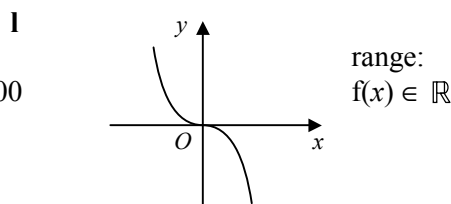
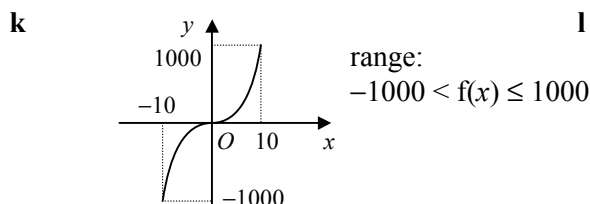
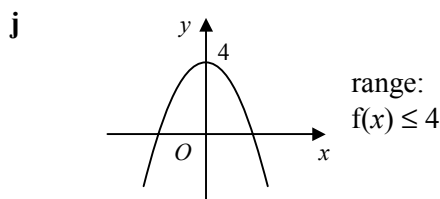
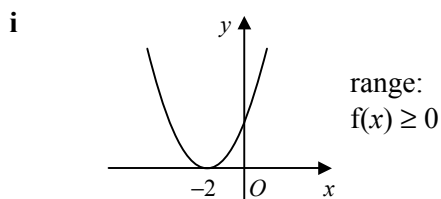
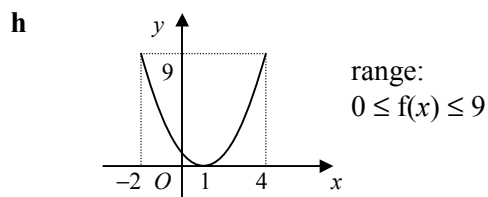
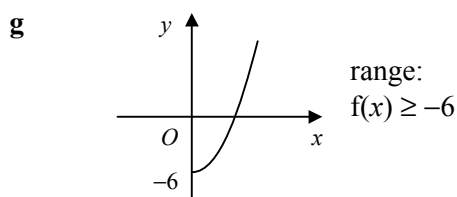
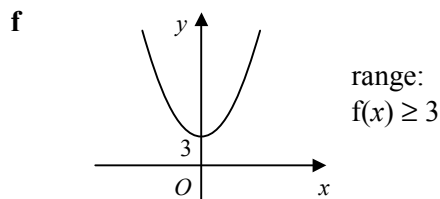
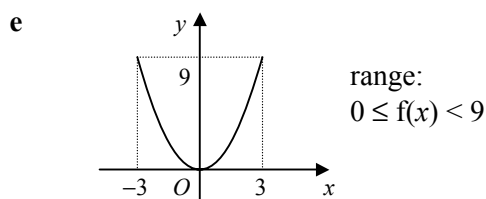
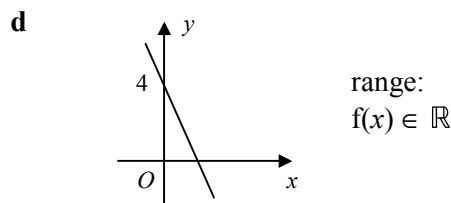
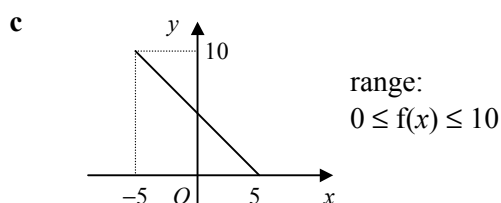
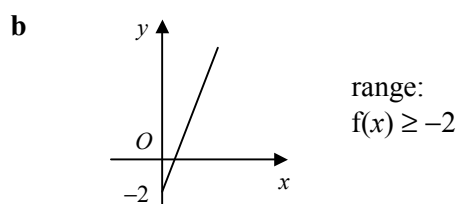
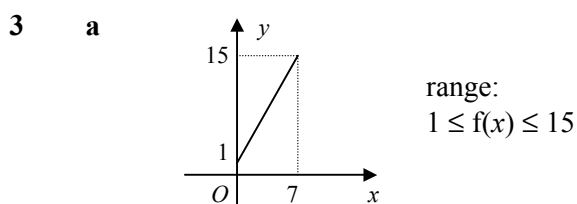
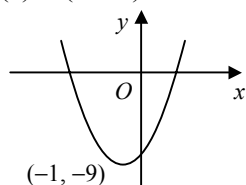


1 **a** 4 **b** 2 **c** 11 **d** -2 **e** -4 **f** -2
 g $\frac{2}{5}$ **h** -3 **i** $\frac{5}{4}$ **j** -8 **k** -4 **l** $\frac{12}{13}$

2 **a** $= \sin \pi$ **b** $= \ln 2$ **c** $= 5$ **d** $= \sin \frac{2\pi}{3}$ **e** $= 3 + 2e^{-1}$ **f** $= \ln \frac{9}{2}$
 $= 0$ $= 0.693$ $= \frac{\sqrt{3}}{2}$ or 0.866 $= 3.74$ $= 1.50$
 g $= 3 + 2e^{1.8}$ **h** $= \ln 1$ **i** $= \sin(0.6 + \frac{\pi}{3})$ **j** $= 3 + 2e^{\frac{1}{3}}$ **k** $= \sin(\frac{\pi}{3} - 2)$ **l** $= \ln \frac{23}{4}$
 $= 15.1$ $= 0$ $= 0.997$ $= 5.79$ $= -0.815$ $= 1.75$

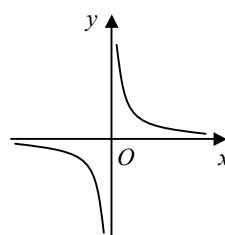


4 a $f(x) = (x+1)^2 - 9 \therefore (-1, -9)$



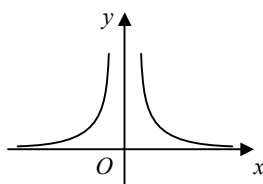
range:
 $f(x) \geq -9$

b



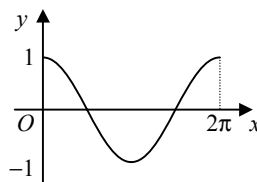
range:
 $f(x) \in \mathbb{R}, f(x) \neq 0$

c



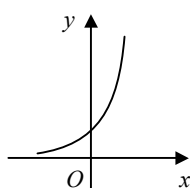
range:
 $f(x) > 0$

d



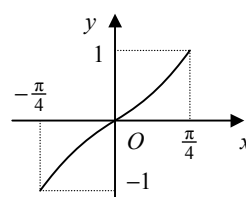
range:
 $-1 \leq f(x) \leq 1$

e



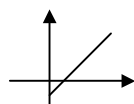
range:
 $f(x) > 0$

f



range:
 $-1 \leq f(x) \leq 1$

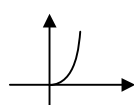
5 a $f(0) = -1, f(7) = 6$
 $\therefore 0 \leq x < 7$



b $f(0) = 4$
 $\therefore x \geq 0$



c $f(0) = 0, f(5) = 125$
 $\therefore 0 \leq x \leq 5$



d $f(\frac{1}{2}) = 2, f(\frac{1}{10}) = 10$
 $\therefore \frac{1}{10} < x < \frac{1}{2}$



6 a $4x + 3 = 9$

$$x = \frac{3}{2}$$

b $x^2 - 7 = 18$

$$x^2 = 25$$

$$x = \pm 5$$

c $\frac{9}{x+2} = 6$

$$6x + 12 = 9$$

$$x = -\frac{1}{2}$$

d $4x + 3 = \frac{9}{x+2}$

$$(4x+3)(x+2) = 9$$

$$4x^2 + 11x - 3 = 0$$

$$(4x-1)(x+3) = 0$$

$$x = -3, \frac{1}{4}$$

e $x^2 - 7 - \frac{x+2}{9} = -\frac{19}{3}$

$$9x^2 - 63 - x - 2 = -57$$

$$9x^2 - x - 8 = 0$$

$$(9x+8)(x-1) = 0$$

$$x = -\frac{8}{9}, 1$$

f $4x + 3 + x^2 - 7 = 0$

$$x^2 + 4x - 4 = 0$$

$$x = \frac{-4 \pm \sqrt{16+16}}{2}$$

$$x = -2 \pm 2\sqrt{2}$$

$$\text{or } -4.83, 0.828 \text{ (3sf)}$$

7 a $f(x) = (x+2)^2 - 4 + 11 = (x+2)^2 + 7$ range: $f(x) \geq 7$

b $f(x) = (x-1)^2 - 1 - 6 = (x-1)^2 - 7$ range: $f(x) \geq -7$

c $f(x) = (2x+3)^2 - 9 + 3 = (2x+3)^2 - 6$ range: $f(x) \geq -6$

d $f(x) = (3x-1)^2 - 1 + 16 = (3x-1)^2 + 15$ range: $f(x) \geq 15$

e $f(x) = 15 - [x^2 + 4x] = 15 - [(x+2)^2 - 4] = 19 - (x+2)^2$ range: $f(x) \leq 19$

