1 USING A CALCULATOR

BBC ACTIVE

Charles Babbage was a 19th century inventor who was frustrated by the errors found in mathematical and astronomical tables calculated by hand. He designed the 'Difference Engine', a machine that would do the calculations correctly. Many people thought the task impossible and he died before he was able to complete it. 150 years later a team at the Science Museum in London finally built the machine.

Objectives

In this chapter you will:

- work out reciprocals, powers, square roots and cube roots using a calculator
- use a calculator to work out complex calculations.

Before you start

You need to be able to:

- write a fraction as a decimal
- work out powers, including squares and cubes
- work out square roots and cube roots
- round to one decimal place
- use the correct order of operations when carrying out a calculation.

1.1 Finding reciprocals

Objective

- You can work out the reciprocal of a number.

Why do this?

Reciprocals are used in engineering and applied maths.

Get Ready

Work out

1. 1 ÷ 4 **2.** 1 ÷ 0.4 **3.** 1 ÷ 25

Key Points

- The **reciprocal** of a number is 1 divided by the number.
  The reciprocal of 2 is $\frac{1}{2}$ (or 0.5). The reciprocal of 3 is $\frac{1}{3}$ (or $0.\dot{3}$).
- To find the reciprocal of a fraction, turn it upside down.
  The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ (or $1\frac{1}{3}$). The reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$ (or 3).
- To work out reciprocals you can use the reciprocal key on a calculator.
  It is usually shown by $\boxed{1/x}$ or $\boxed{x^{-1}}$
- Any number multiplied by its reciprocal gives the answer 1.
- Zero has no reciprocal, because you cannot divide a number by zero.

Example 1 Find the reciprocal of 20.

Method 1

The reciprocal of 20 is $\frac{1}{20}$. ← You could give the answer as a fraction

1 ÷ 2 0 =

The reciprocal of 20 is 0.05. ← or as a decimal.

Method 2

2 0 $1/x$ = ← Use the reciprocal key on your calculator.

The reciprocal of 20 is 0.05.

Exercise 1A

Questions in this chapter are targeted at the grades indicated.

D

1 Find the reciprocals of these numbers.

a 10 **b** 4 **c** 8 **d** 5 **e** 9

2 Find the reciprocals of these fractions.

a $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{2}{3}$ **d** $\frac{5}{6}$ **e** $\frac{3}{10}$

D

3 Use your calculator to find the reciprocals of these numbers.

a 2.5 b 50 c 16 d 80 e 0.2
f 0.5 g 0.05 h 0.125 i 0.04 j 0.01

4 The reciprocal of 1000 is 0.001. What is the reciprocal of 0.001?

5 a Find the reciprocal of 40. b Multiply 40 by its reciprocal.

6 a Find the reciprocal of 100. b Multiply 100 by its reciprocal.

Mixed exercise 1B

D

1 Change these fractions to decimals.

a $\frac{5}{8}$ b $\frac{7}{16}$ c $\frac{5}{9}$ d $\frac{7}{11}$ e $\frac{1}{12}$

2 Which of these fractions can be written as a recurring decimal?

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{9}$

Explain your answer.

3 Lauren says that $\frac{1}{3}$ is equal to 0.3.

Is Lauren correct? Explain your answer.

4 Find the reciprocal of 25.

5 Find the reciprocal of 0.8.

1.2 Interpreting a calculator display

Objective

- You can interpret the answer on a calculator display.

Why do this?

If you are checking a shopping bill you need to be able to interpret the calculator display correctly.

Get Ready

1. Work out the total cost of 4 magazines costing £1.67 each.
2. £375 is shared equally between 5 people. How much does each person get?
3. Work out the total cost of a notebook costing £2.79 and 3 pens costing 65p each.

Key Points

- You need to take care when writing down the answer from a calculator display.
  If you are working in pounds, the calculator display 3.4 means £3.40.
  Answers that are in pounds and pence should always be written with two figures after the decimal point.
- You must make sure that your answer makes sense in the context of the question.
  Sometimes the answer to a problem must be a whole number and if the calculator display shows a decimal you will need to think carefully about whether to round it up or round it down.

Example 2 In a factory, bottles of drink are packed into boxes. Each box holds 8 bottles.
How many of these boxes can be completely filled using 670 bottles?

6 7 0 ÷ 8 =

This gives 83.75.

To the nearest whole number 83.75 rounds to 84 but 84 boxes cannot be completely filled.

So 83 boxes can be completely filled.

ResultsPlus
Examiner's Tip
Always write down the calculation that you are doing.

