16 AVERAGES AND RANGE

BBC ACTIVE

The manager of a supermarket is asked to summarise the amount, in pounds, spent by shoppers at his supermarket. He wishes to give the average amount and an indication of how the actual amounts vary from this. What figures should he give and how can he calculate them?

Objectives

In this chapter you will:

- find the mean, mode and median of a set of data and discuss the advantages and disadvantages of these three measures of average
- find the range of a set of data
- compare distributions
- use stem and leaf diagrams and frequency tables to find the mode, median and range of a set of data
- find the modal class and the interval containing the median of grouped data
- estimate the mean of grouped data.

Before you start

You need to:

- understand frequency tables
- understand grouped data and grouped frequency tables.

16.1 Finding the mode, the median and the mean

Objective

- You can find the mode, the median and the mean of a set of data.

Why do this?

This will help you to understand detailed weather reports where they talk about average rainfall and hours of sunlight.

Get Ready

1. Write these numbers in order of size, smallest first.
   a 325 284 336 296 302
   b 0.6 0.59 0.55 0.625 0.61
2. The frequency table shows the number of times balls of different colours are picked from a bag.

Colour	Red	Yellow	Blue	Green
Frequency	6	10	8	14

   a How many times was a ball picked out?
   b Which ball was picked most?
   c How many more times was a blue ball picked than a red one?

Key Points

- We often describe a numerical data set by giving a single value that is representative of all the values in the set. We call this value an **average**. For example, we might say that 'The average height of members of a certain basketball team is 190 cm.'
- Three different averages are commonly used: the **mode**, the **median** and the **mean**.
- We may also be interested in how values are spread out. A common measure that is used is the **range**.

Finding the mode

Key Point

- The mode of a set of data is the value that occurs most frequently.

Example 1 Find the mode of each of the following sets of data.

a 3 4 6 7 8 7 5
b 13 27 26 32 42 27 8 10 13
c 7 6 9 3 5 8 2
d Black Green Blue Black Red

a Mode = 7 ← 7 occurs twice, the other numbers appear only once.

b Mode = 13 and 27 ← Both 13 and 27 occur twice. There are two modes.

c There is no mode. ← All numbers occur with the same frequency.

d The mode is black. ← Black appears twice. Each other colour only appears once.

Exercise 16A

Questions in this chapter are targeted at the grades indicated.

F

1 Find the mode for each of the following data sets.

a 2 7 8 3 2 9 4 2

b 10 12 14 10 15 12 11 18

c 3 6 8 7 4 9 1 2

d dog cat rabbit dog fish mouse dog

2 Briony did a spelling test every week for ten weeks. Her scores are given below.

11 16 18 16 12 18 17 16 12 16

Find the mode for these data.

3 Bo made a list of the colours of all the cars in a showroom. His list is shown below.

blue silver white red red black silver silver green silver black

Write down the mode for these data.

4 The hourly rates of pay for eight workers are listed below.

£8.20 £8.50 £7.20 £8.80 £9.20 £7.20 £8.20 £9.90

Write down the mode for these data.

Finding the median

Key Points

- The median is the middle value when the data are ordered from the smallest to the largest.
- If there are two middle values in a set of data, the median is halfway between them.

Example 2

Eleven people applied for a job in an office. Their typing speeds, in words per minute, were 36, 42, 58, 47, 43, 42, 38, 48, 52, 46 and 46.

Find the median typing speed.

36 38 42 42 43 **46** 46 47 48 52 58

Arrange the numbers in order, smallest to largest.

The median is 46.

The middle value is 46. There are 5 speeds lower than this and 5 speeds higher.

Example 3

The midday temperatures, in degrees Celsius, at a seaside town were 18, 21, 18, 16, 25, 18, 17, 28, 22 and 19.

Find the median temperature.

16 17 18 18 **18** **19** 21 22 25 28

Put the numbers in order first.

The median is 18.5°C

There are two middle values: 18°C and 19°C.

Halfway between 18 and 19 is 18.5.

Exercise 16B

E

1 Find the median for the following data sets.

a 2 3 3 4 5 6 7 8 8

b 3 5 6 6 8 8 9 10

c 5 5 6 7 7 8

d 4 2 8 6 7 9 4 3 2

e 2 5 6 3 7 8 1 9

E

2 The minimum night-time temperatures (in °C), in February, in seven towns are recorded below.
1.0 1.2 1.4 1.2 1.3 1.2 1.5
Find the median temperature.