- 1 **a** = $g(5) = -3$ **b** = $g(6) = -4$ **c** = $f(5) = 17$ **d** = $h(9) = 86$
 e = $g(-3) = 5$ **f** = $f(-1) = -7$ **g** = $h(-6) = 41$ **h** = $f(\frac{29}{4}) = 26$
- 2 **a** = $f(\ln 20) = 17.0$ **b** = $g(\ln 3) = 0.455$ **c** = $f(\cos 5) = 3.42$ **d** = $g(\cos(-4)) = 0.794$
 e = $g(\frac{43}{4}) = -0.243$ **f** = $h(\cos 6.7) = -0.0895$ **g** = $h(\ln 50) = 1.36$ **h** = $h(0.5) = -0.693$
- 3 **a** = $f(1-3x)$
 = $2(1-3x) + 1$
 $fg : x \rightarrow 3-6x,$
 $x \in \mathbb{R}$
 b = $f(2x+1)$
 = $2(2x+1) + 1$
 $ff : x \rightarrow 4x+3,$
 $x \in \mathbb{R}$
 c = $f(x^2+4)$
 = $2(x^2+4) + 1$
 $fh : x \rightarrow 2x^2+9,$
 $x \in \mathbb{R}$
 d = $h(2x+1)$
 = $(2x+1)^2 + 4$
 $hf : x \rightarrow 4x^2+4x+5,$
 $x \in \mathbb{R}$
 e = $g(x^2+4)$
 = $1-3(x^2+4)$
 $gh : x \rightarrow -3x^2-11,$
 $x \in \mathbb{R}$
 f = $g(1-3x)$
 = $1-3(1-3x)$
 $gg : x \rightarrow 9x-2,$
 $x \in \mathbb{R}$
 g = $h(1-3x)$
 = $(1-3x)^2 + 4$
 $hg : x \rightarrow 9x^2-6x+5,$
 $x \in \mathbb{R}$
 h = $g(2x+1)$
 = $1-3(2x+1)$
 $gf : x \rightarrow -6x-2,$
 $x \in \mathbb{R}$
- 4 **a** = $g(4-x)$
 = e^{4-x}
 $gf : x \rightarrow e^{4-x},$
 $x \in \mathbb{R}$
 b = $h(e^x)$
 = $2(e^x)^2 + 7$
 $hg : x \rightarrow 2e^{2x} + 7,$
 $x \in \mathbb{R}$
 c = $f(2x^2+7)$
 = $4-(2x^2+7)$
 $fh : x \rightarrow -2x^2-3,$
 $x \in \mathbb{R}$
 d = $g(e^x)$
 = e^{e^x}
 $gg : x \rightarrow e^{e^x},$
 $x \in \mathbb{R}$
 e = $g(2x^2+7)$
 = e^{2x^2+7}
 $gh : x \rightarrow e^{2x^2+7},$
 $x \in \mathbb{R}$
 f = $f(4-x)$
 = $4-(4-x)$
 $ff : x \rightarrow x,$
 $x \in \mathbb{R}$
 g = $f(e^x)$
 = $4-e^x$
 $fg : x \rightarrow 4-e^x,$
 $x \in \mathbb{R}$
 h = $h(4-x)$
 = $2(4-x)^2 + 7$
 $hf : x \rightarrow 2x^2-16x+39,$
 $x \in \mathbb{R}$
- 5 **a** $ff(x) = f(5x-3)$
 = $5(5x-3) - 3$
 = $25x - 18$
 $25x - 18 = -8$
 $x = \frac{2}{5}$
 b $hf(x) = h(5x-3)$
 = $\frac{1}{5x-3-2}$
 = $\frac{1}{5x-5}$
 $\frac{1}{5x-5} = 2$
 $1 = 10x - 10$
 $x = \frac{11}{10}$
 c $gf(x) = g(5x-3)$
 = $3(5x-3)^2 + 1$
 = $75x^2 - 90x + 28$
 $75x^2 - 90x + 28 = 28$
 $15x(5x-6) = 0$
 $x = 0, \frac{6}{5}$
 d $hg(x) = h(3x^2+1)$
 = $\frac{1}{3x^2+1-2}$
 = $\frac{1}{3x^2-1}$
 $\frac{1}{3x^2-1} = \frac{1}{2}$
 $3x^2 - 1 = 2$
 $x^2 = 1$
 $x = \pm 1$
 e $fh(x) = f(\frac{1}{x-2})$
 = $\frac{5}{x-2} - 3$
 $\frac{5}{x-2} - 3 = 7$
 $\frac{5}{x-2} = 10$
 $5 = 10x - 20$
 $x = \frac{5}{2}$
 f $fg(x) = f(3x^2+1)$
 = $5(3x^2+1) - 3$
 = $15x^2 + 2$
 $15x^2 + 2 = 32$
 $x^2 = 2$
 $x = \pm\sqrt{2}$
 g $gh(x) = g(\frac{1}{x-2})$
 = $\frac{3}{(x-2)^2} + 1$
 $\frac{3}{(x-2)^2} + 1 = 4$
 $(x-2)^2 = 1$
 $x-2 = \pm 1$
 $x = 1, 3$
 h $hh(x) = h(\frac{1}{x-2})$
 = $\frac{1}{\frac{1}{x-2}-2} = \frac{x-2}{1-2(x-2)}$
 = $\frac{x-2}{5-2x}$
 $\frac{x-2}{5-2x} = -2$
 $x-2 = -10+4x$
 $x = \frac{8}{3}$

6 a $gh(x) = g(e^x)$ $= 3 + 2e^x$ $3 + 2e^x = 9$ $e^x = 3$ $x = \ln 3$ $x = 1.10$	b $fg(x) = f(3 + 2x)$ $= \ln(3 + 2x)$ $\ln(3 + 2x) = 3.6$ $3 + 2x = e^{3.6}$ $x = \frac{1}{2}(e^{3.6} - 3)$ $x = 16.80$	c $hg(x) = h(3 + 2x)$ $= e^{3+2x}$ $e^{3+2x} = 4$ $3 + 2x = \ln 4$ $x = \frac{1}{2}(\ln 4 - 3)$ $x = -0.81$	d $gf(x) = g(\ln x)$ $= 3 + 2 \ln x$ $3 + 2 \ln x = 10.4$ $\ln x = 3.7$ $x = e^{3.7}$ $x = 40.45$
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7 **a** $g(x) > 0$

b $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

8 **a** $= f(4) = 7$

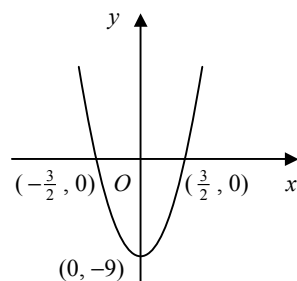
b $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

c $fg(x) = f(x^2) = 4x^2 - 9$



9 **a** $= g(1) = 4$

b $= h(2) = e^3$

c $= g(e^{-3}) = 1$

d $= f(\tan 1) = 74.7 \text{ (3sf)}$

e $= h(\tan 0.2)$
 $= 0.552 \text{ (3sf)}$

f $= f(4 + \ln 7)$
 $= -0.351 \text{ (3sf)}$

g $= h(e^{-\frac{1}{2}})$
 $= 1.24 \text{ (3sf)}$

h $= f(4 + e)$
 $= 0.465 \text{ (3sf)}$

10 **a** $= f(4x + 1)$
 $= 3e^{4x+1} + 2$

b $= g(3e^x + 2)$
 $= 4(3e^x + 2) + 1$

c $= h(3e^x + 2)$
 $= \frac{1}{3e^x + 2 + 1}$

d $= g(4x + 1)$
 $= 4(4x + 1) + 1$

$$fg : x \rightarrow 3e^{4x+1} + 2,$$

 $x \in \mathbb{R}$

$$gf : x \rightarrow 12e^x + 9,$$

 $x \in \mathbb{R}$

$$hf : x \rightarrow \frac{1}{3e^x + 3},$$

 $x \in \mathbb{R}$

$$gg : x \rightarrow 16x + 5,$$

 $x \in \mathbb{R}$

e $= h(4x + 1)$
 $= \frac{1}{4x+1+1}$

$$hg : x \rightarrow \frac{1}{4x+2},$$

 $x \in \mathbb{R}, x \neq -\frac{1}{2}$

f $= g\left(\frac{1}{x+1}\right)$
 $= \frac{4}{x+1} + 1$

$$= \frac{4+x+1}{x+1}$$

 $gh : x \rightarrow \frac{x+5}{x+1},$
 $x \in \mathbb{R}, x \neq -1$

g $= h\left(\frac{1}{x+1}\right)$
 $= \frac{1}{\frac{1}{x+1} + 1}$

$$= \frac{x+1}{1+x+1}$$

 $hh : x \rightarrow \frac{x+1}{x+2},$
 $x \in \mathbb{R}, x \neq -1, -2$

h $= g(16x + 5)$
 $= 4(16x + 5) + 1$

$$ggg : x \rightarrow 64x + 21,$$

 $x \in \mathbb{R}$

11 a $fh(x) = f\left(\frac{x+1}{3}\right)$ b $fg(x) = f(e^{1+2x})$ c $gh(x) = g\left(\frac{x+1}{3}\right)$ d $hh(x) = h\left(\frac{x+1}{3}\right)$

$$= \sqrt{\frac{x+1}{3} + 4}$$

$$= \sqrt{\frac{x+13}{3}}$$

$$\sqrt{\frac{x+13}{3}} = 3$$

$$\frac{x+13}{3} = 9$$

$$x+13 = 27$$

$$x = 14$$

$$= \sqrt{e^{1+2x} + 4}$$

$$\sqrt{e^{1+2x} + 4} = 7$$

$$e^{1+2x} + 4 = 49$$

$$e^{1+2x} = 45$$

$$1+2x = \ln 45$$

$$x = \frac{1}{2}(\ln 45 - 1)$$

$$x = 1.40 \text{ (3sf)}$$

$$= e^{1+\frac{2(x+1)}{3}}$$

$$= e^{\frac{2x+5}{3}}$$

$$e^{\frac{2x+5}{3}} = 11$$

$$\frac{2x+5}{3} = \ln 11$$

$$2x+5 = 3 \ln 11$$

$$x = \frac{1}{2}(3 \ln 11 - 5)$$

$$x = 1.10 \text{ (3sf)}$$

$$= \frac{\frac{x+1}{3} + 1}{3}$$

$$= \frac{x+1+3}{9}$$

$$= \frac{x+4}{9}$$

$$\frac{x+4}{9} = \frac{2}{3}$$

$$3x+12 = 18$$

$$x = 2$$

e $hg(x) = h(e^{1+2x})$ f $hf(x) = h(\sqrt{x+4})$ g $ff(x) = f(\sqrt{x+4})$ h $ghh(x) = g\left(\frac{x+4}{9}\right)$

$$= \frac{e^{1+2x} + 1}{3}$$

$$\frac{e^{1+2x} + 1}{3} = 1.2$$

$$e^{1+2x} = 2.6$$

$$1+2x = \ln 2.6$$

$$x = \frac{1}{2}(\ln 2.6 - 1)$$

$$x = -0.0222 \text{ (3sf)}$$

$$= \frac{\sqrt{x+4} + 1}{3}$$

$$\frac{\sqrt{x+4} + 1}{3} = \frac{1}{2}$$

$$\sqrt{x+4} = \frac{1}{2}$$

$$x+4 = \frac{1}{4}$$

$$x = -3\frac{3}{4}$$

$$= \sqrt{\sqrt{x+4} + 4}$$

$$\sqrt{\sqrt{x+4} + 4} = 3$$

$$\sqrt{x+4} = 5$$

$$x+4 = 25$$

$$x = 21$$

$$= e^{1+\frac{2(x+4)}{9}}$$

$$= e^{\frac{2x+17}{9}}$$

$$e^{\frac{2x+17}{9}} = \frac{1}{2}$$

$$\frac{2x+17}{9} = \ln \frac{1}{2}$$

$$x = \frac{1}{2}(9 \ln \frac{1}{2} - 17)$$

$$x = -11.6 \text{ (3sf)}$$

12 a $h(x) = fg(x)$

b $h(x) = gf(x)$

c $h(x) = gg(x)$

d $h(x) = ff(x)$

e $h(x) = gff(x)$

f $h(x) = gfg(x)$

13 a $j(x) = fg(x)$

b $j(x) = hf(x)$

c $j(x) = gh(x)$

d $j(x) = gg(x)$

e $j(x) = fhg(x)$

f $j(x) = hfg(x)$

14 a $gf(x) = g(5^x - 7)$
 $= 2(5^x - 7) + 3$
 $= 2(5^x) - 11$

$gf: x \rightarrow 2(5^x) - 11, x \in \mathbb{R}$

b $2(5^x) - 11 = 10$

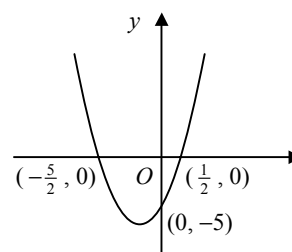
$$5^x = \frac{21}{2}$$

$$x = \frac{\ln \frac{21}{2}}{\ln 5} = 1.46 \text{ (3sf)}$$

15 a $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$
 $gf: x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$

