

$$1 \quad \mathbf{a} = \frac{3x-1}{6(3x-1)} = \frac{1}{6} \quad \mathbf{b} = \frac{3(2x+5)}{4(2x+5)} = \frac{3}{4} \quad \mathbf{c} = \frac{3(y+1)}{(y+6)(y+1)} = \frac{3}{y+6} \quad \mathbf{d} = \frac{(x+5)(x-5)}{(x-2)(x-5)} = \frac{x+5}{x-2}$$

$$\mathbf{e} = \frac{(a+2)(a-3)}{(a+6)(a-3)} = \frac{a+2}{a+6} \quad \mathbf{f} = \frac{x(x+3)}{(2x-1)(x+3)} = \frac{x}{2x-1} \quad \mathbf{g} = \frac{(3t+1)(t-4)}{(t+4)(t-4)} = \frac{3t+1}{t+4} \quad \mathbf{h} = \frac{(3x-2)(2x-3)}{(4x+3)(3x-2)} = \frac{2x-3}{4x+3}$$

$$2 \quad \mathbf{a} = \frac{3x^2}{9(x-1)} \times \frac{4(x-1)}{2x} = \frac{2x}{3} \quad \mathbf{b} = \frac{(x+6)(x-6)}{(x+2)(x+5)} \times \frac{x+2}{x-6} = \frac{x+6}{x+5}$$

$$\mathbf{c} = \frac{n(n+2)}{(n+4)(n+2)} \times \frac{n+4}{n^2} = \frac{1}{n} \quad \mathbf{d} = \frac{4(x-3)}{(x+2)(x-2)} \times \frac{x(x+2)}{(x+1)(x-3)} = \frac{4x}{(x-2)(x+1)}$$

$$\mathbf{e} = \frac{4y^2}{y(2y+1)} \times \frac{(2y+1)(y+5)}{(y+5)(y-3)} = \frac{4y}{y-3} \quad \mathbf{f} = \frac{(x+1)(x-1)}{(2x-1)(x+4)} \times \frac{(3x-1)(2x-1)}{(3x-1)(x-1)} = \frac{x+1}{x+4}$$

$$\mathbf{g} = \frac{10(x-1)}{5(x+3)} \times \frac{(x+3)(x+4)}{(4+x)(1-x)} = -2 \quad \mathbf{h} = \frac{a^2(a-3)}{4a(2a-1)} \times \frac{(2a-1)(a+3)}{(a+3)(a-3)} = \frac{a}{4}$$

$$3 \quad \mathbf{a} = \frac{2(y+4)+7y}{y(y+4)} = \frac{9y+8}{y(y+4)} \quad \mathbf{b} = \frac{2x(x+3)-(x-5)}{(x-5)(x+3)} = \frac{2x^2+5x+5}{(x-5)(x+3)}$$

$$\mathbf{c} = \frac{7-3x^2}{x(x+2)} \quad \mathbf{d} = \frac{2x+5(x-3)}{2(x-3)(x-1)} = \frac{7x-15}{2(x-3)(x-1)}$$

$$\mathbf{e} = \frac{2}{q(q+3)} + \frac{5q}{4(q+3)} = \frac{8+5q^2}{4q(q+3)} \quad \mathbf{f} = \frac{4}{3(x-1)} + \frac{x+2}{x(x-1)} = \frac{4x+3(x+2)}{3x(x-1)} = \frac{7x+6}{3x(x-1)}$$

$$\mathbf{g} = \frac{4}{x+5} + \frac{x}{(x+3)(x+5)} = \frac{4(x+3)+x}{(x+3)(x+5)} = \frac{5x+12}{(x+3)(x+5)}$$

$$\mathbf{h} = \frac{6x}{(x+2)(x-2)} - \frac{3}{x+2} = \frac{6x-3(x-2)}{(x+2)(x-2)} = \frac{3x+6}{(x+2)(x-2)} = \frac{3(x+2)}{(x+2)(x-2)} = \frac{3}{x-2}$$