3 The CO_2 emissions (in thousands of tonnes) for mopeds in Scotland over eight years are shown below.
38 38 39 41 44 40 41 39
Find the median CO_2 emissions.

AO3

4 The annual earnings of six managers in a factory are shown below.
£20 500 £25 700 £30 900 £27 500 £22 000 £26 500

a Work out the median earnings.

b The factory wishes to employ another manager. Recommend a salary range for them to use in an advert.

Calculating the mean

Key Point

- The mean of a set of data is the sum of the values divided by the total number of observations.
  $\text{mean} = \dfrac{\text{sum of the values}}{\text{number of values}}$
- A short way of writing the mean uses the Greek letter sigma Σ to represent the sum of a set of values.
- The mean can be worked out using the statistical functions on a scientific calculator. The method for inputting the data will vary between different scientific calculators, and the instruction leaflet will explain how to enter and process the statistical data.

Example 4 Find the mean of the numbers 2, 4, 6, 6, 8, 9, 8 and 5.

$2 + 4 + 6 + 6 + 8 + 9 + 8 + 5 = 48$ ← Total up the values.

$\text{mean} = \dfrac{48}{8} = 6$ ← Divide by the number of values to get the mean.

Example 5 The weekly consumptions of electricity, in units, during an 8-week period were 340, 350, 340, 355, 340, 345, 340 and 450 units.
a Find the mode. **b** Find the median. **c** Find the mean.

a The mode is 340 units ← 340 occurs four times, each of the other values occurs once.

b 340 340 340 **340** **345** 350 355 450 ← There are two middle values: 340 and 345.

The median is 342.5 units. ← Halfway between is 342.5.

c The total of all values $= 340 + 340 + 340 + 340 + 345 + 350 + 355 + 450 = 2860$ ← Find the total sum.

$\text{mean} = \dfrac{2860}{8} = 357.5$ units ← Divide by the number values.

ResultsPlus
Watch Out!
Some students use the term average – make sure you specify mean, mode or median.

Exercise 16C

E

1 Find the mean of the following data sets.

a 3 5 5 6 8 9

b 2 5 4 7 5 3 8 6

c 7 5 8 9 9 7

2 The number of school meals bought by 10 children in one week were

2 4 3 5 5 4 2 5 5 5

A03

a Work out the mean number of meals.

The school has 250 children.

b Work out an estimate for the number of school meals they should provide each day.

3 The heights, in centimetres, of 11 members of a cricket team were

172 174 190 190 185 186 182 189 190 185 192

a Find the mode.

b Work out the median height.

c Work out the mean height.

4 The numbers of emails received by a sample of six people during one week were

44 72 107 155 214 197

a Find the mode.

b Work out the median number of emails.

c Work out the mean number of emails.

16.2 Knowing the advantages and disadvantages of the three types of average

Objective

- You can discuss the advantages and disadvantages of different measures of average.

Why do this?

You need to be able to determine which type of average to use. For example, if you needed to find out the most popular football team, you would use the mode.

Get Ready

1. For the following data set find

82 54 65 36 48 48 79

a The mode

b The median

c The mean

Key Points

- The three types of averages are mode, median and mean.

  Each of the averages is useful in different situations.

  The table shows which measure of average to use.

Measure	Advantages	Disadvantages
MODE Use the mode when the data is non-numeric or when asked to choose the most popular item.	Extreme values (outliers) do not affect the mode. Can be used with categorical data.	There may be more than one mode. There may not be a mode, particularly if the data set is small.
MEDIAN Use the median to describe the middle of a set of data that does have an extreme value.	Not influenced by extreme values.	Not as popular as mean. Actual value may not exist.
MEAN Use the mean to describe the middle of a set of data that *does not* have an extreme value.	Most popular measure. Can be used for further calculations. Uses all the data.	Affected by extreme values.

Example 6

Shuabur recorded the number of spam emails he received every day for seven days.
Here are his results:

3 7 18 21 24 24 29

a Find the mode, median and mean of these numbers.

b Comment on the results.

a Mode = 24

There are two 24 values.

Median = 21

There are three values greater than 21 and three lower.

Total = 3 + 7 + 18 + 21 + 24 + 24 + 29 = 126

Sum the values and divide by the number of values.

