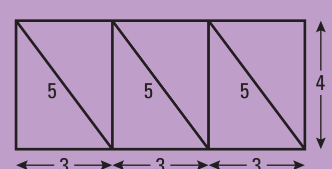
12 PYTHAGORAS' THEOREM

The diagram shows the dimensions of the Parthenon Temple in Athens. By using right-angled triangles as the basis for the construction, Pythagoras' theorem could be used to check that the right angles were accurately constructed. This is an example of maths being used in architecture that dates back to 447 BCE.



Objectives

In this chapter you will:

- understand and use Pythagoras' theorem.

Before you start

You need to know:

- how to square numbers using a calculator
- how to find the square root of numbers using a calculator
- how to work out 4^2, 3.5^2, $4^2 + 6^2$
- how to work out $\sqrt{49}$, $\sqrt{12.25}$.

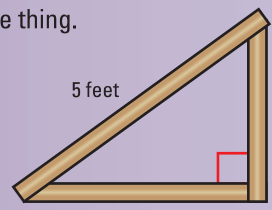
12.1 Finding the length of the hypotenuse of a right-angled triangle

Objectives

- You understand Pythagoras' theorem.
- You can use Pythagoras' theorem to find the hypotenuse.

Why do this?

Three thousand years ago the Egyptians and Babylonians used knotted rope to make a 90° angle using a 3, 4, 5 triangle. They used the right angle in the triangle to make their buildings have square corners. These days, builders sometimes use pieces of wood with length 3 feet, 4 feet and 5 feet to do the same thing.



Get Ready

1. Find the value of

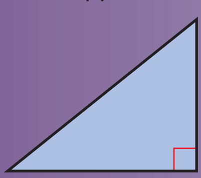
a 4^2 **b** 2.3^2 **c** $7^2 + 24^2$

2. Find the value of

a $\sqrt{25}$ **b** $\sqrt{169}$ **c** $\sqrt{0.25}$

3. Make a copy of these right-angled triangles and mark the hypotenuse (longest side).

a



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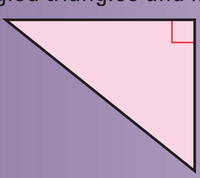

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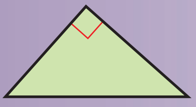

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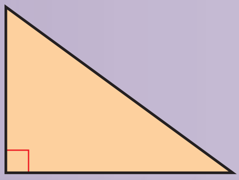
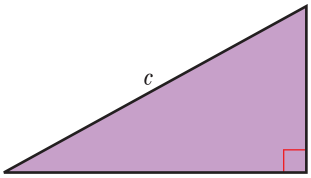


Key Points

- For a right-angled triangle

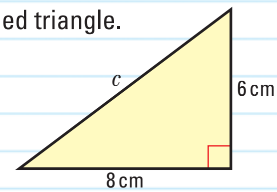
  $c^2 = a^2 + b^2$ or $a^2 + b^2 = c^2$



- To find the length of the **hypotenuse** (c):

  square each of the other sides (a) and (b), add the squares and then square root the sum.

Example 1 Find the length of the hypotenuse in this right-angled triangle.



$c^2 = a^2 + b^2$ — Write down the statement.

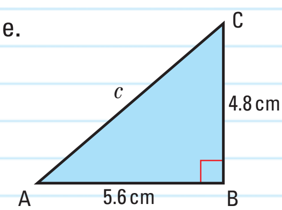
$c^2 = 6^2 + 8^2$ — Put in the values 6 and 8.

$c^2 = 36 + 64$ — Square the 6 and 8. Use the $\boxed{x^2}$ key.

$c^2 = 100$ — Add 36 and 64.

$c = \sqrt{100} = 10$ cm — Square root the 100 to give 10 as the answer. Use the $\boxed{\sqrt{x}}$ key.

Example 2 Find the length of AC in this right-angled triangle.



AC is the hypotenuse.

$c^2 = a^2 + b^2$ — Write down the statement.

$c^2 = 5.6^2 + 4.8^2$ — Put in the values 5.6 and 4.8.

