1 NUMBER

A polyrhythm in drumming is when the drummer plays two different rhythms at the same time, one with each hand. A common polyrhythm is 3 : 2. The lowest common multiple of 2 and 3 is 6, so the drummer counts from 1 to 6. On the first, third and fifth beat he plays with his left hand; on the first and fourth he uses his right. The drum is silent on the sixth beat.

left: 1 2 3 4 5 6

right: 1 2 3 4 5 6

Objectives

In this chapter you will:

- see the importance of place value
- see the usefulness of a number line
- see why whole numbers can be negative as well as positive
- use the rules of addition, subtraction, multiplication and division for combining numbers
- learn some of the language of mathematics connected with numbers.

Before you start

You need to know:

- the addition number bonds to $9 + 9$
- the times tables to 10×10.

1.1 Understanding digits and place value

Objective

- You understand the number system.

Why do this?

To carry out an everyday task such as buying something in a shop you need to understand how the number system works.

Get Ready

1. What numbers come before and after 509?
2. Write down the answer to $7 + 6$.
3. Write down the answer to 7×8.

Key Points

- Although you can keep on counting to very high numbers, you only use ten **digits**, often called figures.

0 1 2 3 4 5 6 7 8 9

- Each digit has a **value** that depends on its position in the number. This is its **place value**.

Example 1

Draw a place value diagram for:

a a three-digit number with a 6 in the tens column

b a five-digit number with a 6 in the thousands column.

Exercise 1A

Questions in this chapter are targeted at the grades indicated.

G

1 Draw a place value diagram and write in:

a a four-digit number with a 4 in the thousands column

b a two-digit number with a 3 in the tens column

c a five-digit number with a 1 in the hundreds column

d a three-digit number with a 9 in the units column

e a four-digit number with a 0 in the tens column

f a five-digit number with a 4 in the hundreds column

g a four-digit number with a 7 in every column except the tens column

h a five-digit number with a 6 in the thousands column and the units column.

2 For each teacher, write down five different numbers that they could be thinking about.

3 Write down the value of the 6 in each of these numbers.

a 63 **b** 3642 **c** 63 214 **d** 2546 **e** 56 345

G

1.2 Reading, writing and ordering whole numbers

Objectives

- You can read and write down whole numbers.
- You can put numbers in order of size.

Why do this?

You need to be able to compare the size of numbers when you want to decide which mobile phone handset is cheaper.

Get Ready

1. Write down the value of the 4 in each of these numbers.

a 46 **b** 2034 **c** 65 403

Key Points

- You can read and write numbers by thinking about the place value of each of the digits.
- You can use your knowledge of place value to put numbers in order of size.

Example 2 Write in figures the numbers shown in these newspaper cuttings.

	Ten thousands	Thousands	Hundreds	Tens	Units	
Two thousand eight hundred and twenty-two		2	8	2	2	or 2822
Thirty-one thousand two hundred and eight	3	1	2	0	8	or 31 208

0 goes in the tens column because there are no tens.

Example 3 Write in words the numbers shown in the place value diagram.

Ten thousands	Thousands	Hundreds	Tens	Units	
		9	8	7	Nine hundred and eighty-seven
1	6	4	1	2	Sixteen thousand four hundred and twelve

In 16 412 the thin space separates thousands from hundreds and makes it easier to read the number.

Example 4 Write the numbers 8400, 6991, 15, 2410, 84 000, 2406 in order of size.
Start with the smallest.

15 is the smallest. ← It has no digits above the tens column.

84 000 is the largest. ← It is the only one with a digit in the ten thousands column.

This leaves 8400, 6991, 2410 and 2406.
2000 is smaller than 6000, which is smaller than 8000.
Lastly, 2406 is smaller than 2410.
The order is 15, 2406, 2410, 6991, 8400, 84 000.

Exercise 1B

G

1 Write these numbers in figures.
- **a** Three hundred and twenty-five
- **b** One thousand seven hundred and eighteen
- **c** Six thousand two hundred and four
- **d** Nineteen thousand four hundred and twenty

2 Write these numbers in words.
a 237 **b** 321 **c** 1792 **d** 6502 **e** 1053

3 Write each set of numbers in order of size, starting with the smallest.
- **a** 183, 235, 190, 73, 179
- **b** 2510, 2015, 970, 2105, 2439
- **c** 30 300, 3033, 3000, 3003, 2998
- **d** 56 762, 59 342, 56 745, 56 321

4 Write the numbers that have been highlighted in blue in figures.
- **a** The numbers of people employed by a local police force are:
  Traffic wardens: sixty-nine
  Civilian support staff: one thousand and ten
  Police officers: two thousand three hundred and six
- **b** The tonnages of three cruise liners are:
  Aurora: seventy-six thousand one hundred and fifty-two
  QE2: seventy thousand three hundred and sixty-three
  Queen Mary 2: one hundred and fifty-one thousand four hundred

G

5 This table gives the populations of five member states of the European Union in 2009. Write the numbers in words.

	Country	Population
a	Czech Republic	10 467 542
b	Cyprus	793 963
c	Estonia	1 340 415
d	France	64 351 000
e	Portugal	10 627 250

6 The table gives the prices of some second-hand cars.

Car	Price
Peugeot 505	£7995
Focus	£11 495
Ka	£4835
Mini	£6549
Sharan	£13 205

a Write down the price of each car in words.
b Rewrite the list in price order, starting with the most expensive.

1.3 The number line

Objectives

- You can see numbers in context.
- You can use a simple number line to increase or decrease numbers and to work out the difference between numbers.