Mean = $\frac{126}{7} = 18$

b The mode of 24 has only one higher value.
This is because the number in the sample is small.
The mean has only two values that are less;
the two low values 3 and 7 have affected it.
The median is unaffected by the two low values.
The median is the best average to represent these data.

Comment on how well the averages represent the numbers as a whole, and give reasons why they might not be representative.

Exercise 16D

1 Jenny decides that she will use the mean to represent the following prices of jumpers.
£4 £5 £3 £7 £5 £42
Write down a reason why the mean is not a good average to use.

A02 A03 E

2 The prices of cars in a showroom were
£5750 £5750 £7750 £7550 £7950 £10 750 £11 550 £14 700
The garage manager puts up a poster saying 'AVERAGE PRICE £5750'.
a Write down the name of the average he is using.
b Is this a fair average to use? Give a reason for your answer.

A02 A03

3 Andy records the number of pieces of junk mail he receives each day for a week.
2 8 2 7 6 17 0
a Find the mode.
b Work out the median.
c Work out the mean.
d Comment on the values of these three averages.

* **4** The following are the times, in minutes, it takes some people to travel to work.
10 10 13 16 17 13 40
Which average is the best to represent these data?
Give a reason for your choice.

D

16.3 Finding the range

Objectives

- You can calculate the range of a set of data.
- You can compare data sets using a measure of average and a measure of range.

Why do this?

If you were trying to negotiate a better rate of pay you might want to find the range of wages that your friends who had similar jobs earned, so that you could make sure you were getting a fair amount.

Get Ready

1. Write down the highest and lowest values in the following data sets.
a 2 7 4 3 5 13
b 22 25 26 19 24 29
c 150 161 152 130 145 156

Key Points

- The range of a set of data is the difference between the highest value and the lowest value. The range tells you how spread out the data are.
  range = highest value − lowest value
- The average and the range together give a description of the frequency distribution of the data.
- To compare the distributions of sets of data you need to give a measure of average and a measure of **spread**.

Example 7

Here are the times, in minutes, that Fiona took going to school in the morning and returning after school in the afternoon.

Going to school: 20, 18, 21, 17, 25, 28, 22, 23
Going home: 14, 17, 23, 28, 24, 12, 24, 32

a Work out the range for each type of journey.
b Which journey time was the most consistent? Give a reason for your answer.

a Range going to school = 28 − 17 = 11 minutes.
Range going home = 32 − 12 = 20 minutes.

Take the smallest value from the largest.

b The time taken to go to school was the most consistent.
The range was smaller.

Don't forget to give a reason.

Example 8

Samples were taken from two machines that filled jars of coffee. The weights of coffee, in grams, were

Machine 1: 187, 192, 195, 198, 200, 200, 203, 205, 210, 210
Machine 2: 193, 194, 196, 199, 200, 200, 202, 204, 205, 207

a Find the mean and range of the data for each machine.
b Comment on these results.

a Machine 1

$$\text{Mean} = \frac{187 + 192 + 195 + 198 + 200 + 200 + 203 + 205 + 210 + 210}{10} = 200\,g$$

$$\text{Range} = 210 - 187 = 23\,g$$

The mean = $\frac{\text{total of values}}{\text{total frequency}}$

Take the smallest value from the largest value.

Machine 2

$$\text{Mean} = \frac{193 + 194 + 196 + 199 + 200 + 200 + 202 + 204 + 205 + 207}{10} = 200\,g$$

$$\text{Range} = 207 - 193 = 14\,g$$

b Both machines filled the jars with the same mean amount of coffee.
Machine 2 had a smaller range. The weights of coffee were less spread out which shows the machine was more consistent.

Exercise 16E

F

AO3

1 Find the range for each of the following sets of data.

a 2 5 7 8 9 20 21

b 3 7 3 5 7 14 5 13

c 112 115 118 117 118 113

2 The scores in a game were

12 14 11 17 23 25 22

Petra says the range is 10. Is she correct?

You must explain your answer.

E

AO3

3 The table gives the exam marks in economics and psychology for a group of students taught by two different teachers.

Student	A	B	C	D	E	F	G	H	I
Economics	72	70	63	87	83	56	88	44	65
Psychology	55	65	57	68	70	55	59	60	62

a Work out the range for each subject.

b The headteacher wants to know which subject had the most consistent marks and also whether students are being stretched. Give a reason for your answer.