range: $gf(x) \geq -9$

b $gf(x) = (2x+5)(2x-1)$



c $gf(x) - 2f(x) = a$

$$4x^2 + 8x - 5 - 2[2(x+1)] = a$$

$$4x^2 + 4x - (a+9) = 0$$

no real roots $\therefore b^2 - 4ac < 0$

$$16 + 16(a+9) < 0$$

$$1 + a + 9 < 0$$

$$a < -10$$

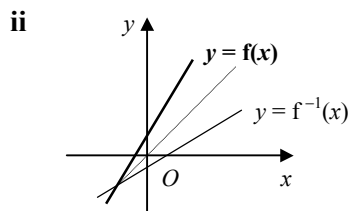
- 1**
- a** $y = 10x + 3$
 swap $x = 10y + 3$
 $y = \frac{x-3}{10}$
 $f^{-1}(x) = \frac{x-3}{10}, x \in \mathbb{R}$
- b** $y = 9 + 2x$
 swap $x = 9 + 2y$
 $y = \frac{x-9}{2}$
 $f^{-1}(x) = \frac{x-9}{2}, x \in \mathbb{R}$
- c** $y = 5 - 6x$
 swap $x = 5 - 6y$
 $y = \frac{5-x}{6}$
 $f^{-1}(x) = \frac{5-x}{6}, x \in \mathbb{R}$
- d** $y = \frac{x+3}{4}$
 swap $x = \frac{y+3}{4}$
 $y = 4x - 3$
 $f^{-1}(x) = 4x - 3, x \in \mathbb{R}$
- e** $y = \frac{1}{3}(2x - 5)$
 swap $x = \frac{1}{3}(2y - 5)$
 $y = \frac{3x+5}{2}$
 $f^{-1}(x) = \frac{3x+5}{2}, x \in \mathbb{R}$
- f** $y = 8 - \frac{3}{5}x$
 swap $x = 8 - \frac{3}{5}y$
 $y = \frac{40-5x}{3}$
 $f^{-1}(x) = \frac{40-5x}{3}, x \in \mathbb{R}$
- 2**
- a** $y = \ln x$
 swap $x = \ln y$
 $y = e^x$
 $f^{-1}(x) = e^x, x \in \mathbb{R}$
- b** $y = \frac{1}{x}$
 swap $x = \frac{1}{y}$
 $y = \frac{1}{x}$
 $f^{-1}(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$
- c** $y = \sqrt[4]{x}$
 swap $x = \sqrt[4]{y}$
 $y = x^4$
 $f^{-1}(x) = x^4, x \in \mathbb{R}, x > 0$
- d** $y = 3x - 4$
 swap $x = 3y - 4$
 $y = \frac{x+4}{3}$
 $f(0) = -4, f(3) = 5$
 $f^{-1}(x) = \frac{x+4}{3}, x \in \mathbb{R}, -4 \leq x < 5$
- e** $y = \frac{1}{x-5}$
 swap $x = \frac{1}{y-5}$
 $y = \frac{1}{x} + 5$
 $f^{-1}(x) = \frac{1}{x} + 5, x \in \mathbb{R}, x \neq 0$
- f** $y = 2 + \frac{1}{x}$
 swap $x = 2 + \frac{1}{y}$
 $y = \frac{1}{x-2}$
 $f^{-1}(x) = \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$

3 a i $y = 2x + 1$

swap $x = 2y + 1$

$$y = \frac{x-1}{2}$$

$$f^{-1}: x \rightarrow \frac{x-1}{2}, x \in \mathbb{R}$$

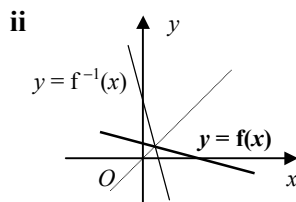


b i $y = \frac{1-x}{5}$

swap $x = \frac{1-y}{5}$

$$y = 1 - 5x$$

$$f^{-1}: x \rightarrow 1 - 5x, x \in \mathbb{R}$$

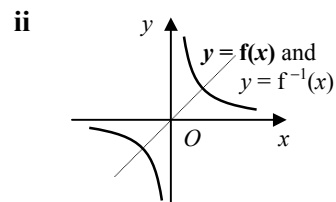


c i $y = \frac{10}{x}$

swap $x = \frac{10}{y}$

$$y = \frac{10}{x}$$

$$f^{-1}: x \rightarrow \frac{10}{x}, x \in \mathbb{R}, x \neq 0$$



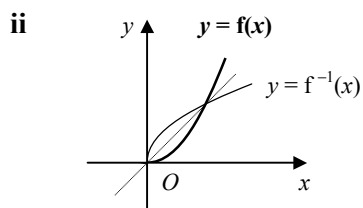
d i $y = x^2$

swap $x = y^2$

$$y = \pm\sqrt{x}$$

(domain of $f \Rightarrow +$)

