

# 14 PERIMETER AND AREA OF 2D SHAPES



Many people need to be able to calculate both the perimeter and area of various shapes in their day-to-day work. For example, the organisers at Glastonbury need to know the area and perimeter of the festival fields so they can hire enough safety barriers.

## Objectives

In this chapter you will:

- learn the difference between the perimeter and area of a shape
- find the perimeter and area of two-dimensional shapes and solve problems involving area.

## Before you start

You should be able to:

- measure the length of a line
- change measurements between millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

# 14.1 Perimeter

## Objectives

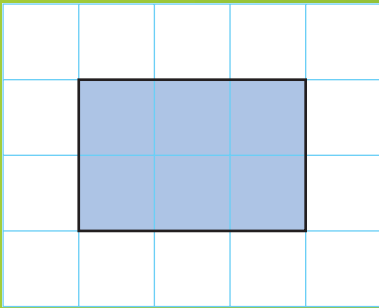
- You can find the perimeter of simple shapes.
- You can find the perimeter of shapes made from squares, rectangles and triangles.

## Why do this?

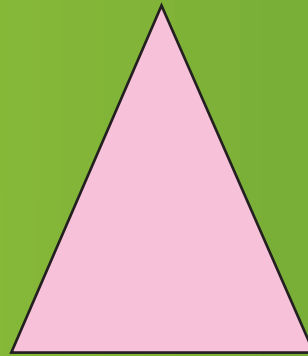
A gardener will need to know the distance around a boundary edge in order to calculate the length of fence needed to surround a garden.

## Get Ready

- What is the length and width of the shaded rectangle drawn on the centimetre grid?



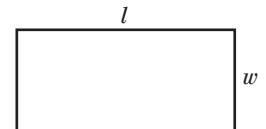
- Measure the lengths of the sides of the following triangle.



## Key Points

- The **perimeter** of a **two-dimensional (2D)** shape is the total distance around the edge of the shape. The examples show you how to work out the perimeter of a variety of shapes.
- To work out the perimeter of a rectangle you can use the following formula.

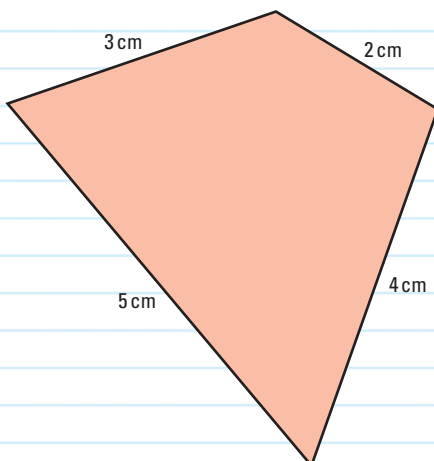
$$\begin{aligned}\text{Perimeter of a rectangle} &= l + w + l + w \\ &= 2l + 2w\end{aligned}$$



$l$  = the length of the rectangle  
 $w$  = the width of the rectangle

## Example 1

What is the perimeter of the following shape?



$$\begin{aligned}\text{Perimeter} &= 3 + 2 + 4 + 5 \\ &= 14 \text{ cm}\end{aligned}$$

Add the lengths of all four sides to find the perimeter.



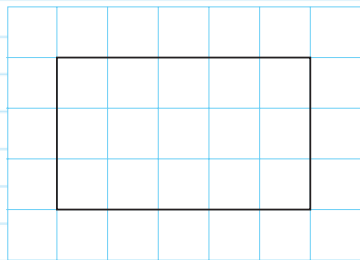
**ResultsPlus**  
**Examiner's Tip**

Always remember to include the units in your answer.



**Example 2**

The diagram shows a rectangle drawn on a centimetre grid.  
Work out the perimeter of the rectangle.



**Method 1**

$$\begin{aligned}\text{Perimeter} &= 5 + 3 + 5 + 3 \\ &= 16 \text{ cm}\end{aligned}$$

Work out the length of each side and add them together.

**Method 2**

$$\begin{aligned}\text{Perimeter} &= 2l + 2w \\ &= (2 \times 5) + (2 \times 3) \\ &= 10 + 6 \\ &= 16 \text{ cm}\end{aligned}$$

Use the formula for the perimeter of a rectangle.

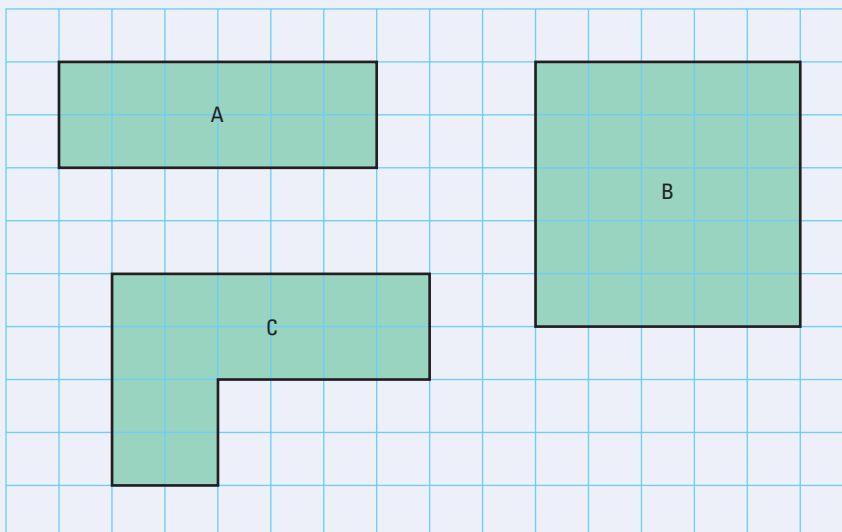


**Exercise 14A**

Questions in this chapter are targeted at the grades indicated.

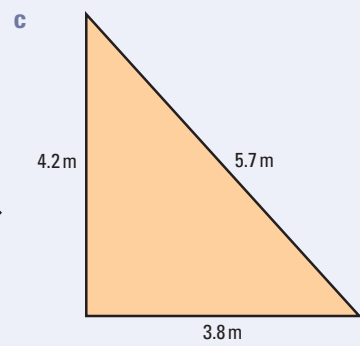
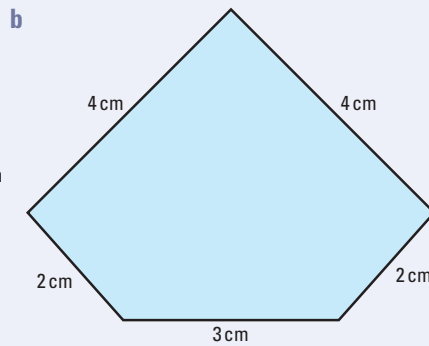
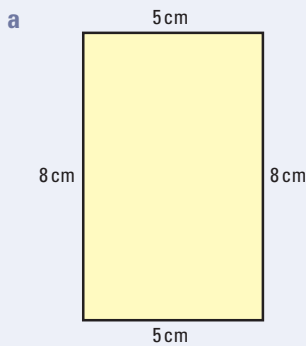
G

- 1** Here are three shapes drawn on a centimetre grid.  
Work out the perimeter of each shape.