$c^2 = 31.36 + 23.04$ — Square the 5.6 and 4.8. Use the $\boxed{x^2}$ key.

$c^2 = 54.4$ — Add 31.36 and 23.04.

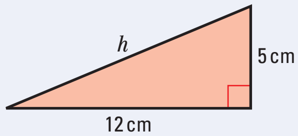
$c = \sqrt{54.4}$
$c = 7.375635566$ cm — Square root the 54.4 to give 7.38 as the answer. Use the $\boxed{\sqrt{x}}$ key.
$c = 7.38$ cm (written to 3 significant figures)

Exercise 12A

Questions in this chapter are targeted at the grades indicated.

1 Calculate the length of the hypotenuse in these right-angled triangles.

a



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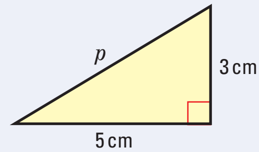

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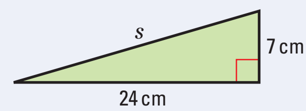

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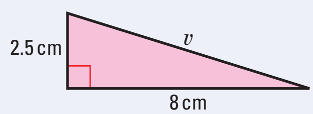

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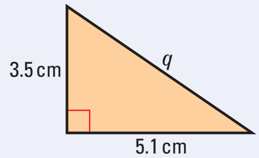

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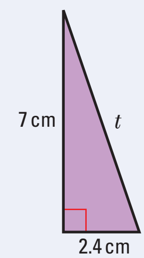

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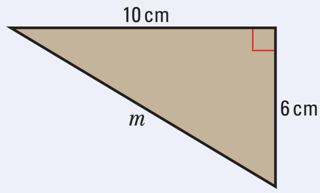

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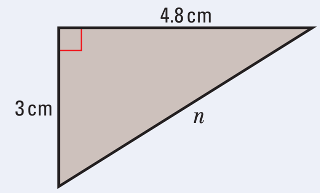

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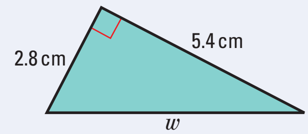
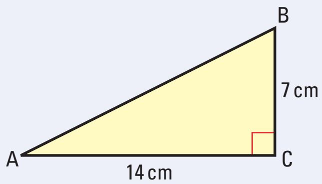


2 Calculate the length of the missing side in these right-angled triangles.

a



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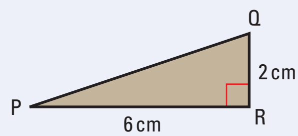

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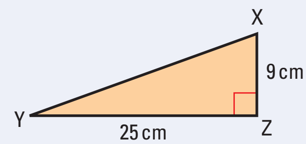

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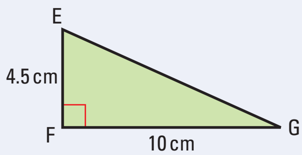

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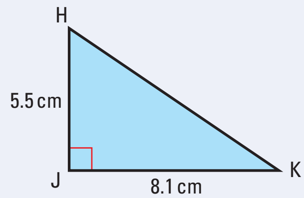

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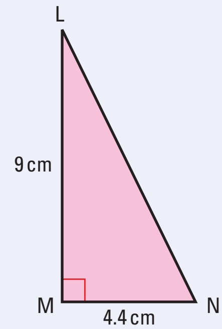

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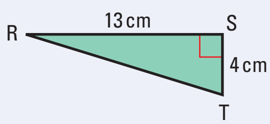

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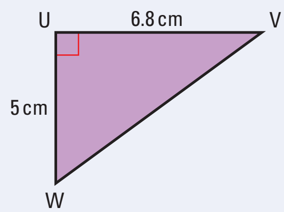

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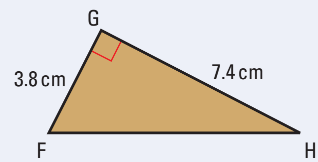
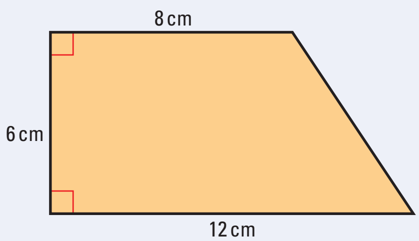


