C3 TRIGONOMETRY — Worksheet A

1 Find to 2 decimal places the value of

a sec 23° **b** cosec 185° **c** cot 251.9° **d** sec (−302°)

2 Find the exact value of

a cosec 30° **b** cot 45° **c** sec 150° **d** cosec 300°

e cot 90° **f** sec 225° **g** cosec 270° **h** cot 330°

i sec 660° **j** cosec (−45°) **k** cot (−240°) **l** sec (−315°)

3 Find to 2 decimal places the value of

a cot 0.56^c **b** cosec 1.74^c **c** sec (-2.07^c) **d** cot 9.8^c

4 Find in exact form, with a rational denominator, the value of

a sec 0 **b** cosec $\frac{\pi}{4}$ **c** cot $\frac{3\pi}{4}$ **d** sec $\frac{4\pi}{3}$

e cosec $\frac{2\pi}{3}$ **f** cot $\frac{7\pi}{2}$ **g** sec $\frac{5\pi}{4}$ **h** cosec $(-\frac{5\pi}{6})$

i cot $\frac{11\pi}{6}$ **j** sec (-4π) **k** cosec $\frac{13\pi}{4}$ **l** cot $(-\frac{7\pi}{3})$

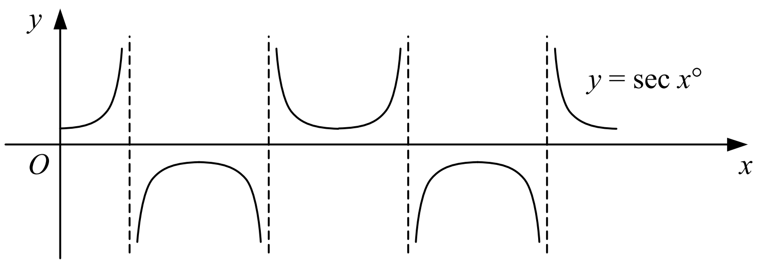
5 Given that $\sin x = \frac{4}{5}$ and that $0 < x < 90°$, find without using a calculator the value of

a $\cos x$ **b** $\tan x$ **c** $\operatorname{cosec} x$ **d** $\sec x$

6 Given that $\cos x = -\frac{5}{13}$ and that $90° < x < 180°$, find without using a calculator the value of

a $\sin x$ **b** $\sec x$ **c** $\operatorname{cosec} x$ **d** $\cot x$

7



The graph shows the curve $y = \sec x°$ in the interval $0 \le x \le 720$.

a Write down the coordinates of the turning points of the curve.

b Write down the equations of the asymptotes.

8 Sketch each pair of curves on the same set of axes in the interval $-180° \le x \le 180°$.

a $y = \sin x$ and $y = \operatorname{cosec} x$ **b** $y = \tan x$ and $y = \cot x$

9 Sketch each of the following curves for x in the interval $0 \le x \le 2\pi$. Show the coordinates of any turning points and the equations of any asymptotes.

a $y = 3 \sec x$ **b** $y = 1 + \operatorname{cosec} x$ **c** $y = \cot 2x$

d $y = \operatorname{cosec} (x - \frac{\pi}{4})$ **e** $y = \sec \frac{1}{3}x$ **f** $y = 3 + 2 \operatorname{cosec} x$

g $y = 1 - \sec 2x$ **h** $y = 2 \cot (x + \frac{\pi}{2})$ **i** $y = 1 + \sec (x - \frac{\pi}{6})$

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10 Solve each equation for x in the interval $0 \le x \le 2\pi$, giving your answers in terms of π.

a $\cot x = 1$ **b** $\sec x = 2$ **c** $\text{cosec } x = \sqrt{2}$ **d** $\cot x = 0$

e $\sec x = -1$ **f** $\text{cosec } x = -2$ **g** $\cot x = -\sqrt{3}$ **h** $\sec x = -\sqrt{2}$

11 Solve each equation for θ in the interval $0 \le \theta \le 360°$, giving your answers to 1 decimal place.

a $\sec \theta = 1.8$ **b** $\text{cosec } \theta = 2.57$ **c** $\cot \theta = 1.06$ **d** $\sec \theta = -2.63$

e $\text{cosec } \theta = 3$ **f** $\cot \theta = -0.94$ **g** $\sec \theta = 1.888$ **h** $\text{cosec } \theta = -1.2$