$$f^{-1}: x \rightarrow \sqrt{x}, x \in \mathbb{R}, x \geq 0$$

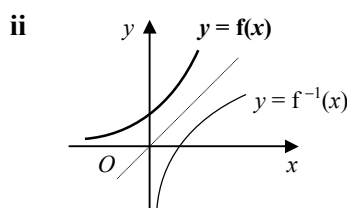


e i $y = e^x$

swap $x = e^y$

$$y = \ln x$$

$$f^{-1}: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$$

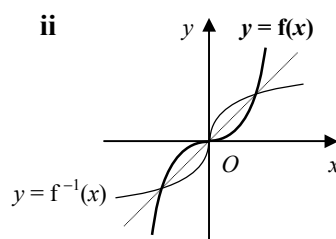


f i $y = x^3$

swap $x = y^3$

$$y = \sqrt[3]{x}$$

$$f^{-1}: x \rightarrow \sqrt[3]{x}, x \in \mathbb{R}$$



4 a $y = 5x + 1$

swap $x = 5y + 1$

$$f^{-1}(x) = y = \frac{x-1}{5}$$

$$\frac{x-1}{5} = 2$$

$$x - 1 = 10$$

$$x = 11$$

d $y = \sqrt{x+2}$

swap $x = \sqrt{y+2}$

$$f^{-1}(x) = y = x^2 - 2$$

$$x^2 - 2 = 3x - 4$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

b $y = \frac{2x-4}{3}$

swap $x = \frac{2y-4}{3}$

$$f^{-1}(x) = y = \frac{3x+4}{2}$$

$$\frac{3x+4}{2} = 7 - x$$

$$3x + 4 = 14 - 2x$$

$$x = 2$$

e $y = \frac{4}{x+3}$

swap $x = \frac{4}{y+3}$

$$f^{-1}(x) = y = \frac{4}{x} - 3$$

$$\frac{4}{x} - 3 = 5(x+1)$$

$$4 - 3x = 5x(x+1)$$

$$5x^2 + 8x - 4 = 0$$

$$(5x-2)(x+2) = 0$$

$$x = -2, \frac{2}{5}$$

c $y = e^x + 2$

swap $x = e^y + 2$

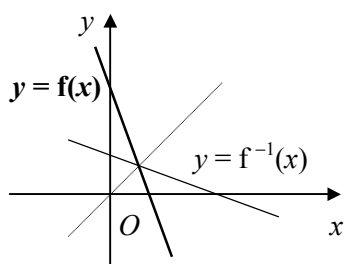
$$f^{-1}(x) = y = \ln(x-2)$$

$$\ln(x-2) = \ln(3x-8)$$

$$x-2 = 3x-8$$

$$x = 3$$

5 a



b $4 - 2x = x$

$x = \frac{4}{3}$

$\therefore (\frac{4}{3}, \frac{4}{3})$

6

a $g \Rightarrow y = \frac{1}{2x+4}$

swap $x = \frac{1}{2y+4}$

$y = \frac{1}{2}(\frac{1}{x} - 4) = \frac{1}{2x} - 2$

$g^{-1}(x) = \frac{1}{2x} - 2, x \in \mathbb{R}, x \neq 0$

range: $g^{-1}(x) \in \mathbb{R}, g^{-1}(x) \neq -2$

b $= g(3 - 2x)$

$= \frac{1}{2(3-2x)+4} = \frac{1}{10-4x}$

$gf(x) = \frac{1}{10-4x}, x \in \mathbb{R}, x \neq \frac{5}{2}$

c $f \Rightarrow y = 3 - 2x$

swap $x = 3 - 2y$

$f^{-1}(x) = y = \frac{3-x}{2}$

$\therefore \frac{1}{10-4x} = \frac{3-x}{2}$

$2 = (3-x)(10-4x)$

$2x^2 - 11x + 14 = 0$

$(2x-7)(x-2) = 0$

$x = 2, \frac{7}{2}$

7 a i $y = 5x + 2$

swap $x = 5y + 2$

$y = \frac{x-2}{5}$

$f^{-1}: x \rightarrow \frac{x-2}{5}, x \in \mathbb{R}$

ii $= f(\frac{1}{x})$

$= \frac{5}{x} + 2$

$fg: x \rightarrow \frac{5}{x} + 2, x \in \mathbb{R}, x \neq 0$

iii $y = \frac{5}{x} + 2$

swap $x = \frac{5}{y} + 2$

$y = \frac{5}{x-2}$

$(fg)^{-1}: x \rightarrow \frac{5}{x-2}, x \in \mathbb{R}, x \neq 2$

b $\frac{x-2}{5} = \frac{5}{x} + 2$

$x(x-2) = 25 + 10x$

$x^2 - 12x - 25 = 0$

$x = \frac{12 \pm \sqrt{144+100}}{2} = 6 \pm \sqrt{61} = -1.81, 13.81$

8 a $y = \frac{1}{2} \ln(4x - 9)$

swap $x = \frac{1}{2} \ln(4y - 9)$

$$4y - 9 = e^{2x}$$

$$y = \frac{1}{4}(e^{2x} + 9)$$

$$f^{-1}: x \rightarrow \frac{1}{4}(e^{2x} + 9), x \in \mathbb{R}$$

b $y = \frac{x-2}{x+5}$

swap $x = \frac{y-2}{y+5}$

$$xy + 5x = y - 2$$

$$y(1-x) = 5x + 2$$

$$y = \frac{5x+2}{1-x}$$

$$f^{-1}: x \rightarrow \frac{5x+2}{1-x}, x \in \mathbb{R}, x \neq 1$$

c $y = e^{0.4x-2}$

swap $x = e^{0.4y-2}$

$$0.4y - 2 = \ln x$$

$$y = \frac{5}{2}(2 + \ln x)$$

$$f^{-1}: x \rightarrow 5 + \frac{5}{2} \ln x, x \in \mathbb{R}, x > 0$$

d $y = \sqrt[3]{x^5 - 3}$

swap $x = \sqrt[3]{y^5 - 3}$

$$y^5 - 3 = x^3$$

$$y = \sqrt[5]{x^3 + 3}$$

$$f^{-1}: x \rightarrow \sqrt[5]{x^3 + 3}, x \in \mathbb{R}$$

e $y = \log_{10}(2 - 7x)$

swap $x = \log_{10}(2 - 7y)$

$$2 - 7y = 10^x$$

$$y = \frac{1}{7}(2 - 10^x)$$

$$f^{-1}: x \rightarrow \frac{1}{7}(2 - 10^x), x \in \mathbb{R}$$

f $y = \frac{4-x}{3x+2}$

swap $x = \frac{4-y}{3y+2}$

$$3xy + 2x = 4 - y$$

$$y(3x+1) = 4 - 2x$$

$$y = \frac{4-2x}{3x+1}$$

$$f^{-1}: x \rightarrow \frac{4-2x}{3x+1}, x \in \mathbb{R}, x \neq -\frac{1}{3}$$

9 a i $y = e^{2x}$

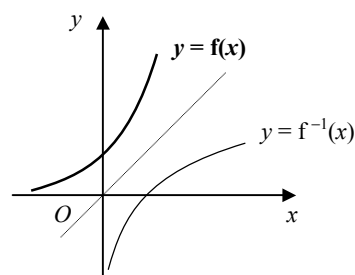
swap $x = e^{2y}$

$$2y = \ln x$$

$$y = \frac{1}{2} \ln x$$

$$f^{-1}: x \rightarrow \frac{1}{2} \ln x, x \in \mathbb{R}, x > 0$$

ii



b i $y = x^2 + 4$

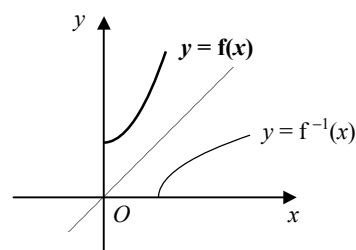
swap $x = y^2 + 4$

$$y = \pm \sqrt{x-4}$$

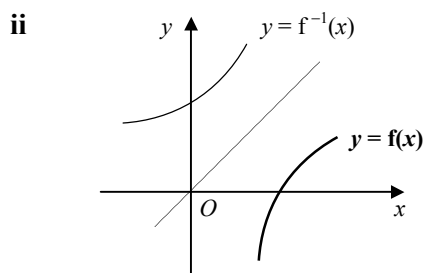
(domain of $f \Rightarrow +$)

$$f^{-1}: x \rightarrow \sqrt{x-4}, x \in \mathbb{R}, x \geq 4$$

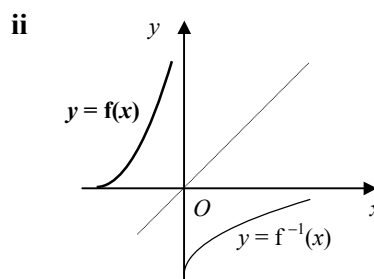
ii



c i $y = \ln(x - 3)$
 swap $x = \ln(y - 3)$
 $y - 3 = e^x$
 $y = e^x + 3$
 $f^{-1} : x \rightarrow e^x + 3, x \in \mathbb{R}$



d i $y = x^2 + 6x + 9 = (x + 3)^2$
 swap $x = (y + 3)^2$
 $y = -3 \pm \sqrt{x}$
 (domain of $f \Rightarrow +$)
 $f^{-1} : x \rightarrow -3 + \sqrt{x}, x \in \mathbb{R}, x \geq 0$



10 a i $f(x) = (x + 3)^2 - 6$
 $x < -3 \therefore \text{range: } f(x) > -6$
ii $y = (x + 3)^2 - 6$
 swap $x = (y + 3)^2 - 6$
 $y = -3 \pm \sqrt{x + 6}$
 (domain of $f \Rightarrow -$)
 $f^{-1}(x) = -3 - \sqrt{x + 6}, x \in \mathbb{R}, x > -6$

b i $f(x) = (x - 2)^2 + 1$
 $x \geq 2 \therefore \text{range: } f(x) \geq 1$
ii $y = (x - 2)^2 + 1$
 swap $x = (y - 2)^2 + 1$
 $y = 2 \pm \sqrt{x - 1}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = 2 + \sqrt{x - 1}, x \in \mathbb{R}, x \geq 1$

c i $f(x) = (x + \frac{5}{2})^2 - \frac{33}{4}$
 $x < -\frac{5}{2} \therefore \text{range: } f(x) > -8\frac{1}{4}$
ii $y = (x + \frac{5}{2})^2 - \frac{33}{4}$
 swap $x = (y + \frac{5}{2})^2 - \frac{33}{4}$
 $y = -\frac{5}{2} \pm \sqrt{x + \frac{33}{4}}$
 (domain of $f \Rightarrow -$)
 $f^{-1}(x) = -\frac{5}{2} - \sqrt{x + \frac{33}{4}}, x \in \mathbb{R}, x > -8\frac{1}{4}$

d i $f(x) = (x - \frac{3}{2})^2 + \frac{11}{4}$
 $2 < x < 4, f(2) = 3, f(4) = 9$
 $\therefore \text{range: } 3 < f(x) < 9$
ii $y = (x - \frac{3}{2})^2 + \frac{11}{4}$
 swap $x = (y - \frac{3}{2})^2 + \frac{11}{4}$
 $y = \frac{3}{2} \pm \sqrt{x - \frac{11}{4}}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = \frac{3}{2} + \sqrt{x - \frac{11}{4}}, x \in \mathbb{R}, 3 < x < 9$

e i $f(x) = 8 - 2x - x^2 = 9 - (x + 1)^2$
 $x \geq -1 \therefore \text{range: } f(x) \leq 9$
ii $y = 9 - (x + 1)^2$
 swap $x = 9 - (y + 1)^2$
 $y = -1 \pm \sqrt{9 - x}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = -1 + \sqrt{9 - x}, x \in \mathbb{R}, x \leq 9$

f i $f(x) = -5(x^2 - 4x) = 20 - 5(x - 2)^2$
 $x > 2 \therefore \text{range: } f(x) < 20$
ii $y = 20 - 5(x - 2)^2$
 swap $x = 20 - 5(y - 2)^2$
 $y = 2 \pm \sqrt{\frac{20 - x}{5}}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = 2 + \sqrt{\frac{20 - x}{5}}, x \in \mathbb{R}, x < 20$

11 a $y = \frac{1}{3}(2x - 5)$

swap $x = \frac{1}{3}(2y - 5)$

$f^{-1}(x) = y = \frac{3x+5}{2}$

$\therefore \frac{3x+5}{2} = \frac{4}{2-x}$

$(3x+5)(2-x) = 8$

$3x^2 - x - 2 = 0$

$(3x+2)(x-1) = 0$

$x = -\frac{2}{3}, 1$

b $y = \ln \frac{x+3}{5}$

swap $x = \ln \frac{y+3}{5}$

$f^{-1}(x) = y = 5e^x - 3$

$\therefore 5e^x - 3 = 10 - 6e^{-x}$

$5e^{2x} - 13e^x + 6 = 0$

$(5e^x - 3)(e^x - 2) = 0$

$e^x = \frac{3}{5}, 2$

$x = \ln \frac{3}{5}, \ln 2$

$x = -0.511, 0.693$ (3sf)

c $y = x^2 - 4$

swap $x = y^2 - 4$

$y = \pm\sqrt{x+4}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = \sqrt{x+4}$

$\therefore \sqrt{x+4} = \frac{x+6}{3}$

$x+4 = \frac{(x+6)^2}{9}$

$9(x+4) = x^2 + 12x + 36$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = -3, 0$

12 a -2

b $\frac{6+b}{6-2} = 4$

$6+b = 16$

$b = 10$

c $y = \frac{x+10}{x-2}$

swap $x = \frac{y+10}{y-2}$

$xy - 2x = y + 10$

$y(x-1) = 2x+10$

$y = \frac{2x+10}{x-1}$

$f^{-1}(x) = \frac{2x+10}{x-1}, x \in \mathbb{R}, x \neq 1$

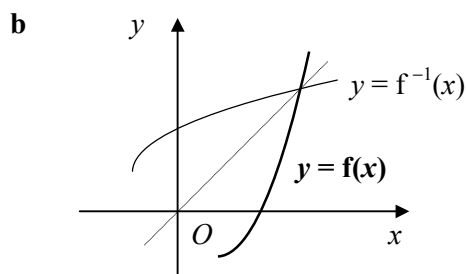
13 a $y = x^2 - 3x = (x - \frac{3}{2})^2 - \frac{9}{4}$

swap $x = (y - \frac{3}{2})^2 - \frac{9}{4}$

$y = \frac{3}{2} \pm \sqrt{x + \frac{9}{4}}$

(domain of $f \Rightarrow +$)

$f^{-1}: x \rightarrow \frac{3}{2} + \sqrt{x + \frac{9}{4}}, x \in \mathbb{R}, x \geq -\frac{9}{4}$



c $g \Rightarrow y = 2x + 3$

swap $x = 2y + 3$

$g^{-1}(x) = y = \frac{x-3}{2}$

$g^{-1}(12) = \frac{9}{2}$

$f^{-1}g^{-1}(12) = f^{-1}(\frac{9}{2})$

$= \frac{3}{2} + \sqrt{\frac{27}{4}}$

$= \frac{3}{2} + \frac{3}{2}\sqrt{3}$

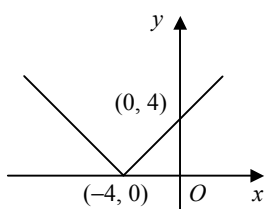
$= \frac{3}{2}(1 + \sqrt{3})$

$\therefore a = 1\frac{1}{2}$

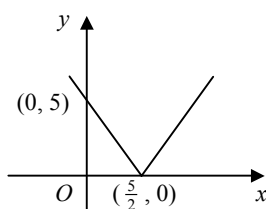
1 a 2 b 1 c 6 d -2 e 4 f -3

2 a $= g(-3)$ b $= f(1)$ c $= f(9)$ d $= g(5)$ e $= g(0)$ f $= f(1)$
 $= 5$ $= 0$ $= 96$ $= 11$ $= 1$ $= 0$