AO2 AO3

3 Find the perimeter of these trapeziums.
Give your answer correct to 3 significant figures.

a



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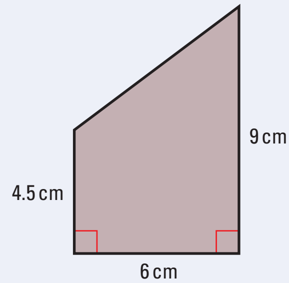
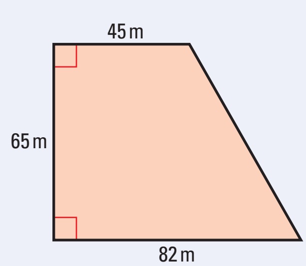


AO2 AO3

4 A farmer wants to fence a field.
The field is in the shape of a trapezium.
Fencing costs £5.50 per metre.
Find the cost of fencing the field.



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12.2 Finding the length of one of the shorter sides of a right-angled triangle

Objective

- You can use Pythagoras' theorem to find one of the shorter sides of a right-angled triangle.

Get Ready

1. Find the value of

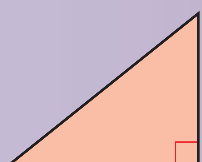
a $5^2 - 4^2$ **b** $5^2 - 3^2$ **c** $5.6^2 - 2.3^2$ **d** $13^2 - 5^2$ **e** $25^2 - 7^2$

2. Find the value of

a $\sqrt{9}$ **b** $\sqrt{16}$ **c** $\sqrt{169}$ **d** $\sqrt{6.25}$ **e** $\sqrt{576}$

3. Make a copy of these right-angled triangles and mark the two shorter sides.

a



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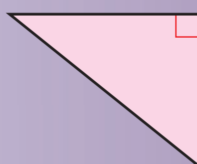

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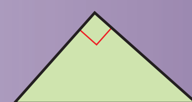

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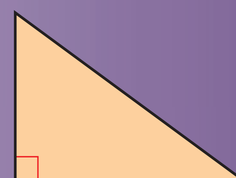
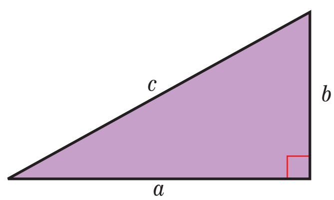


Key Points

- For a right-angled triangle

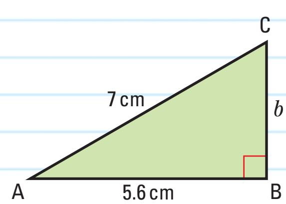
$$c^2 = a^2 + b^2 \quad \text{or} \quad a^2 + b^2 = c^2$$
$$\text{so} \quad a^2 = c^2 - b^2 \quad \text{or} \quad b^2 = c^2 - a^2$$



- To find the length of one of the shorter sides (a or b):
square each of the other sides (c) and (a or b), subtract the squares and then square root the sum.

Example 3

Find the length of BC in this right-angled triangle.



AC is the hypotenuse.
BC is one of the shorter sides.

$c^2 = a^2 + b^2$ ← Write down the statement.

$7^2 = 5.6^2 + b^2$ ← Put in the values 5.6 and 7.

$b^2 = 7^2 - 5.6^2$ ← Square the 5.6 and 7. Use the $\boxed{x^2}$ key.

$b^2 = 49 - 31.36$ ← Subtract 31.36 from 49.