12 Solve each equation for x in the interval $-180 \le x \le 180$

Give your answers to 1 decimal place where appropriate

a $\text{cosec } (x + 30)° = 2$ **b** $\cot (x - 57)° = 1.6$ **c** $\sec 2x° = 2.35$

d $5 - 2 \cot x° = 0$ **e** $\sqrt{3} \sec (x - 60)° = 2$ **f** $2 \text{ cosec } \tfrac{1}{2}x° - 7 = 0$

g $\sec (2x - 18)° = -1.3$ **h** $\text{cosec } 3x° = -3.4$ **i** $\cot (2x + 135)° = 1$

13 Solve each equation for θ in the interval $0 \le \theta \le 360$.

Give your answers to 1 decimal place where appropriate.

a $\text{cosec}^2 \theta° - 4 = 0$ **b** $\sec^2 \theta° - 2 \sec \theta° - 3 = 0$

c $\cot \theta° \text{ cosec } \theta° = 6 \cot \theta°$ **d** $\text{cosec } \theta° = 4 \sec \theta°$

e $2 \cos \theta° = \cot \theta°$ **f** $5 \sin \theta° - 2 \text{ cosec } \theta° = 3$

14 Solve each equation for x in the interval $-\pi \le x \le \pi$.

Give your answers to 2 decimal places.

a $2 \text{ cosec}^2 x + 5 \text{ cosec } x - 12 = 0$ **b** $\sec x = 3 \tan x$

c $3 \sec x = 2 \cot x$ **d** $4 + \tan x = 5 \cot x$

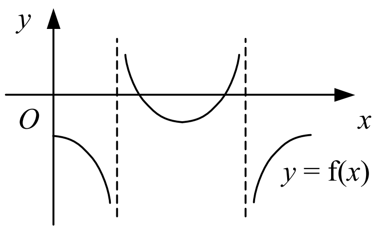
e $\text{cosec } x + 5 \cot x = 0$ **f** $6 \tan x - 5 \text{ cosec } x = 0$

15 Prove each identity.

a $\sec x - \cos x \equiv \sin x \tan x$ **b** $(1 + \cos x)(\text{cosec } x - \cot x) \equiv \sin x$

c $\dfrac{\cot x - \cos x}{1 - \sin x} \equiv \cot x$ **d** $(\sin x + \tan x)(\cos x + \cot x) \equiv (1 + \sin x)(1 + \cos x)$

16



The diagram shows the curve $y = \text{f}(x)$, where

$$\text{f}(x) \equiv 2 \cos x - 3 \sec x - 5, \; x \in \mathbb{R}, \; 0 \le x \le 2\pi.$$

a Find the coordinates of the point where the curve meets the y-axis.

b Find the coordinates of the points where the curve crosses the x-axis.

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1 $$f(x) \equiv \sin x,\ x \in \mathbb{R},\ -\tfrac{\pi}{2} \le x \le \tfrac{\pi}{2}.$$

a State the range of f.

b Define the inverse function $f^{-1}(x)$ and state its domain.

c Sketch on the same diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

2 Find, in radians in terms of π, the value of

a arcsin 0 **b** $\arcsin \frac{1}{\sqrt{2}}$ **c** arcsin (–1) **d** $\arcsin(-\frac{\sqrt{3}}{2})$

3 $$g(x) \equiv \cos x,\ x \in \mathbb{R},\ 0 \le x \le \pi.$$

a Define the inverse function $g^{-1}(x)$ and state its domain.

b Sketch on the same diagram the graphs of $y = g(x)$ and $y = g^{-1}(x)$.

4 $$h(x) \equiv \tan x,\ x \in \mathbb{R},\ -\tfrac{\pi}{2} < x < \tfrac{\pi}{2}.$$

a Define the inverse function $h^{-1}(x)$ and state its domain.

b Sketch on the same diagram the graphs of $y = h(x)$ and $y = h^{-1}(x)$.

5 Find, in radians in terms of π, the value of

a arccos 1 **b** $\arctan \sqrt{3}$ **c** $\arccos \frac{\sqrt{3}}{2}$ **d** $\arcsin(-\frac{1}{2})$

e arctan (–1) **f** arccos (–1) **g** $\arctan(-\frac{1}{\sqrt{3}})$ **h** $\arccos(-\frac{1}{\sqrt{2}})$

6 Find, in radians to 2 decimal places, the value of

a arcsin 0.6 **b** arccos 0.152 **c** arctan 4.7 **d** arcsin (–0.38)

e arccos 0.92 **f** arctan (–0.46) **g** arcsin (–0.506) **h** arccos (–0.75)