Example 3 It takes 22 minutes to fill a water tank.
How many tanks can be filled completely in 10 hours?

10 hours = 10 × 60 = 600 minutes

Convert the hours into minutes so that both measurements are in minutes.

6 0 0 ÷ 2 2 =

27.2727...

So 27 tanks can be completely filled.

Exercise 1C

E

1 The total cost of five adult cinema tickets is £44.50. Work out the cost of one adult cinema ticket.

2 Ryan buys four bars of chocolate costing £1.45 each and one packet of sweets costing £1.30.
Work out the total cost.

3 Hannah buys two magazines costing £3.85 each.
Work out how much change she should get from £10.

4 A garden centre sells plants for £1.90 each. Lee buys 14 plants. Work out the total cost.

5 Tom's company pays him 45p for each mile that he drives his car.
Work out how much money Tom's company pays him when he drives 126 miles.

6 Petrol costs 115.9 pence per litre. Richard buys 38 litres of petrol. How much should Richard pay?

7 Colin needs 160 tiles for a room.
Tiles are sold in boxes. There are 12 tiles in each box.
Work out the least number of boxes of tiles that Colin needs.

8 458 students and teachers are going on a coach trip.
Each coach holds 54 passengers.
Work out the smallest number of coaches needed.

E

9 A beaker holds 225 ml of orange squash.
How many of these beakers can be completely filled using 2000 ml of orange squash?

10 It takes 35 seconds to fill a bucket. How many buckets can be completely filled in 20 minutes?

11 The battery life of a calculator is 420 hours. Work out the battery life in days and hours.

12 A pen costs 38p. Sam has £5. He buys as many pens as he can.
Work out how much change Sam should get from £5.

D

A02 A03

* 13 Raja sees this new monthly plan for a mobile phone.
Raja's current plan gives him 200 minutes and unlimited texts for £25 per month. He wants to find out if he should switch to the new monthly plan.
In September, Raja used 140 minutes and 230 texts.
In October, he used 145 minutes and 190 texts.
In November, he used 135 minutes and 260 texts.
Should Raja switch to the new monthly plan? Explain your answer.

YOU PAY per month	YOU GET per month
£15	**FREE** - 100 minutes **FREE** - 200 texts

Extra minutes: 20p each
Extra texts: 12p each

1.3 Working out powers and roots

Objectives

- You can work out powers using a calculator.
- You can work out square roots and cube roots using a calculator.

Why do this?

Scientists, financial analysts and economists make use of powers and roots.

Get Ready

1. Work out 6^2. **2.** Work out 2^3. **3.** Work out $\sqrt{25}$.

Key Points

- With a scientific calculator you can work out squares using the x^2 key.
(You met square numbers in Unit 2 Section 2.3.)
- Some scientific calculators have an x^3 key for working out cubes.

To work out 3.5^2, key in 3 • 5 x^2 =

To work out 2.7^3, key in 2 • 7 x^3 =

- Scientific calculators have a power (or index) key.

It can be shown by x^y or y^x or $x^{\blacksquare}$ or ^.

To work out 2.7^3, key in 2 • 7 x^y 3 =

- To work out square roots on a calculator use the $\sqrt{}$ key.
- To work out cube roots on a calculator use the $\sqrt[3]{}$ key.

Example 4 Use a calculator to work out $1.2^3 + 6.25$.

Method 1

1 • 2 x^3 =

Or you could key in 1.2 × 1.2 × 1.2 =

The result is 1.728.

1.728 + 6.25 =

Add the result to 6.25.

The answer is 7.978.

Method 2

1 • 2 x^3 + 6 • 2 5 =

Key in the whole calculation.

The answer is 7.978.

Exercise 1D

E

1 Work out

a 2.5^2 **b** 3.2^2 **c** 47^2 **d** 1.8^2 **e** 17.9^2

2 Work out

a 6^3 **b** 22^3 **c** 2.1^3 **d** 3.4^3 **e** 1.5^3

3 Work out

a 2^5 **b** 11^4 **c** 3^5 **d** 12^4 **e** 5^6

4 Work out

a $17^2 + 10$ **b** $2.4^2 + 15$ **c** $19^2 - 322$ **d** $2.7^2 + 5.42$

5 Work out

a $2.1^3 + 1.96$ **b** $1.9^3 + 5.29$ **c** $2.5^3 - 7.29$ **d** $1.4^3 - 1.544$

Example 5 Use a calculator to work out $\sqrt{14.44}$.