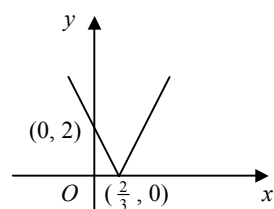
3 a



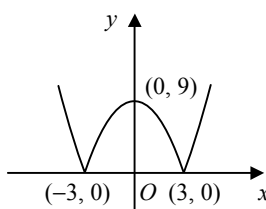
b



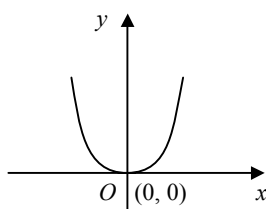
c



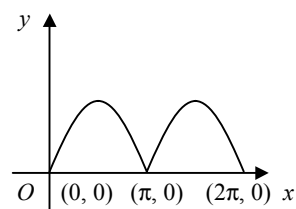
d



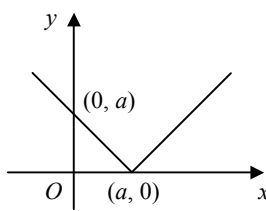
e



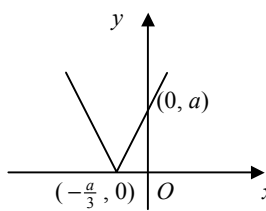
f



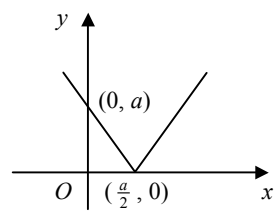
g



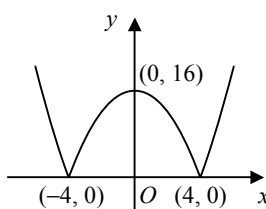
h



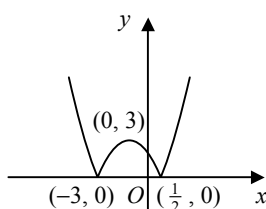
i



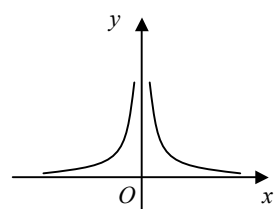
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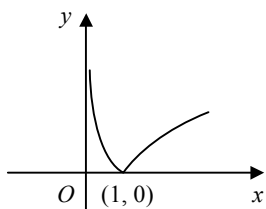
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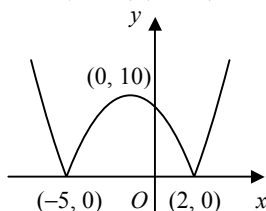
l



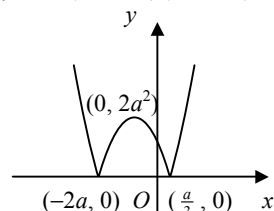
m



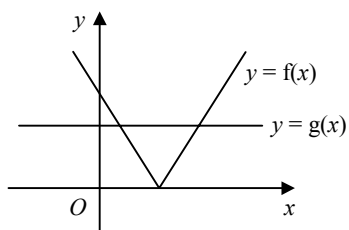
n $y = |(5+x)(2-x)|$



o $y = |(3x-a)(x+2a)|$

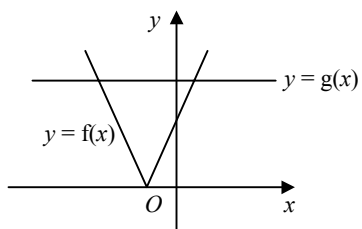


4 a i

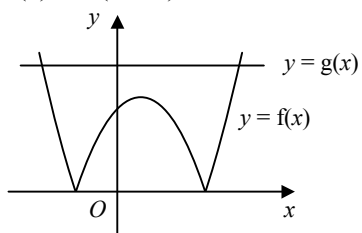


$$\begin{aligned}\text{ii } 2x - 3 &= 2 \Rightarrow x = \frac{5}{2} \\ -(2x - 3) &= 2 \Rightarrow x = \frac{1}{2} \\ \therefore x &= \frac{1}{2}, \frac{5}{2}\end{aligned}$$

c i

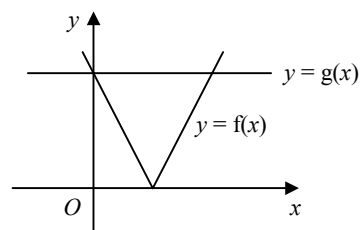


$$\begin{aligned}\text{ii } 4x + 3a &= 5a \Rightarrow x = \frac{1}{2}a \\ -(4x + 3a) &= 5a \Rightarrow x = -2a \\ \therefore x &= -2a, \frac{1}{2}a\end{aligned}$$

e i $f(x) = |(x-2)^2 - 16|$ 

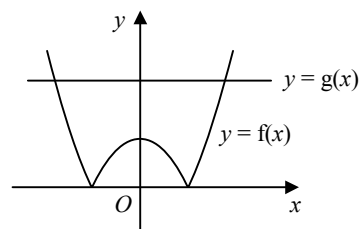
$$\begin{aligned}\text{ii } x^2 - 4x - 12 &= 20 \Rightarrow x^2 - 4x - 32 = 0 \\ &\Rightarrow (x+4)(x-8) = 0 \\ \therefore x &= -4, 8\end{aligned}$$

b i



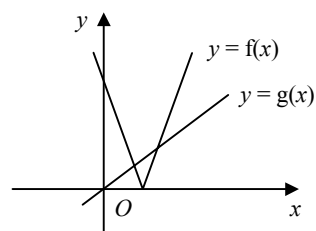
$$\begin{aligned}\text{ii } 7 - 3x &= 7 \Rightarrow x = 0 \\ -(7 - 3x) &= 7 \Rightarrow x = 4\frac{2}{3} \\ \therefore x &= 0, 4\frac{2}{3}\end{aligned}$$

d i



$$\begin{aligned}\text{ii } x^2 - 4 &= 9 \Rightarrow x^2 = 13 \\ \therefore x &= \pm\sqrt{13}\end{aligned}$$

f i

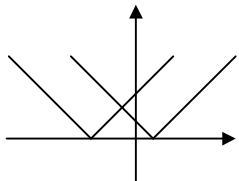


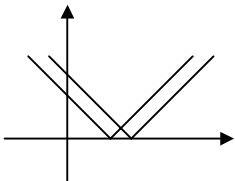
$$\begin{aligned}\text{ii } 2a - 5x &= x \Rightarrow x = \frac{1}{3}a \\ -(2a - 5x) &= x \Rightarrow x = \frac{1}{2}a \\ \therefore x &= \frac{1}{3}a, \frac{1}{2}a\end{aligned}$$

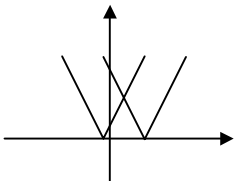
5 a $x - 5 = 3 \Rightarrow x = 8$
 $-(x - 5) = 3 \Rightarrow x = 2$
 $\therefore x = 2, 8$

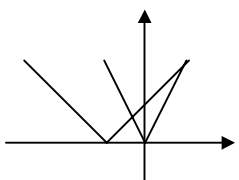
b $x + 1 = 15 \Rightarrow x = 14$
 $-(x + 1) = 15 \Rightarrow x = -16$
 $\therefore x = -16, 14$

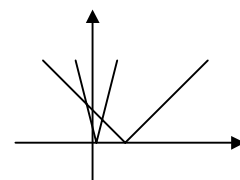
c $2x - 7 = 4 \Rightarrow x = \frac{11}{2}$
 $-(2x - 7) = 4 \Rightarrow x = \frac{3}{2}$
 $\therefore x = \frac{3}{2}, \frac{11}{2}$

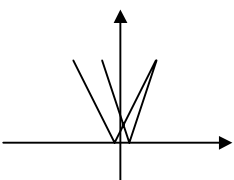
d 
 $-(x - 2) = x + 4 \Rightarrow x = -1$
 $\therefore x = -1$

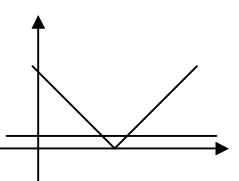
e 
 $x - 5 = 7 - x \Rightarrow x = 6$
 $\therefore x = 6$

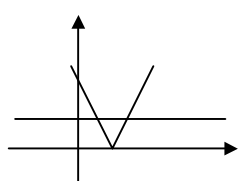
f 
 $2x + 1 = 9 - 2x \Rightarrow x = 2$
 $\therefore x = 2$

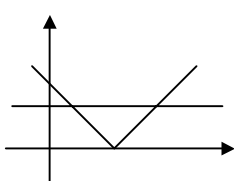
g 
 $x + 3 = 2x \Rightarrow x = 3$
 $x + 3 = -2x \Rightarrow x = -1$
 $\therefore x = -1, 3$

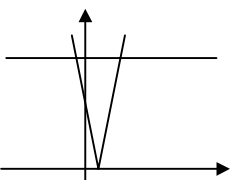
h 
 $4x - 1 = 2 - x \Rightarrow x = \frac{3}{5}$
 $-(4x - 1) = 2 - x \Rightarrow x = -\frac{1}{3}$
 $\therefore x = -\frac{1}{3}, \frac{3}{5}$

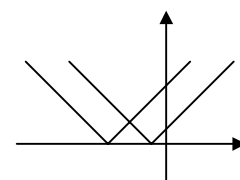
i 
 $3x - 4 = 2x + 3 \Rightarrow x = 7$
 $-(3x - 4) = 2x + 3 \Rightarrow x = \frac{1}{5}$
 $\therefore x = \frac{1}{5}, 7$

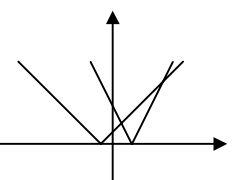
6 a 
 $x - 20 = 2 \Rightarrow x = 22$
 $-(x - 20) = 2 \Rightarrow x = 18$
 $\therefore 18 < x < 22$

b 
 $2x - 11 = 5 \Rightarrow x = 8$
 $-(2x - 11) = 5 \Rightarrow x = 3$
 $\therefore 3 \leq x \leq 8$