Why do this?

You need to be able to read scales you meet, such as rulers, thermometers and speedometers.

Get Ready

Place these numbers in order, starting with the lowest.

1. £5490 £3645 £5250 £4190
2. 10 348 276 462 690 40 280 780 10 524 145 60 424 213
3. 76 152 70 363 150 400

Key Points

- You can use a number line to help you to increase or decrease a number.
- Number lines might be read:
  - from left to right
  - from bottom to top
  - clockwise.

Example 5 Use a number line to: **a** increase 6 by 4 **b** decrease 23 by 8.

Exercise 1C

G

1 Draw a number line from 0 to 30.
Mark these numbers on your number line.
a 6 **b** 23 **c** 15 **d** 0 **e** 29

2 Use a number line from 0 to 25 to
a increase 6 by 3 **b** decrease 15 by 7 **c** increase 11 by 7
d decrease 19 by 13 **e** increase 17 by 8 **f** decrease 24 by 17.

3 For each of these moves, write down the difference between them, and whether it is an increase or decrease.
a 6 to 10 **b** 15 to 21 **c** 10 to 3 **d** 29 to 24 **e** 5 to 27 **f** 19 to 2

1.4 Adding and subtracting

Objective

- You can add and subtract without a calculator.

Why do this?

When you are shopping you might want to know how much you have spent before you get to the checkout. You might not always have a calculator on hand to help you!

Get Ready

1. Use a number line to:
a increase 29 by 33 **b** decrease 34 by 27 **c** find the difference between 20 and 3.

Key Points

- Words that show you have to add numbers are: add, plus, total and sum.
- Words that show you have to take away are: take away, subtract, minus and find the difference.

Example 6 Work out 23 + 693 + 8.

```
   23
  693
+   8
  724
  11
```

Step 1 Put the digits in their correct columns.
Step 2 Add the units column: 3 + 3 + 8 = 14.
Write 4 in the units column and carry the 1 ten into the tens column.
Step 3 Repeat for the tens column: 2 + 9 = 11 plus the 1 that was carried across = 12.
Write 2 in the tens column and carry the 1 into the hundreds column.
Step 4 Add the 6 and the 1 that was carried across: 7.

Exercise 1D

1. Find the total of 26 and 17.
2. Work out 58 plus 22.
3. Work out 236 + 95.
4. In four maths tests, Anna scored 61 marks, 46 marks, 87 marks and 76 marks.
   How many marks did she score altogether?
5. Find the sum of all the single-digit numbers.
6. In a fishing competition, five competitors caught 16 fish, 31 fish, 8 fish, 19 fish and 22 fish.
   Find the total number of fish caught.
7. The number of passengers on a bus was 36 downstairs and 48 upstairs.
   How many passengers were on the bus altogether?
8. On her MP3 player, Lena had 86 pop songs, 58 rock songs and 72 dance songs.
   How many songs did she have in total?
9. Work out 38 + 96 + 127 + 92 + 48.
10. On six days in June, 86, 43, 75, 104, 38 and 70 people went bungee jumping over a gorge.
    How many people jumped in total?

G

Example 7 Work out 423 − 274.

Method 1

```
  3 11 1
  4 2 3
− 2 7 4
  1 4 9
```

Step 1 Put the digits in their correct columns.
Step 2 In the units column try taking 4 from 3 (not possible). So exchange 1 ten for 10 units.
You now have 3 + 10 = 13 units.
Step 3 13 − 4 = 9 in the units column.
Step 4 The tens column has 1 take away 7, which is not possible, so exchange 1 hundred for 10 tens.
Step 5 11 − 7 = 4 in the tens column.
Step 6 Finally there is 3 − 2 = 1 in the hundreds column.

Method 2

Step 1	Count on from 274 to 280 =	6
Step 2	Count on from 280 to 300 =	20
Step 3	Count on from 300 to 400 =	100
Step 4	Count on from 400 to 423 =	23
Step 5	Add	149

Exercise 1E

G

1 Work out 611 − 306.

2 How much is 7260 − 4094?

3 Take 1007 from 2010.

4 In a car boot sale Alistair sells 17 of his 29 CDs.
How many does he have left?

5 When playing darts, James scored 111 with his first three darts, Sunita scored 94 with her first three darts and Nadine scored 75 with her first three darts.
- **a** How many more than Nadine did Sunita score?
- **b** What is the difference between James's and Sunita's scores?
- **c** How many less than James did Nadine score?

A03

6 The winner of the darts match is the first one to reach 501.
James has now scored 413, Sunita 442 and Nadine 368.
- **a** How many does James need to win the game?
- **b** How many more does Sunita need to win the game?
- **c** How many is Nadine short of winning the game?

1.5 Multiplying and dividing

Objective

- You can multiply and divide without a calculator.

Why do this?

If you buy several T-shirts or CDs that are the same price, it is useful if you can work out an estimate of the cost without a calculator.

Get Ready

1. Work out
- **a** 123 + 23 + 23
- **b** 57 − 19 − 19 − 19

Key Points

- Words that show you have to multiply numbers are: product, times and multiply.
- Divide and share are words that show you have to divide.
- You multiply by 10, 100 and 1000 by moving digits 1, 2 or 3 places to the left.
- You divide by 10, 100 and 1000 by moving digits 1, 2 or 3 places to the right.

Example 8 Work out 23 × 10.

