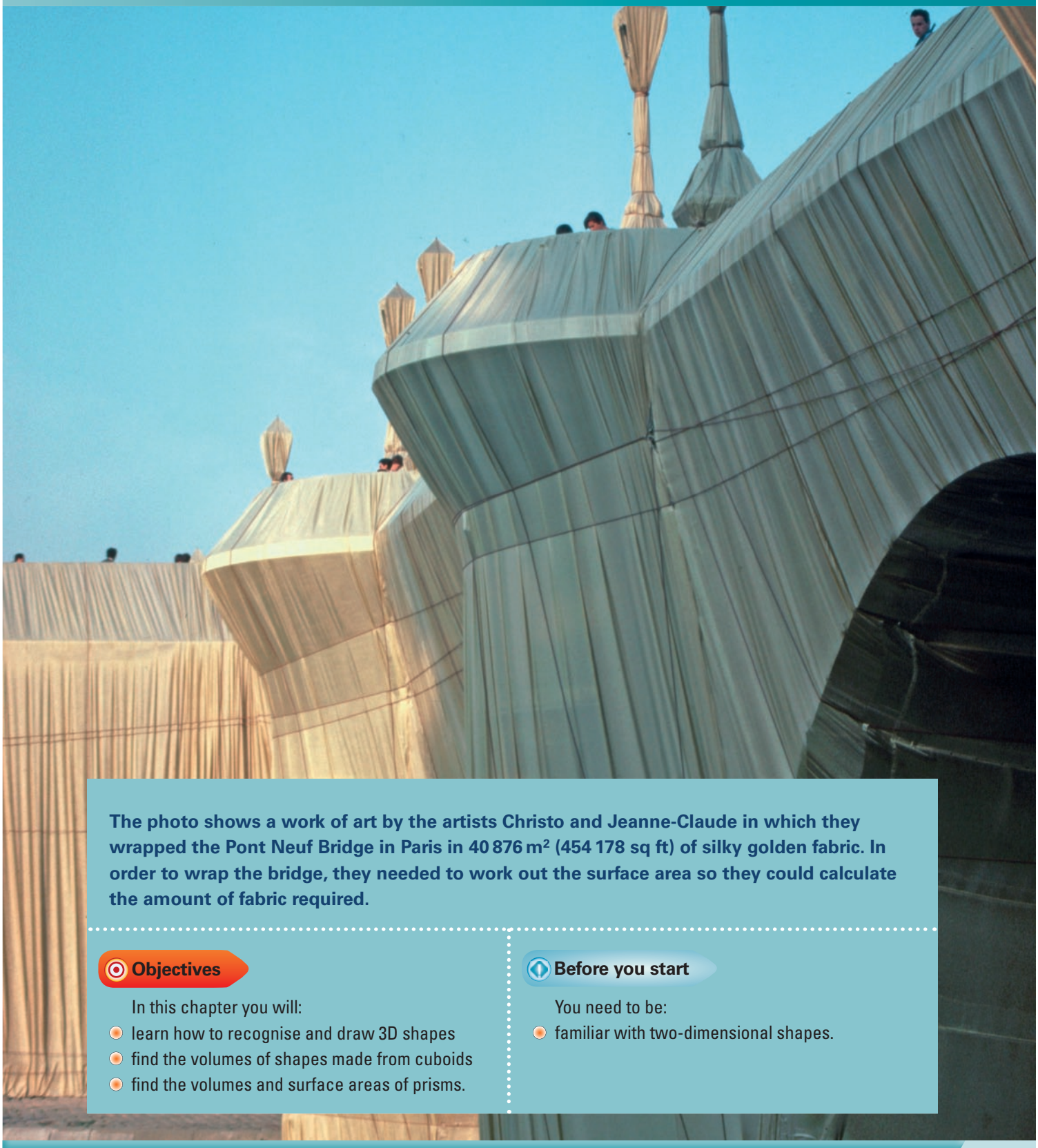
19 THREE-DIMENSIONAL SHAPES



The photo shows a work of art by the artists Christo and Jeanne-Claude in which they wrapped the Pont Neuf Bridge in Paris in 40 876 m² (454 178 sq ft) of silky golden fabric. In order to wrap the bridge, they needed to work out the surface area so they could calculate the amount of fabric required.

