1 **a** $f'(x) = 24 + 6x - 3x^2$

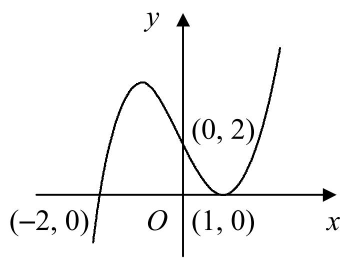
b $24 + 6x - 3x^2 \geq 0$
$x^2 - 2x - 8 \leq 0$
$(x + 2)(x - 4) \leq 0$
$-2 \leq x \leq 4$

2 **a** $\frac{dy}{dx} = 12x^2 + 18x - 12$
SP: $12x^2 + 18x - 12 = 0$
$2x^2 + 3x - 2 = 0$
$(2x - 1)(x + 2) = 0$
$x = -2, \frac{1}{2}$
$\therefore$ $(-2, 26)$ and $(\frac{1}{2}, -\frac{21}{4})$

b $\frac{d^2y}{dx^2} = 24x + 18$
$(-2, 26)$: $\frac{d^2y}{dx^2} = -30$ $\therefore$ maximum
$(\frac{1}{2}, -\frac{21}{4})$: $\frac{d^2y}{dx^2} = 30$ $\therefore$ minimum

3 $= \frac{d}{dx}(x^2 + \frac{1}{2}x^{-1})$
$= 2x - \frac{1}{2}x^{-2}$

4 **a**



b $f(x) = (x + 2)(x^2 - 2x + 1)$
$= x^3 - 2x^2 + x + 2x^2 - 4x + 2$
$= x^3 - 3x + 2$
$f'(x) = 3x^2 - 3$

c $x = 0$ $\therefore$ $y = 2$, grad $= -3$
$\therefore$ $y - 2 = -3(x - 0)$
$y = 2 - 3x$

5 **a** $x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, 1 \quad a < b \;\therefore\; a = -2,\; b = 1$

b $\frac{dy}{dx} = 2x + 1$

grad at $A = -3$

$\therefore$ grad of normal $= \frac{1}{3}$

$\therefore\; y - 0 = \frac{1}{3}(x + 2)$

$3y = x + 2$

$x - 3y + 2 = 0$

c grad at $B = 3$

tangent at B: $y - 0 = 3(x - 1)$

$y = 3x - 3$

at C, $x - 3(3x - 3) + 2 = 0$

$x = \frac{11}{8}$

$\therefore\; C\left(\frac{11}{8}, \frac{9}{8}\right)$

6 **a** $\frac{dy}{dx} = 2x + \frac{3}{2}x^{-\frac{1}{2}}$

b $\frac{d^2y}{dx^2} = 2 - \frac{3}{4}x^{-\frac{3}{2}}$

$\therefore\; 2x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x$

$= 2x(2 - \frac{3}{4}x^{-\frac{3}{2}}) + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$

$= 4x - \frac{3}{2}x^{-\frac{1}{2}} + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$

$= 0$

7 **a** $\frac{dy}{dx} = -4x^{-2}$

grad at $M = -\frac{1}{4}$

$\therefore$ grad of normal $= 4$

$\therefore\; y - 3 = 4(x - 4) \quad [\,y = 4x - 13\,]$

b $4x - 13 = 2 + \frac{4}{x}$

$4x^2 - 15x - 4 = 0$

$(4x + 1)(x - 4) = 0$

$x = 4$ (at M) or $-\frac{1}{4}$

$\therefore\; N\left(-\frac{1}{4}, -14\right)$

8 **a** $\frac{dy}{dx} = 4x - 7$

at A, $y = -5$, grad $= 1$

$\therefore\; y + 5 = 1(x - 2)$

$[\,y = x - 7\,]$

b grad of normal at $B = 1$

$\therefore$ grad of curve at $B = -1$

$\therefore\; 4x - 7 = -1$

$x = \frac{3}{2},\; y = 2(\frac{9}{4}) - 7(\frac{3}{2}) + 1 = -5$

$\therefore\; B\left(\frac{3}{2}, -5\right)$

9 **a** $f'(x) = 2x - 16x^{-2}$

b SP: $2x - 16x^{-2} = 0$

$x^3 = 8$

$x = 2$

$\therefore$ (2, 12)

$f''(x) = 2 + 32x^{-3}$

$f''(2) = 6$

$f''(x) > 0 \;\therefore$ minimum

10 **a** $\frac{dy}{dx} = 3x^2 - 2x + 2$

at $(1, -2)$, grad $= 3$

$\therefore\; y + 2 = 3(x - 1)$

$3x - y - 5 = 0$

b SP when $3x^2 - 2x + 2 = 0$

$b^2 - 4ac = 4 - 24 = -20$

$b^2 - 4ac < 0 \;\therefore$ no real roots

$\therefore$ no stationary points

11 **a** $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$

grad at $P = \frac{1}{4}$

$\therefore\ y - 1 = \frac{1}{4}(x - 4)$

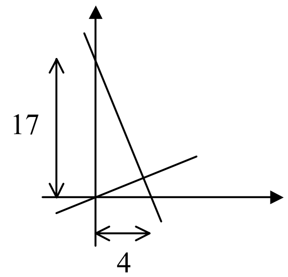
$y = \frac{1}{4}x$ which passes through (0, 0)

b grad of normal = –4

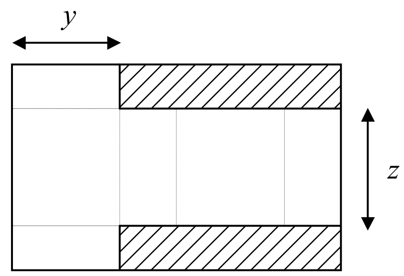
$\therefore\ y - 1 = -4(x - 4)$ $[\,y = 17 - 4x]$

at Q, $x = 0 \Rightarrow y = 17$

$\therefore$ area $= \frac{1}{2} \times 17 \times 4 = 34$



12 **a**



$2x + z = 25$

$2x + 2y = 40$

$\therefore$ length and width $(25 - 2x)$ and $(20 - x)$

b volume $= x(25 - 2x)(20 - x)$

$= x(500 - 65x + 2x^2)$

$= 2x^3 - 65x^2 + 500x$

c $\frac{dV}{dx} = 6x^2 - 130x + 500$

SP: $6x^2 - 130x + 500 = 0$

$2(3x - 50)(x - 5) = 0$

$x = 5, \frac{50}{3}$

$2x < 25 \ \therefore\ x < 12.5$

$\therefore\ x = 5$

d max volume = 1125 cm^3

$\frac{d^2V}{dx^2} = 12x - 130$

when $x = 5$, $\frac{d^2V}{dx^2} = -70$

$\frac{d^2V}{dx^2} < 0 \ \therefore$ maximum

13 **a** $\frac{dy}{dx} = 9 + 6x - 3x^2$

SP: $9 + 6x - 3x^2 = 0$

$-3(x + 1)(x - 3) = 0$

$x = -1, 3$

$\therefore\ (-1, -3)$ and $(3, 29)$

b $\frac{d^2y}{dx^2} = 6 - 6x$

$(-1, -3)$: $\frac{d^2y}{dx^2} = 12 \ \therefore$ minimum

$(3, 29)$: $\frac{d^2y}{dx^2} = -12 \ \therefore$ maximum

c $-3 < k < 29$