H	T	U	
	2	3	
2	3	0	= 230

The 3 moves from the units into the tens column because 3 × 10 = 30.
The 2 moves from the tens into the hundreds column because 20 × 10 = 200.

Example 9 **a** Work out 43 × 6. **b** Work out 256 × 37.

```
  43
×  6
 258
 1
```

3 × 6 = 18 (1 ten and 8 units)
40 × 6 = 240
Total 258

```
   256
×   37
  1792
 7680
 9472
```

Step 1 256 × 7
Step 2 256 × 30
Step 3 add

Exercise 1F

1 Multiply each of these numbers by **i** 10 **ii** 100 **iii** 1000
a 27 **b** 8 **c** 301 **d** 60 **e** 5020

2 Work out
a 35 × 20 **b** 26 × 200 **c** 122 × 30
d 43 × 500 **e** 115 × 40 **f** 214 × 3000

3 Work out
a 65 × 7 **b** 53 × 3 **c** 314 × 6
d 523 × 8 **e** 237 × 5 **f** 399 × 4

4 Work out
a 34 × 12 **b** 65 × 15 **c** 53 × 33 **d** 314 × 16
e 523 × 47 **f** 221 × 64 **g** 146 × 53 **h** 29 × 38

5 Find the product of 71 and 13.

6 How many is 321 multiplied by 14?

7 John travels 17 miles to work and 17 miles back each day.
In July he went to work and back 23 times. How far did he travel altogether?

8 Bocton Football Club hires 23 coaches to take supporters to an away match.
Each coach can take 63 passengers. How many supporters can be taken to the match?

9 Each box of matches contains 43 matches.
How many matches are there in 36 boxes?

10 Tariq buys tins of soup which are packed in cases of 48.
He buys 8 cases. How many tins of soup does he buy?

G

A03

Example 10 Work out 3200 ÷ 100.

Th	H	T	U	
3	2	0	0	
		3	2	= 32

The 3 moves from the thousands into the tens column because 3000 ÷ 100 = 30.

The 2 moves from the hundreds into the units column because 200 ÷ 100 = 2.

Example 11 Work out 256 ÷ 8.

Traditional short division

$$\begin{array}{r} 32 \\ 8\overline{)25^{1}6} \end{array}$$

Step 3 8 into 16 goes 2 times.

Step 1 8 into 2 does not go.

Step 2 8 into 25 goes 3 times remainder 1. Carry remainder into units column.

Example 12 Work out 8704 ÷ 17.

Traditional long division

Step 1
17 divides into 87 five times remainder 2.

Step 2
17 divides into 20 one time remainder 3.

Step 3
17 divides into 34 two times exactly.

$$17\overline{)8704}$$

$$\begin{array}{r} 5 \\ 17\overline{)8704} \\ -\ 85 \\ \hline 2 \end{array} \qquad \begin{array}{r} 51 \\ 17\overline{)8704} \\ -\ 85 \\ \hline 20 \\ -\ 17 \\ \hline 3 \end{array} \qquad \begin{array}{r} 512 \\ 17\overline{)8704} \\ -\ 85 \\ \hline 20 \\ -\ 17 \\ \hline 34 \\ 34 \\ \hline 0 \end{array}$$

So 17 divides into 8704 512 times.

Short division method

$$\begin{array}{r} 512 \\ 17\overline{)87^{2}0^{3}4} \end{array}$$

This is a shorter way of setting out the steps than in the long division method.

Exercise 1G

G

1 Work out

a 3660 ÷ 10 b 4300 ÷ 100 c 9000 ÷ 10 d 87 000 ÷ 1000

2 Work out

a 48 ÷ 2 b 69 ÷ 3 c 56 ÷ 4 d 96 ÷ 6
e 640 ÷ 4 f 565 ÷ 5 g 72 ÷ 4 h 712 ÷ 8
i 828 ÷ 9 j 637 ÷ 7 k 408 ÷ 2 l 1020 ÷ 3

F

3 Work out

a 256 ÷ 16 b 660 ÷ 15 c 512 ÷ 32 d 861 ÷ 21
e 756 ÷ 36 f 1020 ÷ 30 g 1440 ÷ 36 h 7500 ÷ 25

4 a Work out 315 ÷ 15. b How many 50s make 750? c Work out $\frac{680}{17}$.
d Work out 600 divided by 30. e Divide 8 into 112.

5 Five people shared a prize draw win of £2400 equally.
How much did each person receive?

6 In an online computer game tournament, players are put into groups of 24, with the group winners going through to the final. A03
How many finalists will there be if there are
a 240 players b 720 players c 864 players?

7 An aeroplane can hold 18 parachute jumpers at a time. A03
How many trips does the plane have to make for
a 126 jumps b 234 jumps c 648 jumps?

8 A packing case will hold 72 economy-size boxes, 24 large-size boxes or 12 family-size boxes. A03
How many packing cases would be needed to pack
a 864 economy-size boxes b 984 large-size boxes c 960 family-size boxes?

1.6 Rounding

Objective

- You can write numbers to a suitable accuracy.

Why do this?

When you want to estimate a shopping bill, it is useful to be able to round the numbers to make the calculation easier.

Get Ready

Work out the total cost of these.

1. Two T-shirts at £15 each and two pairs of shorts at £4 each
2. Four cookies at 75p each and two cups of coffee at £1 each

Key Points

- There are situations where an estimate of numbers may be good enough.
  - If you are working out how long a journey is going to take, then an answer to the nearest hour may be good enough.
  - If you are recording the number of customers a supermarket has in a day, then the nearest 100 may be good enough.
- **Rounding** means reducing the accuracy of a number by replacing right-hand digits with zeros.
- To round a number, look at the digit before the place value you are rounding to (so for rounding to the nearest 10 you would look at the units digit). If it is less than 5, round down. If it is 5 or more, round up.