Objectives

In this chapter you will:

- learn how to recognise and draw 3D shapes
- find the volumes of shapes made from cuboids
- find the volumes and surface areas of prisms.

Before you start

You need to be:

- familiar with two-dimensional shapes.

19.1 Recognising three-dimensional shapes

Objectives

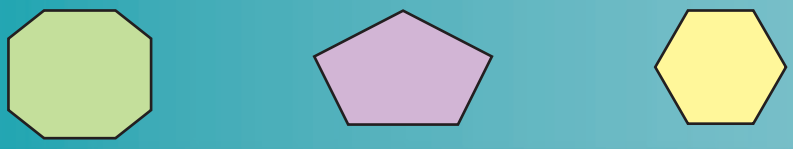
- You can recognise 3D shapes.
- You can count the vertices, edges and faces of 3D shapes.

Why do this?

A designer or architect needs to be able to describe the shape they want to build. The correct names for 3D shapes make it easier to explain what the product will look like.

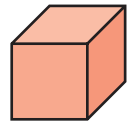
Get Ready

1. Can you remember the names of these two-dimensional shapes?

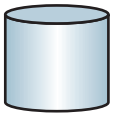


Key Points

- Here are some **three-dimensional** shapes.

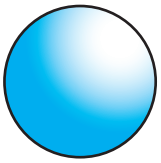


Cube

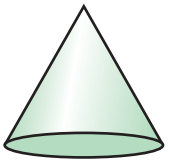


Cylinder

Cuboid



Sphere

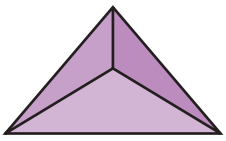


Cone

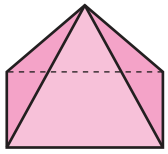
ResultsPlus
Examiner's Tip

You need to know the names of all of these shapes.

- **Pyramids** have a base, which can be any shape, and sloping triangular sides.

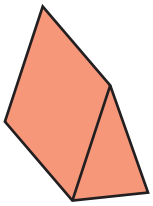


Triangular pyramid called a tetrahedron

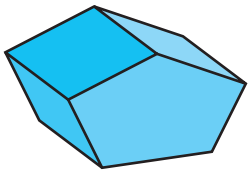


Rectangular-based pyramid

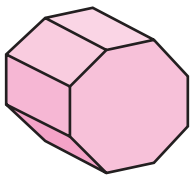
- A **prism** has two parallel faces and a number of rectangular sides joining them.



Triangular prism

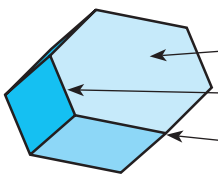


Pentagonal prism



Octagonal prism

- The flat surfaces of a 3D shape are called **faces**.
- The lines where two faces meet are called **edges**.
- The point (corner) at which edges meet is called a **vertex**. The plural of vertex is vertices.



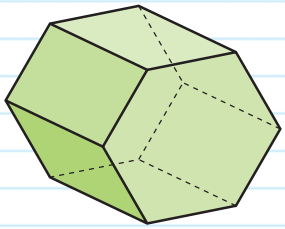
Face (two-dimensional surface).

Edge (where two faces meet).

Vertex (corner where three or more edges meet).

Example 1

a Name this shape.
b How many faces does it have?
c How many edges does it have?
d How many vertices does it have?



Two parallel hexagons
Rectangular sides

a Hexagonal prism
b 8 faces
c 18 edges
d 12 vertices

The parallel faces are called the cross-section of the prism. The cross-section of this prism is a hexagon.

Exercise 19A

Questions in this chapter are targeted at the grades indicated.

