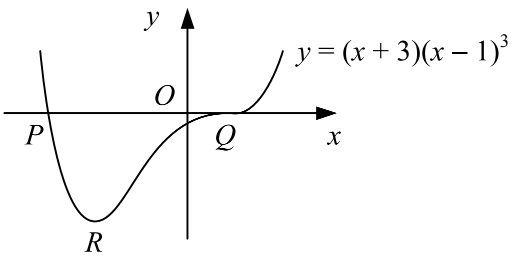


The diagram shows the curve with equation $y = (x + 3)(x - 1)^3$ which crosses the x-axis at the points P and Q and has a minimum at the point R.

a Write down the coordinates of P and Q.

b Find the coordinates of R.

4 Given that $y = x\sqrt{4x+1}$,

a show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x+1}{\sqrt{4x+1}}$,

b solve the equation $\frac{\mathrm{d}y}{\mathrm{d}x} - 5y = 0$.

5 A curve has the equation $y = \frac{2(x-1)}{x^2+3}$ and crosses the x-axis at the point A.

a Show that the normal to the curve at A has the equation $y = 2 - 2x$.

b Find the coordinates of any stationary points on the curve.

6
$$\mathrm{f}(x) \equiv x^{\frac{3}{2}}(x - 3)^3, \ x > 0.$$

a Show that

$$\mathrm{f}'(x) = kx^{\frac{1}{2}}(x - 1)(x - 3)^2,$$

where k is a constant to be found.

b Hence, find the coordinates of the stationary points of the curve $y = \mathrm{f}(x)$.

7
$$\mathrm{f}(x) = x\sqrt{2x+12}, \ x \geq -6.$$

a Find $\mathrm{f}'(x)$ and show that $\mathrm{f}''(x) = \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$.

b Find the coordinates of the turning point of the curve $y = \mathrm{f}(x)$ and determine its nature.

C3 DIFFERENTIATION Worksheet H

1 Differentiate with respect to x

a $\cos x$ **b** $5 \sin x$ **c** $\cos 3x$ **d** $\sin \frac{1}{4}x$

e $\sin (x + 1)$ **f** $\cos (3x - 2)$ **g** $4 \sin (\frac{\pi}{3} - x)$ **h** $\cos (\frac{1}{2}x + \frac{\pi}{6})$

i $\sin^2 x$ **j** $2 \cos^3 x$ **k** $\cos^2 (x - 1)$ **l** $\sin^4 2x$

2 Use the derivatives of $\sin x$ and $\cos x$ to show that

a $\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$ **b** $\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$

c $\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$ **d** $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$

3 Differentiate with respect to t

a $\cot 2t$ **b** $\sec (t + 2)$ **c** $\tan (4t - 3)$ **d** $\operatorname{cosec} 3t$

e $\tan^2 t$ **f** $3 \operatorname{cosec} (t + \frac{\pi}{6})$ **g** $\cot^3 t$ **h** $4 \sec \frac{1}{2}t$

i $\cot (2t - 3)$ **j** $\sec^2 2t$ **k** $\frac{1}{2}\tan (\pi - 4t)$ **l** $\operatorname{cosec}^2 (3t + 1)$

4 Differentiate with respect to x

a $\ln (\sin x)$ **b** $6\mathrm{e}^{\tan x}$ **c** $\sqrt{\cos 2x}$ **d** $\mathrm{e}^{\sin 3x}$

e $2 \cot x^2$ **f** $\sqrt{\sec x}$ **g** $3\mathrm{e}^{-\operatorname{cosec} 2x}$ **h** $\ln (\tan 4x)$

5 Find the coordinates of any stationary points on each curve in the interval $0 \le x \le 2\pi$.

a $y = x + 2 \sin x$ **b** $y = 2 \sec x - \tan x$ **c** $y = \sin x + \cos 2x$

6 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = 1 + \sin 2x$, $x = 0$ **b** $y = \cos x$, $x = \frac{\pi}{3}$

c $y = \tan 3x$, $x = \frac{\pi}{4}$ **d** $y = \operatorname{cosec} x - 2 \sin x$, $x = \frac{\pi}{6}$