Example 13 Round 7650 to the nearest **a** thousand **b** hundred.

a 8000

7650 is nearer 8000 than 7000 on the number line. Rounded to the nearest thousand 7650 is 8000.

b 7700

7650 is exactly halfway between 7600 and 7700. When this happens you round up.

Example 14 Round 314 to the nearest 100.

314 to the nearest 100 is 300.

Look at the tens digit. It is less than 5 so round down.

Exercise 1H

G

1 Round these numbers to the nearest ten.

a 57 **b** 63 **c** 185 **d** 194 **e** 991 **f** 2407

2 Round these numbers to the nearest hundred.

a 312 **b** 691 **c** 2406 **d** 3094 **e** 8777 **f** 29 456

3 Round these numbers to the nearest thousand.

a 2116 **b** 36 161 **c** 28 505 **d** 321 604 **e** 717 171 **f** 2 246 810

F

4

	Length (ft)	Cruising speed (mph)	Takeoff weight (lb)
Airbus A310	153	557	36 095
Boeing 737	94	577	130 000
Saab 2000	89	403	50 265
Dornier 228	54	266	12 566
Lockheed L1011	177	615	496 000

For each of these aircraft, round:

a the length to the nearest ten feet

b the takeoff weight to the nearest hundred pounds

c the cruising speed to the nearest ten mph.

1.7 Negative numbers

Objective

- You can use negative numbers to represent quantities that are less than zero.

Why do this?

Negative numbers are used to record temperatures below zero and distances below sea level.

Get Ready

1. Place the numbers 557, 577, 403, 266 and 615 in order. Start with the smallest.

Key Points

- The further a number is from zero on the left-hand side of a number line, the smaller it is, so −25 is smaller than −3.
- For **negative numbers** you count backwards from zero on the number line.
  - −31°C is 31 degrees below zero.
  - −396 m is 396 m below sea level.

Example 15

Write down: **a** the highest **b** the lowest number in this list.

−19, 7, −10, 0, 4, −3

You can use a number line to help you

a 7
b −19

Exercise 1I

1 For each list of numbers write down **i** the highest and the lowest number
ii the numbers in order, starting with the lowest.

a 5, −10, −3, 0, 4
b −7, −2, −9, −13, 0
c −3, 6, 13, −15, −6
d −13, −2, −20, −21, −5

2 Write down the two missing numbers in each sequence.

a 4, 3, 2, 1, _, _, −2
b 10, 7, 4, 1, _, _, −8
c −13, −9, −5, −1, _, _, 11
d 13, 8, 3, −2, _, _, −17
e 21, 12, 3, −6, _, _, −33
f −13, −10, −7, −4, _, _, 5

3 Use the number line to find the number that is:

a 5 more than 2
b 4 more than −7
c 7 less than 6
d 2 less than −3
e 6 less than 0
f 10 more than −7
g 6 more than −6
h 4 less than −3
i 10 less than 5
j 1 less than −1.

4 What number is:

a 30 more than −70
b 50 less than −20
c 80 greater than −50
d 90 smaller than 60
e 130 smaller than −30
f 70 bigger than 200
g 170 bigger than −200
h 100 bigger than −100
i 140 more than −20
j 200 less than −200?

G

F

F

5 The table gives the highest and lowest temperatures recorded in five cities during one year.

	New York	Brussels	Tripoli	Minsk	Canberra
Highest temperature	27°C	32°C	34°C	28°C	34°C
Lowest temperature	−9°C	−6°C	8°C	−21°C	7°C

a Which city recorded the lowest temperature?
b Which city recorded the biggest difference between its highest and lowest temperatures?
c Which city recorded the smallest difference between its highest and lowest temperatures?

6 The temperature of the fridge compartment of a fridge-freezer is set at 4°C.
The freezer compartment is set at −18°C.
What is the difference between these temperature settings?

This thermometer is showing a temperature of −18°C.

7 The temperature of a shop freezer should be set at −18°C. It is set to −12°C by mistake.
What is the difference between these temperature settings?

1.8 Working with negative numbers

Objective

- You can work with negative numbers to find differences and make changes.

Why do this?

You may wish to find the difference between the temperatures at two ski resorts to see which is colder. To do this you need to work with negative numbers.

Get Ready

1. Which ski resort has the highest temperature?
2. Which resort has the lowest wind chill?

Resort	Kitzbühel	Val d'Isère	Civetta
Max (°C)	−5	−10	−3
Wind chill (°C)	−9	−20	−10

Key Points

- You can add and subtract negative numbers using a number line to help you.

Example 16

At 12 noon the temperature at the top of a mountain was 2°C.
By 6 pm it had fallen by 8°C.
What was the new temperature at 6 pm?

2°C − 8°C = −6°C

ResultsPlus
Examiner's Tip

A number line can help you when working with negative numbers.

Example 17

a The temperature is 5°C. It falls by 8 degrees.
What is the new temperature?

b What is the difference in temperature between 4°C and −4°C?

a −3°C ← From 5°C count 8 degrees down to −3°C.

b 8°C ← From 4°C count to −4°C. There is a difference of 8 degrees between the two temperatures.

Exercise 1J

Use this number line going from −10°C to +10°C to help you with these questions.