7 Solve

a $\arcsin x = \frac{\pi}{4}$ **b** $\arccos x = 0$ **c** $\arctan x = -\frac{\pi}{3}$

d $\arccos 2x = \frac{\pi}{6}$ **e** $\frac{\pi}{4} - \arctan x = 0$ **f** $6 \arcsin x + \pi = 0$

8 Solve each equation, giving your answers to 3 significant figures.

a $\arccos x = 2$ **b** $\arcsin x = -0.7$ **c** $\arctan 3x = 0.96$

d $1 - \arcsin x = 0$ **e** $2 + 3 \arctan x = 0$ **f** $3 - \arccos 2x = 0$

9 $$f(x) \equiv \arccos x - \tfrac{\pi}{3},\ x \in \mathbb{R},\ -1 \le x \le 1.$$

a State the value of $f(-\frac{1}{2})$ in terms of π.

b Solve the equation $f(x) = 0$.

c Define the inverse function $f^{-1}(x)$ and state its domain.

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C3 TRIGONOMETRY Worksheet C

1 Use the identity $\sin^2 x + \cos^2 x \equiv 1$ to obtain the identities

a $1 + \tan^2 x \equiv \sec^2 x$

b $1 + \cot^2 x \equiv \text{cosec}^2 x$

2 **a** Given that $\tan A = \frac{1}{3}$, find the exact value of $\sec^2 A$.

b Given that $\text{cosec } B = 1 + \sqrt{3}$, find the exact value of $\cot^2 B$.

c Given that $\sec C = \frac{3}{2}$, find the possible values of $\tan C$, giving your answers in the form $k\sqrt{5}$.

3 Solve each equation for θ in the interval $0 \le \theta \le 2\pi$ giving your answers in terms of π.

a $3 \sec^2 \theta = 4 \tan^2 \theta$

b $\tan^2 \theta - 2 \sec \theta + 1 = 0$

c $\cot^2 \theta - 3 \text{ cosec } \theta + 3 = 0$

d $\text{cosec}^2 \theta + \cot^2 \theta = 3$

e $\sec^2 \theta + 2 \tan \theta = 0$

f $\text{cosec}^2 \theta - \sqrt{3} \cot \theta - 1 = 0$

4 Solve each equation for x in the interval $-180° \le x \le 180°$.
Give your answers to 1 decimal place where appropriate.

a $\tan^2 x - 2 \sec x - 2 = 0$

b $2 \text{ cosec}^2 x + 2 = 9 \cot x$

c $\text{cosec}^2 x + 5 \text{ cosec } x + 2 \cot^2 x = 0$

d $3 \tan^2 x - 3 \tan x + \sec^2 x = 2$

e $\tan^2 x + 4 \sec x - 2 = 0$

f $2 \cot^2 x + 3 \text{ cosec}^2 x = 4 \cot x + 3$

5 Solve each equation for x in the interval $0 \le x \le 360°$.

a $\cot^2 2x + \text{cosec } 2x - 1 = 0$

b $8 \sin^2 x + \sec x = 8$

c $3 \text{ cosec}^2 x - 4 \sin^2 x = 1$

d $9 \sec^2 x - 8 = \text{cosec}^2 x$

6 Prove each of the following identities.

a $\text{cosec}^2 x - \sec^2 x \equiv \cot^2 x - \tan^2 x$

b $(\cot x - 1)^2 \equiv \text{cosec}^2 x - 2 \cot x$

c $(\cos x - 2 \sec x)^2 \equiv \cos^2 x + 4 \tan^2 x$

d $\sec^2 x - \sin^2 x \equiv \tan^2 x + \cos^2 x$

e $(\tan x + \cot x)^2 \equiv \sec^2 x + \text{cosec}^2 x$

f $(\sin x - \sec x)^2 \equiv \sin^2 x + (\tan x - 1)^2$

g $\sec^2 x + \text{cosec}^2 x \equiv \sec^2 x \text{ cosec}^2 x$

h $\sec^4 x + \tan^4 x \equiv 2 \sec^2 x \tan^2 x + 1$

7 Prove that there are no real values of x for which

$$4 \sec^2 x - \sec x + 2 \tan^2 x = 0.$$

8 **a** Prove the identity

$$\text{cosec } x \sec x - \cot x \equiv \tan x.$$

b Hence, or otherwise, find the values of x in the interval $0 \le x \le 360°$ for which

$$\text{cosec } x \sec x = 3 + \cot x,$$

giving your answers to 1 decimal place.