√ 1 4 • 4 4 =

On some calculators you first key in 14.44 and then press the square root key.

$\sqrt{14.44} = +3.8$

Calculators always give the positive square root.

Exercise 1E

E

1 Work out

a $\sqrt{529}$ b $\sqrt{289}$ c $\sqrt{1156}$ d $\sqrt{625}$

2 Work out

a $\sqrt{2.56}$ b $\sqrt{22.09}$ c $\sqrt{13.69}$ d $\sqrt{88.36}$

3 Work these out, giving your answers correct to one decimal place.

a $\sqrt{150}$ b $\sqrt{80}$ c $\sqrt{124}$ d $\sqrt{240}$

4 Work out

a $\sqrt[3]{216}$ b $\sqrt[3]{729}$ c $\sqrt[3]{343}$ d $\sqrt[3]{1728}$

ResultsPlus
Examiner's Tip

Write down all the figures on your calculator display before you round your answer to one decimal place. (See Unit 2 Section 3.7 on Rounding.)

5 Work these out, giving your answers correct to one decimal place.

a $\sqrt[3]{40}$ b $\sqrt[3]{200}$ c $\sqrt[3]{120}$ d $\sqrt[3]{84}$

6 Work out

a $\sqrt{51.84} + 4.8$ b $\sqrt{841} - 21.3$ c $\sqrt[3]{9.261} - 1.9$ d $\sqrt[3]{1.728} + 1.8$

D

AO2 AO3

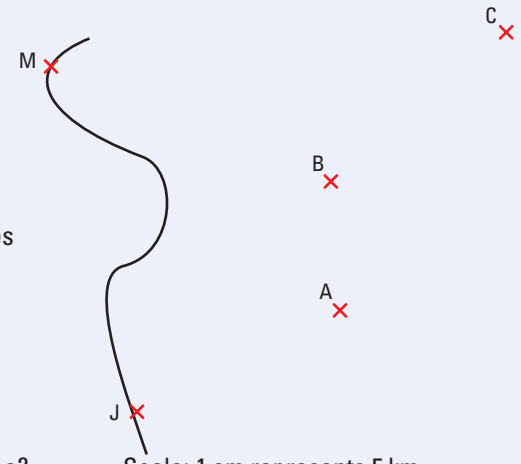
7 How far can you see?

To work out the distance, in kilometres, you can see:

1 Find the height, in metres, of your eyes above sea level.
2 Multiply this height by 13.
3 Find the square root of the answer.

James is standing on a cliff, at J. His eyes are 20 metres above sea level.
Matthew is standing in a lighthouse, M. His eyes are 50 metres above sea level.

Which of the three boats, A, B and C, should James be able to see?
Which of the three boats should Matthew be able to see?



Scale: 1 cm represents 5 km.

1.4 Using a calculator to work out complex calculations

Objective

- You can use a calculator to work out complex calculations.

Why do this?

Engineers, architects, computer programmers and accountants all need to be able to work out complex calculations using a calculator.

Get Ready

1. Work out 1.5×4^2.
2. Work out $3.5 + 2.5 \times 4$.
3. Work out $\frac{20 - 10}{5}$.

Key Points

- All scientific calculators carry out mathematical operations in the same order.
  This order is sometimes abbreviated to BIDMAS (see Unit 2 Section 8.4).
  You need to know about BIDMAS in order to use a scientific calculator properly.
- On most scientific calculators you can key in calculations in the order in which they are written down.
  To work out $(1.27 + 3.8)^2 \times 3.5$, for example, you would key in
  [( ] [1] [•] [2] [7] [+] [3] [•] [8] [ )] [x^2] [×] [3] [•] [5] [=]
- When you are doing a division calculation you must remember to divide by ALL of the denominator.
  To work out $\frac{14.5}{1.32 + 1.28}$, for example, you must divide 14.5 by the result of 1.32 + 1.28.
- Most scientific calculators have the negative sign [(−)]. To enter the number −6, for example, key in [(−)] [6].

Example 6 Work out the value of $\frac{16.3 + 7.82}{7.2 - 4.7}$.

Method 1

[1] [6] [•] [3] [+] [7] [•] [8] [2] [=] ← Key in the numerator.

The result is 24.12.

[7] [•] [2] [−] [4] [•] [7] [=] ← Key in the denominator.

The result is 2.5.

[2] [4] [•] [1] [2] [÷] [2] [•] [5] [=] ← Divide the first result by the second result.

The value is 9.648.