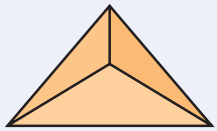
G

1 Name this shape.

2 This 3D shape has a circular base.
Name the 3D shape.

3 A prism has a base which has five sides. What type of prism is it?

4 This pyramid has a special name. Name this pyramid.



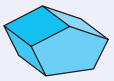
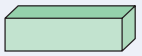
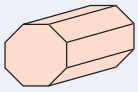
5 Look at the picture. Copy and complete the table with as many 3D shapes as you can find.

	Shape	Object
1	sphere	football
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

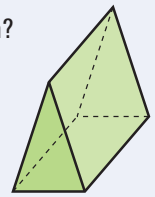
Exercise 19B

F

1 Copy and complete the table below.

	Shape	Faces	Edges	Vertices
A	Cube 			
B	Pentagonal prism 			
C	Triangular prism 			
D	Square-based pyramid 			
E	Cuboid 			
F	Tetrahedron 			
G	Octagonal prism 			

2 What is the shape of the cross-section of this prism?



E A03

3 Draw a sketch of a prism with a pentagonal cross-section.

4 A pyramid has six triangular faces.

a What is the shape of its other face? **b** What type of pyramid is it?

19.2 Isometric paper

Objective

- You can draw 3D shapes using isometric paper.

Why do this?

A designer making a 3D container must be able to make an accurate 3D drawing of it.

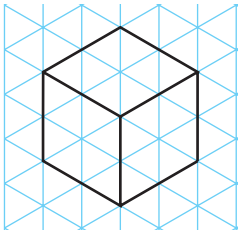
Get Ready

Draw these shapes.

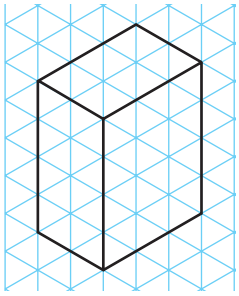
1. A cube **2.** A triangular prism **3.** A square-based pyramid

Key Points

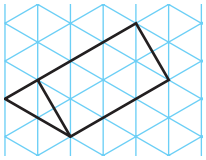
- **Isometric paper** will help you to make scale drawings of three-dimensional objects.
- Isometric paper must be the right way up – **vertical** lines down the page and no **horizontal** lines.



This cube has sides of 2 cm.



This cuboid has height 4 cm, length 3 cm and width 2 cm.



This prism has a triangular face.

Shapes can be joined together.

ResultsPlus
Examiner's Tip

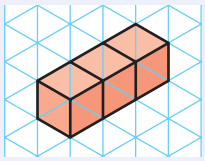
Draw the shape at an angle as if you are looking from the bottom-right corner.

Exercise 19C

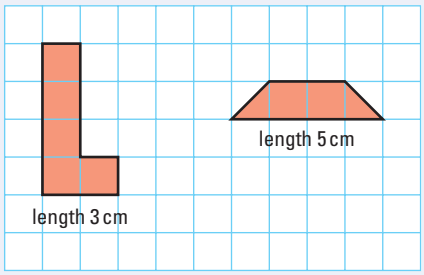
1 On isometric paper draw a cube of side 3 cm.

2 Use isometric paper to draw a cuboid with height 2 cm, width 4 cm and length 3 cm.

3 The diagram shows a shape made up of three cubes.
On isometric paper draw a different shape made up of the same three cubes.



4 Use isometric paper to make full-sized drawings of these prisms.



E

19.3 Volume of a prism

Objectives

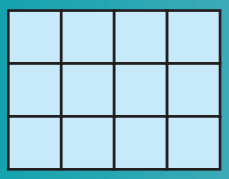
- You can find the volumes of shapes made from cuboids.
- You can find the volumes of prisms.

Why do this?

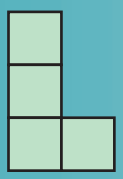
Food manufacturers need to know the volume of food containers, for example, cereal packets or drink cartons.

Get Ready

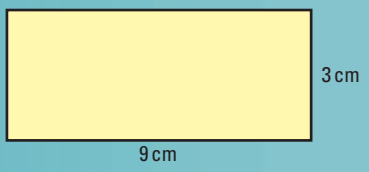
1. What is the area of this rectangle?
Each small square has side 1 cm.