$$\mathbf{i} = \frac{5t+12}{(2t+1)(t+3)} - \frac{4}{2t+1} = \frac{5t+12-4(t+3)}{(2t+1)(t+3)} = \frac{t}{(2t+1)(t+3)}$$

$$4 \quad \mathbf{a} = \frac{x(x-5)}{6(x-5)} = \frac{x}{6} \quad \mathbf{b} = \frac{(4+x)(4-x)}{(x+4)(x-2)} = \frac{4-x}{x-2}$$

$$\mathbf{c} = \frac{2(x-3)(x+1)}{3(x-1)(x-3)} = \frac{2(x+1)}{3(x-1)} \quad \mathbf{d} = \frac{x(x+1)(x-1)}{(2x+1)(x-1)} = \frac{x(x+1)}{2x+1}$$

$$\mathbf{e} = \frac{x(3-x)}{2(x+3)(x-3)} = -\frac{x}{2(x+3)} \quad \mathbf{f} = \frac{x(x+2)(x-1)}{(3x-2)(x+2)} = \frac{x(x-1)}{3x-2}$$

$$\mathbf{g} = \frac{(2-x)(1+3x)}{(2x+5)(x-2)} = -\frac{3x+1}{2x+5} \quad \mathbf{h} = \frac{(x^2-1)(x^2-4)}{(x+1)(x-2)} = \frac{(x+1)(x-1)(x+2)(x-2)}{(x+1)(x-2)} = (x-1)(x+2)$$

$$\begin{aligned}
 5 \quad \mathbf{a} &= \frac{10(x+1)(x-1)}{5(x+2)} \times \frac{(x+2)(x+4)}{(x+1)(x+4)} = 2(x-1) & \mathbf{b} &= \frac{t(t-2)}{(2t+3)(t-2)} \times \frac{(3t+2)(2t+3)}{(3t+2)(3t-2)} = \frac{t}{3t-2} \\
 \mathbf{c} &= \frac{2(x+1)(x+5)}{(4x-3)(x-1)} \times \frac{x(4x-3)}{4x(x+5)} = \frac{x+1}{2(x-1)} & \mathbf{d} &= \frac{(4x-3)(2x+3)}{(2x+3)^2} \times \frac{x(2x+3)}{2(3-4x)} = -\frac{x}{2} \\
 \mathbf{e} &= \frac{(x^2+1)(x^2+5)}{(x+3)(x-3)} \times \frac{2x(x-3)}{4(x^2+1)} = \frac{x(x^2+5)}{2(x+3)} & \mathbf{f} &= \frac{(y^2+4)(y+2)(y-2)}{(5y-1)(y+2)} \times \frac{(5y-1)^2}{y^2+4} = (y-2)(5y-1)
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \mathbf{a} &= \frac{5}{(x+1)(x-1)} - \frac{1}{2(x+1)} \\
 &= \frac{10 - (x-1)}{2(x+1)(x-1)} \\
 &= \frac{11-x}{2(x+1)(x-1)} & \mathbf{b} &= \frac{3x}{(x+2)(x-2)} - \frac{4}{(2x-1)(x+2)} \\
 & & &= \frac{3x(2x-1) - 4(x-2)}{(x+2)(x-2)(2x-1)} \\
 & & &= \frac{6x^2 - 7x + 8}{(x+2)(x-2)(2x-1)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= \frac{4}{(x+3)(x-1)} + \frac{1}{(x-1)(x-2)} \\
 &= \frac{4(x-2) + (x+3)}{(x+3)(x-1)(x-2)} \\
 &= \frac{5x-5}{(x+3)(x-1)(x-2)} \\
 &= \frac{5}{(x+3)(x-2)} & \mathbf{d} &= \frac{x+1}{(x+5)(x-5)} + \frac{2}{x(x+5)} \\
 & & &= \frac{x(x+1) + 2(x-5)}{x(x+5)(x-5)} \\
 & & &= \frac{x^2 + 3x - 10}{x(x+5)(x-5)} \\
 & & &= \frac{(x+5)(x-2)}{x(x+5)(x-5)} \\
 & & &= \frac{x-2}{x(x-5)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} &= \frac{2x-1}{(x+2)^2} + \frac{x}{3(x+2)} \\
 &= \frac{3(2x-1) + x(x+2)}{3(x+2)^2} \\
 &= \frac{x^2 + 8x - 3}{3(x+2)^2} & \mathbf{f} &= \frac{1}{x-3} + \frac{3}{x(x-3)} + \frac{x}{(x-3)^2} \\
 & & &= \frac{x(x-3) + 3(x-3) + x^2}{x(x-3)^2} \\
 & & &= \frac{2x^2 - 9}{x(x-3)^2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{g} &= \frac{x}{(x+2)(x-3)} + \frac{2}{(2x-1)(x+2)} \\
 &= \frac{x(2x-1) + 2(x-3)}{(x+2)(x-3)(2x-1)} \\
 &= \frac{2x^2 + x - 6}{(x+2)(x-3)(2x-1)} \\
 &= \frac{(2x-3)(x+2)}{(x+2)(x-3)(2x-1)} \\
 &= \frac{2x-3}{(x-3)(2x-1)} & \mathbf{h} &= \frac{1}{x^2} - \frac{1}{x(3x-2)} + \frac{3}{2(3x-2)} \\
 & & &= \frac{2(3x-2) - 2x + 3x^2}{2x^2(3x-2)} \\
 & & &= \frac{3x^2 + 4x - 4}{2x^2(3x-2)} \\
 & & &= \frac{(3x-2)(x+2)}{2x^2(3x-2)} \\
 & & &= \frac{x+2}{2x^2}
 \end{aligned}$$