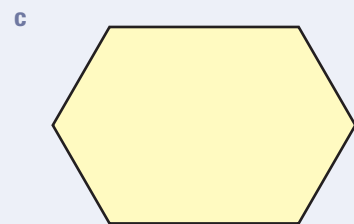
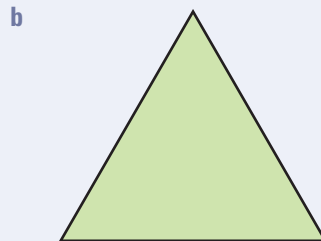
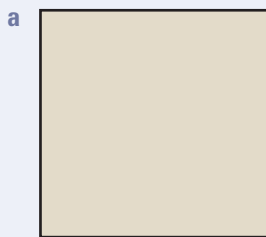




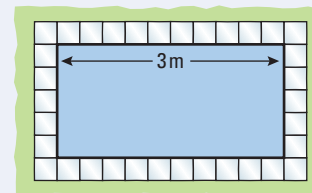
- 2 Work out the perimeters of the following shapes.



- 3 These shapes are drawn accurately. Find the perimeter of each shape.

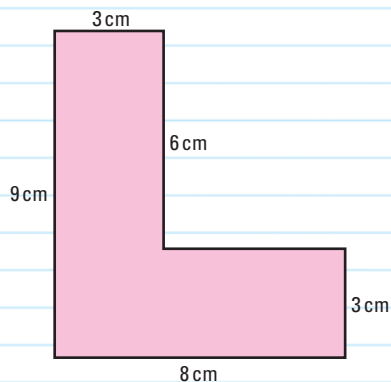


- 4 Jenny has a rectangular pond in her garden.  
The length of the pond is 3 m.  
The width of the pond is half the length of the pond.  
She wants to put a low fence around the edge of the pond.  
What length of fencing does Jenny need?



### Example 3

Work out the perimeter of the following shape.



**ResultsPlus**  
Examiner's Tip

Make sure that you know the lengths of all the sides before you work out the perimeter.

$$\begin{aligned}\text{Missing length} &= 8 - 3 \\ &= 5 \text{ cm}\end{aligned}$$

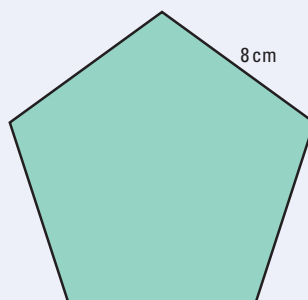
First, find the missing length.

$$\begin{aligned}\text{Perimeter} &= 3 + 6 + 5 + 3 + 8 + 9 \\ &= 34 \text{ cm}\end{aligned}$$

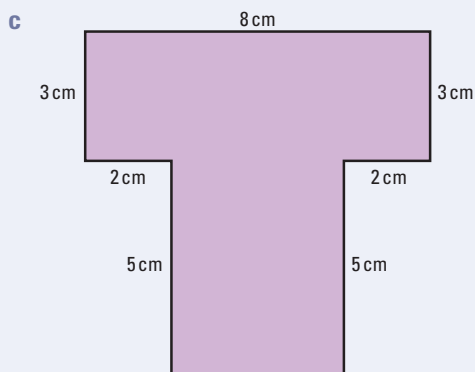
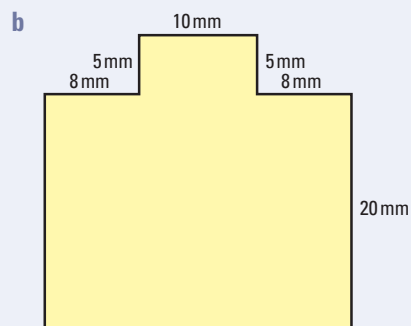
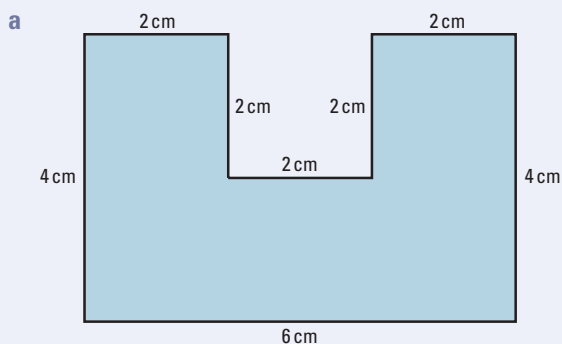
Then, add the lengths of all six sides.

**Exercise 14B**

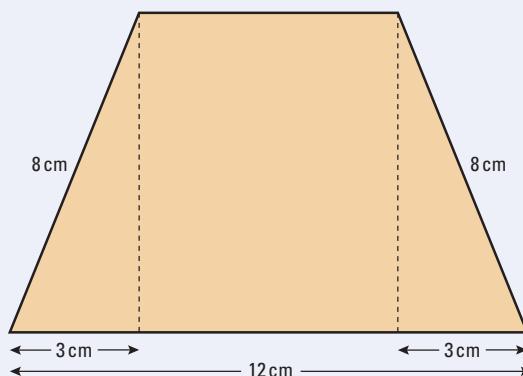
- 1** Each side of a regular pentagon has length 8 cm.  
Work out the perimeter of the pentagon.



- 2** Work out the perimeters of the following three shapes.



- 3** Work out the perimeter of this trapezium.



- 4** The perimeter of an equilateral triangle is 21.6 cm.  
Work out the length of each side.

## 14.2 Area

### Objectives

- You can find the area of simple shapes by counting squares.
- You can find the area of more complicated shapes by counting squares.

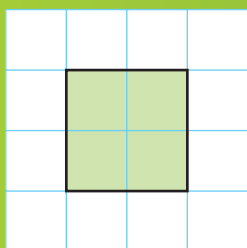
### Why do this?

It would be useful to know the area of a wall you were going to paint, so that you could buy the right amount of paint for the job.