D

AO3

4 Samples were taken from two machines filling bottles of mineral water.

The amounts of water (m*l*) in the bottles were

Machine 1: 30 29 30 30 29 32

Machine 2: 30 29 29 34 30 28

a Find the range for each machine.

b Find the mean for each machine.

c Comment on your answers to parts **a** and **b**.

AO3

* **5** The heights of two teams of footballers (cm) were

Max Rangers: 170 172 180 190 184 179 176 183 186 190 170

Red United: 179 190 187 170 180 182 163 188 181 190 179

Calculate the mean and range for each team and compare and contrast the frequency distributions of the heights of the two teams.

16.4 Using stem and leaf diagrams to find averages and range

Objectives

- You can use a stem and leaf diagram to find the mode and median of a set of data.
- You can use a stem and leaf diagram to find the range.

Why do this?

A stem and leaf diagram is a good way to put data into order.

Get Ready

Write these numbers in order of size, from smallest to largest.

1. 35 42 26 58 **2.** 152 151 154 153 **3.** 0.5 0.2 0.1 0.4 **4.** 0.1 0.11 0.12 0.9

Key Point

- A **stem and leaf diagram** makes it easy to find the mode, the median and the range of a set of data.

Example 9

Here are the times, in minutes, taken by 12 people to complete a crossword puzzle.
35, 48, 42, 35, 38, 56, 34, 28, 52, 18, 43, 27

a Write these data as a stem and leaf diagram.
b Write down the mode of these data.
c Find the median of these data.
d Work out the range of these data.

a 18 28 27 35 35 38 34 48 42 43 56 52

First write down the numbers whose tens digit is 1.
Then write down those numbers whose tens digits are 2, 3, 4 and finally 5.
The digit that each number begins with is called the stem.
The following digit is called the leaf.

Stem	Leaf			
1	8			
2	8	7		
3	5	5	8	4
4	8	2	3	
5	6	2		

Under stem write the numbers 1 to 5. Opposite each stem write the leaves. Don't worry about the order. This gives you an unordered stem and leaf diagram.

Key: 1|8 stands for 18 minutes

Stem	Leaf			
1	8			
2	7	8		
3	4	5	5	8
4	2	3	8	
5	2	6		

Order the leaves to give an **ordered stem and leaf diagram**.

b The mode is 35 minutes.

35 appears twice, the other values only once.

c The median is $\frac{35 + 38}{2} = 36.5$ minutes.

You can find the middle numbers by counting in from each end.

d The range is $56 - 18 = 38$ minutes.

The largest and smallest values are the first leaf and the last leaf. The range is the difference between them.

Exercise 16F

D

1 Here is an unordered stem and leaf diagram.

0	1	6	3	4	2		
1	2	6	2	4	2		
2	4	2	5	7	3	9	
3	3	0	2	5	8	4	6
4	7	3	6	2			

Key:
1|2 stands for 12

Draw this as an ordered stem and leaf diagram.

D

2 Keith has a job that involves driving to different shops each day.
Here are the distances, in kilometres, that he drove during April.

8	10	21	17	9	31	22	6	9	15
17	22	17	14	39	25	26	18	12	27

Draw a stem and leaf diagram to represent these data.

3 Here is a stem and leaf diagram showing the numbers of cars sold by a garage group over each of a number of weeks.

0	9								
1	2	6	7	8					
2	1	3	5	5	5	6	6	6	6
3	2	2	2	4	6	7			
4	0								

Key:
2|1 stands for 21 cars

a Write down the number of weeks represented in this diagram.

b Write down the mode for these data.

c Find the median number of cars.

d Work out the range of these data.

4 Here is a list of the number of minutes patients had to wait to see a dentist during one day at a dental surgery.

AO2

10	5	23	8	14	16	3	2	12	24	22	7
15	18	23	30	23	16	16	20	3	5	2	18

Draw a suitable diagram that the dentist could use to

a display these data

b show a measure of spread

c show a measure of average.

16.5 Using frequency tables to find averages for discrete data

Objective

- You can use a frequency table to find averages.

Why do this?

If your dance club was debating whether to move location it could work out the change to average journey time as this might affect membership levels.

Get Ready

1. When a die was thrown 30 times the number 6 came up 5 times.

a What was the frequency of the number 6?

b What was the sum of all the 6s added together?