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1 **a** Write down the identities for $\sin (A + B)$ and $\cos (A + B)$.

b Use these identities to obtain similar identities for $\sin (A - B)$ and $\cos (A - B)$.

c Use the above identities to obtain similar identities for $\tan (A + B)$ and $\tan (A - B)$.

2 Express each of the following in the form $\sin \alpha$, where α is acute.

a $\sin 10° \cos 30° + \cos 10° \sin 30°$ **b** $\sin 67° \cos 18° - \cos 67° \sin 18°$

c $\sin 62° \cos 74° + \cos 62° \sin 74°$ **d** $\cos 14° \cos 39° - \sin 14° \sin 39°$

3 Express as a single trigonometric ratio

a $\cos A \cos 2A - \sin A \sin 2A$ **b** $\sin 4A \cos B - \cos 4A \sin B$

c $\dfrac{\tan 2A + \tan 5A}{1 - \tan 2A \tan 5A}$ **d** $\cos A \cos 3A + \sin A \sin 3A$

4 Find in exact form, with a rational denominator, the value of

a $\sin 15°$ **b** $\sin 165°$ **c** $\operatorname{cosec} 15°$ **d** $\cos 75°$

e $\cos 15°$ **f** $\sec 195°$ **g** $\tan 75°$ **h** $\operatorname{cosec} 105°$

5 Find the maximum value that each expression can take and the smallest positive value of x, in degrees, for which this maximum occurs.

a $\cos x \cos 30° + \sin x \sin 30°$ **b** $3 \sin x \cos 45° + 3 \cos x \sin 45°$

c $\sin x \cos 67° - \cos x \sin 67°$ **d** $4 \sin x \sin 108° - 4 \cos x \cos 108°$

6 Find the minimum value that each expression can take and the smallest positive value of x, in radians in terms of π, for which this minimum occurs.

a $\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3}$ **b** $2 \cos x \cos \frac{\pi}{6} - 2 \sin x \sin \frac{\pi}{6}$

c $\cos 4x \cos x + \sin 4x \sin x$ **d** $6 \sin 2x \cos 3x - 6 \sin 3x \cos 2x$

7 Given that $\sin A = \frac{4}{5}$, $0 < A < 90°$ and that $\cos B = \frac{2}{3}$, $0 < B < 90°$, find without using a calculator the value of

a $\tan A$ **b** $\sin B$ **c** $\cos (A + B)$ **d** $\sin (A + B)$

8 Given that $\operatorname{cosec} C = \frac{5}{3}$, $0 < C < 90°$ and that $\sin D = \frac{5}{13}$, $90° < D < 180°$, find without using a calculator the value of

a $\cos C$ **b** $\cos D$ **c** $\sin (C - D)$ **d** $\sec (C - D)$

9 Solve each equation for θ in the interval $0 \le \theta \le 360$.

Give your answers to 1 decimal place where appropriate.

a $\sin \theta° \cos 15° + \cos \theta° \sin 15° = 0.4$ **b** $\dfrac{\tan 2\theta° - \tan 60°}{1 + \tan 2\theta° \tan 60°} = 1$

c $\cos (\theta - 60)° = \sin \theta°$ **d** $2 \sin \theta° + \sin (\theta + 45)° = 0$

e $\sin (\theta + 30)° = \cos (\theta - 45)°$ **f** $3 \cos (2\theta + 60)° - \sin (2\theta - 30)° = 0$

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10 Find the value of k such that for all real values of x

$$\cos\left(x + \tfrac{\pi}{3}\right) - \cos\left(x - \tfrac{\pi}{3}\right) \equiv k \sin x.$$

11 Prove each identity.

a $\cos x - \cos\left(x - \tfrac{\pi}{3}\right) \equiv \cos\left(x + \tfrac{\pi}{3}\right)$

b $\sin\left(x - \tfrac{\pi}{6}\right) + \cos x \equiv \sin\left(x + \tfrac{\pi}{6}\right)$

12 **a** Use the identity for $\sin(A + B)$ to express $\sin 2A$ in terms of $\sin A$ and $\cos A$.

b Use the identity for $\cos(A + B)$ to express $\cos 2A$ in terms of $\sin A$ and $\cos A$.

c Hence, express $\cos 2A$ in terms of

i $\cos A$ **ii** $\sin A$

d Use the identity for $\tan(A + B)$ to express $\tan 2A$ in terms of $\tan A$.