Method 2

[( ] [1] [6] [•] [3] [+] [7] [•] [8] [2] [ )] [÷] [( ] [7] [•] [2] [−] [4] [•] [7] [ )] [=]

Put brackets around the numerator and around the denominator.

ResultsPlus
Examiner's Tip

If you work out the numerator and the denominator separately, make sure you write down the value of each.

Exercise 1F

E

1 Work out
a $(5.2 + 2.7)^2$ **b** $(12.4 - 9.71)^2$ **c** $(2.43 + 1.87)^3$ **d** $(5.1 - 3.7)^3$

2 Work out
a $12^2 + 13^2$ **b** $34^2 + 6^3$ **c** $12^3 - 23^2$ **d** $38^2 - 18^2$

3 Work out
a $\sqrt{17.8 + 13.56}$ **b** $\sqrt{415 - 159}$ **c** $\sqrt[3]{129 - 65}$ **d** $\sqrt[3]{1.85 + 1.525}$

D

4 Work out the value of each of these.
Write down all the figures on your calculator display.

a $\dfrac{4.78 - 1.42}{0.84}$ **b** $\dfrac{48.88}{3.62 + 5.78}$

c $\dfrac{12.24 \times 2.5}{6.8}$ **d** $\dfrac{35.36}{12.6 - 5.8}$

5 Work out the value of each of these.
Write down all the figures on your calculator display.

a $\dfrac{13.2 - 6.84}{2.8 + 3.41}$ **b** $\dfrac{5.6 \times 8.1}{12.5 - 3.9}$

c $\dfrac{4.37 \times 6.52}{2.8 + 7.19}$ **d** $\dfrac{17.6 + 9.82}{23.6 - 5.94}$

Chapter review

- The **reciprocal** of a number is 1 divided by the number.
- To find the reciprocal of a fraction, turn it upside down.
- To work out reciprocals you can use the reciprocal key on a calculator.
  It is usually shown by $\boxed{1/x}$ or $\boxed{x^{-1}}$.
- Any number multiplied by its reciprocal gives the answer 1.
- Zero has no reciprocal, because you cannot divide a number by zero.
- You need to take care when writing down the answer from a calculator display.
  Answers that are in pounds and pence should always be written with two figures after the decimal point.
- You must make sure that your answer makes sense in the context of the question.
  Sometimes the answer to a problem must be a whole number and if the calculator display shows a decimal you will need to think carefully about whether to round it up or round it down.
- With a scientific calculator you can work out squares using the $\boxed{x^2}$ key.
- Some scientific calculators have an $\boxed{x^3}$ key for working out cubes.
- Scientific calculators have a power (or index) key.
  It can be shown by $\boxed{x^y}$ or $\boxed{y^x}$ or $\boxed{x^{\blacksquare}}$ or $\boxed{\wedge}$.
- To work out square roots on a calculator use the $\boxed{\sqrt{\ }}$ key.
- To work out cube roots on a calculator use the $\boxed{\sqrt[3]{\ }}$ key.
- All scientific calculators carry out mathematical operations in the same order. This order is sometimes abbreviated to BIDMAS. You need to know about BIDMAS in order to use a scientific calculator properly.
- On most scientific calculators you can key in calculations in the order in which they are written down.
- When you are doing a division calculation you must remember to divide by ALL of the denominator.
- Most scientific calculators have the negative sign $\boxed{(-)}$.

Review exercise

E

1 Christine buys a calculator costing £3.99, a pencil case costing £1.65 and two rulers costing 28p each. She pays with a £10 note. How much change should she get from her £10 note?

2 Shares in a company cost £6.23 each. Tauqeer has £500. He buys as many shares as he can. Work out how many shares Tauqeer can buy.

3 A milk crate holds 24 bottles. Amraiz has 357 bottles of milk. Work out how many milk crates he can fill completely.

4 The table below shows the cost of each of three calculators.

Quicksum	£2.30
Basic	£2.15
Easycalc	£2.90

a Emily buys one Quicksum calculator and two Easycalc calculators. She pays with a £10 note. How much change should she get?

b Mrs Windsor wants to buy some Basic calculators. She has £60 to spend. Work out the greatest number of Basic calculators she can buy.

