



General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2008 examination - June series

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$f\left(-\frac{1}{3}\right) = 27 \times \left(-\frac{1}{3}\right)^3 - 9 \times \left(-\frac{1}{3}\right) + 2$ $= -1 + 3 + 2 = 4$	M1 A1	2	Use of $\pm \frac{1}{3}$ or complete division with integer remainder M1 remainder = 4 indicated A1
(b)(i)	$f\left(-\frac{2}{3}\right) = -8 + 6 + 2 = 0$	B1	1	AG
(b)(ii)	$f(x) = (3x + 2)(ax^2 + bx + c)$ $a = 9 \quad c = 1$ $x^2 \text{ term } 3b + 2a = 0$ or $x \text{ term } 3c + 2b = -9$ $b = -6$ or (could be shown as) $9x^2 - 6x + 1$	B1 M1 A1		$(3x + 2)$ or $\left(x + \frac{2}{3}\right)$ is a factor PI quadratic factor; find coefficients; 2 correct equate coefficients and solve for b correct quadratic factor or a , b , and c correct or use division or factor theorem to seek another factor (see alternative methods at end of scheme)
	$f(x) = (3x + 2)(3x - 1)(3x - 1)$	A1	4	SC (see alternative methods at end of scheme)
(b)(iii)	$9x^2 + 3x - 2 = (3x - 1)(3x + 2)$ $\frac{27x^3 - 9x + 2}{9x^2 + 3x - 2} = 3x - 1$	M1 A1		factorise denominator correctly or complete division simplified result indicated
	Total		9	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\frac{dx}{dt} = 4 \quad \frac{dy}{dt} = -\frac{1}{2t^2}$ $\frac{dy}{dx} = -\frac{1}{2t^2} \times \frac{1}{4}$ $t = \frac{1}{2} \quad \frac{dy}{dx} = -\frac{1}{2}$	M1 A1 M1 A1	 4	differentiate. 4; at^{-2} seen both derivatives correct use chain rule candidates' $\frac{dy}{dt} \div \frac{dx}{dt}$ CSO
(b)	gradient of normal = 2 $(x, y) = (5, 0) \quad \frac{y}{x-5} = 2$	B1F M1 A1F	 3	F if gradient $\neq \pm 1$ calculate and use (x, y) on normal F on gradient of normal ACF
(c)	$x-3=4t \quad \text{or} \quad y+1=\frac{1}{2t}$ $(x-3)(y+1)=2$	B1 M1 A1	 3	or $t = \frac{x-3}{4}$ or $\frac{1}{t} = 2(y+1)$ eliminate t ; allow one error accept $y = \frac{1}{\frac{2(x-3)}{4}} - 1$ ACF SC allow marks for part (c) if done in part (a)
	Total		10	
3(a)	$\sin(x+2x) = \sin x \cos 2x + \cos x \sin 2x$ $= \sin x(1-2\sin^2 x) + \cos x(2\sin x \cos x)$ $= \sin x(1-2\sin^2 x) + 2\sin x(1-\sin^2 x)$ $= 3\sin x - 2\sin^3 x - 2\sin^3 x$ $= 3\sin x - 4\sin^3 x$	M1 B1B1 A1 A1	 5	double angles; ACF ISW condone missing x all in $\sin x$, correct expression CSO AG
(b)	$\sin^3 x = a \sin x + b \sin 3x$ $\int \sin^3 x \, dx = -a \cos x - \frac{b}{3} \cos 3x$ $\int \sin^3 x \, dx = \frac{1}{4} \left(-3 \cos x + \frac{1}{3} \cos 3x \right) (+C)$	M1 A1F A1	 3	attempt to solve for $\sin^3 x$ where $a \neq 0$ and $b \neq 0$ either integral correct F on a, b CAO alternative method by parts (see end of mark scheme)
	Total		8	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$(1-x)^{\frac{1}{4}} = 1 + \frac{1}{4}(-x) + \frac{1}{2} \times \frac{1}{4} \left(-\frac{3}{4}\right)(-x)^2$ $= 1 - \frac{1}{4}x - \frac{3}{32}x^2$	M1 A1	2	$1 \pm \frac{1}{4}x + kx^2$ equivalent fractions or decimals
(a)(ii)	$(81-16x)^{\frac{1}{4}} = 81^{\frac{1}{4}} \left(1 - \frac{16}{81}x\right)^{\frac{1}{4}}$ $= k \left(1 - \frac{1}{4} \times \frac{16}{81}x - \frac{3}{32} \left(\frac{16}{81}x\right)^2\right)$ $= 3(\quad)$ $= 3 - \frac{4}{27}x - \frac{8}{729}x^2$	B1 M1 A1	3	x replaced by $\frac{16}{81}x$ or start binomial again condone one error (missing bracket; x or x^2 ; sign error) CSO AG use of $(a+bx)^n$ ignoring hence (see end of mark scheme)
(b)	$3 - \frac{4}{27} \times \frac{1}{16} - \frac{8}{729} \left(\frac{1}{16}\right)^2$ $= 2.9906979$	M1 A1	2	use $x = \frac{1}{16}$ seven decimal places only
Total			7	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\cos \alpha = \frac{3}{5}$	B1	1	ACF