7 a $x(2x - 5) - 2(2x - 5) = 3x$

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

$$x = 1, 5$$

c $\frac{20}{(2x+1)(x+2)} + 1 = \frac{10}{2x+1}$

$$20 + 2x^2 + 5x + 2 = 10(x + 2)$$

$$2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$x = \frac{1}{2}, 2$$

e $5 + \frac{1}{(x+3)(x+2)} = \frac{11}{x+3}$

$$5(x^2 + 5x + 6) + 1 = 11(x + 2)$$

$$5x^2 + 14x + 9 = 0$$

$$(5x + 9)(x + 1) = 0$$

$$x = -\frac{9}{5}, -1$$

b $\frac{2}{(x+1)(x-1)} + \frac{3}{x+1} = 1$

$$2 + 3(x - 1) = x^2 - 1$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0, 3$$

d $2y(y + 3) - y(y + 5) = 2(y + 5)(2y - 1)$

$$2y^2 + 6y - y^2 - 5y = 4y^2 + 18y - 10$$

$$3y^2 + 17y - 10 = 0$$

$$y = \frac{-17 \pm \sqrt{289 + 120}}{6} = \frac{-17 \pm \sqrt{409}}{6}$$

$$y = -6.20, 0.537 \text{ (3sf)}$$

f $\frac{3}{(1-2x)^2} - \frac{10}{(1+2x)(1-2x)} = \frac{5}{1+2x}$

$$3(1 + 2x) - 10(1 - 2x) = 5(1 - 4x + 4x^2)$$

$$10x^2 - 23x + 6 = 0$$

$$(10x - 3)(x - 2) = 0$$

$$x = \frac{3}{10}, 2$$

8 $f(x) = \frac{7x-15}{x(x-5)} - \frac{4}{x-5}$

$$= \frac{7x-15-4x}{x(x-5)} = \frac{3x-15}{x(x-5)}$$

$$= \frac{3(x-5)}{x(x-5)} = \frac{3}{x} \quad [k = 3]$$

9 $f(x) = \frac{x-5}{(3x-1)(x+2)} + \frac{2}{3x-1}$

$$= \frac{x-5+2(x+2)}{(3x-1)(x+2)} = \frac{3x-1}{(3x-1)(x+2)}$$

$$= \frac{1}{x+2}$$

10 $f(x) - \frac{1}{f(x)} = \frac{x+2}{x-2} - \frac{x-2}{x+2}$

$$= \frac{(x+2)^2 - (x-2)^2}{(x-2)(x+2)}$$

$$= \frac{x^2 + 4x + 4 - (x^2 - 4x + 4)}{x^2 - 4}$$

$$= \frac{8x}{x^2 - 4}$$

11 a $= \frac{2(x+2)+3}{(x+2)(x+5)} = \frac{2x+7}{(x+2)(x+5)}$

b $\frac{2x+7}{(x+2)(x+5)} = \frac{1}{3}$

$$3(2x + 7) = (x + 2)(x + 5)$$

$$x^2 + x - 11 = 0$$

$$x = \frac{-1 \pm \sqrt{1+44}}{2} = \frac{-1 \pm \sqrt{45}}{2} = -3.85, 2.85$$

12 $\frac{3}{2(2x+1)} - \frac{5}{(2x+1)^2} = 2$

$$3(2x + 1) - 10 = 4(4x^2 + 4x + 1)$$

$$16x^2 + 10x + 11 = 0$$

