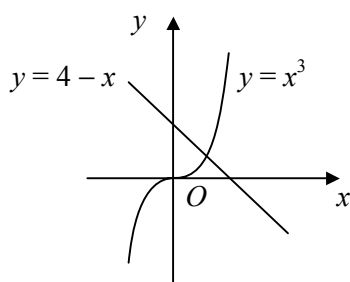


- 1**
- a** $f(1) = -3$ $f(2) = 7$
sign change, $f(x)$ continuous \therefore root
- c** $f(-6) = -0.995$ $f(-5) = 0.0135$
sign change, $f(x)$ continuous \therefore root
- e** $f(0.4) = -0.351$ $f(0.5) = 0.25$
sign change, $f(x)$ continuous \therefore root
- b** $f(0.5) = 2.89$ $f(1) = -0.298$
sign change, $f(x)$ continuous \therefore root
- d** $f(2.1) = -1.60$ $f(2.2) = 0.226$
sign change, $f(x)$ continuous \therefore root
- f** $f(10) = 6.00$ $f(11) = -9.00$
sign change, $f(x)$ continuous \therefore root
- 2**
- a** $f(0) = -4$
 $f(3) = 17.8$
 $f(1) = -6$
 $f(2) = -0.243$
 $\therefore N = 2$
- b** $f(1) = -12$
 $f(5) = 5.65$
 $f(3) = -0.704$
 $f(4) = 2.55$
 $\therefore N = 3$
- c** $f(0) = 15$
 $f(-2) = -57$
 $f(-1) = 9$
 $\therefore N = -2$
- d** $f(0) = -1.63$
 $f(1) = 3$
 $\therefore N = 0$
- e** $f(0) = 1$
 $f(-5) = -2.87$
 $f(-4) = -2.25$
 $f(-3) = 0.473$
 $\therefore N = -4$
- f** $f(0) = -6$
 $f(4) = -1.58$
 $f(5) = -0.454$
 $f(6) = 0.684$
 $\therefore N = 5$
- 3**
- a** let $f(x) = x^3 - 12 + \frac{x}{4}$
 $f(2) = -3.5$ $f(3) = 15.75$
sign change, $f(x)$ continuous \therefore root
- c** let $f(x) = 10 \ln 3x - 5 + 7x^2$
 $f(0.47) = -0.0178$ $f(0.48) = 0.259$
sign change, $f(x)$ continuous \therefore root
- e** let $f(x) = 4^x - 3x - 10$
 $f(-4) = 2.00$ $f(-3) = -0.984$
sign change, $f(x)$ continuous \therefore root
- b** let $f(x) = 12e^x - 9 + 4x$
 $f(-1) = -8.59$ $f(0) = 3$
sign change, $f(x)$ continuous \therefore root
- d** let $f(x) = \sin 4x - 7e^x$
 $f(-6.5) = -0.773$ $f(-6) = 0.888$
sign change, $f(x)$ continuous \therefore root
- f** let $f(x) = \tan\left(\frac{1}{2}x\right) - 2x + 1$
 $f(2.6) = -0.598$ $f(2.7) = 0.0552$
sign change, $f(x)$ continuous \therefore root
- 4**
- a** $f(1) = -1$
 $f(2) = 12.5$
 $f(1.1) = -0.809$
 $f(1.2) = -0.426$
 $f(1.3) = 0.164$
 $\therefore a = 12$
- b** $f(2) = -0.303$
 $f(3) = 0.292$
 $f(2.5) = -0.00553$
 $f(2.6) = 0.0537$
 $\therefore a = 25$
- c** $f(-2) = -41$
 $f(-1) = 3$
 $f(-1.1) = 0.715$
 $f(-1.2) = -1.96$
 $\therefore a = -12$
- d** $f(11) = 0.723$
 $f(12) = -0.177$
 $f(11.7) = 0.0362$
 $f(11.8) = -0.0425$
 $\therefore a = 117$
- e** $f(5) = 1.19$
 $f(6) = -1.13$
 $f(5.5) = 0.928$
 $f(5.8) = 0.256$
 $f(5.9) = -0.246$
 $\therefore a = 58$
- f** $f(-3) = 6.42$
 $f(-2) = -15.0$
 $f(-2.7) = 2.60$
 $f(-2.6) = 1.03$
 $f(-2.5) = -0.75$
 $\therefore a = -26$