7 Differentiate with respect to x

a $x \sin x$ **b** $\frac{\cos 2x}{x}$ **c** $\mathrm{e}^x \cos x$ **d** $\sin x \cos x$

e $x^2 \operatorname{cosec} x$ **f** $\sec x \tan x$ **g** $\frac{x}{\tan x}$ **h** $\frac{\sin 2x}{\mathrm{e}^{3x}}$

i $\cos^2 x \cot x$ **j** $\frac{\sec 2x}{x^2}$ **k** $x \tan^2 4x$ **l** $\frac{\sin x}{\cos 2x}$

8 Find the value of $\mathrm{f}'(x)$ at the value of x indicated in each case.

a $\mathrm{f}(x) = \sin 3x \cos 5x$, $x = \frac{\pi}{4}$ **b** $\mathrm{f}(x) = \tan 2x \sin x$, $x = \frac{\pi}{3}$

c $\mathrm{f}(x) = \frac{\ln(2\cos x)}{\sin x}$, $x = \frac{\pi}{3}$ **d** $\mathrm{f}(x) = \sin^2 x \cos^3 x$, $x = \frac{\pi}{6}$

9 Find an equation for the normal to the curve $y = 3 + x \cos 2x$ at the point where it crosses the y-axis.

10 A curve has the equation $y = \frac{2 + \sin x}{1 - \sin x}$, $0 \le x \le 2\pi$, $x \ne \frac{\pi}{2}$.

a Find and simplify an expression for $\frac{dy}{dx}$.

b Find the coordinates of the turning point of the curve.

c Show that the tangent to the curve at the point P, with x-coordinate $\frac{\pi}{6}$, has equation

$$y = 6\sqrt{3}\,x + 5 - \sqrt{3}\,\pi.$$

11 A curve has the equation $y = e^{-x} \sin x$.

a Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the exact coordinates of the stationary points of the curve in the interval $-\pi \le x \le \pi$ and determine their nature.

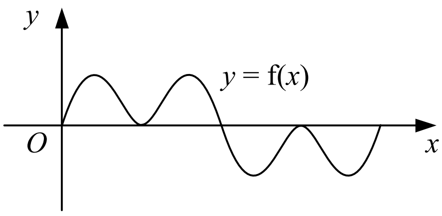
12 The curve C has the equation $y = x \sec x$.

a Show that the x-coordinate of any stationary point of C must satisfy the equation

$$1 + x \tan x = 0.$$

b By sketching two suitable graphs on the same set of axes, deduce the number of stationary points C has in the interval $0 \le x \le 2\pi$.

13



The diagram shows the curve $y = f(x)$ in the interval $0 \le x \le 2\pi$, where

$$f(x) \equiv \cos x \sin 2x.$$

a Show that $f'(x) = 2 \cos x\,(1 - 3 \sin^2 x)$.

b Find the x-coordinates of the stationary points of the curve in the interval $0 \le x \le 2\pi$.

c Show that the maximum value of $f(x)$ in the interval $0 \le x \le 2\pi$ is $\frac{4}{9}\sqrt{3}$.

d Explain why this is the maximum value of $f(x)$ for all real values of x.

14 A curve has the equation $y = \operatorname{cosec}\,(x - \frac{\pi}{6})$ and crosses the y-axis at the point P.

a Find an equation for the normal to the curve at P.

The point Q on the curve has x-coordinate $\frac{\pi}{3}$.

b Find an equation for the tangent to the curve at Q.

The normal to the curve at P and the tangent to the curve at Q intersect at the point R.

c Show that the x-coordinate of R is given by $\frac{8\sqrt{3} + 4\pi}{13}$.

1 A curve has the equation $x = \sqrt{y}$.

a Write down $\frac{dx}{dy}$ in terms of y.

b Express the equation of the curve in the form $y = f(x)$.

c Write down $\frac{dy}{dx}$ in terms of x.

d Hence verify that for this curve, $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

2 Verify the relationship $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ when

a $y = e^{2x-1}$,

b $y = x^3 + 2$,

c $x = \sqrt{\ln y}$.

3 Find expressions for $\frac{dy}{dx}$ in terms of y in each case.

a $x = y^2 + 3$ **b** $x = (y-1)^3$ **c** $x = \tan y$

d $x = \ln(3y + 2)$ **e** $x = \sin^2 y$ **f** $x = \frac{y-2}{e^y}$

4 The curve C has the equation $x = y^3 - 4y^2$.

a Find $\frac{dx}{dy}$ in terms of y.

b Find an equation for the tangent to C at the point on the curve with y-coordinate 3.