$b^2 = 17.64$
$b = \sqrt{17.64}$ ← Square root the 17.64 to give 4.2 as the answer. Use the $\boxed{\sqrt{x}}$ key.
$b = 4.2$ cm

Exercise 12B

C

1 Calculate the length of the shorter side marked with a letter in these right-angled triangles.

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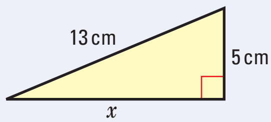

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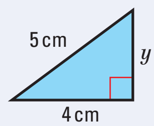

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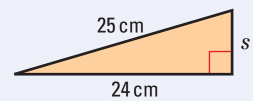

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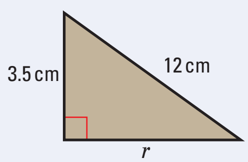

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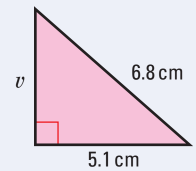

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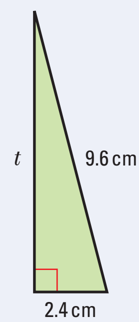

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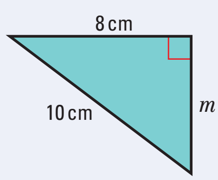

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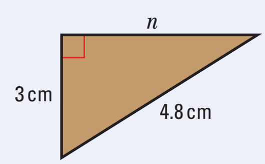

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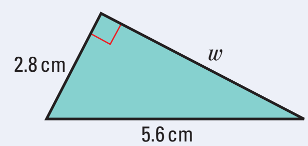

2 Calculate the length of the missing side in these right-angled triangles.

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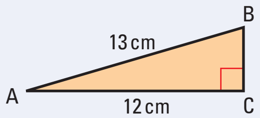

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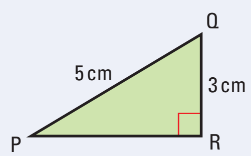

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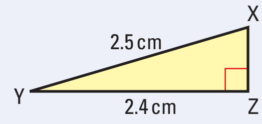

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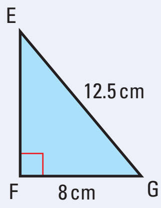

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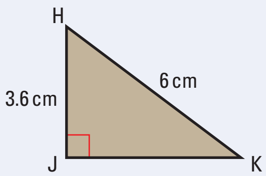

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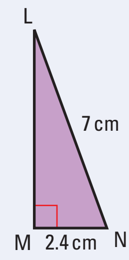

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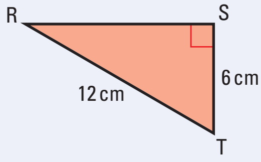

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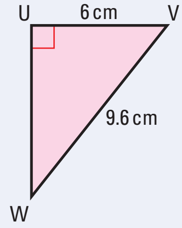

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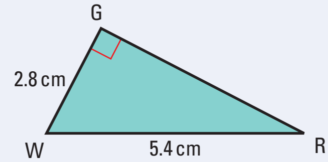

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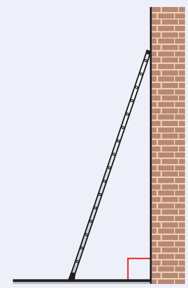
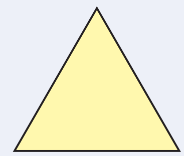
3 A 7.5 metre-long ladder leans against a vertical wall.
The foot of the ladder is 1.5 metres from the base of the wall.
How far up the wall does the ladder reach?



4 A farmer has a field in the shape of an equilateral triangle.
Each side of the field is of length 500 metres.
He sells the field at 50p per square metre.
How much money does he sell the field for?

A03



12.3 Checking to see if a triangle is right-angled or not

Objective

- If you know the lengths of all the sides of a triangle, you can use Pythagoras' theorem to show whether the triangle is right-angled or not.

Why do this?

In buildings, engineers need to check whether an angle is a right angle or not to ensure that the walls, doors or windows are straight.

Get Ready

Which of these are correct?

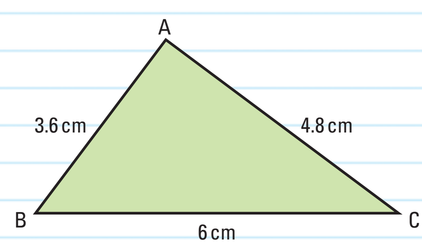
1. $5.4^2 + 3.6^2 = 42.12$ **2.** $3.5^2 + 3.1^2 = 19.86$ **3.** $4.8^2 + 3.2^2 = 33.28$

Key Point

- If the length of the longest side of a triangle squared is equal to the sum of the squares of the other two sides then the triangle has a right angle.