2. What is the area of this shape?
Each small square has side 1 cm.

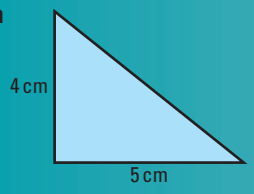


3. Use the formula
area = length × width
to find the area of this shape.

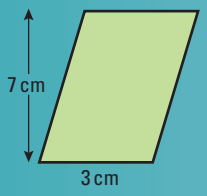


4. Find the area of these two-dimensional shapes.

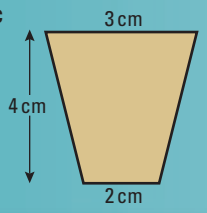
a



b

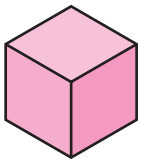


c



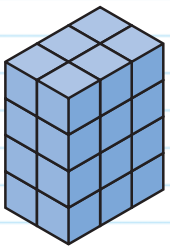
Key Points

- The **volume** of a 3D shape is the amount of space it takes up.
The diagram shows a cube of side 1 cm. Its volume is 1 cm^3.



- The volume of a 3D shape with measurements in centimetres is the number of centimetre cubes it contains.
- The volume of a prism is the area of the **cross-section** × its length.

Example 2 The diagram shows a cuboid made from centimetre cubes.
Find the volume of the cuboid.



Remember some of the cubes are hidden. Try building the shape using multilink.

Multiply the length by the width (3 × 2) to find the number of cubes (6) in each layer.

There are 6 centimetre cubes in each layer.

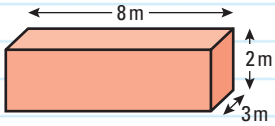
Multiply this by the height (4) to give the number of cubes altogether (24).

There are 4 layers.

So the volume of the cuboid = length × width × height. When the lengths are measured in metres, volume is measured in m^3.

The volume of the cuboid is $6 \times 4 = 24$ cm^3.

Example 3 Work out the volume of the cuboid shown below.



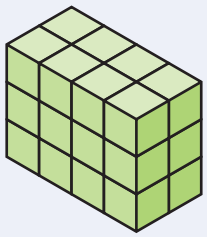
Volume of the cuboid = length × width × height
Volume = 8 × 3 × 2 m³ = 48 m³

ResultsPlus
Examiner's Tip
Remember, you need to specify the units in all of your answers.

Exercise 19D

G

1 The diagrams below show prisms that have been made from centimetre cubes.
Find the volume of each prism.

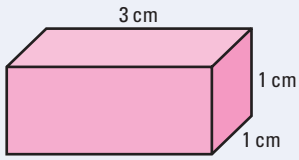
a


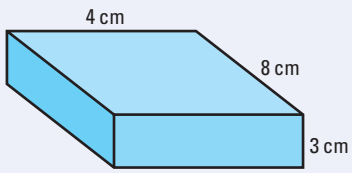
b


c


E

2 Work out the volumes of these cuboids.

a


b


3 Work out the volume of the following cuboids.
a length 7 m, width 8 m and height 4 m
b length 17 mm, width 12 mm and height 3 mm

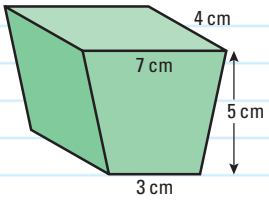
4 Work out the volume of a cube of side 7 cm.

5 A cuboid has volume 150 cm³. Its width is 3 cm and its height is 10 cm.
Find the length of the cuboid.

6 A cereal packet was measured and found to have height 35 cm, width 20 cm and depth 10 cm.
Give the dimensions of a box that will hold 24 cereal packets.

A03

Example 4 Work out the volume of this prism.



ResultsPlus
Examiner's Tip
If you can't remember how to use the formula, you can split the trapezium into a rectangle and two triangles.