5 Given that $y = \ln(ax + b)$, where a and b are constants,

a express x as a function of y,

b find $\frac{dx}{dy}$ in terms of y.

c Hence, prove that $\frac{d}{dx}[\ln(ax + b)] = \frac{a}{ax+b}$.

6 A curve has the equation $y = 3^x$.

a Express the equation of the curve in the form $x = f(y)$.

b Find $\frac{dx}{dy}$ in terms of y.

c Hence, find $\frac{dy}{dx}$ in terms of x.

d Find an equation for the tangent to the curve at the point (2, 9).

1 Find an equation for the tangent to the curve with equation

$$y = (3 - x)^{\frac{3}{2}}$$

at the point on the curve with x-coordinate -1. **(4)**

2 **a** Sketch the curve with equation $y = 3 - \ln 2x$. **(2)**

b Find the exact coordinates of the point where the curve crosses the x-axis. **(2)**

c Find an equation for the tangent to the curve at the point on the curve where $x = 5$. **(4)**

This tangent cuts the x-axis at A and the y-axis at B.

d Show that the area of triangle OAB, where O is the origin, is approximately 7.20 **(3)**

3 Differentiate with respect to x

a $(3x - 1)^4$, **(2)**

b $\dfrac{x^2}{\sin 2x}$. **(3)**

4 The area of the surface of a boulder covered by lichen, A cm^2, at time t years after initial observation, is modelled by the formula

$$A = 2e^{0.5t}.$$

Using this model,

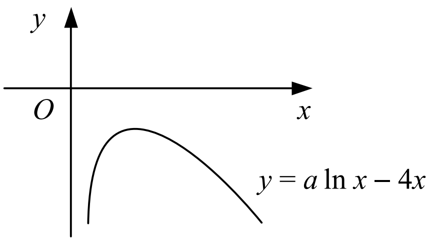
a find the area of lichen on the boulder after three years, **(2)**

b find the rate at which the area of lichen is increasing per day after three years, **(4)**

c find, to the nearest year, how long it takes until the area of lichen is 65 cm^2. **(2)**

d Explain why the model cannot be valid for large values of t. **(1)**

5



The diagram shows the curve with equation $y = a \ln x - 4x$, where a is a positive constant.

Find, in terms of a,

a the coordinates of the stationary point on the curve, **(4)**

b an equation for the tangent to the curve at the point where $x = 1$. **(3)**

Given that this tangent meets the x-axis at the point (3, 0),

c show that $a = 6$. **(2)**

6 Given that $y = e^{2x} \sin x$,

a find $\dfrac{dy}{dx}$, **(2)**

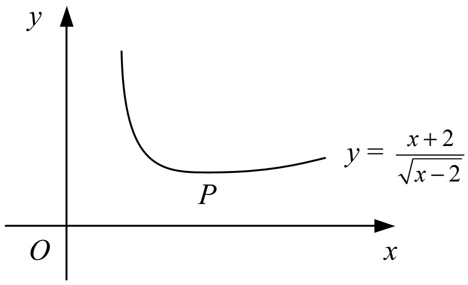
b show that $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 0$. **(3)**

7 A curve has the equation $x = \tan^2 y$.

a Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{x}(x+1)}$. **(5)**

b Find an equation for the normal to the curve at the point where $y = \frac{\pi}{4}$. **(3)**

8



The diagram shows the curve $y = \frac{x+2}{\sqrt{x-2}}$, $x > 2$, which has a minimum point at P.

a Find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(3)**

b Find the coordinates of P. **(2)**

The point Q on the curve has x-coordinate 3.

c Show that the normal to the curve at Q has equation

$$2x - 3y + 9 = 0.$$ **(3)**

9 A curve has the equation $y = \mathrm{e}^x(x - 1)^2$.

a Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(3)**

b Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^x(x^2 + 2x - 1)$. **(2)**

c Find the exact coordinates of the turning points of the curve and determine their nature. **(4)**

d Show that the tangent to the curve at the point where $x = 2$ has the equation

$$y = \mathrm{e}^2(3x - 5).$$ **(3)**

10 The curve with equation $y = \frac{1}{2}x^2 - 3\ln x$, $x > 0$, has a stationary point at A.

a Find the exact x-coordinate of A. **(3)**

b Determine the nature of the stationary point. **(2)**

c Show that the y-coordinate of A is $\frac{3}{2}(1 - \ln 3)$. **(2)**

d Find an equation for the tangent to the curve at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