Example 4

Prove that the triangle ABC is right-angled.



$6^2 = 36$

Square the longest side. $6^2 = 36$

$3.6^2 + 4.8^2$
$= 12.96 + 23.04$
$= 36$

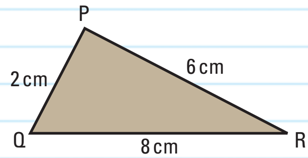
Square the other two sides.
$3.6^2 = 12.96$
$4.8^2 = 23.04$
Add them to get 36.

Since $6^2 = 3.6^2 + 4.8^2$ ABC must be a right-angled triangle.

Both are equal so this proves that the triangle is right-angled because the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Example 5

Josie says that triangle PQR is right-angled because $8^2 = (6 + 2)^2$.
Josie is wrong. Explain why.



Square the longest side.
$8^2 = 64$

$8^2 = 64$
$6^2 + 2^2$
$= 36 + 4$
$= 40$

Square the other two sides.
$6^2 = 36$
$2^2 = 4$

Since $8^2 \neq 6^2 + 2^2$ ABC is not a right-angled triangle.

64 and 40 are not equal so triangle is not right-angled.

Exercise 12C

C

AO3 **1** Check if these triangles have right angles.

- **a** Triangle ABC where AB = 5 cm, BC = 12 cm and CA = 13 cm
- **b** Triangle PQR where PQ = 5 cm, QR = 10 cm and RP = 12 cm
- **c** Triangle XYZ where XY = 4 cm, YZ = 6 cm and ZX = 7 cm
- **d** Triangle FGH where FG = 3.5 cm, GH = 4.5 cm and HF = 5.5 cm
- **e** Triangle RST where RS = 6 cm, ST = 8 cm and TR = 10 cm
- **f** Triangle JKL where JK = 5 cm, KL = 5 cm and LJ = 7 cm

AO3 **2** Jenny says that triangle PQR is right-angled because $12^2 = (6 + 6)^2$.
Jenny is wrong. Explain why.

AO3 **3** Jason says that triangle PQR is not right-angled because $10^2 \neq (6 + 8)^2$.
Jason is wrong. Explain why.

AO3 **4** A acute-angled triangle has three acute angles.
An obtuse-angled triangle has two acute angles and one obtuse angle.
Investigate whether these triangles are right-angled, acute-angled or obtuse-angled.

- **a** Triangle ABC where AB = 5 cm, BC = 10 cm and CA = 10 cm
- **b** Triangle PQR where PQ = 5 cm, QR = 8 cm and RP = 12 cm
- **c** Triangle XYZ where XY = 4 cm, YZ = 6 cm and ZX = 7 cm
- **d** Triangle FGH where FG = 3.5 cm, GH = 4.5 cm and HF = 6.5 cm
- **e** Triangle RST where RS = 7.5 cm, ST = 10 cm and TR = 12.5 cm
- **f** Triangle JKL where JK = 5 cm, KL = 5 cm and LJ = 5 cm

12.4 Finding the length of a line segment

Objective

- You can use Pythagoras' theorem to find the length of the line segment between two coordinates.

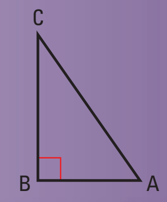
Why do this?

Sat navs use Pythagoras' theorem to calculate the shortest distance between two places.

Get Ready

Find the lengths of the missing sides in these right-angled triangles, ABC.