5

a



b $x^3 + x - 4 = 0 \Rightarrow x^3 = 4 - x$
the graphs $y = x^3$ and $y = 4 - x$

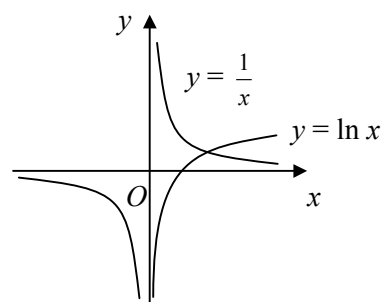
intersect at exactly one point

\therefore one real root

c let $f(x) = x^3 + x - 4$
 $f(1) = -2$
 $f(1.5) = 0.875$
sign change, $f(x)$ continuous \therefore root

6

a



b $x \ln x - 1 = 0 \Rightarrow x \ln x = 1 \Rightarrow \ln x = \frac{1}{x}$

the graphs $y = \ln x$ and $y = \frac{1}{x}$

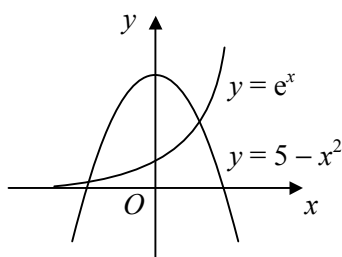
intersect at exactly one point

\therefore one real root

c $f(1) = -1$
 $f(2) = 0.386$
 $\therefore 1 < \alpha < 2$
 $\therefore n = 1$

7

a



b $e^x + x^2 - 5 = 0 \Rightarrow e^x = 5 - x^2$
the graphs $y = e^x$ and $y = 5 - x^2$
intersect at two points,
one for $x < 0$ and one for $x > 0$
 \therefore one negative and one
positive real root

c let $f(x) = e^x + x^2 - 5$
 $f(-3) = 4.05$
 $f(-2) = -0.865$
sign change, $f(x)$ continuous \therefore root

d $f(1) = -1.28$
 $f(2) = 6.39$
 $f(1.2) = -0.240$
 $f(1.3) = 0.359$
 $\therefore 1.2 < \alpha < 1.3$
 $\therefore n = 12$

1 a $9 + 4x - 2x^3 = 0$

$$x^3 = 2x + 4.5$$

$$x = \sqrt[3]{2x + 4.5}$$

$$\therefore x_{n+1} = \sqrt[3]{2x_n + 4.5}$$

$$x_1 = 2.040828$$

$$x_2 = 2.047342$$

$$x_3 = 2.048377 = 2.0484 \text{ (4dp)}$$

c $\tan x - 5x + 13 = 0$

$$\tan x = 5x - 13$$

$$x = \arctan(5x - 13)$$

$$\therefore x_{n+1} = \arctan(5x_n - 13)$$

$$x_1 = -1.518213$$

$$x_2 = -1.522270$$

$$x_3 = -1.522317 = -1.5223 \text{ (4dp)}$$

b $e^x - 8x + 5 = 0$

$$e^x = 8x - 5$$

$$x = \ln(8x - 5)$$

$$\therefore x_{n+1} = \ln(8x_n - 5)$$

$$x_1 = 2.944439$$

$$x_2 = 2.920767$$

$$x_3 = 2.910508 = 2.9105 \text{ (4dp)}$$

d $\ln x + \sqrt{x} + 1.4 = 0$

$$\ln x = -(\sqrt{x} + 1.4)$$

$$x = e^{-(\sqrt{x} + 1.4)}$$

$$\therefore x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$$

$$x_1 = 0.165299$$

$$x_2 = 0.164216$$

$$x_3 = 0.164436 = 0.1644 \text{ (4dp)}$$

2 a $e^{2x-1} - 6x = 0$

$$e^{2x-1} = 6x$$

$$2x - 1 = \ln 6x$$

$$x = \frac{1}{2}(\ln 6x + 1)$$

$$\therefore x_{n+1} = \frac{1}{2}(\ln 6x_n + 1), \quad a = \frac{1}{2}, \quad b = 6$$