13 Solve each equation for x in the interval $0 \le x \le 360°$.
Give your answers to 1 decimal place where appropriate.

a $\cos 2x + \cos x = 0$

b $\sin 2x + \cos x = 0$

c $2\cos 2x = 7\sin x$

d $11\cos x = 4 + 3\cos 2x$

e $\tan 2x - \tan x = 0$

f $\sec x - 4\sin x = 0$

g $5\sin 4x = 2\sin 2x$

h $2\sin^2 x - \cos 2x - \cos x = 0$

14 Prove each identity.

a $(\cos x + \sin x)^2 \equiv 1 + \sin 2x$

b $\tan x\,(1 + \cos 2x) \equiv \sin 2x$

c $\dfrac{2\sin x}{2\cos x - \sec x} \equiv \tan 2x$

d $\tan x + \cot x \equiv 2\operatorname{cosec} 2x$

e $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$

f $(\cos x + \sin x)(\operatorname{cosec} x - \sec x) \equiv 2\cot 2x$

g $\dfrac{1 - \sin 2x}{\operatorname{cosec} x - 2\cos x} \equiv \sin x$

h $\cos 3x \equiv 4\cos^3 x - 3\cos x$

15 Use the double angle identities to prove that

a $\cos x \equiv 2\cos^2 \dfrac{x}{2} - 1$

b $\sin^2 \dfrac{x}{2} \equiv \tfrac{1}{2}(1 - \cos x)$

16 **a** Given that $\cos A = \tfrac{7}{9}$, $0 < A < 90°$, find the exact value of $\sin \dfrac{A}{2}$ without using a calculator.

b Given that $\cos B = -\tfrac{3}{8}$, $90° < B < 180°$, find the value of $\cos \dfrac{B}{2}$, giving your answer in the form $k\sqrt{5}$.

17 Prove each identity.

a $\dfrac{2}{1 + \cos x} \equiv \sec^2 \dfrac{x}{2}$

b $\dfrac{1 + \cos x}{1 - \cos x} \equiv \cot^2 \dfrac{x}{2}$

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1 **a** Write down the identities for $\sin (A + B)$ and $\sin (A - B)$.

b Hence, express $2 \sin A \cos B$ in terms of $\sin (A + B)$ and $\sin (A - B)$.

c Use the identities for $\cos (A + B)$ and $\cos (A - B)$ to obtain similar expressions for $2 \cos A \cos B$ and $2 \sin A \sin B$.

2 Express each of the following as the sum or difference of trigonometric functions.

a $2 \sin 30° \cos 10°$ **b** $2 \cos 36° \cos 18°$

c $\cos 49° \sin 25°$ **d** $2 \sin 3A \sin A$

e $2 \cos 5A \sin 2A$ **f** $4 \cos 3A \cos B$

g $\sin A \cos 6B$ **h** $2 \cos A \sin (A + 40°)$

3 **a** Use the identity for $2 \sin A \cos B$ to prove that

$$\sin P + \sin Q \equiv 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}.$$

b Obtain similar identities for

i $\sin P - \sin Q$

ii $\cos P + \cos Q$

iii $\cos P - \cos Q$

4 Express each of the following as the product of trigonometric functions.

a $\cos 25° + \cos 15°$ **b** $\sin 84° - \sin 30°$

c $\sin 5A + \sin A$ **d** $\cos A - \cos 2A$

e $\cos 2A - \cos 4B$ **f** $\sin (A + 30°) + \sin (A + 60°)$

g $2 \cos A + 2 \cos 3A$ **h** $\sin (A + 2B) - \sin (3A - B)$

5 Solve each equation for x in the interval $0 \le x \le \pi$.

Give your answers to 2 decimal places where appropriate.

a $\sin 3x - \sin x = 0$ **b** $\cos x = \cos 4x$

c $2 \sin x \sin 5x = \cos 4x$ **d** $8 \cos (x + \frac{\pi}{3}) \sin (x + \frac{\pi}{6}) = 1$

e $\sin x + \sin \frac{x}{2} = 0$ **f** $\cos 3x + \cos x = \cos 2x$

6 Solve each equation for x in the interval $0 \le x \le 180°$.

a $2 \cos 2x \cos 3x - \cos x = 0$ **b** $\sin 3x - \sin 2x = 0$

c $\sin 4x + \sin 2x = \sin 3x$ **d** $\cos 2x = \cos (x - 60°)$

e $\cos 5x \sin x + \sin 4x = 0$ **f** $\sin x + \sin 3x = \cos x + \cos 3x$

7 Prove each identity.

a $\sin x + \sin 2x + \sin 3x \equiv \sin 2x (2 \cos x + 1)$

b $\dfrac{\cos x - \cos 3x}{\cos x + \cos 3x} \equiv \tan x \tan 2x$

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1 Express each of the following in the form $R\cos(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.
Give the values of R and α correct to 1 decimal place where appropriate.