(a)(ii)	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ $= \frac{3}{5} \cos \beta + \frac{4}{5} \sin \beta$	M1 A1	2	ACF
(a)(iii)	$\sin \beta = \frac{12}{13}$ $\cos(\alpha - \beta) = \frac{63}{65}$	B1 B1	2	$\frac{63}{65}$ NMS B1B1
(b)(i)	$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $2 \tan x = 1 - \tan^2 x$ $\tan^2 x + 2 \tan x - 1 = 0$	M1 A1	2	CSO AG
(b)(ii)	$\tan x = \frac{-2 \pm \sqrt{4+4}}{2}$ $= -1 \pm \sqrt{2}$ $2x = 45^\circ \Rightarrow x = 22\frac{1}{2}^\circ$ is acute $\Rightarrow \tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$	M1 A1 E1	3	must solve quadratic equation by formula or by completing the square condone one slip $\pm \sqrt{2}$ required explain selection of positive root
	Total		10	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{2}{(x^2 - 1)} = \frac{A}{x-1} + \frac{B}{x+1}$ $2 = A(x+1) + B(x-1)$ $x=1 \quad x=-1$ $A=1 \quad B=-1$	M1 m1 A1	3	use two values of x or equate coefficients and solve $A + B = 0$ and $A - B = 2$ both A and B
(b)	$\int \frac{2}{x^2 - 1} dx = p \ln(x-1) + q \ln(x+1)$ $= \ln(x-1) - \ln(x+1)$	M1 A1F	2	ln integrals F on A and B condone missing brackets
(c)	$\int \frac{dy}{y} = \int \frac{2}{3(x^2 - 1)} dx$ $\ln y = \frac{1}{3}(\ln(x-1) - \ln(x+1)) \quad (+C)$ $(3, 1) \quad \ln 1 = \frac{1}{3}(\ln 2 - \ln 4) + C$ $3 \ln y = \ln(x-1) - \ln(x+1) - (\ln 2 - \ln 4)$ $3 \ln y = \left(\ln \left(\frac{x-1}{x+1} \right) + \ln 2 \right)$ $\ln y^3 = \ln \left(\frac{2(x-1)}{x+1} \right)$ $y^3 = \frac{2(x-1)}{x+1}$	M1 A1 A1F m1 A1	5	separate and attempt to integrate on one side left hand side F from part (b) on right hand side use (3, 1) to attempt to find a constant CSO AG
	Total		10	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$AB^2 = (5-3)^2 + (3-2)^2 + (0-1)^2$ $AB = \sqrt{30}$	M1 A1	2	use $\pm(\overrightarrow{OB} - \overrightarrow{OA})$ in sum of squares of components allow one slip in difference accept 5.5 or better
(b)	$\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} = 2 + 3 = 5$ $\cos \theta = \frac{5}{\sqrt{30}\sqrt{10}}$ $\theta = 73^\circ$	M1 A1 B1F M1 A1	5	$\pm \overrightarrow{AB} \bullet$ direction l evaluated condone one component error 5 or -5 F on either of candidates' vectors use $ a b \cos \theta = a \bullet b$; values needed CAO (condone 73.2, 73.22 or 73.22...)
(c)	$\overrightarrow{AC} = \begin{bmatrix} 5+\lambda \\ 3 \\ -3\lambda \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+\lambda \\ 5 \\ -1-3\lambda \end{bmatrix}$ $(2+\lambda)^2 + 5^2 + (-1-3\lambda)^2 = 30$ $10\lambda^2 + 10\lambda = 0$ $(\lambda = 0 \text{ or } \lambda = -1)$ $(\lambda = 0 \Rightarrow (5, 3, 0) \text{ is } B)$ $\lambda = -1 \Rightarrow C \text{ is } (4, 3, 3)$	M1 A1 m1 A1 A1	5	for $\overrightarrow{OC} - \overrightarrow{OA}$ or $\overrightarrow{OA} - \overrightarrow{OC}$ with \overrightarrow{OC} in terms of λ condone one component error condone $\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$
Total			12	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$p \frac{dx}{dt} = q$	M1		where p and q are functions
	$\frac{dx}{dt} = -kx$	A1	2	in any correct combination
(a)(ii)	$-500 = -k 20000$ or $500 = k 20000$	M1		condone sign error or missing 0 k can be on either side of the equation
	$k = \frac{5}{200}$ ($= 0.025$)	A1	2	CSO both (a)(i) and (a)(ii)
(b)(i)	$A = 1300$	B1	1	
(b)(ii)	$100 > Ae^{-0.05t}$	M1		condone = for >; condone 99 for 100
	$\ln\left(\frac{100}{A}\right) > -0.05t$	m1		take logs correctly condone 0.5
	$t > 51.3$	A1		or by trial and improvement (see end of mark scheme)
	population first exceeds 1900 in 2059	A1F	4	F if M1 m1 earned and $t > 0$ following A
	Total		9	
	TOTAL		75	