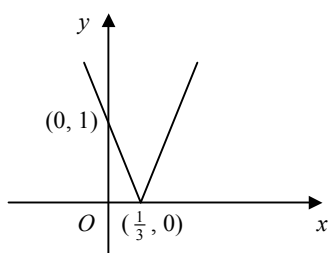
c 
 $x - 17 = 12 \Rightarrow x = 29$
 $-(x - 17) = 12 \Rightarrow x = 5$
 $\therefore x < 5 \text{ or } x > 29$

d 
 $5x - 22 = 40 \Rightarrow x = 12\frac{2}{5}$
 $-(5x - 22) = 40 \Rightarrow x = -3\frac{3}{5}$
 $\therefore -3\frac{3}{5} < x < 12\frac{2}{5}$

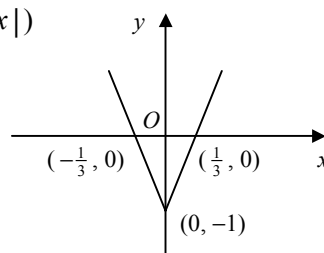
e 
 $x + 4 = -(x + 1) \Rightarrow x = -\frac{5}{2}$
 $\therefore x \leq -\frac{5}{2}$

f 
 $x + 2 = 2x - 5 \Rightarrow x = 7$
 $x + 2 = -(2x - 5) \Rightarrow x = 1$
 $\therefore 1 < x < 7$

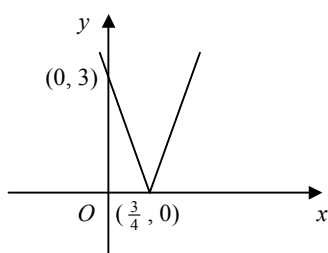
7 a $y = |f(x)|$



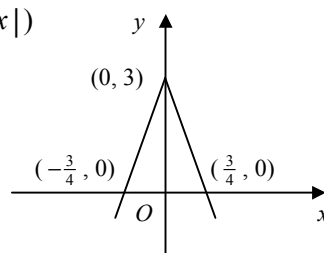
$y = f(|x|)$



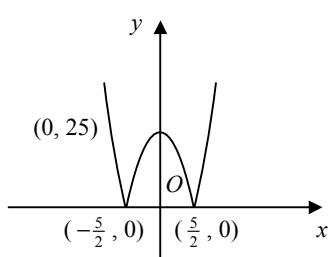
b $y = |f(x)|$



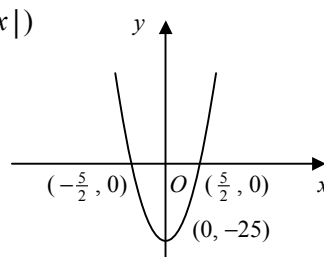
$y = f(|x|)$



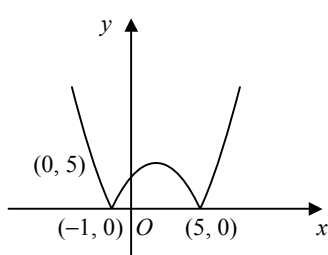
c $y = |f(x)|$



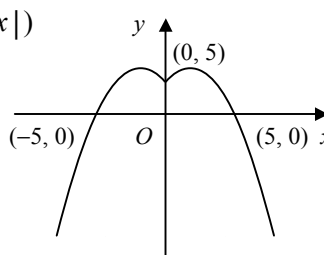
$y = f(|x|)$



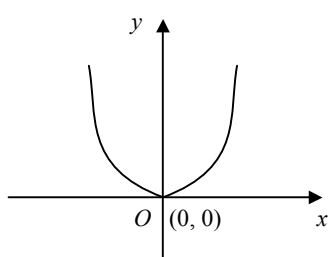
d $y = |f(x)|$



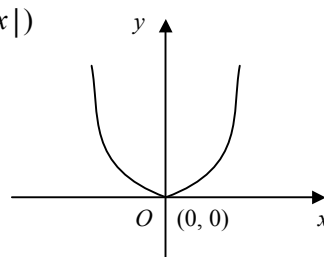
$y = f(|x|)$



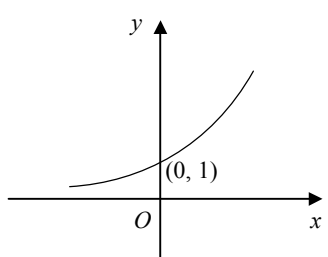
e $y = |f(x)|$



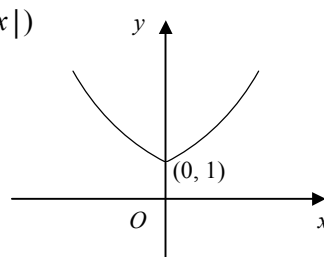
$y = f(|x|)$



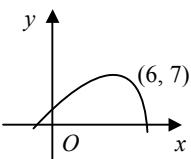
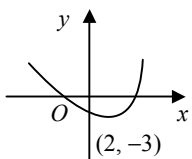
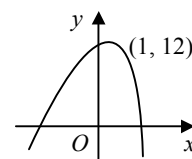
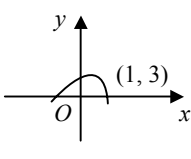
f $y = |f(x)|$



$y = f(|x|)$

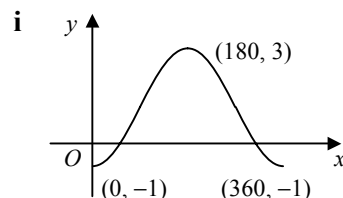
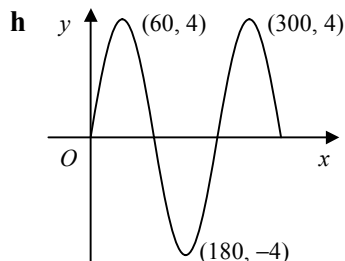
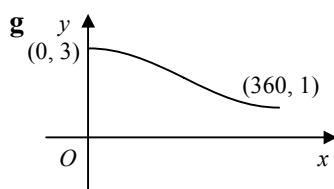
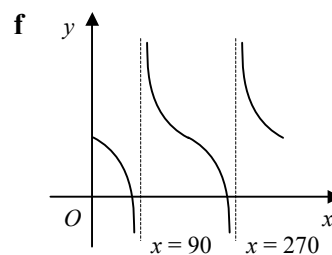
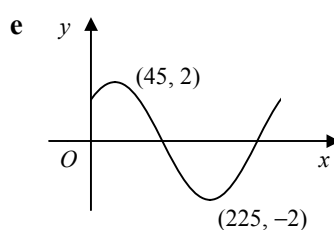
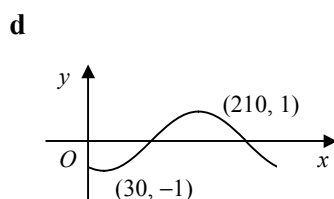
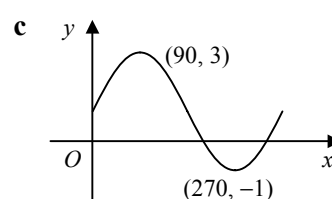
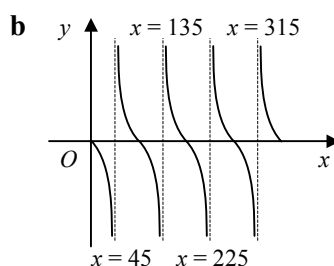
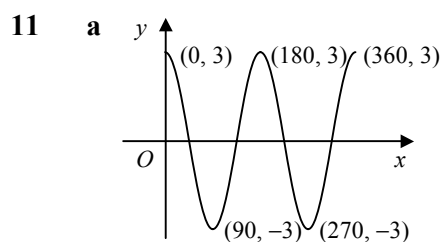
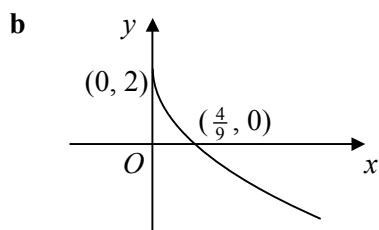


Note: For this worksheet especially, there may be alternative correct answers

- 1 a translated 3 units in negative x -direction and translated 2 units in positive y -direction
 b reflected in the y -axis and stretched by a factor of 2 in y -direction
 c translated 1 unit in positive x -direction and stretched by a factor of 3 in y -direction
 d reflected in the x -axis and then translated 4 units in positive y -direction
- 2 a $= (x + 3)^2 - 9 + 2 = (x + 3)^2 - 7$
 b translation by 3 units in negative x -direction and translation by 7 units in negative y -direction
- 3 a $y = 2[2(x - 3) + 7] \Rightarrow y = 4x + 2$
 b $y = 2[3e^{(x-3)}] \Rightarrow y = 6e^{x-3}$
 c $y = 2[(x - 3)^2 - 3(x - 3) + 1] \Rightarrow y = 2x^2 - 18x + 38$
 d $y = 2\left[\frac{1}{(x-3)}\right] \Rightarrow y = \frac{2}{x-3}$
- 4 a stretch by a factor of $\frac{1}{3}$ in x -direction and reflection in the x -axis (either first)
 b reflection in the y -axis and translation by 5 units in positive y -direction (either first)
 c translation by 4 units in negative x -direction and stretch by a factor of 3 in y -direction (either first)
 d stretch by a factor of 3 in y -direction, then translation by 2 units in positive y -direction
- 5 a  b  c  d 
- 6 first $\Rightarrow y = (x + 2)^2 + 4(x + 2) - 2 \Rightarrow y = x^2 + 8x + 10$
 second $\Rightarrow y = 3[x^2 + 8x + 10] \Rightarrow y = 3x^2 + 24x + 30$
 third $\Rightarrow y = 3(-x)^2 + 24(-x) + 30 \Rightarrow y = 3x^2 - 24x + 30$
- 7 a $= 2[x^2 - 2x] + 7 = 2[(x - 1)^2 - 1] + 7 = 2(x - 1)^2 + 5$
 b translation by 5 units in negative y -direction,
 then stretch by a factor of $\frac{1}{2}$ in y -direction,
 then translation by 1 unit in negative x -direction
- 8 a $f'(x) = 3x^2 - 6x$
 SP: $3x^2 - 6x = 0$
 $3x(x - 2) = 0$
 $x = 0, 2$
 $\therefore (0, 4)$ and $(2, 0)$
 b i $(0, -8)$ and $(2, 0)$ ii $(0, 7)$ and $(4, 3)$ iii $(2, 1)$ and $(4, 0)$