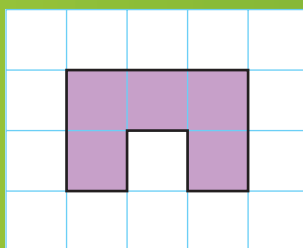
### Get Ready

1. How many squares do you need to cover each of the shaded shapes?

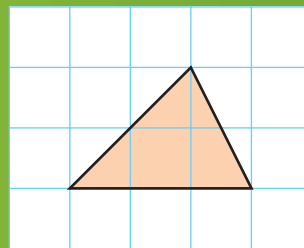
a



b

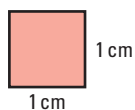


c



### Key Points

- The **area** of a two-dimensional (2D) shape is a measure of the amount of space inside the shape.
- The area of a square with sides of length 1 cm is 1 square centimetre.



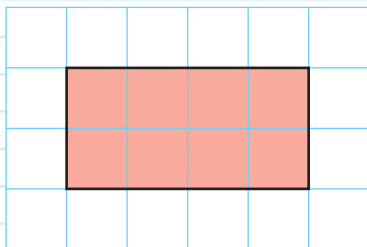
This is written as  $1 \text{ cm}^2$ .

- Other units of area include square millimetres ( $\text{mm}^2$ ), square metres ( $\text{m}^2$ ) and square kilometres ( $\text{km}^2$ ).



### Example 4

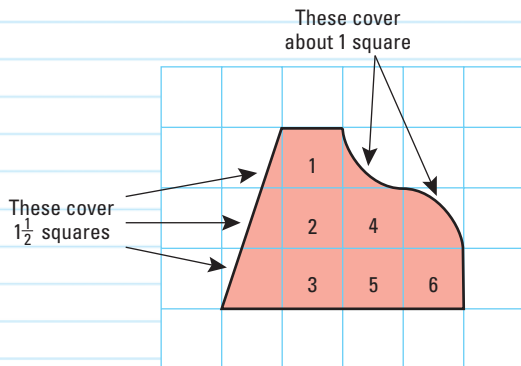
The following diagram shows a rectangle drawn on a centimetre grid. Find the area of the rectangle.



Count the number of squares.  
Each square has area of  $1 \text{ cm}^2$ .  
Area = 6 squares =  $6 \text{ cm}^2$ .

**Example 5**

Estimate the area of the shaded shape drawn on the centimetre grid.



Number of whole squares = 6

Count the whole squares.

Number of part squares =  $1 + 1\frac{1}{2}$

Estimate the number of squares covered by the other parts.

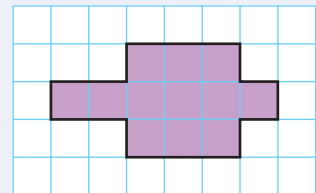
$$= 6 + 1 + 1\frac{1}{2} \\ = 8\frac{1}{2} \text{ cm}^2$$

Add your answers. State the units.

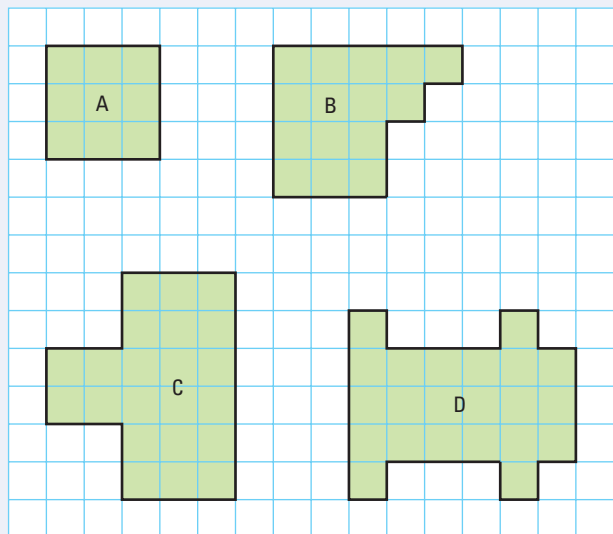
**Exercise 14C**

G

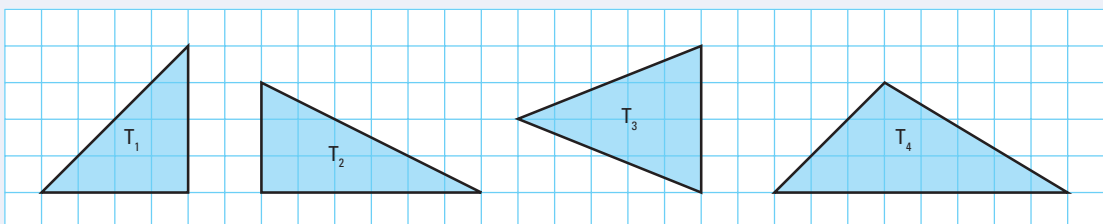
- 1 Find the area of the shape shown on the centimetre grid. Give the units with your answer.



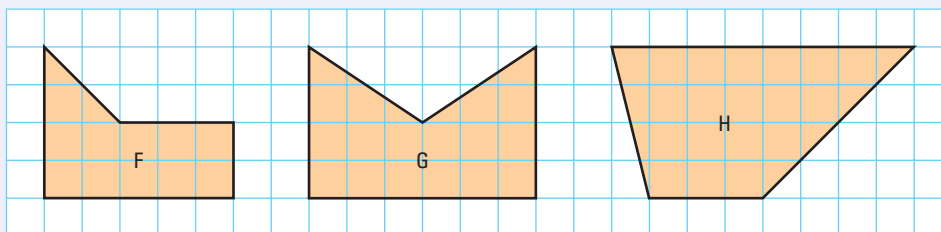
- 2 The diagram shows four shapes drawn on a centimetre grid. Find the area of each shape.



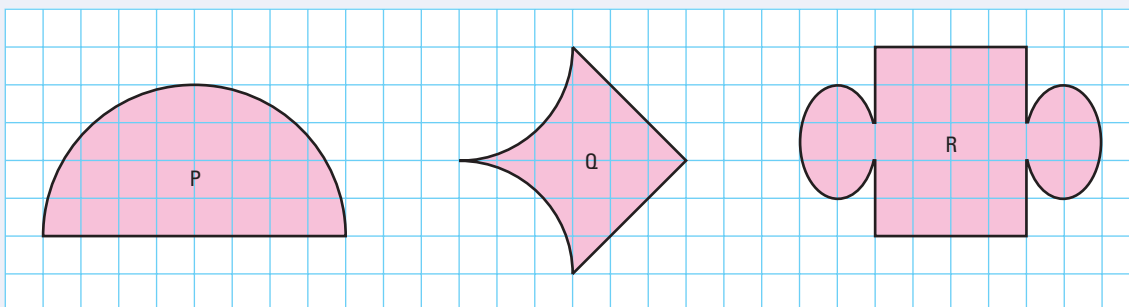
- 3 Find the area of each of the four triangles,  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$ , drawn on the centimetre grid.