$$x_1 = 1.661194$$

$$x_2 = 1.649648$$

$$x_3 = 1.646161 = 1.646 \text{ (3dp)}$$

c $2x^3 - 6x - 11 = 0$

$$2x^3 = 6x + 11$$

$$x^2 = 3 + \frac{11}{2x}$$

$$x = \pm \sqrt{3 + \frac{11}{2x}}$$

$$\therefore x_{n+1} = \sqrt{3 + \frac{5.5}{x_n}}, \quad a = 3, \quad b = 5.5$$

$$x_1 = 2.397916$$

$$x_2 = 2.300795$$

$$x_3 = 2.321740 = 2.322 \text{ (3dp)}$$

b $\frac{2}{x} + \cos x - 3 = 0$

$$\frac{2}{x} = 3 - \cos x$$

$$2 = x(3 - \cos x)$$

$$x = \frac{2}{3 - \cos x}$$

$$\therefore x_{n+1} = \frac{2}{3 - \cos x_n}, \quad a = 2, \quad b = 3$$

$$x_1 = 0.868322$$

$$x_2 = 0.849657$$

$$x_3 = 0.854789 = 0.855 \text{ (3dp)}$$

d $15 \ln(x + 3) - 4x = 0$

$$\ln(x + 3) = \frac{4}{15}x$$

$$x + 3 = e^{\frac{4}{15}x}$$

$$x = e^{\frac{4}{15}x} - 3$$

$$\therefore x_{n+1} = e^{\frac{4}{15}x_n} - 3, \quad a = \frac{4}{15}, \quad b = -3$$

$$x_1 = -2.486583$$

$$x_2 = -2.484743$$

$$x_3 = -2.484490 = -2.484 \text{ (3dp)}$$

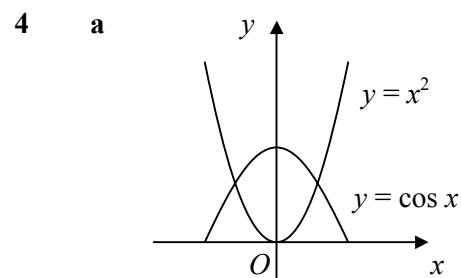
- 3**
- a** $x_1 = 0.428135$
 $x_2 = 0.433865$
 $x_3 = 0.431107$
 $x_4 = 0.432437$
 $x_5 = 0.431796$
 $\therefore \text{root} = 0.432 \text{ (3dp)}$
 $f(0.4315) = -0.00465$
 $f(0.4325) = 0.00457$
 sign change, $f(x)$ continuous $\therefore \text{root}$
- b** $x_1 = 0.474342$
 $x_2 = 0.470474$
 $x_3 = 0.469923$
 $\therefore \text{root} = 0.47 \text{ (2sf)}$
 $f(0.465) = -0.00428$
 $f(0.475) = 0.00463$
 sign change, $f(x)$ continuous $\therefore \text{root}$
- c** $x_1 = 5.892685$
 $x_2 = 5.859202$
 $x_3 = 5.850013$
 $x_4 = 5.847607$
 $x_5 = 5.846985$
 $x_6 = 5.846825$
 $\therefore \text{root} = 5.85 \text{ (3sf)}$
 $f(5.845) = 0.00658$
 $f(5.855) = -0.0305$
 sign change, $f(x)$ continuous $\therefore \text{root}$
- d** $x_1 = 3.731246$
 $x_2 = 3.724839$
 $x_3 = 3.726145$
 $x_4 = 3.725879$
 $\therefore \text{root} = 3.726 \text{ (3dp)}$
 $f(3.7255) = 0.000672$
 $f(3.7265) = -0.000912$
 sign change, $f(x)$ continuous $\therefore \text{root}$
- 4**
- a** $x_1 = -3.192595$
 $x_2 = -3.188214$
 $x_3 = -3.185620$
 $x_4 = -3.184084$
 $x_5 = -3.183174$
 $x_6 = -3.182635$
 $\therefore \text{root} = -3.18 \text{ (2dp)}$
- b** $x^5 - 10x^3 + 4 = 0$
 $4 = 10x^3 - x^5 = x^3(10 - x^2)$
 $x^3 = \frac{4}{10 - x^2}$
 $x = \sqrt[3]{\frac{4}{10 - x^2}} \therefore a = 4, b = 10$
- c** $x_1 = 0.763143$
 $x_2 = 0.751692$
 $x_3 = 0.751231$
 $x_4 = 0.751212$
 $\therefore \text{root} = 0.751 \text{ (3dp)}$
- 5**
- a** $\arcsin 2x - 0.5x - 0.7 = 0$
 $\arcsin 2x = 0.5x + 0.7$
 $2x = \sin(0.5x + 0.7)$
 $x = 0.5 \sin(0.5x + 0.7) \therefore a = 0.5, b = 0.5, c = 0.7$
- b** $x_1 = 0.391663$
 $x_2 = 0.390365$
 $x_3 = 0.390162$
 $x_4 = 0.390130$
 $\therefore \text{solution} = 0.390 \text{ (3dp)}$