$$b^2 - 4ac = 100 - 704 = -604$$

$$b^2 - 4ac < 0 \therefore \text{no real roots}$$

13 $= \frac{6x-(x+5)}{x(x+5)} \div \frac{x-1}{(x+5)(x-5)}$

$$= \frac{5x-5}{x(x+5)} \times \frac{(x+5)(x-5)}{x-1}$$

$$= \frac{5(x-1)}{x(x+5)} \times \frac{(x+5)(x-5)}{x-1}$$

$$= \frac{5(x-5)}{x}$$

1 a

$$\begin{array}{r}
 3x^2 + 2x - 1 \\
 x-4 \overline{) 3x^3 - 10x^2 - 9x + 15} \\
 \underline{3x^3 - 12x^2} \\
 2x^2 - 9x \\
 \underline{2x^2 - 8x} \\
 -x + 15 \\
 -x + 4 \\
 \hline
 11
 \end{array}$$

quotient: $3x^2 + 2x - 1$, remainder: 11

b

$$\begin{array}{r}
 x^2 - 5x - 3 \\
 2x-1 \overline{) 2x^3 - 11x^2 - x + 3} \\
 \underline{2x^3 - x^2} \\
 -10x^2 - x \\
 \underline{-10x^2 + 5x} \\
 -6x + 3 \\
 -6x + 3 \\
 \hline
 0
 \end{array}$$

quotient: $x^2 - 5x - 3$, remainder: 0

c

$$\begin{array}{r}
 2x^2 - x + 6 \\
 2x+5 \overline{) 4x^3 + 8x^2 + 7x + 32} \\
 \underline{4x^3 + 10x^2} \\
 -2x^2 + 7x \\
 \underline{-2x^2 - 5x} \\
 12x + 32 \\
 \underline{12x + 30} \\
 2
 \end{array}$$

quotient: $2x^2 - x + 6$, remainder: 2

d

$$\begin{array}{r}
 -2x^2 - 6x + 4 \\
 3x+2 \overline{) -6x^3 - 22x^2 + 0x + 1} \\
 \underline{-6x^3 - 4x^2} \\
 -18x^2 + 0x \\
 \underline{-18x^2 - 12x} \\
 12x + 1 \\
 \underline{12x + 8} \\
 -7
 \end{array}$$

quotient: $-2x^2 - 6x + 4$, remainder -7

2

a let $f(x) = x^3 + 4x^2 + x - 6$
 $f(-2) = -8 + 16 - 2 - 6 = 0$
 $\therefore (x+2)$ is a factor

b

$$\begin{array}{r}
 x^2 + 2x - 3 \\
 x+2 \overline{) x^3 + 4x^2 + x - 6} \\
 \underline{x^3 + 2x^2} \\
 2x^2 + x \\
 \underline{2x^2 + 4x} \\
 -3x - 6 \\
 -3x - 6 \\
 \hline
 0
 \end{array}$$

$$\begin{aligned}
 \therefore x^3 + 4x^2 + x - 6 \\
 &= (x+2)(x^2 + 2x - 3) \\
 &= (x+2)(x+3)(x-1)
 \end{aligned}$$

$$\begin{aligned}
 \text{c } &= \frac{(x+2)(x+3)(x-1)}{(x+3)(x-3)} \\
 &= \frac{(x+2)(x-1)}{x-3}
 \end{aligned}$$