The cross-section is a trapezium. ← Decide on the shape of the cross-section.

area of a trapezium $= \frac{1}{2}(a + b) \times l$

$= \frac{1}{2} \times (3 + 7) \times 5$ ← Put in the lengths you know.

$= 25\ cm^2$ ← Don't forget the units.

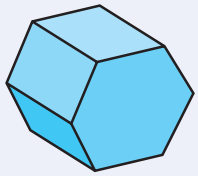
Volume $= 25 \times 4$

$= 100\ cm^3$

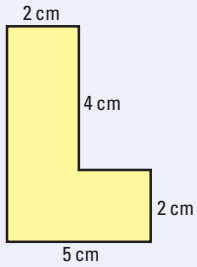
Exercise 19E

C

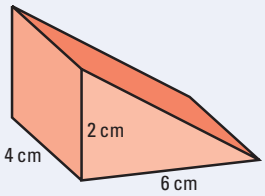
1 The diagram shows a prism.
The cross-section of this prism is a hexagon with an area of 5 m^2.
If the length of the prism is 3 m, what is the volume of the prism?



2 The diagram shows the cross-section of a prism of length 8 cm.
Work out the volume of the prism.

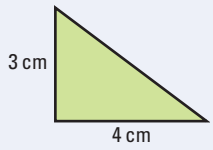


3 The diagram shows a prism with a right-angled triangle as its cross-section.
Work out the volume of the prism.



A02 A03

4 A metal bar of length 30 cm has a square cross-section of side 5 cm.
The metal is to be recast into triangular rods of length 10 cm. The cross-section of the rods is a right-angled triangle as shown.



a Work out the volume of the metal bar.
b Work out the volume of each rod.
c Work out the maximum number of rods that can be made from the bar.
d Work out how much metal is left over.

5 A triangular prism has length 12 cm. The triangular face has base 8 cm and height 9 cm.
Calculate the volume of the prism.

19.4 Surface area of a prism

Objective

- You can find the surface area of a prism.

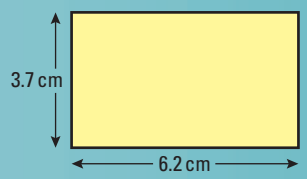
Why do this?

The surface area is the amount of space on an object available for design, information or advertising.

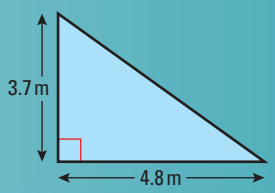
Get Ready

1. Find the area of these two-dimensional shapes.

a

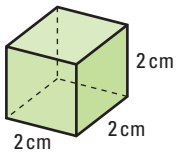


b

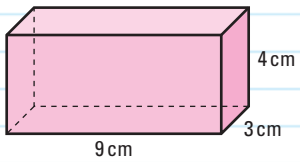


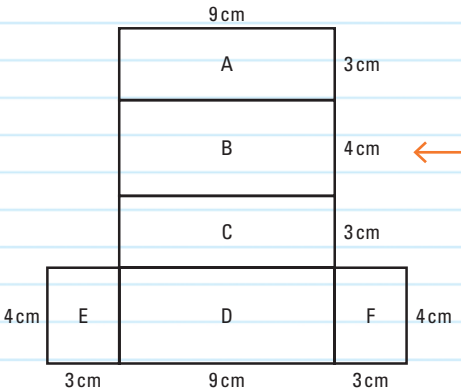
Key Points

- The **surface area** of a prism is the area of the net that can be used to build the shape.
- Area is measured in square units – mm^2, cm^2, m^2 and km^2 are common.
- Each of the six faces of a cube is a square, so the surface area of a cube is 6 times the area of one face.
- The area of each square face of this cube is $2 \times 2 = 4\ cm^2$.
  So the surface area of the cube is $6 \times 4 = 24\ cm^2$.



Example 5 Work out the surface area of this cuboid.





Sketch the net of the cuboid and label with the lengths you know. Label the shapes you will find the area of with a letter.