- 1 a let $f(x) = x^3 - 7x - 11$
 $f(3) = -5$
 $f(4) = 25$
 sign change, $f(x)$ continuous \therefore root
 b $x_1 = 3.230712$
 $x_2 = 3.225651$
 $x_3 = 3.226479 = 3.23$ (2dp)

- 3 a $f(0.4) = -0.809$
 $f(0.5) = 0.307$
 sign change, $f(x)$ continuous \therefore root
 $\therefore 0.4 < \alpha < 0.5$
 b $x_1 = 0.468857$
 $x_2 = 0.463841$
 $x_3 = 0.465157$
 $x_4 = 0.464810$
 $\therefore \alpha = 0.465$ (3dp)

- 5 a $f(1.4) = 3.65$
 $f(1.5) = -0.205$
 sign change, $f(x)$ continuous \therefore root
 b $e^{5-2x} - x^5 = 0 \Rightarrow x^5 = e^{5-2x}$
 $\Rightarrow x = (e^{5-2x})^{\frac{1}{5}}$
 $\Rightarrow x = e^{1-\frac{2}{5}x}, k = \frac{2}{5}$
 c $x_1 = 1.491825$
 $x_2 = 1.496711$
 $x_3 = 1.493789 = 1.494$ (3dp)

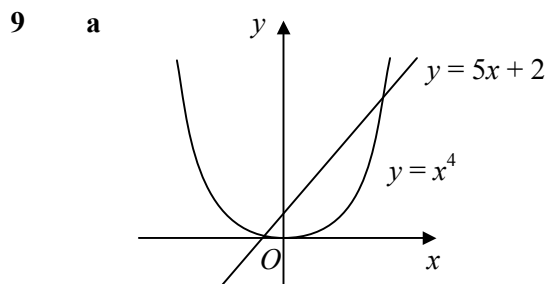
- 2 a $f(4) = -2.29$ (3sf)
 $f(5) = 0.829$ (3sf)
 b sign change, $f(x)$ continuous \therefore root
 c $4 \operatorname{cosec} x - 5 + 2x = 0$
 $2x = 5 - 4 \operatorname{cosec} x$
 $x = 2.5 - \frac{2}{\sin x}, a = 2.5, b = -2$
 d $x_1 = 4.545973$
 $x_2 = 4.528018$
 $x_3 = 4.534481 = 4.534$ (3dp)



- b $\cos x - x^2 = 0 \Rightarrow \cos x = x^2$
 the graphs $y = \cos x$ and $y = x^2$ intersect at 2 points, one for $x < 0$ and one for $x > 0$
 \therefore one negative and one positive real root
 c let $f(x) = \cos x - x^2$
 $f(0.8) = 0.0567$
 $f(0.9) = -0.188$
 sign change, $f(x)$ continuous \therefore root
 d $x_1 = 0.834690$
 $x_2 = 0.819395$
 $x_3 = 0.826235$
 $x_4 = 0.823195$
 $x_5 = 0.824550$
 \therefore root = 0.82 (2dp)
- 6 a $f(1.3) = -0.341$
 $f(1.4) = 0.383$
 sign change, $f(x)$ continuous \therefore root
 b $x_1 = 1.331571$
 $x_2 = 1.354168$
 $x_3 = 1.346907$
 $x_4 = 1.349261$
 c 1.35 (3sf)
 d diverges leading to \ln of a -ve which is not real