a $\cos x° + \sin x°$ **b** $3\cos x° + 4\sin x°$

c $2\sin x° + \cos x°$ **d** $\cos x° + \sqrt{3}\sin x°$

2 Express each of the following in the given form, where $R > 0$ and $0 < \alpha < 90$.
Give the exact value of R and the value of α correct to 1 decimal place.

a $5\cos x° - 12\sin x°$, $R\cos(x + \alpha)°$ **b** $4\sin x° + 2\cos x°$, $R\sin(x + \alpha)°$

c $\sin x° - 7\cos x°$, $R\sin(x - \alpha)°$ **d** $8\cos 2x° - 15\sin 2x°$, $R\cos(2x + \alpha)°$

3 Express each of the following in the given form, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Give the exact value of R and the value of α correct to 2 decimal places where appropriate.

a $3\sin x - 2\cos x$, $R\sin(x - \alpha)$ **b** $3\cos x + \sqrt{3}\sin x$, $R\cos(x - \alpha)$

c $8\sin 3x + 6\cos 3x$, $R\sin(3x + \alpha)$ **d** $\cos x + \frac{1}{2}\sin x$, $R\cos(x - \alpha)$

4 Find the maximum value that each expression can take and the smallest positive value of x, in degrees, for which this occurs.

a $24\sin x - 7\cos x$ **b** $4\cos 2x + 4\sin 2x$

c $3\cos x - 5\sin x$ **d** $5\sin 3x + \cos 3x$

5 **a** Express $3\sin x° - 3\cos x°$ in the form $R\sin(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.

b Hence, describe two transformations that would map the graph of $y = \sin x°$ onto the graph of $y = 3\sin x° - 3\cos x°$.

6 By first expressing each curve in an appropriate form, sketch each of the following for x in the interval $0 \le x \le 360°$, showing the coordinates of any turning points.

a $y = 12\cos x + 5\sin x$ **b** $y = \sin x - 2\cos x$

c $y = 2\sqrt{3}\cos x - 6\sin x$ **d** $y = 9\sin x + 4\cos x$

7 **a** Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

b Solve the equation $\sqrt{3}\cos x - \sin x = 1$ for x in the interval $0 \le x \le 2\pi$, giving your answers in terms of π.

8 Solve each equation for x in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places.

a $6\sin x + 8\cos x = 5$ **b** $2\cos x - 2\sin x = 1$

c $7\sin x - 24\cos x - 10 = 0$ **d** $3\cos x + \sin x + 1 = 0$

e $\cos 2x + 4\sin 2x = 3$ **f** $5\sin x - 8\cos x + 7 = 0$

9 Solve each equation for x in the interval $-180° \le x \le 180°$, giving your answers to 1 decimal place where appropriate.

a $\sin x + \cos x = 1$ **b** $4\cos x - \sin x + 2 = 0$

c $\cos\frac{x}{2} + 5\sin\frac{x}{2} - 4 = 0$ **d** $6\sin x = 5 - 3\cos x$

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1 Find all values of x in the interval $0 \leq x \leq 360°$ for which

$$\tan^2 x - \sec x = 1.$$ **(6)**

2 **a** Express $2\cos x° + 5\sin x°$ in the form $R\cos(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.

Give the values of R and α to 3 significant figures. **(4)**

b Solve the equation

$$2\cos x° + 5\cos x° = 3,$$

for values of x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. **(4)**

3 **a** Solve the equation

$$\pi - 6\arctan 2x = 0,$$

giving your answer in the form $k\sqrt{3}$. **(4)**

b Find the values of x in the interval $0 \leq x \leq 360°$ for which

$$2\sin 2x = 3\cos x,$$

giving your answers to an appropriate degree of accuracy. **(6)**

4 **a** Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$\sin P - \sin Q \equiv 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}.$$ **(4)**

b Hence, or otherwise, find the values of x in the interval $0 \leq x \leq 180°$ for which

$$\sin 4x = \sin 2x.$$ **(6)**

5 **a** Prove the identity

$$(2\sin\theta - \operatorname{cosec}\theta)^2 \equiv \operatorname{cosec}^2\theta - 4\cos^2\theta, \quad \theta \neq n\pi, \ n \in \mathbb{Z}.$$ **(3)**

b **i** Sketch the curve $y = 3 + 2\sec x$ for x in the interval $0 \leq x \leq 2\pi$.