1 Find the number of degrees between each pair of temperatures.

a −3°C, 2°C
b −4°C, −1°C
c 2°C, 8°C
d −6°C, 4°C
e 7°C, −3°C
f 1°C, 9°C
g −3°C, −8°C
h −7°C, 6°C

2 Find the new temperature after:

a a 2° rise from −4°C
b a 7° fall from 4°C
c 8°C falls by 15°
d −4°C rises by 7°
e −5°C rises by 8°
f 4°C falls by 10°
g −3°C falls by 6°.

F

1.9 Calculating with negative numbers

Objective

- You know the rules to use when working with positive and negative numbers

Why do this?

Using rules is quicker than using a number line.

Get Ready

1. Write down the value of

a an 8° rise from −10°C **b** an 8° fall from −2°C

Key Points

- Adding a negative number is the same as subtracting a **positive number**.
- Subtracting a negative number is the same as adding a positive number.
- The rules for multiplying and dividing are:
  - if the signs are the same the result is positive
  - if the signs are different the result is negative.

Example 18 Work out

a $2 - +3$ **b** $-3 - -2$ **c** $4 + -2$ **d** $-3 + +1$

a $2 - +3 = -1$

2 − +3 is the same as 2 + −3. Start at 2 and go down 3 to get to −1.

b $-3 - -2 = -1$

3 − −2 is the same as −3 + +2. Start at −3 and go up 2 to get to −1.

c $4 + -2 = 2$

4 + −2 is the same as 4 − 2. Start at 4 and go up 2 to get to +2.

d $-3 + +1 = -2$

Start at −3 and go up 1 to get to −2.

Exercise 1K

F

AO3

1 Work out

a $-4 + -3$ **b** $9 - +5$ **c** $8 - -2$ **d** $5 + +4$

e $-7 - -6$ **f** $-2 + +4$ **g** $6 + -8$ **h** $-3 - +7$

2 A diver dives to a depth of −27 metres. A second diver dives to a depth of −16 metres. What is the difference in the depths of the dives?

3 The temperature at the Arctic Circle is recorded as −18°C one night. The following day it rises by 6°C. What is the temperature during the day?

Example 19 Work out

a 15×-3 b -7×-3 c -6×4 d $-12 \div -2$

a $+15 \times -3 = -45$ ← signs are different

b $-7 \times -3 = +21$ ← signs are the same

c $-6 \times +4 = -24$ ← signs are different

d $-12 \div -2 = +6$ ← signs are the same

Where no sign is given the number is positive.

Exercise 1L

Work out these multiplications and divisions.

1 a $+3 \times -1$ b $+24 \div -8$ c $+4 \div +1$
d $+2 \times +6$ e $-12 \div +3$ f $-3 \times +4$

2 a $-9 \times +10$ b $-32 \div -8$ c $-20 \div -4$
d $-2 \times +7$ e $+10 \div -5$ f -3×-4

3 a $-5 \times +4$ b $-16 \div -8$ c $-4 \times +5$
d $-18 \div -3$ e $+18 \div +2$ f $-6 \times +7$

4 a -8×-3 b $-30 \div +2$ c $-16 \div +4$
d -3×-9 e $+5 \times -8$ f $+24 \div +8$

5 a $-50 \div -5$ b $-7 \times +8$ c $+6 \times +6$
d -3×-7 e $-9 \div +3$ f $-7 \times +6$

F

1.10 Factors, multiples and prime numbers

Objectives

- You can recognise types of numbers and know their properties.
- You can separate the prime factors that make up a number.

Why do this?

Credit card companies use prime numbers to protect your card details when you shop online.

Get Ready

1. Write down the value of

a 7×6 b 9×7 c 8×8 d 3×6

Key Points

- Even numbers are whole numbers that divide exactly by 2.
- Any number that ends in 2, 4, 6, 8 or 0 is even.
- Odd numbers do not divide exactly by 2 and always end in 1, 3, 5, 7 or 9.
- The **factors** of a number are whole numbers that divide exactly into the number.
  They include 1 and the number itself.
- **Multiples** of a number are the results of multiplying the number by a positive whole number.
- A **common multiple** is a number that is a multiple of two or more numbers.
- A **prime number** is a whole number greater than 1 whose only factors are 1 and the number itself.
- A **prime factor** is a factor that is also a prime number.
  For example, the prime factors of 18 are 2 and 3.
- A **common factor** is a number that is a factor of two or more numbers.
- A number can be written as a product of its prime factors.

Example 20 Separate 3, 11, 14, 22, 23, 36, 39, 40, 52, 57, 60 into odd and even numbers.

14, 22, 36, 40, 52 and 60 end in an even number (2, 4, 6, 8, 0). These are the even numbers.
3, 11, 23, 39 and 57 end in an odd number (1, 3, 5, 7, 9). These are the odd numbers.

Exercise 1M

G

1 Write down all the even numbers from this list.
42, 18, 37, 955, 1110, 73 536, 500 000

2 Write down all the odd numbers from this list.
105, 537, 9216, 811, 36 225, 300 000

3 Write down the next two even numbers after:
a 12 **b** 28 **c** 196.

4 Write down the odd number that comes before:
a 5 **b** 31 **c** 200.

5 Write down all the even four-digit numbers that can be made using only the numbers on the cards below.

4 7 6 5

AO3

6 Using the digits 2, 7, 3 and 8 only once each,
a write down the largest even number that can be made
b write down the smallest odd number that can be made.