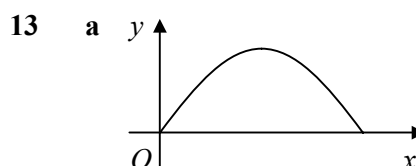
- 9 a stretch by factor of 3 in y -direction,
then reflection in x -axis,
then translation by 2 units in +ve y -dir'n

- 10 a 180°
b $(0, 1)$
c $(90, 3)$ and $(270, 3)$



- 12 a 60°
b $\frac{360^\circ}{k}$

- 14 a max. value 4 $\therefore a = 4$
max. occurs at $x = 45 \therefore b = 2$
b $(135, -4)$



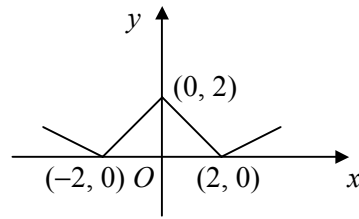
- b $(\pi, 2)$
c $2 \sin \frac{1}{2}x = \sqrt{2}$
 $\sin \frac{1}{2}x = \frac{1}{\sqrt{2}}$
 $\frac{1}{2}x = \frac{\pi}{4}, \pi - \frac{\pi}{4}$
 $= \frac{\pi}{4}, \frac{3\pi}{4}$
 $x = \frac{\pi}{2}, \frac{3\pi}{2}$

1 a $= (x - 4)^2 - 16 + 18$
 $= (x - 4)^2 + 2$

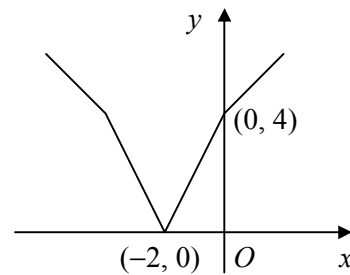
b vertex $= (4, 2)$
 $\therefore \text{dist.} = \sqrt{4^2 + 2^2}$
 $= \sqrt{20} = 2\sqrt{5}$

c translation by 4 units in +ve x -direction
translation by 2 units in +ve y -direction
(either first)

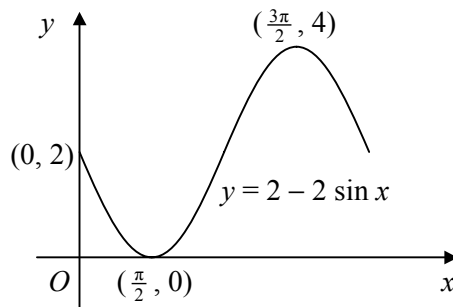
2 a



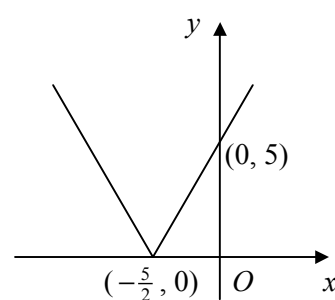
b



3



4 a



b $f(-4) = |-3| = 3$
 $ff(-4) = f(3) = |11| = 11$

c $-\frac{5}{2}$

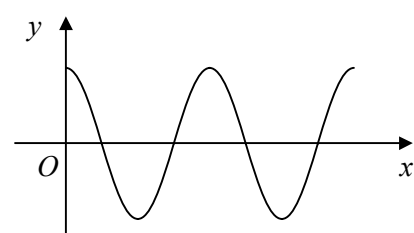
5

a $(0, 2) \therefore a = 2$
 $(90, 7) \therefore 7 = 2 + b \therefore b = 5$

b $2 + 5 \sin x = 0$
 $\sin x = -0.4$
 $x = 180 + 23.6, 360 - 23.6$
 $x = 203.6^\circ, 336.4^\circ$ (1dp)

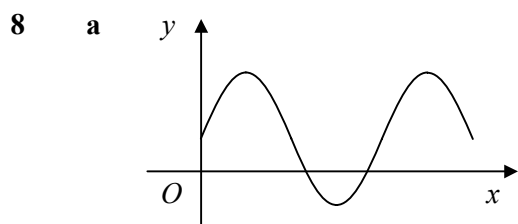
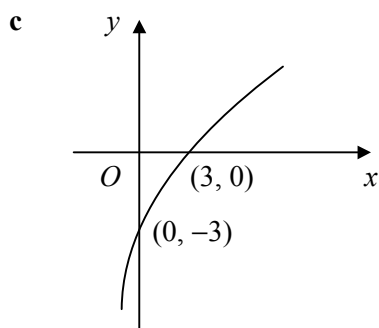
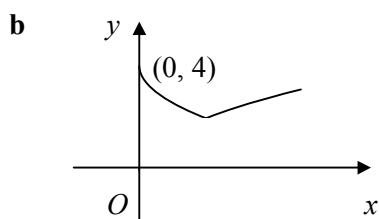
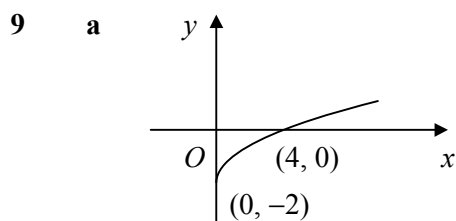
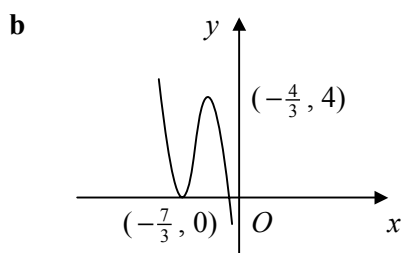
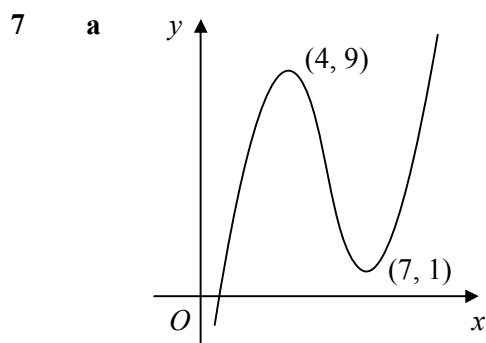
6

a



b $(45, 0), (135, 0), (225, 0), (315, 0)$

c $(0, 3), (90, -3), (180, 3), (270, -3), (360, 3)$



b $(30^\circ, 1.5), (90^\circ, -0.5), (150^\circ, 1.5)$

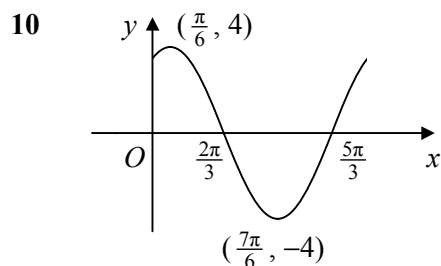
c $\frac{1}{2} + \sin 3x = 0$

$$\sin 3x = -\frac{1}{2}$$

$$3x = 180 + 30, 360 - 30$$

$$= 210, 330$$

$$x = 70^\circ, 110^\circ$$



11 a $y = \frac{3x-5}{x-2}$

swap $x = \frac{3y-5}{y-2}$

$$xy - 2x = 3y - 5$$

$$y(x-3) = 2x-5$$

$$y = \frac{2x-5}{x-3}$$

$$f^{-1}(x) = \frac{2x-5}{x-3}, x \in \mathbb{R}, x \neq 3$$

b $f(x) = 4 \Rightarrow x = f^{-1}(4) = 3$

c $f(x) = \frac{3(x-2)+1}{x-2} = 3 + \frac{1}{x-2}$

$$\therefore a = 3, b = 1$$

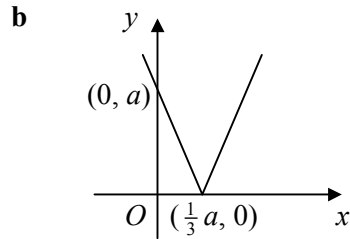
d translation by 2 in the positive x -direction and translation by 3 in the positive y -direction (either first)

1 a $f(1) = 2 + \log_4 1 = 2$
 $ff(1) = f(2) = 2 + \log_4 2$
 $= 2 + \frac{1}{2} = \frac{5}{2}$

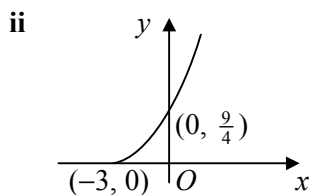
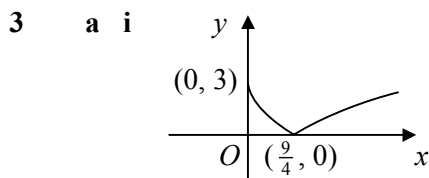
b $2 + \log_4 x = 0$
 $\log_4 x = -2$
 $x = 4^{-2} = \frac{1}{16}$

c $y = 2 + \log_4 x$
 swap $x = 2 + \log_4 y$
 $\log_4 y = x - 2$
 $y = 4^{x-2}$
 $f^{-1}(x) = 4^{x-2}, x \in \mathbb{R}$

2 a $f(-2a) = |-7a| = 7a$
 $ff(-2a) = f(7a) = |20a| = 20a$



c $3x - a = x \Rightarrow x = \frac{1}{2}a$
 $-(3x - a) = x \Rightarrow x = \frac{1}{4}a$
 $\therefore x = \frac{1}{4}a, \frac{1}{2}a$



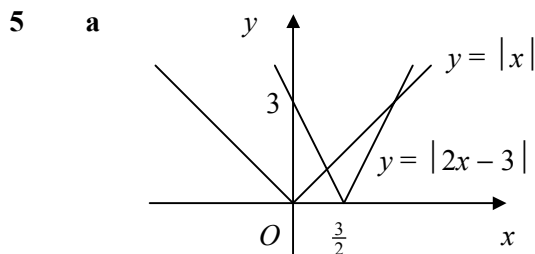
b $f(0) = b = -3$
 $f(\frac{9}{4}) = \frac{3}{2}a - 3 = 0$
 $\therefore a = 2, b = -3$

c $y = 2x^{\frac{1}{2}} - 3$
 swap $x = 2y^{\frac{1}{2}} - 3$
 $y^{\frac{1}{2}} = \frac{1}{2}(x+3)$
 $f^{-1}(x) = \frac{1}{4}(x+3)^2, x \in \mathbb{R}, x \geq -3$