3 a let $f(x) = 2x^3 - 5x^2 + 13x - 15$
 $f(\frac{3}{2}) = \frac{27}{4} - \frac{45}{4} + \frac{39}{2} - 15 = 0$
 $\therefore (2x - 3)$ is a factor

b

$$\begin{array}{r}
 x^2 - x + 5 \\
 2x - 3 \overline{) 2x^3 - 5x^2 + 13x - 15} \\
 \underline{2x^3 - 3x^2} \\
 - 2x^2 + 13x \\
 \underline{- 2x^2 + 3x} \\
 10x - 15 \\
 \underline{10x - 15} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 2x^3 - 5x^2 + 13x - 15 \\
 = (2x - 3)(x^2 - x + 5)
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6} \\
 = \frac{(2x - 3)(x^2 - x + 5)}{(2x - 3)(x - 2)} \\
 = \frac{x^2 - x + 5}{x - 2}
 \end{aligned}$$

4 a $x - 1$

b

$$\begin{array}{r}
 x^2 + x + 1 \\
 x - 1 \overline{) x^3 + 0x^2 + 0x - 1} \\
 \underline{x^3 - x^2} \\
 x^2 + 0x \\
 \underline{x^2 - x} \\
 x - 1 \\
 \underline{x - 1} \\
 0
 \end{array}$$

$$\therefore x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$\begin{aligned}
 \therefore \frac{x^3 - 1}{x^2 + x - 2} &= \frac{(x - 1)(x^2 + x + 1)}{(x - 1)(x + 2)} \\
 &= \frac{x^2 + x + 1}{x + 2}
 \end{aligned}$$

5 $\frac{2x+5}{x+3} = \frac{2(x+3)-1}{x+3} = 2 - \frac{1}{x+3}$
 $\therefore A = 2, B = -1$

6 a $= \frac{(x+1)+1}{x+1} = 1 + \frac{1}{x+1}$

b $= \frac{(x-2)+5}{x-2} = 1 + \frac{5}{x-2}$

c $= \frac{-(1-x)+1}{1-x} = -1 + \frac{1}{1-x}$

d $= \frac{2(x+2)-3}{x+2} = 2 - \frac{3}{x+2}$

e $= \frac{\frac{1}{2}(2x-1) - \frac{1}{2}}{2x-1} = \frac{1}{2} - \frac{1}{2(2x-1)}$

f $= \frac{-2(3+2x)+7}{3+2x} = -2 + \frac{7}{3+2x}$

7

a

$$x^2 + x + 2 \overline{) \begin{array}{r} 1 \\ x^2 + 3x + 5 \\ \underline{x^2 + x + 2} \\ 2x + 3 \end{array}}$$

quotient: 1, remainder: $2x + 3$

c

$$x^2 + 3x - 1 \overline{) \begin{array}{r} 1 \\ x^2 + 0x + 7 \\ \underline{x^2 + 3x - 1} \\ -3x + 8 \end{array}}$$

quotient: 1, remainder: $-3x + 8$

e

$$x^2 + x - 2 \overline{) \begin{array}{r} x - 3 \\ x^3 - 2x^2 - 5x + 8 \\ \underline{x^3 + x^2 - 2x} \\ -3x^2 - 3x + 8 \\ \underline{-3x^2 - 3x + 6} \\ 2 \end{array}}$$