Area of rectangle A $= 3 \times 9 = 27\ cm^2$
Area of rectangle B $= 4 \times 9 = 36\ cm^2$
Area of rectangle C $= 3 \times 9 = 27\ cm^2$
Area of rectangle D $= 4 \times 9 = 36\ cm^2$
Area of rectangle E $= 3 \times 4 = 12\ cm^2$
Area of rectangle F $= 3 \times 4 = 12\ cm^2$
Total surface area $= 150 cm^2$

ResultsPlus Examiner's Tip

Show the examiner evidence that you can find the area of each shape. Marks will be awarded for this step!

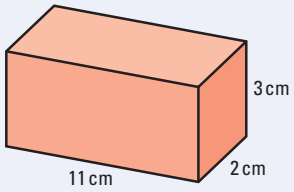
ResultsPlus Examiner's Tip

Remember units of area are cm^2.

Exercise 19F

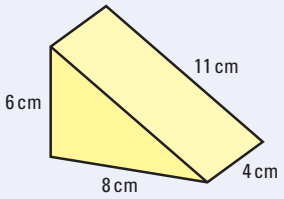
E

1 Find the surface area of this cuboid.



A02

2 The diagram shows a piece of cheese.
Work out the surface area of the cheese.

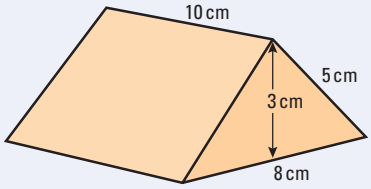


A02

3 A cereal packet in the shape of a cuboid has height 35 cm, width 20 cm and depth 20 cm.
Work out the surface area of the cereal packet.

D A02

4 The diagram shows a plastic part from a child's toy.
Work out the surface area of the part.



C A02 A03

5 A cube has a surface area of 24 cm^2.
Work out the length of the side of the cube.

A03

6 A block of icing is sold in the shape of a prism.
The cross section of the prism is in the shape of a square.
The length of the prism is twice the length of the side of the square.
The surface area of the prism is 90 cm^2.
Work out the length of the side of the square.

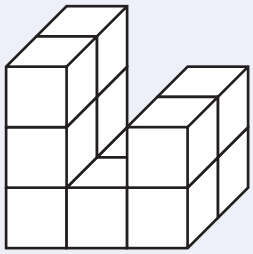
Chapter review

- Some examples of **three-dimensional** shapes are: **cube**, **cylinder**, **cuboid**, **sphere** and **cone**.
- **Pyramids** have a base which can be any shape, and sloping triangular sides.
- A **prism** has two parallel faces and a number of rectangular sides joining them.
- The flat surfaces of a 3D shape are called **faces**.
- The lines where two faces meet are called **edges**.
- The point (corner) at which edges meet is called a **vertex**. The plural of vertex is vertices.
- **Isometric paper** will help you to make scale drawings of three-dimensional objects.
- Isometric paper must be the right way up – **vertical** lines down the page and no **horizontal** lines.
- The **volume** of a 3D shape is the amount of space it takes up.
- The volume of a 3D shape with measurements in centimetres is the number of centimetre cubes it contains.
- The volume of a prism is the area of the **cross-section** × its length.
- The **surface area** of a prism is the area of the net that can be used to build the shape.

Review exercise

G

1 Here is a solid prism made from centimetre cubes.

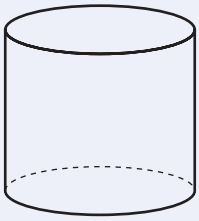


Work out the volume of the solid prism.

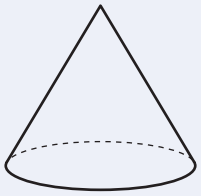
Nov 2008

2 Write down the mathematical name of each of these two 3D shapes.

i



ii



Nov 2008

F

3 Find the volume of this prism.

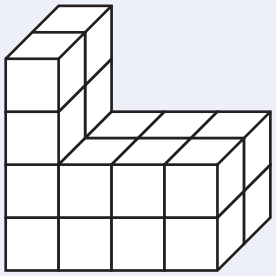
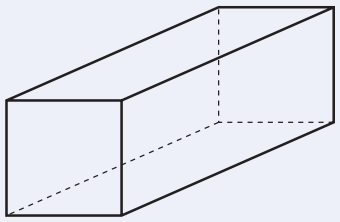


Diagram **NOT** accurately drawn

represents 1 cm^3

June 2008

4 Here is a diagram of a cuboid.

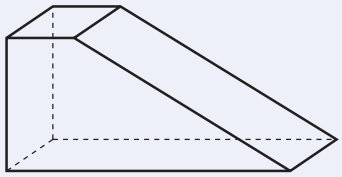


Write down the number of

i faces ii edges iii vertices.