- 7 a $f'(x) = 6x^2 + 4$
 b for all real x , $x^2 \geq 0$
 $\Rightarrow 6x^2 + 4 > 0$
 $\therefore f(x)$ increasing for all x
 $\therefore y = f(x)$ only crosses x -axis once
 so exactly 1 real root
 c $f(1.2) = -0.744$
 $f(1.3) = 0.594$
 sign change, $f(x)$ continuous \therefore root
 d $x_1 = 1.280579$
 $x_2 = 1.246945$
 $x_3 = 1.261203$
 $x_4 = 1.255199$
 \therefore root $= 1.26$ (2dp)
 e $f(1.255) = -0.0267$
 $f(1.265) = 0.109$
 sign change, $f(x)$ continuous \therefore root

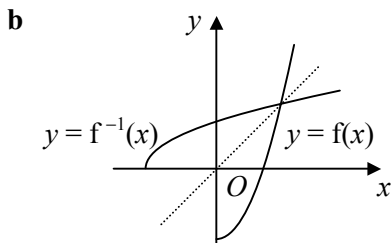
- 8 a $3x + \ln x - x^2 = x \Rightarrow \ln x = x^2 - 2x$
 $\Rightarrow x = e^{x^2 - 2x}$
 b let $f(x) = 2x + \ln x - x^2$
 $f(0.4) = -0.276$
 $f(0.5) = 0.0569$
 sign change, $f(x)$ continuous \therefore root
 c $f(2.3) = 0.143$
 $f(2.4) = -0.0845$
 sign change, $f(x)$ continuous \therefore root
 d $x_1 = 0.472367$
 $x_2 = 0.485973$
 $x_3 = 0.479134$
 $x_4 = 0.482537$
 \therefore x -coord of $A = 0.48$ (2dp)
 e $f(0.475) = -0.0201$
 $f(0.485) = 0.0112$
 sign change, $f(x)$ continuous \therefore root



- b $x^4 - 5x - 2 = 0 \Rightarrow x^4 = 5x + 2$
 the graphs $y = x^4$ and $y = 5x + 2$ intersect
 at 2 points, one for $x < 0$ and one for $x > 0$
 \therefore one negative and one positive real root
 c $x_1 = 1.821160$
 $x_2 = 1.825524$
 $x_3 = 1.826420$
 $x_4 = 1.826603 = 1.827$ (3dp)
 d $x^4 - 5x - 2 = 0 \Rightarrow x^4 - 5x = 2$
 $\Rightarrow x(x^3 - 5) = 2$
 $\Rightarrow x = \frac{2}{x^3 - 5}$, $a = 2$, $b = -5$
 e $x_1 = -0.394945$
 $x_2 = -0.395132$
 $x_3 = -0.395125$
 \therefore root $= -0.3951$ (4dp)

- 1 a $\frac{dy}{dx} = e^x + 2x$
 b at A, $x = 0 \therefore y = -3$, grad = 1
 $\therefore y = x - 3$
 c SP: $e^x + 2x = 0$
 let $f(x) = e^x + 2x$
 $f(-0.4) = -0.130$
 $f(-0.3) = 0.141$
 sign change, $f(x)$ continuous \therefore root
 \therefore x-coord of B in interval $[-0.4, -0.3]$
 d $x_1 = -0.34694$
 $x_2 = -0.35126$
 $x_3 = -0.35169$
 $x_4 = -0.35173$
 \therefore x-coord of B = -0.352 (3dp)
- 2 a $f(0) = 0.279$
 $f(5) = -4.10$
 $f(1) = 0.266$
 $f(3) = -2.44$
 $f(2) = -0.853$
 $\therefore k = 1$
 b $x_0 = 1$
 $x_1 = 1.2684$
 $x_2 = 1.3106$
 $x_3 = 1.3106$
- 3 a area of segment = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$
 $= \frac{1}{2}r^2(\theta - \sin\theta)$
 $\therefore \frac{1}{2}r^2\sin\theta = 4 \times \frac{1}{2}r^2(\theta - \sin\theta)$
 $\sin\theta = 4(\theta - \sin\theta)$
 $\sin\theta = 4\theta - 4\sin\theta$
 $4\theta - 5\sin\theta = 0$
 b $\theta_1 = 1.11401$
 $\theta_2 = 1.12184$
 $\theta_3 = 1.12613$
 $\theta_4 = 1.12844$
 $\theta_5 = 1.12968$
 $\therefore \theta = 1.13$ (2dp)
- 4 a $e^{x^2} - x - 3 = 0$
 $e^{x^2} = x + 3$
 $x^2 = \ln(x + 3)$
 $x = \sqrt{\ln(x + 3)} \therefore a = 1, b = 3$
 b e.g. $x_0 = 1.5$
 $x_1 = 1.226408$
 $x_2 = 1.200563$
 $x_3 = 1.198006$
 $x_4 = 1.197752$
 $x_5 = 1.197727$
 \therefore solution = 1.198 (3dp)