1. AC where AB = 3.2 cm and BC = 4.3 cm
2. BC where AC = 7.5 cm and AB = 2.9 cm
3. AB where AC = 9.8 cm and BC = 1.8 cm

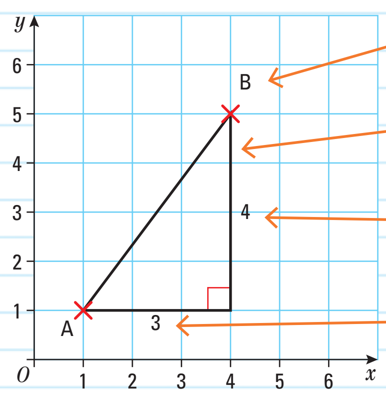


Key Points

- To find the distance between two points on a coordinate grid:
  - subtract the x-coordinates and square
  - subtract the y-coordinates and square
  - add the results
  - square root the answer.

Example 6

A has coordinates (1, 1). B has coordinates (4, 5).
Find the length of the line segment AB.



A has coordinates (1, 1)
B has coordinates (4, 5)

Make a right-angled triangle by drawing in the vertical and horizontal.

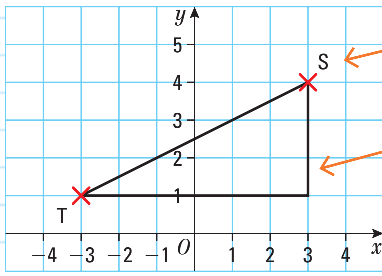
To find the vertical distance you subtract the y-coordinates $5 - 1 = 4$

To find the horizontal distance you subtract the x-coordinates $4 - 1 = 3$

Square $3^2 + 4^2$
Add $9 + 16$
Square root $\sqrt{25} = 5$

AB = 5 units.

Example 7 Find the length of the line segment ST.



S has coordinates (3, 4)
T has coordinates (−3, 1)

Make a right-angled triangle by drawing in the vertical and horizontal.

To find the horizontal distance you subtract the x-coordinates $3 - -3 = 3 + 3 = 6$

To find the vertical distance you subtract the y-coordinates $4 - 1 = 3$

Square $6^2 + 3^2$
Add $36 + 9$
Square root $\sqrt{45} = 6.708$

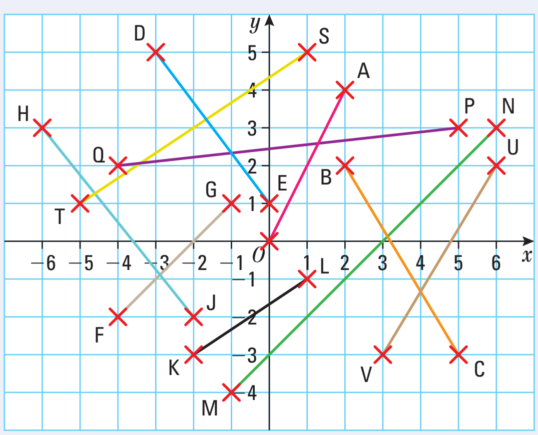
ST = 6.71 units (to 2 d.p.).

Exercise 12D

C

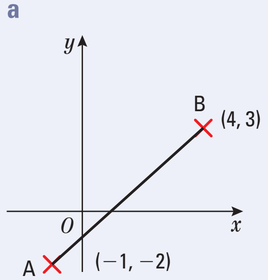
1 Work out the length of each of the line segments shown on the grid.

a OA	**b** BC	**c** DE	**d** FG	**e** HJ
f KL	**g** MN	**h** PQ	**i** ST	**j** UV