- 4 Find the area of each of the three shapes, F, G and H, drawn on the centimetre grid.



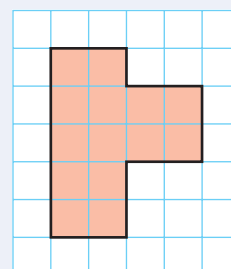
- 5 Estimate the area of each of the three shapes, P, Q and R, drawn on the centimetre grid.



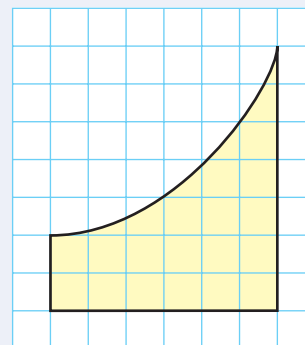
### Mixed exercise 14D

- 1 This shape has been drawn on a centimetre grid.

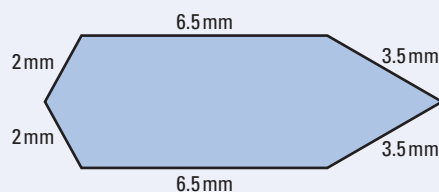
- Find the perimeter of the shape.
- Find the area of the shape.



- 2 Estimate the area of this shape that has been drawn on a centimetre grid.



- 3 Work out the perimeter of the shape below.



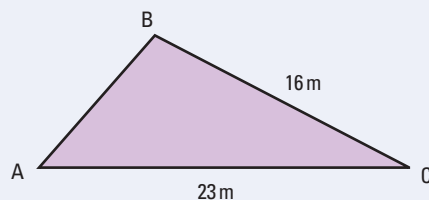
G

G

E



- E**
- 4 The perimeter of triangle ABC is 52 m.  
What is the length of side AB?



## 14.3 Finding areas using formulae

### Objectives

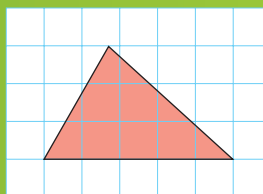
- You can find the areas of rectangles, triangles and parallelograms.
- You can use the formula to find the area of a trapezium.

### Why do this?

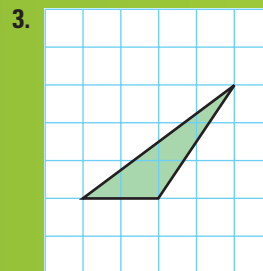
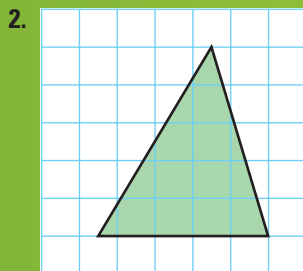
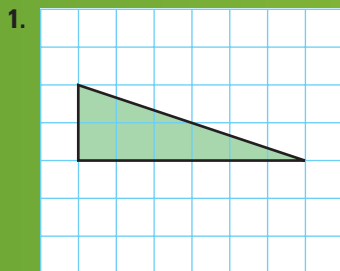
It might be necessary for your school to find out the area of a field, so that they can work out if it is big enough for an athletics track.

### Get Ready

The diagram shows a triangle drawn on a centimetre grid.  
The length of the base is 5 cm.  
The vertical height is 3 cm.



State the length of the base and the vertical height of these triangles.

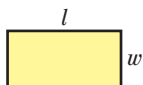


### Key Points

- To work out the area of a rectangle, square, triangle and parallelogram use the following formulae.

#### Rectangle

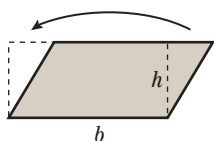
Area of a rectangle  
= length  $\times$  width  
=  $l \times w$



#### Parallelogram

You can cut a triangle off a parallelogram and put it on the other side to make a rectangle, so:

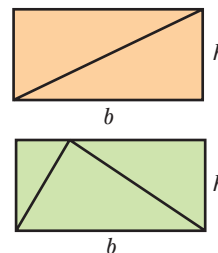
Area of a parallelogram  
= base  $\times$  vertical height  
=  $b \times h$



#### Triangle

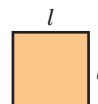
The area of a triangle is half the area of a rectangle that surrounds it.

Area of a triangle  
=  $\frac{1}{2} \times$  base  $\times$  vertical height  
=  $\frac{1}{2} \times b \times h$

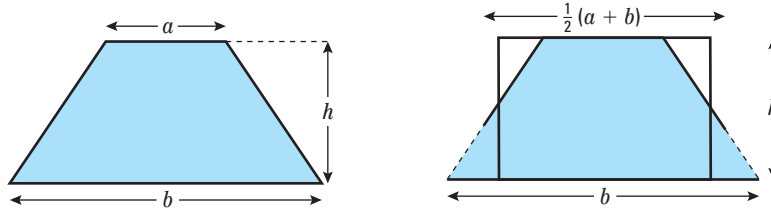


#### Square

Area of a square  
= length  $\times$  length  
=  $l \times l$   
=  $l^2$



- The area of a trapezium is worked out by finding the average of the lengths of the parallel sides and multiplying by the distance between them.



A formula is used to work out the area of a trapezium:

Area =  $\frac{1}{2} \times$  sum of the lengths of the parallel sides  $\times$  distance between the parallel sides

$$A = \frac{1}{2}(a + b)h$$



### Example 6

Work out the area of this rectangle.

Area of rectangle

= length  $\times$  width

=  $l \times w$

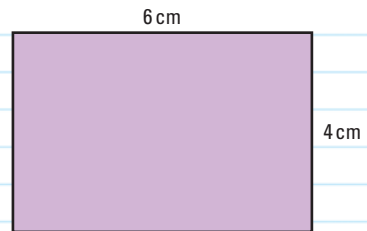
=  $6 \times 4$

=  $24 \text{ cm}^2$

Choose the formula to use.

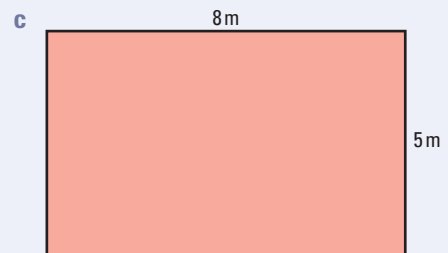
Put in the values of  $l$  and  $w$ .

Remember to include the units.

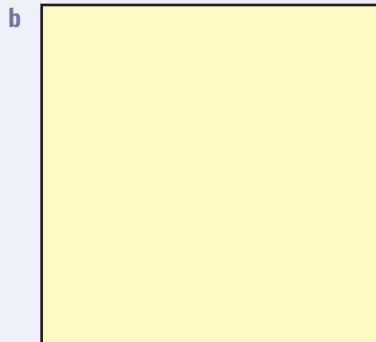


### Exercise 14E

- 1 Find the area of the following rectangles.  
Remember to give the units with your answer.