Key Points

- The mode is the number that has the highest frequency.
- The median is the number that is the middle value or halfway between the middle values if there are two of them.
- The mean is the sum of the values divided by the number of values (the number of values is the total frequency).
- For discrete data in a frequency table:

  mean $= \dfrac{\sum f \times x}{\sum f}$ where f is the frequency, x is the variable and $\sum$ means 'the sum of'.

Example 10

The table shows information about the number of children in a sample of families.

Number of children in family	1	2	3	4	5
Frequency	3	11	9	5	6

a Write down the mode of these data.
b Find the median of these data.
c Work out the mean of these data.

a The mode is 2 children.

2 has the highest frequency, which is 11.

b

Number of children in family x	Frequency f	Frequency × number of children $f \times x$
1	3	3
2	11	22
3	9	27
4	5	20
5	6	30
Total	34	102

There are 11 families with 2 children in each so the number of children is 11 × 2 = 22 children.

The total number of families is 34.

The total number of children is the sum of all the $f \times x$ values.

The total frequency is 34 so the median will be between the 17th and 18th values.

There will be 16 values either side.

There are 3 families with 1 child in them.

3 lies between the 14th and 23rd values.

There are 3 + 11 = 14 families with 2 or 1 children.

There are 3 + 11 + 9 = 23 families with 3, 2 or 1 children.

The 17th and 18th values must both be 3.

The median is 3.

c The mean is $\dfrac{102}{34} = 3$ children.

Mean $= \dfrac{\text{Total number of children}}{\text{Total number of families}} = \dfrac{\sum f \times x}{\sum f}$

Exercise 16G

AO3 D

1 A council wanted to provide extra parking on an estate.
They asked a sample of households how many cars they had.
The results are shown in the frequency table.

Number of cars	Frequency
0	0
1	10
2	7
3	6
4	2

a Write down the mode of these data.
b Find the median number of cars.
c Work out the mean number of cars.
d Recommend the number of extra parking places they should provide. You must explain your answer.

2 A sample of a tomato crop was taken and each tomato was weighed.
The weights to the nearest 5 g are shown in the frequency table.

Weight of tomatoes	Frequency
55	2
60	5
65	10
70	6
75	2

a Write down the mode of these data.
b Find the median weight of the tomatoes.
c Work out the mean weight of the tomatoes.

3 In an experiment with peas the number of peas per pod was recorded.
The results are shown in the frequency table.

Number of peas per pod	Frequency
1	0
2	0
3	3
4	7
5	11
6	12
7	15
8	12
9	10

a Write down the mode of these data.
b Find the median number of peas per pod.
c Work out the mean number of peas per pod.

16.6 Working with grouped data

Objectives

- You can find the modal class for grouped data.
- You can find a class interval containing the median of grouped data.

Why do this?

If you are collecting continuous data, such as times taken to swim 100 m, you need to have class intervals in your frequency table.

Get Ready

1. Write whether each of the following types of data is continuous or discrete.
   - **a** Number of items in a shopping basket
   - **b** Size of feet
   - **c** Waist size
2. Write down whether each statement is true or false.
   - **a** $10 > 12$ **b** $0.1 < 0.2$ **c** $0.17 > 0.6$

Key Points

- If you do not know the exact data in each class interval, you cannot give an exact value for the mode or the median and can only estimate the mean value.
- The class interval with the highest frequency is called the **modal class**.
- You can only write down the class interval in which the median falls.

Example 11

The frequency table gives information about the number of letters, l, in a sample of people's surnames.

Class interval	Frequency
3 to 5	1
6 to 8	3
9 to 11	5
12 to 13	4
14 to 15	2

This is discrete data. No whole number appears in two classes.

There will be $3 + 1 = 4$ values less than or equal to 8.

a Find the modal class.
b Find the class into which the median falls.

Look for the class with the highest frequency.

a The modal class is 9 to 11.
b There are 15 names in total so the median will be the 8th value.
There are $3 + 1 = 4$ names that are less than 9 letters long and $5 + 3 + 1 = 9$ names that are less than 12 letters long, so the median is in the class interval 9–11.

Example 12

The frequency table gives information about the lengths, l in mm, of leaves from a certain plant.

Class interval	Frequency f
$20 \leqslant l < 25$	6
$25 \leqslant l < 30$	8
$30 \leqslant l < 35$	13
$35 \leqslant l < 40$	14
$40 \leqslant l < 45$	9

This is continuous data.