quotient: $x - 3$, remainder: 2

g

$$3x^2 + 4 \overline{) \begin{array}{r} x + 2 \\ 3x^3 + 6x^2 - 2x + 5 \\ \underline{3x^3 + 0x^2 + 4x} \\ 6x^2 - 6x + 5 \\ \underline{6x^2 + 0x + 8} \\ -6x - 3 \end{array}}$$

quotient: $x + 2$, remainder: $-6x - 3$

8

a

$$x^2 + 3x - 4 \overline{) \begin{array}{r} x + 2 \\ x^3 + 5x^2 + 7x - 13 \\ \underline{x^3 + 3x^2 - 4x} \\ 2x^2 + 11x - 13 \\ \underline{2x^2 + 6x - 8} \\ 5x - 5 \end{array}}$$

 $\therefore = x + 2$ remainder $5x - 5$

b

$$\begin{aligned} \text{LHS} &= x + 2 + \frac{5x-5}{x^2+3x-4} \\ &= x + 2 + \frac{5(x-1)}{(x-1)(x+4)} \\ &= x + 2 + \frac{5}{x+4} \end{aligned}$$

b

$$x^2 - x - 4 \overline{) \begin{array}{r} 2 \\ 2x^2 + 3x - 8 \\ \underline{2x^2 - 2x - 8} \\ 5x \end{array}}$$

quotient: 2, remainder: $5x$

d

$$x^2 + 2 \overline{) \begin{array}{r} 3 \\ 3x^2 - x - 4 \\ \underline{3x^2 + 0x + 6} \\ -x - 10 \end{array}}$$

quotient: 3, remainder: $-x - 10$

f

$$x^2 - 5x + 1 \overline{) \begin{array}{r} 2x + 3 \\ 2x^3 - 7x^2 + 0x + 1 \\ \underline{2x^3 - 10x^2 + 2x} \\ 3x^2 - 2x + 1 \\ \underline{3x^2 - 15x + 3} \\ 13x - 2 \end{array}}$$

quotient: $2x + 3$, remainder: $13x - 2$

h

$$2x^2 - 5x - 2 \overline{) \begin{array}{r} 3x + 7 \\ 6x^3 - x^2 - 44x - 6 \\ \underline{6x^3 - 15x^2 - 6x} \\ 14x^2 - 38x - 6 \\ \underline{14x^2 - 35x - 14} \\ -3x + 8 \end{array}}$$

quotient: $3x + 7$, remainder: $-3x + 8$

9

a

$$x^2 + 2x - 15 \overline{) \begin{array}{r} x - 4 \\ x^3 - 2x^2 - 21x + 70 \\ \underline{x^3 + 2x^2 - 15x} \\ -4x^2 - 6x + 70 \\ \underline{-4x^2 - 8x + 60} \\ 2x + 10 \end{array}}$$

$$\begin{aligned} \therefore f(x) &= x - 4 + \frac{2x+10}{x^2+2x-15} \\ &= x - 4 + \frac{2(x+5)}{(x+5)(x-3)} \\ &= x - 4 + \frac{2}{x-3} \end{aligned}$$

b

$$\begin{aligned} \frac{3x-7}{x-3} &= \frac{3(x-3)+2}{x-3} = 3 + \frac{2}{x-3} \\ \therefore f(x) &= \frac{3x-7}{x-3} \\ \Rightarrow x - 4 + \frac{2}{x-3} &= 3 + \frac{2}{x-3} \\ x - 4 &= 3 \\ x &= 7 \end{aligned}$$

$$\begin{aligned}
 1 \quad &= \frac{6}{(x+3)(x-3)} - \frac{7}{(2x+1)(x-3)} \\
 &= \frac{6(2x+1) - 7(x+3)}{(x+3)(x-3)(2x+1)} = \frac{5x-15}{(x+3)(x-3)(2x+1)} \\
 &= \frac{5(x-3)}{(x+3)(x-3)(2x+1)} = \frac{5}{(x+3)(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad f(x) &= \frac{3}{2x+3} - \frac{x+9}{(2x+3)(x+4)} \\
 &= \frac{3(x+4) - (x+9)}{(2x+3)(x+4)} \\
 &= \frac{2x+3}{(2x+3)(x+4)} = \frac{1}{x+4}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad a \quad &= \frac{1}{x-6} - \frac{2}{(x+6)(x-6)} = \frac{(x+6)-2}{(x-6)(x+6)} \\
 &= \frac{x+4}{(x-6)(x+6)}
 \end{aligned}$$