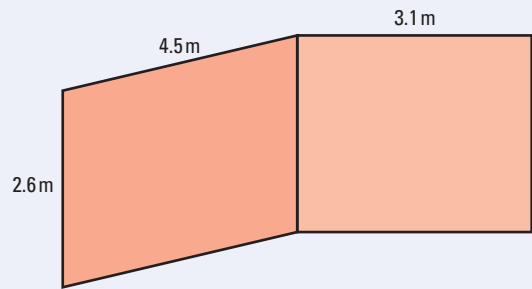


- 2 The following three squares are accurately drawn. Find the area of each one.



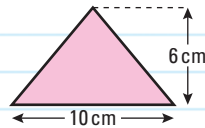
E

- 3 A decorator wants to paint two rectangular walls. The walls are 4.5 m by 2.6 m and 3.1 m by 2.6 m. What is the total area of the two walls to be painted?



**Example 7**

Work out the area of this triangle.



Area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{vertical height}$$

Choose the formula to use.

$$= \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 6$$

Put in the values of  $b$  and  $h$ .

$$= 30 \text{ cm}^2$$

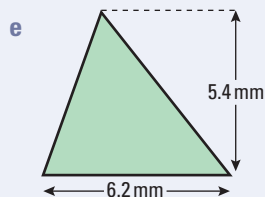
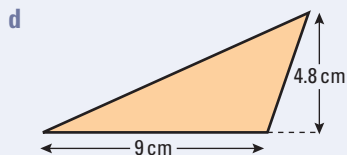
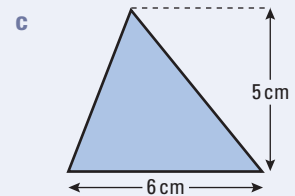
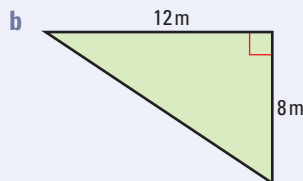
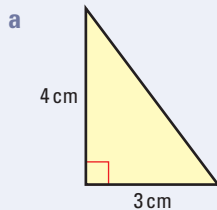
Remember to include the units.



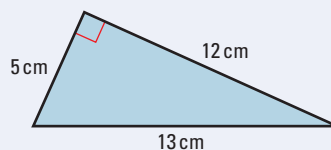
**Exercise 14F**

D

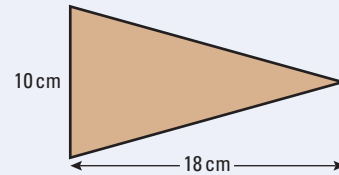
- 1 Find the area of the following triangles.



- 2 Find the area of this triangle.



- 3 A company makes flags in this shape.  
It makes 50 identical flags.  
Work out the area of fabric used to make these flags.



**Example 8** Find the shaded area in the company logo.

$$\begin{aligned}\text{Area of parallelogram} &= \text{base} \times \text{vertical height} \\ &= b \times h \\ &= 3 \times 6 \\ &= 18 \text{ cm}^2\end{aligned}$$

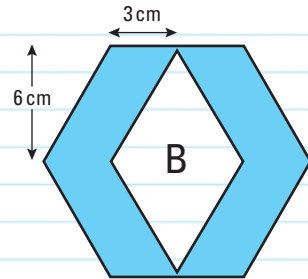
Work out the area of one of the shaded parallelograms.

Put in the values of  $b$  and  $h$ .

Remember to include the units.

$$\begin{aligned}\text{Area shaded blue} &= 4 \times 18 \\ &= 72 \text{ cm}^2.\end{aligned}$$

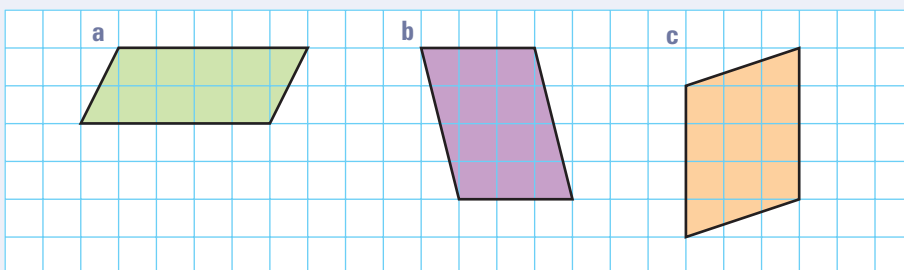
Multiply by 4 to find the total area shaded blue.



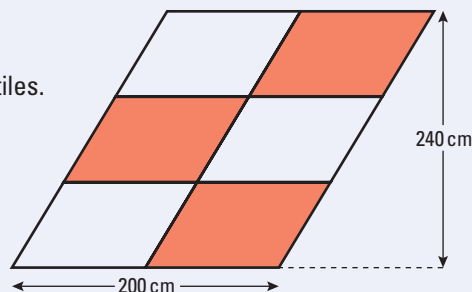
Remember to use the vertical height, not the slant height, when working out the area of a parallelogram.

### Exercise 14G

- 1 Find the areas of the parallelograms drawn on the centimetre grid.



- 2 A tiler creates the following pattern using parallelogram-shaped tiles.  
Work out the total area covered by the red tiles.



**Example 9**

Work out the area of this trapezium.

Area of trapezium

$$= \frac{1}{2} \times \text{sum of parallel sides} \times \text{distance between parallel sides}$$

$$= \frac{1}{2} (a + b)h$$

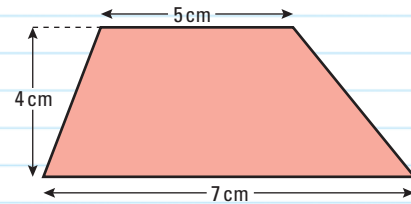
Choose the formula to use.

$$= \frac{1}{2} \times (5 + 7) \times 4$$

Put in the values of  $a$ ,  $b$  and  $h$ .

$$= \frac{1}{2} \times 12 \times 4$$

$$= 24 \text{ cm}^2$$

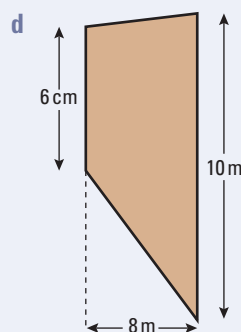
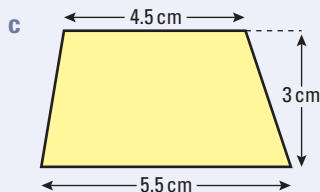
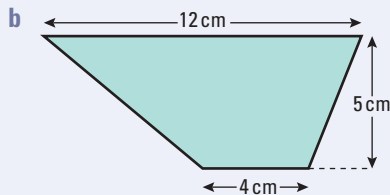
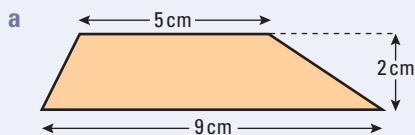


**Exercise 14H**

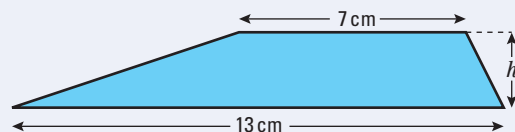
1 Copy and complete the table to find the area of each trapezium.