11
$$\mathrm{f}(x) = \frac{6x}{(x-1)(x+2)} - \frac{2}{x-1}.$$

a Show that $\mathrm{f}(x) = \frac{4}{x+2}$. **(5)**

b Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point with x-coordinate 2, giving your answer in the form $ax + by = c$, where a, b and c are integers. **(4)**

1 The curve C has equation $y = \frac{1}{4x} - \ln x$.

a Find the gradient of C at the point $(1, \frac{1}{4})$. **(3)**

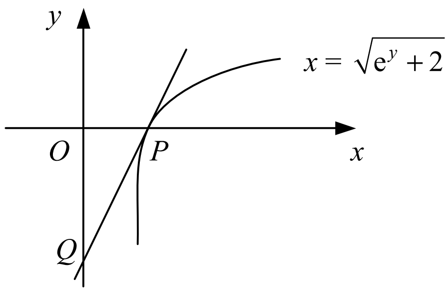
b Find an equation for the normal to C at the point $(1, \frac{1}{4})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

2 A curve has the equation $y = xe^{-2x}$.

a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **(4)**

b Find the exact coordinates of the turning point of the curve and determine its nature. **(4)**

3



The diagram shows the curve $x = \sqrt{e^y + 2}$ which crosses the x-axis at the point P.

a Find the coordinates of P. **(1)**

b Find $\frac{dx}{dy}$ in terms of y. **(2)**

The tangent to the curve at P crosses the y-axis at the point Q.

c Show that the area of triangle OPQ, where O is the origin, is $3\sqrt{3}$. **(5)**

4 A rock contains a radioactive substance which is decaying.

The mass of the rock, m grams, at time t years after initial observation is given by

$$m = 600 + 80e^{-0.004t}.$$

a Find the percentage reduction in the mass of the rock over the first 100 years. **(3)**

b Find the value of t when $m = 640$. **(2)**

c Find the rate at which the mass of the rock will be decreasing when $t = 150$. **(3)**

5 Differentiate with respect to x

a $\sqrt{\sin x + \cos x}$, **(3)**

b $\ln\left(\frac{x-1}{2x+1}\right)$. **(3)**

6 A curve has the equation $y = (2x - 3)^5$.

a Find an equation for the tangent to the curve at the point $P\ (1, -1)$. **(4)**

Given that the tangent to the curve at the point Q is parallel to the tangent at P,

b find the coordinates of Q. **(3)**

7 A curve has the equation $y = \frac{2}{x^2 - 5}$.

a Find the coordinates of the stationary point of the curve. **(4)**

b Show that the tangent to the curve at the point with x-coordinate 3 has the equation

$$3x + 4y - 11 = 0.$$ **(3)**

8 $$\mathrm{f} : x \to a\mathrm{e}^x + a, \quad x \in \mathbb{R}.$$

Given that a is a positive constant,

a sketch the graph of $y = \mathrm{f}(x)$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. **(2)**

b Find the inverse function f^{-1} in the form $\mathrm{f}^{-1} : x \to \ldots$ and state its domain. **(4)**

c Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point on the curve with x-coordinate 1. **(4)**

9 **a** Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\mathrm{cosec}^2\, x.$$ **(4)**

b Show that the curve with equation

$$y = \mathrm{e}^x \cot x$$

has no turning points. **(5)**

10 A curve has the equation $y = (2 + \ln x)^3$.

a Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

b Find, in exact form, the coordinates of the stationary point on the curve. **(3)**

c Show that the tangent to the curve at the point with x-coordinate e passes through the origin. **(3)**

11 $$\mathrm{f} : x \to \ln(9 - x^2), \quad -3 < x < 3.$$

a Find $\mathrm{f}'(x)$. **(2)**

b Find the coordinates of the stationary point of the curve $y = \mathrm{f}(x)$. **(2)**

c Show that the normal to the curve $y = \mathrm{f}(x)$ at the point with x-coordinate 1 has equation

$$y = 4x - 4 + 3\ln 2.$$ **(4)**

12 A botanist is studying the regeneration of an area of moorland following a fire.

The total biomass in the area after t years is denoted by M tonnes and two models are proposed for the growth of M.

Model A is given by

$$M = 900 - \frac{1500}{3t + 2}.$$

Model B is given by

$$M = 900 - \frac{1500}{2 + 5\ln(t + 1)}.$$

For each model, find

a the value of M when $t = 3$, **(2)**

b the rate at which the biomass is increasing when $t = 3$. **(6)**