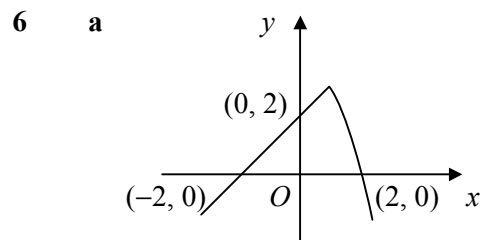
4 a $ff(x) = f(\frac{x+2}{x-1}) = \frac{\frac{x+2}{x-1}+2}{\frac{\frac{x+2}{x-1}-1}$
 $= \frac{x+2+2(x-1)}{x+2-(x-1)}$
 $= \frac{3x}{3} = x$

b $f^{-1}(x) = \frac{x+2}{x-1}, x \in \mathbb{R}, x \neq 1$

c $gf(x) = 2f(x) - 3 = 0$
 $\therefore f(x) = \frac{x+2}{x-1} = \frac{3}{2}$
 $2(x+2) = 3(x-1)$
 $2x+4 = 3x-3$
 $x = 7$



b $x = 2x - 3 \Rightarrow x = 3$
 $x = -(2x - 3) \Rightarrow x = 1$
 $\therefore x = 1, 3$



b $f(3) = 4 - 9 = -5$
 $ff(3) = f(-5) = -5 + 2 = -3$
 c $x + 2 = 1 \Rightarrow x = -1$
 $4 - x^2 = 1 \Rightarrow x^2 = 3$
 $x \geq 1 \therefore x = \sqrt{3}$
 $\therefore x = -1, \sqrt{3}$

7 a i $y = kx + 2$
 swap $x = ky + 2$
 $y = \frac{x-2}{k}$
 $f^{-1}(x) = \frac{x-2}{k}, x \in \mathbb{R}$

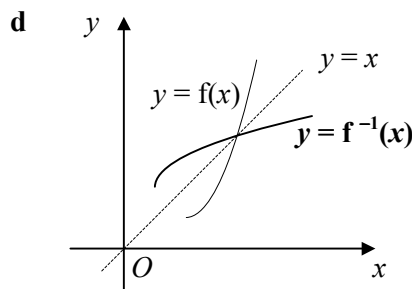
ii $fg(x) = f(x - 3k)$
 $= k(x - 3k) + 2$
 $fg(x) = kx - 3k^2 + 2, x \in \mathbb{R}$

b $4 = 7k - 3k^2 + 2$
 $3k^2 - 7k + 2 = 0$
 $(3k - 1)(k - 2) = 0$
 $k = \frac{1}{3}, 2$

8 a $f(x) = (x - 2)^2 - 4 + 5 = (x - 2)^2 + 1$
 b $f(x) \geq 1$
 c $y = (x - 2)^2 + 1$

swap $x = (y - 2)^2 + 1$

$y = 2 \pm \sqrt{x - 1}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = 2 + \sqrt{x - 1}, x \in \mathbb{R}, x \geq 1$

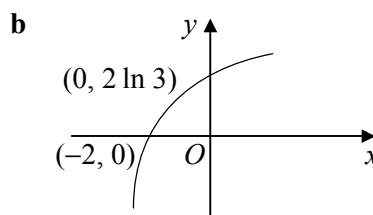


the graphs are reflections of each other
 in the line $y = x$

9 a $f(-2) = 8$
 $gf(-2) = g(8) = 15\frac{7}{8}$
 b $fg(x) = f(2x - \frac{1}{x}) = (2x - \frac{1}{x})^2 + 4$
 $= 4x^2 - 4 + \frac{1}{x^2} + 4$
 $fg(x) = 4x^2 + \frac{1}{x^2}$

c $4x^2 + \frac{1}{x^2} = 5$
 $4x^4 - 5x^2 + 1 = 0$
 $(4x^2 - 1)(x^2 - 1) = 0$
 $x^2 = \frac{1}{4}, 1$
 $x = \pm \frac{1}{2}, \pm 1$

10 a $y = e^{\frac{1}{2}x} - 3$
 swap $x = e^{\frac{1}{2}y} - 3$
 $\frac{1}{2}y = \ln(x + 3)$
 $y = 2 \ln(x + 3)$
 $f^{-1}(x) = 2 \ln(x + 3), x \in \mathbb{R}, x > -3$



c $g(4) = \ln 9 = 2 \ln 3$
 $fg(4) = f(2 \ln 3) = 0$
 d $2 \ln(x + 3) = \ln(x + 5)$
 $(x + 3)^2 = x + 5$
 $x^2 + 5x + 4 = 0$
 $(x + 4)(x + 1) = 0$
 $x > -3 \therefore x = -1$

1 a $3 + \ln(x+2) \geq 3$

$$\ln(x+2) \geq 0$$

$$x+2 \geq 1$$

$$x \geq -1$$

$$\therefore k = -1$$

b $y = 3 + \ln(x+2)$

$$\text{swap } x = 3 + \ln(y+2)$$

$$y+2 = e^{x-3}$$

$$y = e^{x-3} - 2$$

$$f^{-1}(x) = e^{x-3} - 2, \quad x \in \mathbb{R}, \quad x \geq 3$$

c $3 + \ln(x+2) = 4 + \ln(x-1)$

$$\ln(x+2) - \ln(x-1) = 1$$

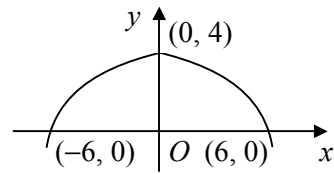
$$\frac{x+2}{x-1} = e$$

$$x+2 = e(x-1)$$

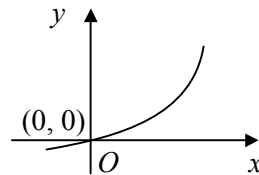
$$x(e-1) = e+2$$

$$x = \frac{e+2}{e-1}$$

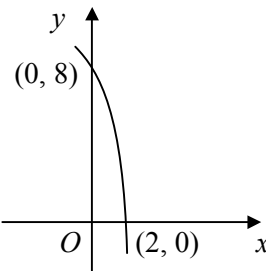
2 a



b



c



3 a $fg(x) = f\left(\frac{3}{x}\right) = \frac{\frac{3}{x}}{\frac{3}{x}+2} = \frac{3}{3+2x}$

$$\therefore \frac{3}{3+2x} = 4$$

$$3+2x = \frac{3}{4}$$

$$x = -\frac{9}{8}$$

b $y = \frac{x}{x+2}$

$$\text{swap } x = \frac{y}{y+2}$$

$$x(y+2) = y$$

$$2x = y(1-x)$$

$$y = \frac{2x}{1-x}$$

$$f^{-1}(x) = \frac{2x}{1-x}, \quad x \in \mathbb{R}, \quad x \neq 1$$

c $\frac{x}{x+2} = \frac{2x}{1-x}$

$$x(1-x) = 2x(x+2)$$

$$3x^2 + 3x = 0$$

$$3x(x+1) = 0$$

$$x = -1, 0$$

4 a $f(x) = (x-1)^2 - 1 - 9 = (x-1)^2 - 10$

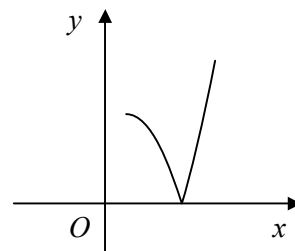
for f^{-1} to exist, f must be one-one

$$\therefore \text{min value of } k = 1$$

b $f^{-1}(x) = 4 \Rightarrow x = f(4)$

$$\therefore x = 16 - 8 - 9 = -1$$

c



d $x^2 - 2x - 9 = 6$

$$x^2 - 2x - 15 = 0$$

$$(x+3)(x-5) = 0$$

$$x \geq 1 \quad \therefore x = 5$$

$$-(x^2 - 2x - 9) = 6$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x \geq 1 \quad \therefore x = 3$$

$$\therefore x = 3, 5$$

5 a $f(1) = 2 - 3 = -1$

$ff(1) = f(-1) = 2 + 3 = 5$

b $y = 2 - \frac{3}{x}$ swap $x = 2 - \frac{3}{y}$

$$\frac{3}{y} = 2 - x$$

$$y = \frac{3}{2-x}$$

$$f^{-1}(x) = \frac{3}{2-x}, x \in \mathbb{R}, x \neq 2$$

c $gf(x) = 1 \Rightarrow (2 - \frac{3}{x})^2 = 1$

$$2 - \frac{3}{x} = \pm 1$$

$$\frac{3}{x} = 1, 3$$

$$x = 1, 3$$

6 a $f(\ln 9) = f(2 \ln 3) = e^{\ln 3} - 2 = 3 - 2 = 1$

b $f(x) > -2$

c $y = e^{\frac{1}{2}x} - 2$ swap $x = e^{\frac{1}{2}y} - 2$

$$\frac{1}{2}y = \ln(x+2)$$

$$y = 2 \ln(x+2)$$

$$f^{-1}(x) = 2 \ln(x+2), x \in \mathbb{R}, x > -2$$

d $gf(x) = (e^{\frac{1}{2}x} - 2)^2 + 4(e^{\frac{1}{2}x} - 2)$

$$= e^x - 4e^{\frac{1}{2}x} + 4 + 4e^{\frac{1}{2}x} - 8$$

$$gf(x) = e^x - 4$$

e $e^x - 4 + 1 = 0$

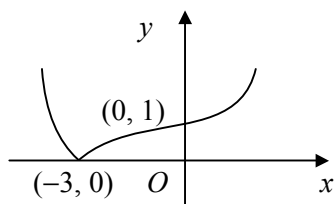
$$e^x = 3$$

$$x = \ln 3$$

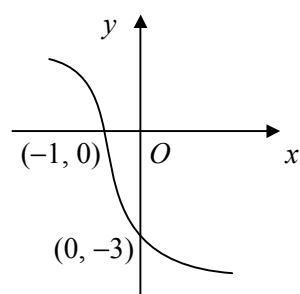
7 a each value of $f(x)$ corresponds

to a unique value of x

b i



ii



8 a $f(x) = \frac{5+(2x-3)}{(x+1)(2x-3)} = \frac{2x+2}{(x+1)(2x-3)}$

$$= \frac{2(x+1)}{(x+1)(2x-3)} = \frac{2}{2x-3}$$

b $f(2) = 2$

$$\therefore \text{range: } 0 < f(x) \leq 2$$

c $y = \frac{2}{2x-3}$ swap $x = \frac{2}{2y-3}$

$$2y - 3 = \frac{2}{x}$$

$$f^{-1}(x) = \frac{1}{x} + \frac{3}{2}, x \in \mathbb{R}, 0 < x \leq 2$$

d $fg(x) = \frac{2}{2(\frac{1}{x-2})-3} = \frac{2(x-2)}{2-3(x-2)} = \frac{2x-4}{8-3x}$

$$\therefore \frac{2x-4}{8-3x} = \frac{2}{3}$$

$$6x - 12 = 16 - 6x$$

$$x = \frac{7}{3}$$