F AO3

7 A postman has letters to deliver to house numbers
16, 9, 4, 3, 22, 17, 14, 8, 16, 3, 12, 14, 17, 1, 42, 15, 16, 22, 9, 23, 31, 15 and 12.
He is going to walk down the side with even numbers first, starting at number 2, and come back on the opposite side with odd numbers. Arrange the house numbers in order for him.

factors multiple common multiple prime number prime factor common factor

Example 21 Find the factors of 12.

12 can be made from 1×12, 2×6, 3×4. The factors of 12 are 1, 2, 3, 4, 6 and 12.

Example 22 Write down the multiples of 3 that are between 20 and 29.

$7 \times 3 = 21$ is the first multiple of 3 after 20.
Then comes $21 + 3 = 24$ and $24 + 3 = 27$.
As the next multiple of 3 is 30 the answer is 21, 24 and 27.

ResultsPlus
Watch Out!

Students sometimes get confused between factors and multiples – remember multiples are from multiplying.

Exercise 1N

G

1 Write down all the factors of the following numbers.

a 15 **b** 20 **c** 24 **d** 18 **e** 13 **f** 90

2 List the first five multiples of the following numbers.

a 2 **b** 5 **c** 10 **d** 7 **e** 13

3 Write down three multiples of 10 that are larger than 50.

4 Write down the numbers in the cloud that are:

a factors of 24
b multiples of 5
c factors of 16
d multiples of 3.

5 Find the two prime numbers that are between 30 and 40.

6 Find the next prime number after 91.

F

7 Find the largest number with a factor of 4 that can be made using the digits 3, 2, 4, 5. AO3

8 A florist has 216 roses. AO3
She makes these into bunches with an equal number of roses.
Each bunch has more than 10 roses.
Find a possible number for the roses in her bunches:

a if all the roses are used
b if 6 roses are left over.

Example 23 Find the common factors of 12 and 18.

The factors of 12 are 1, 2, 3, 4, 6, 12.
The factors of 18 are 1, 2, 3, 6, 9, 18.
1, 2, 3 and 6 are all factors of both 12 and 18.
They are the common factors of 12 and 18.

Example 24 Write 36 as a product of its prime factors.

Method 1

$36 = 2 \times 18$
$= 2 \times 2 \times 9$
$= 2 \times 2 \times 3 \times 3$

In this method, use 2 as often as possible, then move onto 3, then 5 and work through the prime numbers in order.

Method 2

This is called a factor tree.

$36 = 2 \times 3 \times 2 \times 3$

Exercise 1O

F

1 Find the common factors of:

a 4 and 6 b 10 and 15 c 24 and 36
d 10, 30 and 60 e 16 and 24 f 15 and 40
g 12 and 28 h 30 and 42 i 18 and 25.

2 Find all the prime factors of the following numbers.

a 30 b 25 c 42 d 39 e 105

3 Write these numbers as products of their prime factors.

a 45 b 36 c 28 d 80 e 72

1.11 Finding lowest common multiple (LCM) and highest common factor (HCF)

Objectives

- You can find the highest common factor (HCF).
- You can find the lowest common multiple (LCM).

Why do this?

If you know the number of seats per row at a cinema, you can use multiples to work out the total number of seats.

Get Ready

1. Write these numbers as products of their prime factors.

a 99 b 324 c 175

Key Points

- The **lowest common multiple (LCM)** is the lowest multiple that is common to two or more numbers.
- The **highest common factor (HCF)** is the highest factor that is common to two or more numbers.

Example 25

a Find the highest common factor (HCF) of 36 and 24.
b Find the lowest common multiple (LCM) of 6 and 8.

a $36 = 2 \times 2 \times 3 \times 3$
$24 = 2 \times 2 \times 2 \times 3$
$2 \times 2 \times 3 = 12$
12 is the HCF

Write each number in prime factor form.
Pick out the factors common to both numbers.

b 6: 6, 12, 18, 24, 30, 36, 42, 48, 54
8: 8, 16, 24, 32, 40, 48, 56
There are two common multiples so far, but 24 is the lower.
The LCM of 6 and 8 is 24.

Write a list of multiples for each number.
The LCM is the lowest number that appears in both lists.

Exercise 1P

1 Find the highest common factor of:
a 4 and 8 **b** 9 and 12 **c** 18 and 24
d 14 and 30 **e** 21 and 35.

2 Find the lowest common multiple of:
a 3 and 4 **b** 4 and 6 **c** 12 and 15
d 36 and 16 **e** 50 and 85.

3 Find the LCM and HCF of:
a 12 and 18 **b** 120 and 180 **c** 24 and 84
d 91 and 130 **e** 72 and 96 **f** 40 and 60.

4 Two lighthouses off the Cornish coast can be recognised by the different intervals between their flashes. One flashes every 24 seconds and the other every 40 seconds. A ship's captain sees them flash at the same time. How long will it be before this happens again?

5 The light on a motorway service vehicle flashes every 20 seconds. The light on a tractor flashes every 10 seconds. As they pass each other on a road, the lights flash together. How long will it be before this happens again?

D

C

AO2

AO2

1.12 Finding square numbers and cube numbers

Objective

- You can calculate the square and cube of a whole number.

Why do this?

Square and cube numbers are found in everyday life. There are 8×8 squares on a chessboard, and $3 \times 3 \times 3$ cubes on a Rubik's Cube.