$$\begin{aligned}
 b \quad &\frac{x+4}{(x-6)(x+6)} = \frac{1}{2} \\
 2(x+4) &= x^2 - 36 \\
 x^2 - 2x - 44 &= 0 \\
 x &= \frac{2 \pm \sqrt{4+176}}{2} = \frac{2 \pm 6\sqrt{5}}{2} = 1 \pm 3\sqrt{5}
 \end{aligned}$$

$$4 \quad a \quad f(5) = 250 - 125 - 115 - 10 = 0$$

$\therefore (x-5)$ is a factor of $f(x)$

$$\begin{array}{r}
 b \quad \begin{array}{r} 2x^2 + 5x + 2 \\ x-5 \overline{) 2x^3 - 5x^2 - 23x - 10} \\ \underline{2x^3 - 10x^2} \\ 5x^2 - 23x \\ \underline{5x^2 - 25x} \\ 2x - 10 \\ \underline{2x - 10} \end{array}
 \end{array}$$

$$\begin{aligned}
 \therefore f(x) &= (x-5)(2x^2 + 5x + 2) \\
 &= (x-5)(2x+1)(x+2) \\
 \therefore \frac{f(x)}{2x^2 - 9x - 5} &= \frac{(x-5)(2x+1)(x+2)}{(2x+1)(x-5)} \\
 &= x+2
 \end{aligned}$$

$$5 \quad \frac{x+6}{(x+3)(x+6)} + \frac{x-p}{x+7} = 0$$

$$(x+7) + (x-p)(x+3) = 0$$

$$x^2 + (4-p)x + 7 - 3p = 0$$

real, equal roots $\therefore b^2 - 4ac = 0$

$$(4-p)^2 - 4(7-3p) = 0$$

$$p^2 + 4p - 12 = 0$$

$$(p+6)(p-2) = 0$$

$$p = -6, 2$$

$$\begin{aligned}
 6 \quad &= \frac{1}{3x-1} - \frac{3x}{(3x-1)^2} - \frac{1}{x(3x-1)} \\
 &= \frac{x(3x-1) - 3x^2 - (3x-1)}{x(3x-1)^2} \\
 &= \frac{1-4x}{x(3x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad a \quad i \quad &= \frac{7(x+2)}{(2+x)(2-x)} = \frac{7}{2-x} \\
 ii \quad &= \frac{(2x-7)(x+4)}{3x(x+4)} = \frac{2x-7}{3x}
 \end{aligned}$$

$$\begin{aligned}
 b \quad &\frac{7}{2-x} = \frac{2x-7}{3x} \\
 21x &= (2-x)(2x-7) \\
 x^2 + 5x + 7 &= 0 \\
 b^2 - 4ac &= 25 - 28 = -3 \\
 b^2 - 4ac < 0 &\therefore \text{no real roots}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad a \quad &\frac{1}{2} - \frac{1}{t-2} = \frac{4}{t^2-2t} - \frac{1}{2} \\
 \therefore \frac{4}{t^2-2t} + \frac{1}{t-2} &= 1
 \end{aligned}$$

$$\begin{aligned}
 b \quad &\frac{4}{t(t-2)} + \frac{1}{t-2} = 1 \\
 4+t &= t^2 - 2t \\
 t^2 - 3t - 4 &= 0 \\
 (t+1)(t-4) &= 0 \\
 t &= -1, 4 \\
 \text{but if } t &= 4, \text{ common difference} = 0 \\
 \therefore t &= -1, \text{ first term} = -\frac{1}{3}
 \end{aligned}$$