ii Write down the coordinates of the point where the curve meets the y-axis.

iii Find the coordinates of the points where the curve crosses the x-axis in this interval. **(7)**

6 **a** Find the exact values of R and α, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, for which

$$\cos x - \sin x \equiv R\cos(x + \alpha).$$ **(3)**

b Using the identity

$$\cos X + \cos Y \equiv 2\cos\frac{X+Y}{2}\cos\frac{X-Y}{2},$$

or otherwise, find in terms of π the values of x in the interval $[0, 2\pi]$ for which

$$\cos x + \sqrt{2}\cos(3x - \tfrac{\pi}{4}) = \sin x.$$ **(7)**

7 **a** Prove the identity

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \quad x \neq \frac{n\pi}{2}, \ n \in \mathbb{Z}.$$ **(4)**

b Hence, for x in the interval $0 \leq x \leq 2\pi$, solve the equation

$$\cot 2x + \operatorname{cosec} 2x = 6 - \cot^2 x,$$

giving your answers correct to 2 decimal places. **(6)**

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8 **a** Prove that for all real values of x

$$\cos (x + 30)° + \sin x° \equiv \cos (x - 30)°. \qquad \textbf{(4)}$$

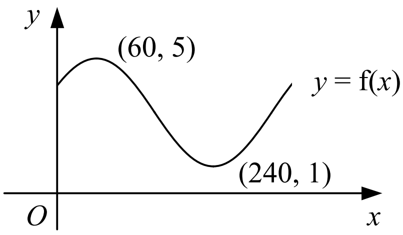
b Hence, find the exact value of $\cos 75° - \cos 15°$, giving your answer in the form $k\sqrt{2}$. **(3)**

c Solve the equation

$$3 \cos (x + 30)° + \sin x° = 3 \cos (x - 30)° + 1$$

for x in the interval $-180 \le x \le 180$. **(4)**

9



The diagram shows the curve $y = \text{f}(x)$ where

$$\text{f}(x) \equiv a + b \sin x° + c \cos x°, \; x \in \mathbb{R}, \; 0 \le x \le 360,$$

The curve has turning points with coordinates (60, 5) and (240, 1) as shown.

a State, with a reason, the value of the constant a. **(2)**

b Find the values of k and α, where $k > 0$ and $0 < \alpha < 90$, such that

$$\text{f}(x) = a + k \sin (x + \alpha)°. \qquad \textbf{(3)}$$

c Hence, or otherwise, find the exact values of the constants b and c. **(3)**

10 **a** Prove the identity

$$\frac{1 - \cos x}{1 + \cos x} \equiv \tan^2 \frac{x}{2}, \quad x \ne (2n + 1)\pi, \; n \in \mathbb{Z}. \qquad \textbf{(4)}$$

b Use the identity in part **a** to

i find the value of $\tan^2 \frac{\pi}{12}$ in the form $a + b\sqrt{3}$, where a and b are integers,

ii solve the equation

$$\frac{1 - \cos x}{1 + \cos x} = 1 - \sec \frac{x}{2},$$

for x in the interval $0 \le x \le 2\pi$, giving your answers in terms of π. **(9)**

11 **a** Prove that there are no real values of x for which

$$6 \cot^2 x - \text{cosec}\, x + 5 = 0. \qquad \textbf{(4)}$$

b Find the values of y in the interval $0 \le y \le 180°$ for which

$$\cos 5y = \cos y. \qquad \textbf{(6)}$$

12 **a** Use the identities for $\cos (A + B)$ and $\cos (A - B)$ to prove that

$$\sin A \sin B \equiv \tfrac{1}{2}[\cos (A - B) - \cos (A + B)]. \qquad \textbf{(2)}$$

b Hence, or otherwise, find the values of x in the interval $0 \le x \le \pi$ for which

$$4 \sin (x + \tfrac{\pi}{3}) = \text{cosec}\, (x - \tfrac{\pi}{6}),$$

giving your answers as exact multiples of π. **(7)**

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1 **a** Solve the equation

$$2 \sec x - 3 \operatorname{cosec} x = 0,$$

for x in the interval $-180° \leq x \leq 180°$. **(4)**

b Find all values of θ in the interval $0 \leq \theta \leq 2\pi$ for which

$$\cot^2 \theta - \cot \theta + \operatorname{cosec}^2 \theta = 4.$$ **(6)**

2 For values of θ in the interval $0 \leq \theta \leq 360°$, solve the equation

$$2 \sin (\theta + 30°) = \sin (\theta - 30°).$$ **(6)**

3 **a** Given that $\sin A = 2 - \sqrt{3}$, find in the form $a + b\sqrt{3}$ the exact value of

i $\operatorname{cosec} A$,

ii $\cot^2 A$. **(5)**

b Solve the equation

$$3 \cos 2x - 8 \sin x + 5 = 0,$$

for values of x in the interval $0 \leq x \leq 360°$, giving your answers to 1 decimal place. **(5)**