There will be $6 + 8 = 14$ values less than 30.

a Find the modal class.

b Find the class into which the median falls.

a The modal class is $35 \leqslant l < 40$.

b There are 50 leaves so the median will be halfway between the 25th and 26th values.
There are 14 values less than 30 and 27 less than 35.
The median falls in the class $30 \leqslant l < 35$.

Exercise 16H

E

1 A group of students were asked how many times they visited a library in a term.
The results are shown in the frequency table.

Class interval	0 to 2	3 to 5	6 to 8	9 to 11
Frequency	0	6	10	6

a Write down the modal class.

b Find the class into which the median falls.

2 A group of students did a mental arithmetic test.
The results are shown in the frequency table.

Class interval	1 to 5	6 to 10	11 to 15	16 to 20
Frequency	1	9	15	5

a Write down the modal class.

b Find the class into which the median falls.

3 The frequency table gives the diameter, d in mm, of 48 balls of lead used in a quality control investigation.

a Write down the modal class.

b Find the class into which the median falls.

Class interval	Frequency
$0.7 \leqslant d < 0.9$	2
$0.9 \leqslant d < 1.1$	4
$1.1 \leqslant d < 1.3$	16
$1.3 \leqslant d < 1.5$	12
$1.5 \leqslant d < 1.7$	14

16.7 Estimating the mean of grouped data

Objective

- You can estimate the mean of grouped data.

Why do this?

If data is collected and arranged in a grouped frequency table, you will not have exact data values, so you won't be able to calculate the mean exactly. Using the middle values of the class intervals provides good estimates to work with.

Get Ready

1. Which number is halfway between:

a 6 and 8 **b** 56 and 64 **c** 75 and 76 **d** 0.75 and 0.85 **e** 100 000 and 150 000?

Key Point

- An estimate for the mean of grouped data can be found by using the midpoint of the class interval and the formula $\frac{\sum fx}{\sum f}$ where f is the frequency and x is the class midpoint.

Example 13 Work out an estimate for the mean length of the people's surnames given in Example 11.

Class interval	Frequency f	Class midpoint x	$f \times x$
3 to 5	1	4	4
6 to 8	3	7	21
9 to 11	5	10	50
12 to 14	4	13	52
15 to 17	2	16	32
Totals	15		159

The middle value of the class 3–5 is 4.

The middle value of the class 6–8 is 7. The three people in the class 6–8 might not all have surnames 7 letters long. This is why it is an estimated mean.

$$\text{Estimated mean} = \frac{\sum f \times x}{\sum f} = \frac{159}{15} = 10.6$$

You can now use the formula.

Example 14 Work out an estimate for the mean length of the leaves given in Example 12.

Class interval	Frequency f	Class midpoint x	$f \times x$
$20 \leqslant l < 25$	6	22.5	135
$25 \leqslant l < 30$	8	27.5	220
$30 \leqslant l < 35$	13	32.5	422.5
$35 \leqslant l < 40$	14	37.5	525
$40 \leqslant l < 45$	9	42.5	382.5
Totals	50		1685

$$\text{Estimated mean} = \frac{1685}{50} = 33.7 \text{ mm.}$$

ResultsPlus
Examiner's Tip

Remember to use the class midpoint when estimating the average.

Exercise 16I

1 In a healthy eating investigation the canteen supervisor at Conville College recorded the numbers of packets of crisps bought per month by a sample of students.
The results are shown in the frequency table.

Find an estimate for the mean number of crisps.

Class interval	Frequency f
1 to 3	1
4 to 6	9
7 to 9	15
10 to 12	5

AO2 C

2 A store is worried about the reliability of its lift. It records the number of times it breaks down each week over a period of 28 weeks.
The results are shown in the frequency table.
Find an estimate for the mean number of breakdowns.

Class interval	Frequency f
0 to 1	20
2 to 3	3
4 to 5	4
6 to 7	1

AO2

3 Emma recorded the length of time (t), in minutes, each of her business phone calls took over a period of one month.
The results are shown in the frequency table.

Find an estimate for the mean time for her business phone calls.