Get Ready

1. Find these.

a 11×11 **b** 13×13 **c** $3 \times 3 \times 3$

Key Points

- A **square number** is the result of multiplying a whole number by itself.
  4×4 can be written as the square of 4, 4 squared or 4^2. (For more on indices, see Section 9.1.)
- You need to be able to recall the squares of all whole numbers up to 15×15.
- A **cube number** comes from multiplying a number by itself and then multiplying the result by the original number.
  $4 \times 4 \times 4$ can be written as the cube of 4, 4 cubed or 4^3.
- You need to be able to recall the cubes of 2, 3, 4, 5 and 10.

Example 26 Find **a** the first four square numbers, **b** the first four cube numbers.

a The first four square numbers are
$1 \times 1 = 1,\ 2 \times 2 = 4,\ 3 \times 3 = 9,\ 4 \times 4 = 16.$

b The first four cube numbers are
$1 \times 1 \times 1 = 1,\ 2 \times 2 \times 2 = 8,\ 3 \times 3 \times 3 = 27,\ 4 \times 4 \times 4 = 64.$

ResultsPlus
Watch Out!

Students often double rather than square or multiply by 3 rather than cube.

Exercise 1Q

G

1 Work out the square of the following numbers.

a 3 **b** 6 **c** 10 **d** 2 **e** 20

F

2 Work out the cube of the following numbers.

a 2 **b** 5 **c** 7 **d** 20 **e** 12

3 Work out

a 5 squared **b** the cube of 6 **c** 9^2 **d** 3^3

Chapter review

- Although you can keep on counting, you only use ten **digits**, often called figures.
- Each digit has a **value** that depends on its position in the number. This is its **place value**.
- You can read and write numbers by thinking about the place value of each of the digits.
- You can use your knowledge of place value to put numbers in order of size.

- You can use a number line to help you to increase or decrease a number.
- Number lines might be read:
  - from left to right
  - from bottom to top
  - clockwise.
- Words that show you have to add numbers are: add, plus, total and sum.
- Words that show you have to take away are: take away, subtract, minus and find the difference.
- Words that show you have to multiply are: product, times and multiply.
- Divide and share are words that show you have to divide.
- You multiply by 10, 100 or 1000 by moving digits 1, 2 or 3 places to the left.
- You divide by 10, 100 or 1000 by moving digits 1, 2 or 3 places to the right.
- There are situations where an estimate of numbers could be good enough.
- **Rounding** means reducing the accuracy of a number by replacing right-hand digits with zeros.
- To round a number, look at the digit before the place value you are rounding to (so for rounding to the nearest 10 you would look at the units digit). If it is less than 5, round down. If it is 5 or more, round up.
- The further a number is from zero on the left-hand side of a number line, the smaller it is.
- For **negative numbers** you count backwards from zero on the number line.
- You can add and subtract negative numbers using a number line to help you.
- Adding a negative number is the same as subtracting a **positive number**.
- Subtracting a negative number is the same as adding a positive number.
- When multiplying or dividing numbers:
  - if the signs are the same the result is positive
  - if the signs are different the result is negative.
- Even numbers are numbers that divide by 2. They always end in 2, 4, 6, 8 or 0.
- Odd numbers are numbers that do not divide by 2. They always end in 1, 3, 5, 7 or 9.
- The **factors** of a number are whole numbers that divide exactly into the number. They include 1 and the number itself.
- Factors are pairs of numbers which, when multiplied together, give the number.
- **Multiples** of a number are the results of multiplying the number by a positive whole number.
- A **common multiple** is a number that is a multiple of two or more numbers.
- A **prime number** is a whole number greater than 1 whose only factors are 1 and the number itself.
- A **prime factor** is a factor that is also a prime number.
- A **common factor** is a number that is a factor of two or more numbers.
- A number can be written as a product of its prime factors.
- The **lowest common multiple (LCM)** is the lowest multiple that is common to two or more numbers.
- The **highest common factor (HCF)** is the highest factor that is common to two or more numbers.
- A **square number** is the result of multiplying a whole number by itself.
- You need to be able to recall the squares of all whole numbers up to 15×15.
- A **cube number** comes from multiplying a whole number by itself and then multiplying the result by the original number.
- You need to be able to recall the cubes of 2, 3, 4, 5 and 10.

G

Review exercise

1 Draw a place value diagram and write in a number with five digits and a 2 in the thousands column and a 3 in the units column.

2 Write down the value of the 5 in the following numbers.

a 651 **b** 5302 **c** 253 101 **d** 10 050 **e** 175

3 Write down these numbers in words.

a 3723 **b** 107 **c** 2007 **d** 15 071

4 Write these numbers in figures.

a twenty-one thousand two hundred and thirty-one
b five hundred and seven
c seventy thousand two hundred and three

5 This table shows the number of people who were seriously injured in road accidents in a part of Britain.

Year	2001	2002	2003	2004	2005
Number	37 346	33 645	31 456	29 788	26 466

In which year were:

a the smallest number of people seriously injured
b more than 35 000 seriously injured
c between 30 000 and 32 000 seriously injured
d fewer than 28 000 seriously injured?

6 A shop's takings for March were £34 176.
The takings for April were £58 358.
Work out the total takings for the two months.

7 A school has 1321 students. 738 are boys. How many are girls?

8 Using only the numbers in the cloud, write down:

a all the multiples of 6
b all the square numbers
c all the factors of 12
d all the cube numbers.

9 4, 5, 8, 9, 12, 14, 16, 20, 27, 35, 36, 37
From the numbers in the list write down:

a the odd numbers **b** the multiples of 3 **c** the factors of 48
d the prime numbers **e** the square numbers **f** the cube numbers.