4 $$\mathrm{f} : x \to \tfrac{\pi}{2} + 2 \arcsin x, \quad x \in \mathbb{R}, \quad -1 \leq x \leq 1.$$

a Find the exact value of $\mathrm{f}(\tfrac{1}{2})$. **(2)**

b State the range of f. **(2)**

c Sketch the curve $y = \mathrm{f}(x)$. **(2)**

d Solve the equation $\mathrm{f}(x) = 0$. **(3)**

5 **a** Express $2 \sin x - 3 \cos x$ in the form $R \sin (x - \alpha)$, where $R > 0$ and $0 < \alpha < \tfrac{\pi}{2}$.

Give the values of R and α to 3 significant figures. **(4)**

b State the minimum value of $2 \sin x - 3 \cos x$ and the smallest positive value of x for which this minimum occurs. **(3)**

c Solve the equation

$$2 \sin 2x - 3 \cos 2x + 1 = 0,$$

for x in the interval $0 \leq x \leq \pi$, giving your answers to 2 decimal places. **(5)**

6 **a** Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 2 \cos^2 \frac{x}{2} - 1.$$ **(3)**

b Solve the equation

$$\frac{\sin x}{1 + \cos x} = 3 \cot \frac{x}{2},$$

for values of x in the interval $0 \leq x \leq 360°$. **(7)**

7 **a** Prove the identity

$$\text{cosec}\,\theta - \sin\theta \equiv \cos\theta\cot\theta,\quad \theta \neq n\pi,\ n \in \mathbb{Z}\,. \qquad \textbf{(3)}$$

b Find the values of x in the interval $0 \le x \le 2\pi$ for which

$$2\sec x + \tan x = 2\cos x,$$

giving your answers in terms of π. **(6)**

8 **a** Sketch on the same diagram the curves $y = 3\sin x°$ and $y = 1 + \text{cosec}\,x°$ for x in the interval $-180 \le x \le 180$. **(4)**

b Find the x-coordinate of each point where the curves intersect in this interval, giving your answers correct to 1 decimal place. **(6)**

9 **a** Prove the identity

$$(1 - \sin x)(\sec x + \tan x) \equiv \cos x,\quad x \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}. \qquad \textbf{(4)}$$

b Find the values of y in the interval $0 \le y \le \pi$ for which

$$2\sec^2 2y + \tan^2 2y = 3,$$

giving your answers in terms of π. **(6)**

10 **a** Express $4\sin x° - \cos x°$ in the form $R\sin(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.
Give the values of R and α to 3 significant figures. **(4)**

b Show that the equation

$$2\,\text{cosec}\,x° - \cot x° + 4 = 0 \qquad \text{(I)}$$

can be written in the form

$$4\sin x° - \cos x° + 2 = 0. \qquad \textbf{(2)}$$

c Using your answers to parts **a** and **b**, solve equation (I) for x in the interval $0 \le x \le 360$. **(4)**

11 **a** Use the identities

$$\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$$

and

$$\cos(A - B) \equiv \cos A\cos B + \sin A\sin B$$

to prove that

$$\cos P + \cos Q \equiv 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}. \qquad \textbf{(4)}$$

b Find, in terms of π, the values of x in the interval $0 \le x \le 2\pi$ for which

$$\cos x + \cos 2x + \cos 3x = 0. \qquad \textbf{(7)}$$

12 **a** Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

b Given that the function f is defined by

$$\text{f}(\theta) \equiv 1 - 3\cos 2\theta - 4\sin 2\theta,\quad \theta \in \mathbb{R},\ 0 \le \theta \le \pi,$$

i state the range of f,

ii solve the equation $\text{f}(\theta) = 0$. **(6)**

c Find the coordinates of the turning points of the curve with equation $y = \dfrac{2}{3\cos x + 4\sin x}$ in the interval $[0, 2\pi]$. **(3)**