Class interval	Frequency
$0 \leqslant t < 5$	9
$5 \leqslant t < 10$	10
$10 \leqslant t < 15$	9
$15 \leqslant t < 20$	7
$20 \leqslant t < 25$	5

AO2

Chapter review

- The **mode** of a set of data is the value that occurs most frequently.
- The **median** is the middle value when the data are ordered from the smallest to the largest.
- If there are two middle values in a set of data the median is halfway between them.
- The **mean** of a set of data is the sum of the values divided by the total number of observations.

  $$\text{mean} = \frac{\text{sum of the values}}{\text{number of values}}$$
- **Range** = highest value − lowest value
- To compare the distributions of sets of data you need to give a measure of **average** and a measure of **spread**.
- A **stem and leaf diagram** makes it easy to find the mode, median and range of a set of data.
- For discrete data in a frequency table:

  $$\text{mean} = \frac{\sum f \times x}{\sum f}$$ where f is the frequency, x is the variable and $\sum$ means 'the sum of'.
- For grouped data:
  - the class interval with the highest frequency is called the **modal class**
  - you can only write down the class interval in which the median falls
  - an estimate for the mean of grouped data can be found by using the midpoint of the class interval.

Review exercise

F

1 Mary threw a dice 24 times.
Here are the 24 scores.

3 5 3 4 1 2 4 5 6 2 3 4
3 1 4 3 2 3 5 5 3 4 2 1

a Complete the frequency table.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		

b Write down the mode.

ResultsPlus
Exam Question Report

Most candidates did very well on part **a** of this question as they knew to write tallies and then show the frequencies.

March 2007

E

2 The weekly incomes of five people are shown in the table.

a Work out the mean income per week.

b Work out the median income per week.

c Work out the range of weekly incomes.

d How much more did Mrs Chown earn than Mrs Basingi?

Name	Income
Mr Rahman	£420
Mrs Basingi	£365
Mr Clarke	£400
Mr Abson	£280
Mrs Chown	£430

3 Here are ten numbers.

7 6 8 4 5 9 7 3 6 7

a Work out the range.

b Work out the mean.

Nov 2008

4 Jason collected some information about the heights of 19 plants.
This information is shown in the stem and leaf diagram.

1	1	2	3	4		
2	3	3	5	9	9	
3	0	2	2	6	6	7
4	1	1	4	8		

Key:
4|8 means 48 mm

Find the median.

Nov 2008

5 Peter rolled a 6-sided dice ten times.
Here are his scores.

3 2 4 6 3 3 4 2 5 4

a Work out the median of his scores.

b Work out the mean of his scores.

c Work out the range of his scores.

June 2007

6 Here are the weights, in kg, of 8 people.

63 65 65 70 72 86 90 97

a Write down the mode of the 8 weights.

b Work out the range of the weights.

June 2007

7 Five positive numbers have a mode of 5, a median of 5 and a mean of 4.
Write down as many possible combinations of five numbers that give these statistics as you can.

AO3 E

8 The stem and leaf diagram shows the ages, in years, of all the workers in a small factory.

AO2 D

2	0	2	2	5	7				
3	3	4	4	4	4	5			
4	5	6	6	6	6	8	9	9	9
5	2	4	4	6	7	9			
6	0	2	5						

Key:
4|5 stands for 45 years

a Work out the number of workers.
b Write down the mode of these data.
c Find the median of these data.
d Work out the range of these data.

9 A group of girls went to a college dance. They each bought a new dress.
The costs of the dresses were

AO3

£22 £22 £22 £28 £32 £36 £40 £40 £45 £180

a Write down the mode of these data.
b Find the median price
c Work out the mean price.
d Which of the three averages worked out in parts **a**, **b** and **c** best describes the price the girls paid? Give a reason for your answer.

10 Samples of apples were taken from two trees. One was an eating apple tree and the other was a cooking apple tree.
The weights, in grams, of the apples were

Eating apple	135	135	140	138	142	150	132
Cooking apple	140	136	150	160	138	162	150

a Find the means and ranges for these data.
b Compare the frequency distributions of the weights of the two types of apple.

AO3

11 Zoe recorded the weights, in kilograms, of 15 people.
Here are her results.

87 51 46 77 74 58 68 78
48 63 52 64 79 60 66

a Complete the ordered stem and leaf diagram to show these results.
b Write down the number of people with a weight of more than 70 kg.
c Work out the range of the weights.

4	
5	
6	
7	
8	

March 2009

D

12 Zach has 10 CDs.
The table gives some information about the number of tracks on each CD.

a Write down the mode.

b Work out the mean.

Number of tracks	Frequency	
11	1	
12	3	
13	0	
14	2	
15	4	

June 2009

ResultsPlus
Exam Question Report

81% of students answered part **b** of this question poorly.