	$a$	$b$	$h$	Area
Trapezium 1	4 cm	6 cm	3 cm	
Trapezium 2	10 cm	12 cm	5 cm	
Trapezium 3	9 m	7 m	6 m	
Trapezium 4	5 m	10 m	4 m	

2 Work out the area of each trapezium.



3 A trapezium has an area of  $40 \text{ cm}^2$ .  
The two parallel sides have lengths 7 cm and 13 cm.  
The distance between the two parallel sides is  $h$  cm.  
Work out the value of  $h$ .



## 14.4 Problems involving areas

### Objective

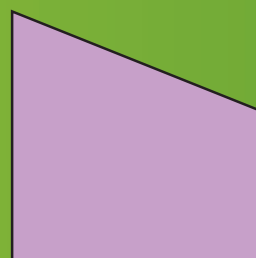
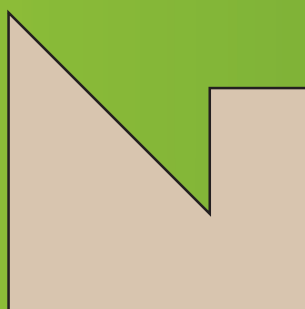
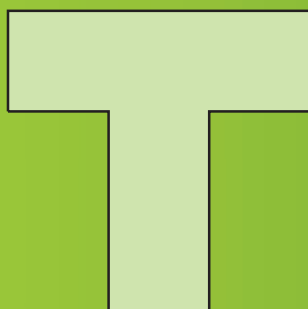
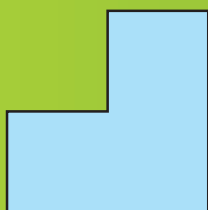
- You can find the area of a more complicated shape by splitting it up into simple shapes.

### Why do this?

Garden patios are not always simple shapes. A gardener will need to work out the area of a new patio to know how many tiles to buy.

### Get Ready

- Here are some shapes. Copy them and draw lines to show how each one can be split up into squares, rectangles and triangles.



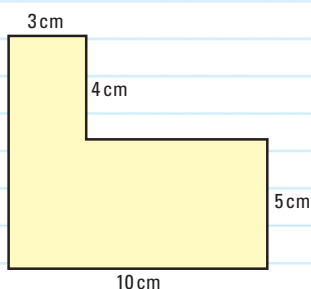
### Key Point

- To find the area of more complicated shapes you will need to split the shape into a number of simpler shapes such as rectangles, squares, triangles or parallelograms. You can then find the area of each part and add these areas together to find the total area.

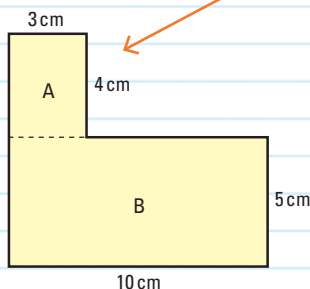
The following examples will show you how to do this.

### Example 10

Find the area of this shape.



Split the shape into two rectangles, A and B



$$\begin{aligned}\text{Area of rectangle A} \\ &= 3 \times 4 \\ &= 12 \text{ cm}^2\end{aligned}$$

Work out the area of rectangle A.

$$\begin{aligned}\text{Area of rectangle B} \\ &= 10 \times 5 \\ &= 50 \text{ cm}^2\end{aligned}$$

Work out the area of rectangle B.

$$\begin{aligned}\text{Total area} &= 12 + 50 \\ &= 62 \text{ cm}^2\end{aligned}$$

Add the two areas to find the total area of the shape.



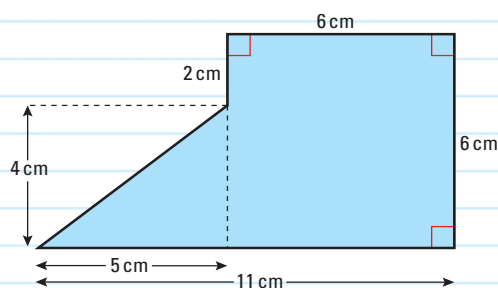
**ResultsPlus**  
Examiner's Tip

Remember to show how you split up the shape. You might get marks for this in the exam.



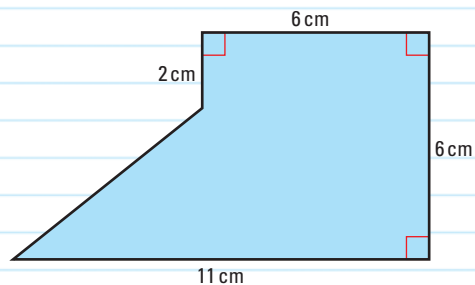
**Example 11**

Work out the area of this shape.



Area of shape  
= area of triangle  
+ area of square

Split into simpler shapes.



Area of triangle

$$= \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 5 \times 4$$

$$= 10 \text{ cm}^2$$

Work out any unknown lengths you need and mark them on the diagram. Then calculate the area of the triangle.

Area of square

$$= l^2$$

$$= 6^2$$

$$= 36 \text{ cm}^2$$

Calculate the area of the square.

$$\text{Total area} = 10 + 36$$

$$= 46 \text{ cm}^2$$

Add the two areas to find the total area of the shape.

**Example 12**

Julie wants to make a rectangular patio in her garden.

She needs to cover an area which measures 5 m by 3.5 m with square paving stones.

Each paving stone measures 50 cm by 50 cm and costs £3.99.

Work out the cost of the paving stones Julie will need.

**Method 1**



Draw a diagram to help you understand the question. Convert your units so they are all the same.