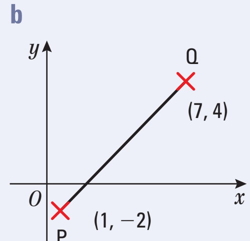


2 Work out the lengths of each of these line segments.

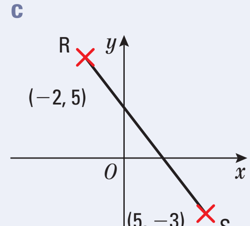
a



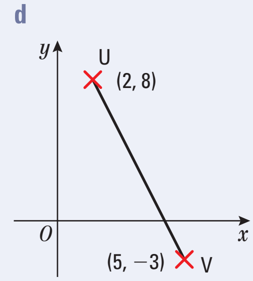
b



c



d

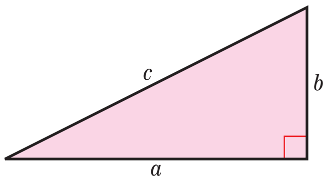


3 Work out the lengths of each of these line segments.

- **a** AB when A is $(-1, -1)$ and B is $(9, 9)$
- **b** PQ when P is $(2, -4)$ and Q is $(-6, 9)$
- **c** ST when S is $(5, -8)$ and T is $(-2, 1)$
- **d** CD when C is $(1, 7)$ and D is $(-7, 2)$
- **e** UV when U is $(-2, 3)$ and V is $(6, -8)$
- **f** GH when G is $(-2, -6)$ and H is $(7, 3)$

Chapter review

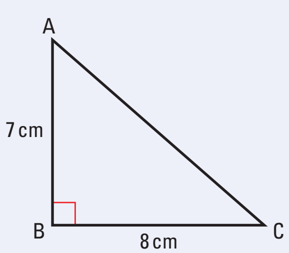
- For a right-angled triangle:
  $c^2 = a^2 + b^2$ or $a^2 + b^2 = c^2$



- To find the length of the **hypotenuse** (c):
  square each of the other sides (a) and (b), add the squares and then square root the sum.
- To find the length of one of the shorter sides (a or b):
  square each of the other sides (c) and (a or b), subtract the squares and then square root the sum.
- If the length of the longest side of a triangle squared is equal to the sum of the squares of the other two sides then the triangle has a right angle.
- To find the distance between two points on a coordinate grid:
  - subtract the x-coordinates and square
  - subtract the y-coordinates and square
  - add the results
  - square root the answer.

Review exercise

1 ABC is a right-angled triangle.



AB = 7 cm,
BC = 8 cm.
Work out the length of AC.
Give your answer correct to 2 decimal places.

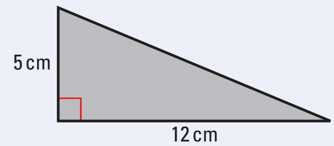
June 2008

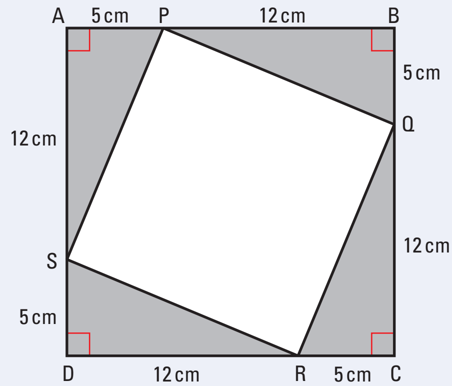
ResultsPlus
Exam Question Report

83% of students answered this question poorly. The most common wrong answer was 15, obtained by either adding the two sides $(7 + 8)$ or subtracting $(8^2 - 7^2)$.

C A02 A03

2 **a** Work out the area of the triangle.

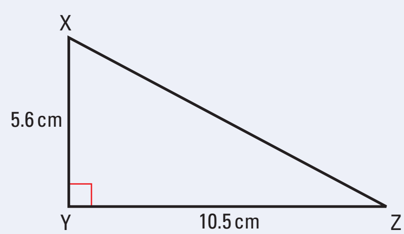




4 copies of the triangle and the quadrilateral PQRS are used to make the square ABCD.

b Work out the area of the quadrilateral PQRS. *Nov 2007*

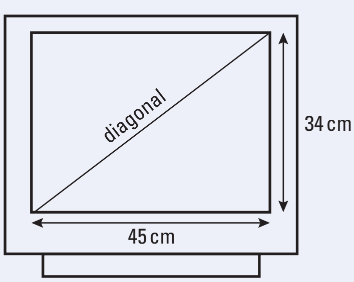
3 In the triangle XYZ
XY = 5.6 cm
YZ = 10.5 cm
angle XYZ = 90°
Work out the length of XZ.