June 2008

5 Here is a diagram of a 3D prism.



Write down the number of

i faces ii edges iii vertices.

Nov 2007

E

6 Work out the volume of the cuboid.

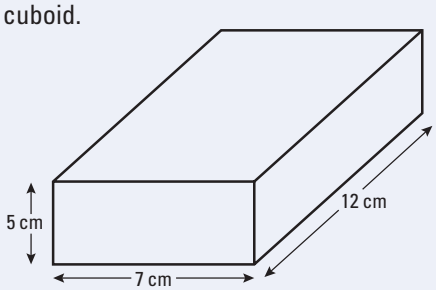
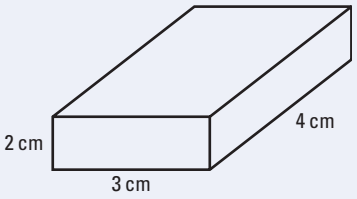


Diagram **NOT** accurately drawn

Nov 2008

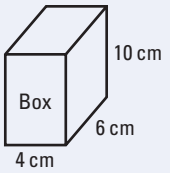
7 The diagram shows a solid cuboid.
On a triangular isometric grid, make an accurate full-size drawing of the cuboid.



June 2007

D

8 Cereal boxes are packed into cartons.
A cereal box measures 4 cm by 6 cm by 10 cm.
A carton measures 20 cm by 30 cm by 60 cm.
The carton is completely filled with cereal boxes.
Work out the number of cereal boxes that will completely fill **one** carton.



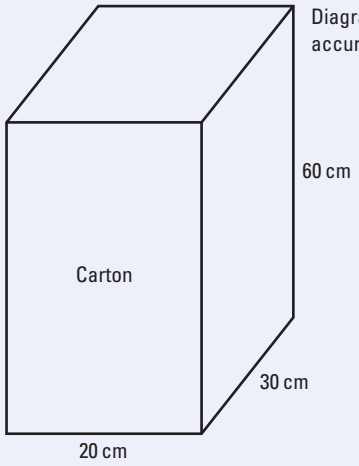


Diagram **NOT** accurately drawn

Nov 2007

C

9 Here is a triangular prism.
Calculate the volume of the prism.

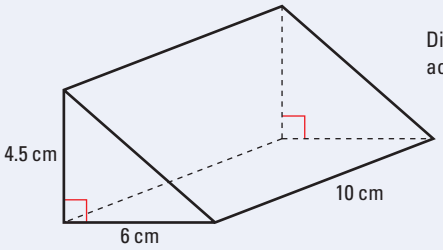


Diagram **NOT** accurately drawn

June 2007

10 A solid cube has sides of length 5 cm.
Work out the total surface area of the cube.
State the units of your answer.

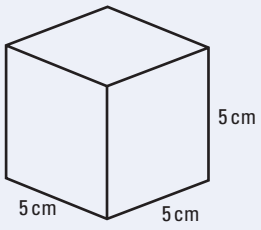


Diagram **NOT** accurately drawn

Nov 2009

C

11 Work out the total surface area of the triangular prism.

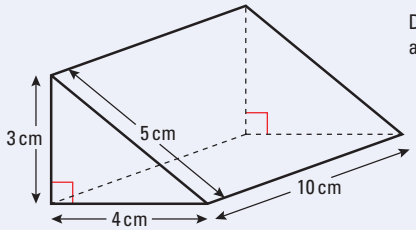


Diagram **NOT** accurately drawn

May 2008