Work out the area of the patio.

$$\text{Area of patio} = 500 \times 350 = 175\,000 \text{ cm}^2$$

$$\text{Area of a paving stone} = 50 \times 50 = 2500 \text{ cm}^2$$

$$\text{Number of paving stones needed} = \frac{175\,000}{2500}$$

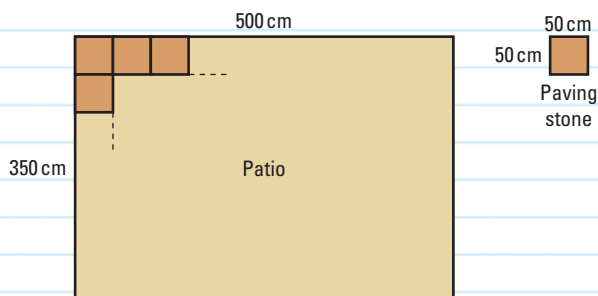
$$= 70$$

Work out the area of a paving stone. Divide the area of the patio by the area of a paving stone to calculate the required number of paving stones.

$$\text{Total cost of paving stones} = £3.99 \times 70$$

$$= £279.30$$

Multiply the cost of 1 paving stone by the number needed.

**Method 2**

Divide the length of the patio by the length of a paving stone.

$$\text{Number of stones needed in each row} = 500 \div 50 = 10$$

Divide the width of the patio by the width of a paving stone.

$$\text{Number of rows needed} = 350 \div 50 = 7$$

Multiply the number of stones in each row by the number of rows needed.

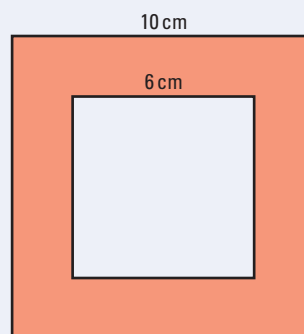
$$\text{Number of paving stones needed} = 10 \times 7 = 70$$

Multiply the cost of 1 paving stone by the number needed.

$$\begin{aligned} \text{Total cost of paving stones} &= £3.99 \times 70 \\ &= £279.30 \end{aligned}$$

**Exercise 14I**

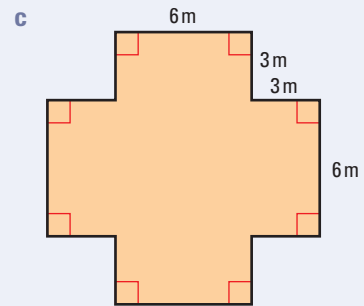
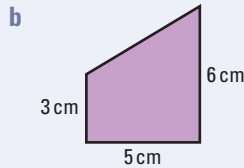
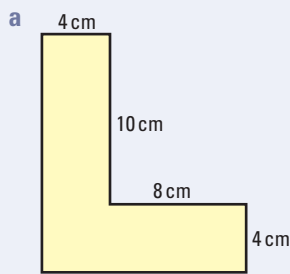
- 1 A square piece of card has sides of length 10 cm. A hole is cut from the card. The hole is a square of side 6 cm.
  - a Work out the area of the large square.
  - b Work out the area of the small square.
  - c Work out the area of the card left.



- 2 The floor of the hall in a house is a 225 cm by 150 cm rectangle. Tiles which are squares of side 15 cm are used to tile the floor. Work out how many tiles are needed.
- 3 Liam wants to replace the carpet in his room. The floor of the room is a rectangle measuring 4 metres by 3 metres. The carpet he wants to buy costs £8.65 per square metre. Work out how much it will cost Liam to buy enough carpet to cover the floor.
- 4 Libby wants to buy some grass seed so that she can sow a new lawn in her garden. She wants the lawn to be a rectangle measuring 3.2 metres by 2.5 metres. She needs 35 grams of lawn seed for every square metre of lawn. One box of lawn seed contains 250 g.
  - a How many boxes of lawn seed will Libby need to buy?
  - b How much lawn seed will be left over?

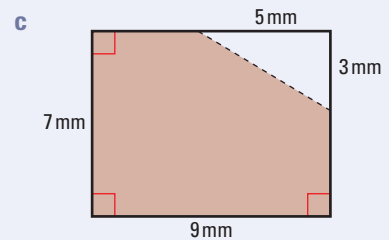
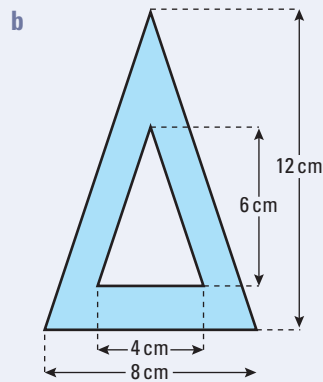
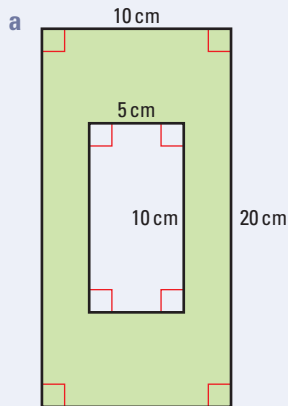
A02  
A03

- 5 Find the area of the following shapes.



A02  
A03

- 6 Work out the shaded area in each diagram.



## Chapter review

- The **perimeter** of a **2D shape** is the total distance around the edge of the shape.
- The perimeter of a rectangle can be found using the formula  

$$\text{Perimeter of a rectangle} = l + w + l + w$$

$$= 2l + 2w$$
- The **area** of a 2D shape is the amount of space inside the shape.
- The area of a 2D shape can be found by counting squares or by using the formulae:
  - Area of rectangle = length  $\times$  width  

$$= l \times w$$
  - Area of square = length  $\times$  length  

$$= l^2$$
  - Area of triangle =  $\frac{1}{2} \times \text{base} \times \text{vertical height}$   

$$= \frac{1}{2} \times b \times h$$
  - Area of parallelogram = base  $\times$  vertical height  

$$= b \times h$$
  - Area of trapezium =  $\frac{1}{2} \times \text{sum of the lengths of the parallel sides} \times \text{distance between the parallel sides}$   

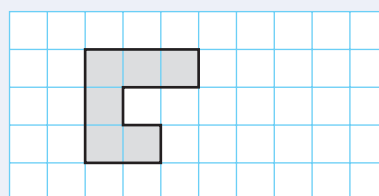
$$= \frac{1}{2}(a + b)h$$
- To find the area of more complicated shapes you will need to split the shape into a number of simpler shapes such as rectangles, squares, triangles or parallelograms. You can then find the area of each part and add these areas together to find the total area.



## Review exercise

- 1 Here is a shaded shape on a grid of centimetre squares.

- a Find the perimeter of the shaded shape.  
b Find the area of the shaded shape.



Nov 2008

- 2 Here is a rectangle.

- a Work out the perimeter of the rectangle.  
b Work out the area of the rectangle.