5 Work out

a 2.9^2 **b** 12^3 **c** 3.7^2 **d** 2.2^3

6 Work out

a $\sqrt{51.84}$ **b** $\sqrt{784}$ **c** $\sqrt[3]{512}$ **d** $\sqrt[3]{15.625}$

7 Work these out, giving your answers correct to one decimal place.

a $2.8^2 + \sqrt{34}$ **b** $\sqrt{56} - 2.3^2$ **c** $4.7^2 - \sqrt{28}$ **d** $3.8^2 - \sqrt{50}$

8 Jonathan buys a can of cola and a roll.

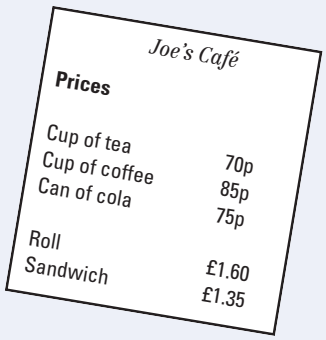
a Work out the total cost.

Sachin buys a cup of tea, a cup of coffee and 2 sandwiches.

b Work out the total cost.

Kim buys a can of cola, a cup of coffee and a sandwich. She pays with a £5 note.

c Work out how much change she should get.



June 2007

9 Cans of drink are put into packs of 24. How many packs can be filled from 750 cans of drink?

10 There are 1230 students in a school. All the students go on a trip. Each bus can take 48 students. How many buses are needed?

11 Plain tiles cost 28p each.
Patterned tiles cost £9.51 each.
Julie buys 450 plain tiles and 15 patterned tiles.
Work out the total cost of the tiles.

Nov 2007

12 Use a calculator to work out

$\sqrt{2.56} + 8.4$

Nov 2008

D

13 Work out

a $(3.7 + 2.64)^2$ **b** $\sqrt{17 + 25.25}$ **c** $(2.1 + 2.8)^2 \times 1.2$

14 Work out the value of each of these.
Write down all the figures on your calculator display.

a $\dfrac{5.68 - 1.52^2}{0.83}$ **b** $\dfrac{1}{3.58^2 - 2.87}$ **c** $\dfrac{8.7 + 5.92}{16.3 - 4.56}$

15 Work out the value of each of these.
Write down all the figures on your calculator display.

a $\dfrac{\sqrt{3.96 + 1.8}}{7.625 - 3.48}$ **b** $\sqrt{\dfrac{4.92 + 3.48}{9.2 - 3.75}}$

* **16** To work out a person's daily calorie requirement you can use one of these rules.

Gender	Daily calorie requirement
Female	655 + (9.6 × weight in kg) + (1.8 × height in cm) − (4.7 × age in years)
Male	66 + (13.7 × weight in kg) + (5 × height in cm) − (6.8 × age in years)

The table below shows some information about four people.

Name	Gender	Age (years)	Weight (kg)	Height (cm)
Sophie	F	32	68	165
Chelsea	F	47	55	175
Kenny	M	27	98	191
Hassan	M	38	117	182

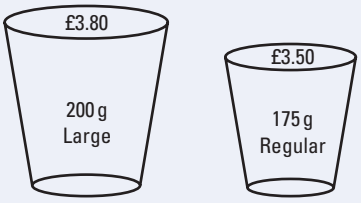
Work out the recommended daily calorie intake for each person.
Which person has the greatest daily calorie requirement?
Which person has the smallest daily calorie requirement?

17 Find the reciprocal of **a** $\frac{5}{8}$ **b** 2.5 **c** $\frac{1}{4}$

18 Use your calculator to work out $\dfrac{22.4 \times 14.5}{8.5 \times 3.2}$

Write down all the figures on your calculator display. *June 2007*

* **19**



A03

A large tub of popcorn costs £3.80 and holds 200 g.
A regular tub of popcorn costs £3.50 and holds 175 g.
Rob says that the 200 g large tub is the better value for money.
Linda says that the 175 g regular tub is the better value for money.
Who is correct?
Explain the reasons for your answer.
You must show all your working. *June 2006*

D

20 Work out $\dfrac{4.6 + 3.85}{3.2^2 - 6.51}$

Write down all the numbers on your calculator display.

June 2009

A02 A03

21 Salma has £1.55.

She wants to buy a burger and fries.

a What are the different combinations that can she buy?

Mark buys 2 double burgers with cheese, 1 large fries and 1 large cola.

He pays with a £10 note

b He gets the best price.
What change should he get?

Ben's Burger Bar

Burgers

Single Burger	£0.85
Single Burger with Cheese	£0.95
Double Burger	£1.55
Double Burger with Cheese	£1.70

Fries

Regular	£0.65
Large	£0.99

Cola

Regular	£0.85
Large	£1.10

Meal Deals

Regular

Single Burger with Cheese regular Fries and regular Cola £2.09

Large

Double Burger with Cheese large Fries and large Cola £3.49