Nov 2007

A02

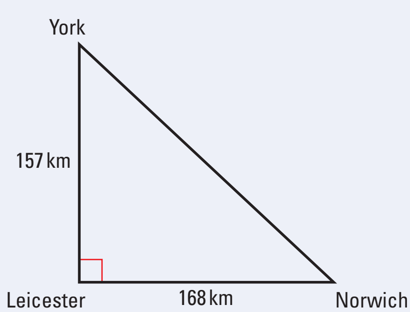
4



A rectangular television screen has a width of 45 cm and a height of 34 cm.
Work out the length of the diagonal of the screen.
Give your answer correct to the nearest centimetre. *June 2007*

A02

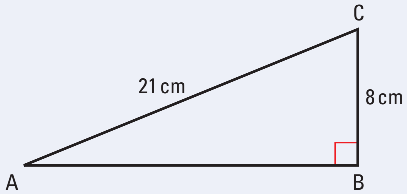
5 The diagram shows three cities.
Norwich is 168 km due east of Leicester.
York is 157 km due north of Leicester.
Calculate the distance between Norwich and York.
Give your answer correct to the nearest kilometre.



Nov 2006

C

6



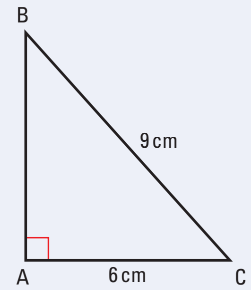
In triangle ABC
angle ABC = 90°
BC = 8 cm
AC = 21 cm
Work out the length of AB.
Give your answer correct to 3 significant figures.

ResultsPlus
Exam Question Report

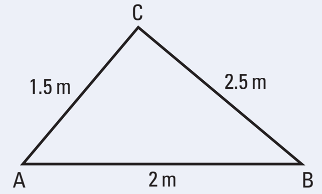
82% of students answered this question poorly. The majority of students did not recognise that they were required to use Pythagoras' theorem.

March 2007

7 ABC is a right-angled triangle.
AC = 6 cm
BC = 9 cm
Work out the length of AB.
Give your answer correct to 3 significant figures.



* **8** Prove that triangle ABC is a right-angled triangle. A03



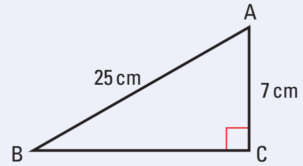
9 A has coordinates (1, 7). B has coordinates (6, 8).
Find the length of the line segment AB.
Give your answer correct to 2 decimal places.

10 P has coordinates (9, 3). Q has coordinates (2, 0).
Find the length of the line segment PQ.
Give your answer correct to 1 decimal place.

June 2006

11 Paul flies his helicopter from Ashwell to Birton. A03
He flies due west from Ashwell for 4.8 km. He then flies due south for 7.4 km to Birton.
Calculate the shortest distance between Ashwell and Birton.

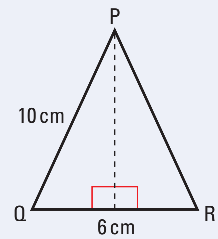
12 ABC is a right-angled triangle. A02
Calculate the area of the triangle ABC.



C

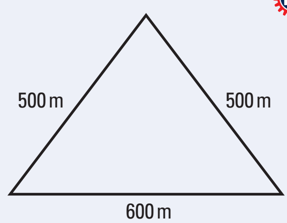
AO2

13 PQR is an isosceles triangle with PQ = PR.
PQ = PR = 10 cm
QR = 6 cm
Calculate the area of triangle PQR.



AO3

14 A farmer has a field in the shape of an isosceles triangle.
The sides of the field have lengths 500 metres, 500 metres and 600 metres.
He sells the field at 45p per square metre.
For how much money does he sell the field?



15 A is the point with coordinates (2, 5).
B is the point with coordinates (8, 13).
Calculate the length AB.

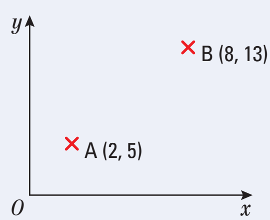


Diagram **NOT** accurately drawn

2006