Diagram **NOT**  
accurately drawn

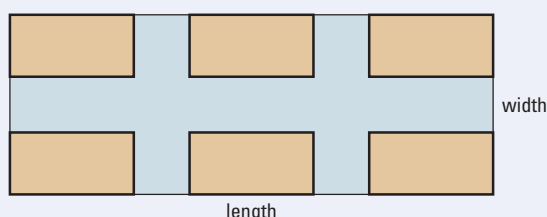
Nov 2008

- 3 A carpet 60 cm wide is to be used to cover a rectangular floor measuring 4 metres by 9 metres. Calculate the length of carpet needed.

- 4 Office regulations say the gap between the desks should be 900 mm to allow for wheelchair users. A desk has a length of 2 m and a width of 1 m.

Diagram **NOT**  
accurately drawn

Six of these desks are arranged as shown in the diagram below.  
The gap between each desk is 900 mm.

Diagram **NOT**  
accurately drawn

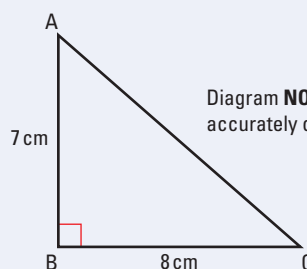
What is the total length and the total width of the office space needed for these desks?

- 5 ABC is a right-angled triangle.

AB = 7 cm,

BC = 8 cm.

Work out the area of the triangle.

Diagram **NOT**  
accurately drawn

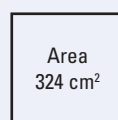
June 2008

- 6 a Work out the area of this rectangle.

Diagram **NOT**  
accurately drawn

A square has an area of  $324 \text{ cm}^2$ .

- b Work out the length of one side of the square.

Diagram **NOT**  
accurately drawn

June 2007

G

F

E

A03

D

D  
A03

- 7 The diagram shows Rob's patio. All the corners are right angles. The patio is made up of square paving stones each 50 cm by 50 cm. Work out how many of these paving stones are needed to tile Rob's patio.

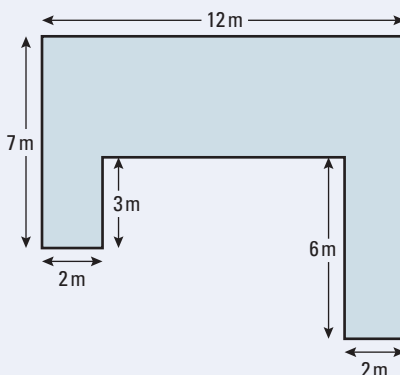


Diagram **NOT** accurately drawn



A03

- 8 A room has four interior walls.

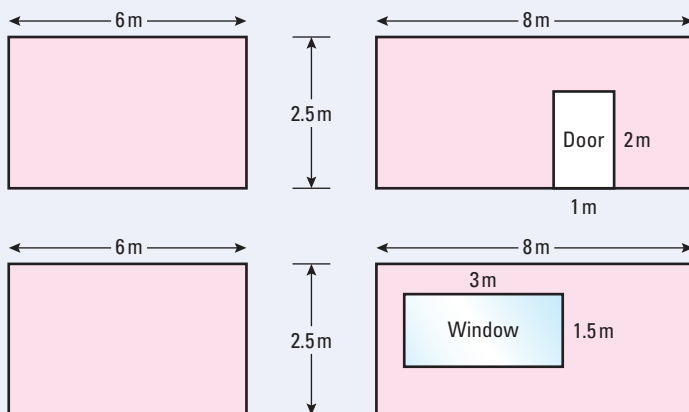


Diagram **NOT** accurately drawn



Alesha paints the walls with emulsion paint. She does not paint the door. A 3 litre tin of emulsion paint covers  $30 \text{ m}^2$  of wall. Work out how many 3 litre tins she needs to buy. Show all your working.

C  
A02

9

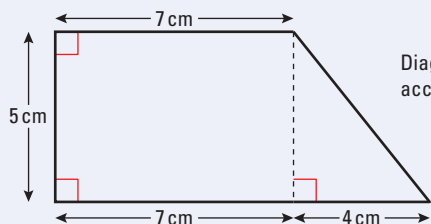


Diagram **NOT** accurately drawn

Work out the area of the shape.

Nov 2008

A02

- 10 The diagram shows a rectangle inside a triangle. The triangle has a base of 12 cm and a height of 10 cm. The rectangle is 5 cm by 3 cm. Work out the area of the region shown shaded in the diagram.

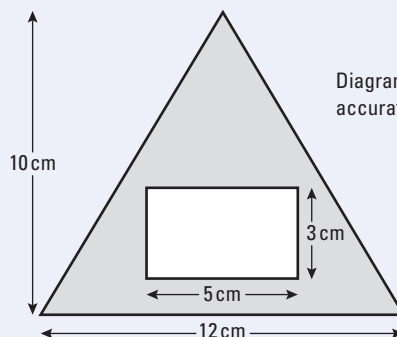


Diagram **NOT** accurately drawn

Nov 2006

11

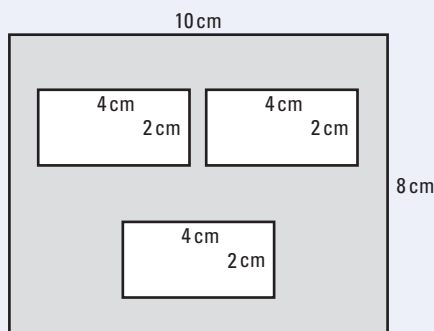


Diagram **NOT**  
accurately drawn

The diagram shows 3 small rectangles inside a large rectangle.  
The large rectangle is 10 cm by 8 cm.  
Each of the 3 small rectangles is 4 cm by 2 cm.  
Work out the area of the region shown shaded in the diagram.

June 2007

12

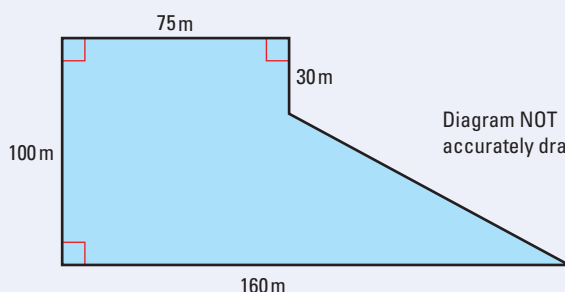


Diagram **NOT**  
accurately drawn

The diagram shows the plan of a field.  
The farmer sells the field for £3 per square metre.  
Work out the total amount of money the farmer should get.

March 2007



### ResultsPlus Exam Question Report

84% of students answered this question poorly.  
Some candidates confused perimeter with area,  
and had trouble with the area of a triangle.

A02  
A03