10 Using the numbers 4, 1, 6, 7 no more than once each, make a number which is

a a multiple of 3 **b** a multiple of 4.

G

11 Cans of cola are delivered in packs of 24.
Copy and complete the table.

Number of packs	Number of cans
1	24
2	
3	
4	
5	

If you need at least 75 cans of cola, how many packs should you order?

12

cube	multiple	factor	product

Use a word from the box to complete this sentence correctly.

12 is a ..... of 36

AO2

13 Dan writes down the numbers from 2 to 30.
He crosses out all the multiples of 2.
He crosses out all the multiples of 3 and then all the multiples of 5.

a How many numbers have been crossed out more than once?
b How many numbers have not been crossed out at all?

14 Write down all the multiples of 15 that are less than 100.

15 Round these numbers to the nearest multiple of 10 given in the brackets.

a 27 (10) **b** 349 (100) **c** 2047 (100)
d 78 939 (10) **e** 7 813 076 (million) **f** 83.7 (10)

F

AO3

16 There are 376 passengers on a train.
At its first stop 27 passengers get off.
295 passengers get on.
How many passengers are now on the train?

17 Peter buys 273 stamps costing 38p each.
How much do they cost altogether?

18 There are 14 winners in a lottery.
They share the winnings of £10 332 equally.
How much does each get?

19 **a** The temperature during an Autumn morning went up from −3°C to 6°C.
By how many degrees did the temperature rise?
b During the afternoon the temperature fell by 11°C from 6°C.
What was the temperature at the end of the afternoon?

20 Work out

a $3 - 7$ **b** $-3 + 5$ **c** $-11 - 4$ **d** $4 - (-6)$ **e** $(-5) + (+3)$

21 Work out

a $+3 \times -7$ **b** -4×-5 **c** $16 \div -2$ **d** $-15 \div -3$ **e** $-28 \div +4$

F

AO3 **22** Find three different numbers each below 10 which have a sum of 20.

23

Diagram **NOT** accurately drawn

Here is a picture of a stick. The stick is in three parts, A, B and C.
The total length of the stick is 30 cm.
The length of part A is 16 cm.
The length of part B is the same as the length of part C.
Work out the length of part C.

May 2009 adapted

AO3 **24**

In this set of squares, each number in a square is obtained by adding the two numbers immediately underneath.
What number should go in the top square?

AO2 **25** The table shows the cost of two different models of the Eiffel Tower.

Small	£2.40
Large	£4.50

Pierre buys 10 small models and 5 large models.
He pays with a £50 note.
Work out how much change he should get.

AO2 **26**

City	Temperature
Cardiff	−2°C
Edinburgh	−4°C
Leeds	2°C
London	−1°C
Plymouth	5°C

The table gives information about the temperatures at midnight in 5 cities.

a Write down the lowest temperature.
b Work out the difference in temperature between Cardiff and Plymouth.
c Which city has a temperature halfway between London and Plymouth?

May 2009

AO3 **27** Jill says
'If you multiply any two prime numbers together, the answer will always be an odd number'.
Write down an example to show that Jill is wrong.

June 2006

28 Work out 10^3.

29 Charlie writes down the numbers from 1 to 50. Ben puts a red spot on all the even numbers. Alex puts a blue spot on all the multiples of 3.

a What is the largest number that has both a red and a blue spot?

b How many numbers have neither a blue nor a red spot?

Sophie puts a green spot on all the multiples of 5.

c How many numbers have exactly two coloured spots on them?

F AO3

30 Two whole numbers are each less than 10 and greater than 0. Work out the greatest possible difference between their product and their sum.

E AO2 AO3

31 Here is some information about the coaches in a coach hire company.

AO2 AO3

Type of coach	Number of passengers	Number of coaches available
Small	14	5
Medium	32	3
Large	44	2
Touring	56	3

Jim hires two medium and two large coaches to take people to a show.

a Work out the total number of passengers that could go to the show.

Becky wants to take 300 people to a show.

b Work out the smallest number of coaches she would need to hire from the company.

32 Perfume bottles are sold in boxes which measure 4 cm by 5 cm by 20 cm.
They are supplied to shops in cartons containing 12 bottles.
Work out two arrangements of the bottles in these cartons. (There are lots of sensible answers.)

D AO2 AO3

33 A buzzer buzzes every 4 seconds and a bell rings every 6 seconds. The buzzer and the bell start at the same time. How many times in the first minute will they make a sound at the same instant?

AO3

34 **a** Express 108 as the product of its prime factors.

b Find the highest common factor of 108 and 24.

C

35 **a** Express the following numbers as products of their prime factors.

i 60 **ii** 96

b Find the highest common factor of 60 and 96.

c Work out the lowest common multiple of 60 and 96.

36 Doughnuts are sold in packs of 8. Cakes are sold in packets of 6.
What is the smallest number of packs of doughnuts and the smallest number of packets of cakes that can be bought so that the number of doughnuts is equal to the number of cakes?

AO2 AO3

37 A chocolate company wishes to produce a presentation box of 36 chocolates for Valentine's Day.
It decides that a rectangular shaped box is the most efficient, but needs to decide how to arrange the chocolates.
How many different possible arrangements are there:

a using one layer **b** using two layers **c** using three layers?

Which one do you think would look best?

AO2 AO3

38 A car's service book states that the air filter must be replaced every 10 000 miles and the diesel fuel filter every 24 000 miles. After how many miles will both need replacing at the same time?

AO3