- 5 a $y = x^2 - 9$
 swap $x = y^2 - 9$
 $y = \pm \sqrt{x+9}$
 (domain $\Rightarrow +$)
 $f^{-1}(x) = \sqrt{x+9}$, $x \in \mathbb{R}$, $x \geq -9$
 range: $f^{-1}(x) \geq 0$



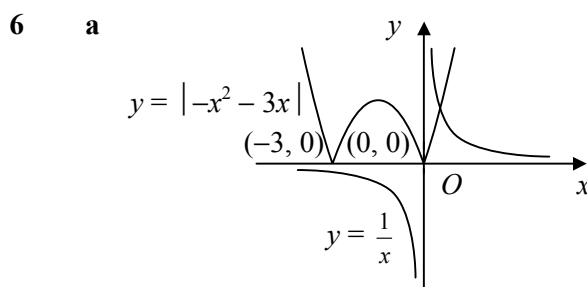
- c let $h(x) = f^{-1}(x) + g(x) = \sqrt{x+9} + x^3$
 $h(-2) = -5.35$
 $h(-1) = 1.83$
 sign change, $h(x)$ continuous \therefore root
 d $x_1 = -1.41421$, $x_2 = -1.40174$,
 $x_3 = -1.40212$, $x_4 = -1.40211$
 \therefore root $= -1.402$ (3dp)

- 7 a at A, $x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x = 0$
 let $f(x) = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x$
 $f(4) = -2$, $f(5) = 14.2$
 sign change, $f(x)$ continuous \therefore root
 $\therefore 4 < \alpha < 5$

- b $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$
 at B, $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$
 let $g(x) = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$
 $g(2) = -0.990$, $g(3) = 5.12$
 sign change, $g(x)$ continuous \therefore root
 $\therefore 2 < \beta < 3$

- c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$
 $5x^2 - 3 - 14x^{\frac{1}{2}} = 0$
 $x^2 = 0.6 + 2.8x^{\frac{1}{2}}$
 $x > 0 \therefore x = \beta$ is a soln to $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$

- d $x_1 = 2.158144$
 $x_2 = 2.171031$
 $x_3 = 2.173853$
 $x_4 = 2.174470$
 $x_5 = 2.174604$
 $\therefore \beta = 2.175$ (4sf)



- b $-(-x^2 - 3x) = \frac{1}{x}$
 $x^2 + 3x = \frac{1}{x}$
 $x^3 + 3x^2 = 1$
 $x^3 + 3x^2 - 1 = 0$
 c $x_1 = 0.57735$
 $x_2 = 0.52871$
 $x_3 = 0.53234$
 $x_4 = 0.53207$
 \therefore x-coord of P $= 0.532$ (3dp)

- 8 a $\frac{dy}{dx} = 3 - \frac{1}{x}$
 grad $= 2$
 \therefore grad of normal $= -\frac{1}{2}$
 $\therefore y - 3 = -\frac{1}{2}(x - 1)$
 $[y = \frac{7}{2} - \frac{1}{2}x]$

- b $3x - \ln x = \frac{7}{2} - \frac{1}{2}x$

$$6x - 2 \ln x = 7 - x$$

$$2 \ln x - 7x + 7 = 0$$

- c $2 \ln x = 7x - 7$
 $\ln x = \frac{7}{2}(x - 1)$
 $x = e^{\frac{7}{2}(x-1)} \therefore k = \frac{7}{2}$

- d $x_1 = 0.173774$
 $x_2 = 0.055477$
 $x_3 = 0.036669$
 $x_4 = 0.034333$
 $x_5 = 0.034053$
 \therefore x-coord of Q $= 0.034$ (3dp)

- e let $f(x) = 2 \ln x - 7x + 7$
 $f(0.0335) = -0.027$
 $f(0.0345) = 0.025$
 sign change, $f(x)$ continuous \therefore root