MPC4 (cont)

Alternative methods permitted in the mark scheme

Q	Solution	Marks	Total	Comments
1(b)(ii)	<p>ALTERNATIVE METHOD 1</p> <p>$(3x + 2)$ is a factor</p> <p>use factor theorem</p> <p>$f\left(\frac{1}{3}\right) = 0 \Rightarrow (3x - 1)$ is a factor</p> <p>$f(x) = (3x + 2)(3x - 1)(ax + b)$</p> <p>$f(x) = (3x + 2)(3x - 1)(3x - 1)$</p> <p>ALTERNATIVE METHOD 2</p> <p>$(3x + 2)$ is a factor</p> <p>divide $27x^3 - 9x + 2$ by $(3x + 2)$</p> <p>$9x^2 - 6x + 1$</p> <p>$f(x) = (3x + 2)(3x - 1)(3x - 1)$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>4</p> <p>4</p>	<p>PI</p> <p>use factor theorem or algebraic division to find another factor</p> <p>PI by division</p> <p>complete division to $ax^2 + bx + c$</p>
1(b)(ii)	<p>SPECIAL CASE</p> <p>$(3x + 2)(3x - 1)(ax + b)$</p>		2	
2(a)	<p>$y = \frac{2}{x-3} - 1$ and differentiate</p> <p>$\frac{dy}{dx} = \frac{-2}{(x-3)^2}$</p> <p>$x = 5$</p> <p>$\frac{dy}{dx} = \frac{-2}{(5-3)^2}$</p> <p>$\frac{dy}{dx} = -\frac{1}{2}$</p>	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	4	<p>differentiate expression in y and x</p> <p>correct</p> <p>find and therefore use x (and y)</p>

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(b)	<p>ALTERNATIVE METHOD 1</p> $\int \sin^3 x dx = \int \sin^2 x \sin x dx$ $= -\sin^2 x \cos x - \int -2 \cos x \sin x \cos x dx$ $= -\sin^2 x \cos x - \frac{2}{3} \cos^3 x \quad (+C)$ <p>ALTERNATIVE METHOD 2</p> $\int \sin^3 x dx = \int \sin^2 x d(-\cos x)$ $= \int -(1 - \cos^2 x) d(\cos x)$ $= -\cos x + \frac{1}{3} \cos^3 x \quad (+C)$ <p>ALTERNATIVE METHOD 3</p> $\int \sin x \sin^2 x dx$ $\int \sin x (1 - \cos^2 x) dx$ $= -\cos x + \frac{1}{3} \cos^3 x \quad (+C)$	<p>M1</p> <p>A2</p> <p>M1</p> <p>A2</p> <p>M1</p> <p>A2</p>	<p>3</p> <p>3</p> <p>3</p>	<p>identify parts and attempt to integrate</p> <p>condone sign error</p> <p>this form and attempt to integrate</p>
4(a)(ii)	$(81 - 16x)^{\frac{1}{4}} = 81^{\frac{1}{4}} + \frac{1}{4} 81^{-\frac{3}{4}} (-16x) + \frac{1}{4} \left(-\frac{3}{4}\right) \frac{1}{2} 81^{-\frac{7}{4}} (-16x)^2$ $= \left(3 - \frac{4}{27}x - \frac{8}{729}x^2\right)$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>3</p>	<p>using $(a + bx)^n$ from FB</p> <p>condone one error</p> <p>CSO completely correct</p>
8(b)(ii)	<p>$t = 51 \rightarrow 101.5$</p> <p>$t = 52 \rightarrow 96.6$</p> <p>$\Rightarrow 51 < t < 52$</p> <p>population first exceeds 1900 in 2059</p>	<p>M1</p> <p>A3</p>	<p>4</p>	<p>$t = 51$ or $t = 52$ considered</p> <p>CAO</p>