13 Here are the ages, in years, of 15 teachers.

35 52 42 27 36
23 31 41 50 34
44 28 45 45 53

Draw an ordered stem and leaf diagram to show this information. You must include a key. *May 2008 adapted*

14 Explain why the sentence 'The majority of spiders in this country have more than the average number of legs' can be true. You need to state which average is being used.

15 Ali found out the number of rooms in each of 40 houses in a town.
He used the information to complete the frequency table.

Number of rooms	Frequency	
4	4	
5	7	
6	10	
7	12	
8	5	
9	2	

Ali said that the mode is 9. Ali is wrong.

a Explain why.

b Calculate the mean number of rooms. *Nov 2007*

AO3

16 A group of university students did a maths test. The table shows their scores.

Males	42	22	65	42	70	50	45
Females	25	90	55	26	95	50	87

Using your understanding of averages and range, compare the males' and the females' scores.

17 One teacher is responsible for the distribution of milk at break time in each school.
There are p classes and each class is sent $x + 2$ bottles of milk, where x is the number of children present in a class.
The teacher uses $\Sigma x + 2p$ to work out how many bottles are needed.

AO2 AO3 D

a Is $\Sigma x + 2p$ an expression, an equation or a formula?

There are 80 children in Banjo school.
The numbers attending Banjo school one day are shown in the table.

Class	1	2	3	4
Numbers attending	20	18	18	20

b Work out the number of bottles of milk required for that day.

c The dairy send Banjo school 84 bottles of milk per day.
Discuss whether or not you think this is a suitable number.

* **18** Class 5A take six maths tests every year, each one out of 100. Meena has a mean score of 64 marks per test for the first five tests of the year. Her parents have promised her a bicycle if she can achieve a mean score of 70. What mark would she have to get in the sixth test to achieve this mean score?

AO3

19 A small factory pays salaries to 8 workers, a manager and an owner.
The salaries they earn are shown in the table.

AO3

	Salary
Workers	£10 000
Manager	£40 000
Owner	£180 000

Depending on the average you use, the average wage of people in the factory is vastly different.
If you were negotiating for a higher salary for the workers, which average would you use?
If you were negotiating for the management to keep the salaries low, which average would you use?
Explain your answers.

20 The hourly wages, in pounds, of the employees in a factory were recorded.
The results are shown in the frequency table.

C

Hourly wage £s	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21
Frequency	5	20	20	10	5

a Write down the modal class of these data.

b Find the class interval that contains the median hourly wage.

c Estimate the mean hourly wage.

C

21 Oliver measured the heights (h), in cm, of the leek plants in his garden.
Here are his results.

Class interval	Frequency
$25 \leqslant h < 27$	5
$27 \leqslant h < 29$	10
$29 \leqslant h < 31$	13
$31 \leqslant h < 33$	15
$33 \leqslant h < 35$	7

ResultsPlus
Exam Question Report

91% of students scored poorly on this question because they did not use the midpoint of the range to find the mean of grouped data.

a Write down the modal class.

b Find the class into which the median height falls.

c Work out an estimate for the mean height of the leeks.

22 Josh asked 30 students how many minutes they each took to get to school.
The table shows some information about his results.
Work out an estimate for the mean number of minutes taken by the 30 students.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	6
$10 < t \leqslant 20$	11
$20 < t \leqslant 30$	8
$30 < t \leqslant 40$	5

Nov 2008

23 Vanessa made 80 phone calls last month.
The table gives information about the length of the calls.

Length of call (t minutes)	Frequency	
$0 < t \leqslant 10$	20	
$10 < t \leqslant 20$	32	
$20 < t \leqslant 30$	14	
$30 < t \leqslant 40$	9	
$40 < t \leqslant 50$	5	

ResultsPlus
Exam Question Report

86% of students answered this question poorly.

Work out an estimate for the mean length of the calls.

March 2008

24 80 people work in Joe's factory.
The table shows some information about the annual pay of these 80 workers.

Annual pay (£x)	Number of workers
$10\,000 < x \leqslant 14\,000$	32
$14\,000 < x \leqslant 16\,000$	24
$16\,000 < x \leqslant 18\,000$	16
$18\,000 < x \leqslant 20\,000$	6
$20\,000 < x \leqslant 40\,000$	2

a Write down the modal class interval.

b Find the class interval that contains the median.

June 2007