Dynamics

Dynamics is the study of moving objects. In previous chapters we have considered the sum total of forces on a body (resultant) and our next consideration is the movement that these forces bring about. Forces of friction, thrust, gravity and tension will be considered and the subsequent motion will be measured by the application of constant acceleration equations and the equation of motion. As in most mechanics questions a certain amount of modeling will have to be used in our working.

Our first area of study of moving objects will involve the application of Newton's laws.

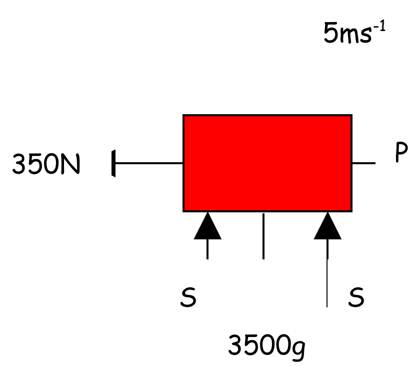
Newton's Laws

Newton's First Law

A body will remain at rest, or will continue to move with constant velocity, unless external forces force it to do otherwise.
A change in state of motion of a body is caused by a force. The unit of force is the Newton, (N).

Example 1

A body of mass 3500Kg moves horizontally at a constant speed of $5ms^{-1}$ subject to the forces shown. Find P and S.



There is no vertical motion therefore:

$$2S = 3500g$$

$$S = 1750g$$

The horizontal acceleration is zero, therefore:

$$P = 350N$$

<u>Newton's Second Law</u>

The force F applied to a particle is proportional to the product of mass of the particle and the acceleration produced.
A force of 1N produces an acceleration of $1ms^{-2}$ in a body of mass 1kg. Newton's Second Law is summarized by the equation:

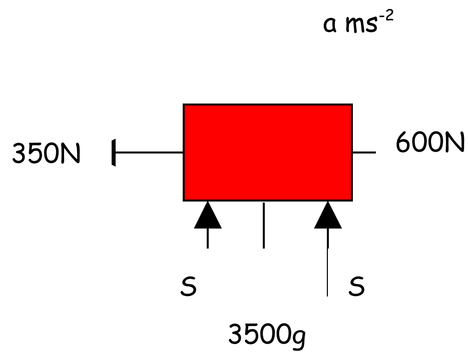
$$\mathbf{F = ma}$$

this is often termed **the equation of motion**.

It is vitally important to realise that F is the overall resultant and not Friction (F_R).

Example 2

If the object in Example 1 is slightly modified to take account of the fact that there is a pushing force 600N, calculate the acceleration.



The resultant of the two horizontal forces is 250N pushing the object to the right. So by setting up an equation of motion:

$$F = ma$$

$$250 = 3500 \times a$$

$$a = 0.07 ms^{-2}$$

Questions A

1 Find the resultant force that will bring about an acceleration of 4 ms^{-2} for a particle of mass 3.75kg

2 A particle of mass 4.5kg is acted upon by forces $(6\mathbf{i} + 3\mathbf{j})$N and $(-2\mathbf{i} + 7\mathbf{j})$N. Calculate the acceleration of the particle in vector form.

3 A toy train of mass 1.25kg is pulled across a horizontal floor by a horizontal string. The tension in the string is 1.2N. Calculate the acceleration and the distance traveled by the train in the first 4 seconds.

4 A car of mass 750Kg experiences a resistive force of RN while being brought to rest in 9 seconds form a speed of 18ms^{-1}. Calculate the magnitude of the force.

5 A car of mass 900Kg is under constant resistance to motion of 500N. Find the value of the engine force required to bring about an acceleration of 1.25ms^{-2}.

6 A train of mass 4000kg produces a driving force of 3000N. If the train experiences constant resistance to motion of 1200N calculate the acceleration.

7 A stone of mass 1.25kg is dropped into a viscous liquid and fall vertically through it with an acceleration of 4.8ms^{-2}. Find the resistive force acting against the stone.

Vertical Motion

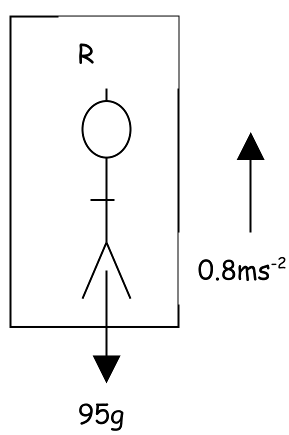
If a particle is falling in the earth's atmosphere then it will accelerate at $9.8ms^{-2}$. By Newton's first law the particle must be experiencing a force and the only force present is the weight, so by considering the equation of motion:

$$F = ma$$

$$\text{Weight} = mg$$

<u>Example 3</u>

A man of mass 95kg is traveling up in a lift. Given that the acceleration of the lift is $0.8ms^{-2}$, find the force exerted on the man by the floor of the lift.
Calculate the same force when the lift is descending with the same acceleration.



Taking up to be positive and setting up an equation of motion:

$$F = ma$$

$$R - 95g = 95a$$

$$R - 931 = 95 \times 0.8$$

$$R = 1007\text{N}$$

When the lift is traveling in the opposite direction let **down** be positive so the equation of motion becomes:

$$95g - R = 95a$$

$$931 - R = 95 \times 0.8$$

$$R = 855\text{N}$$

It is worth pointing out that the reaction from the floor is less than the weight of the man when the lift is descending (and vice versa).

The next example introduces constant acceleration equations into the problem.

Example 4

A ball of mass 0.9 kg falls from a height of 22m above horizontal ground. The ball reaches the ground after t seconds. The ball sinks into the ground a distance of 1.9cm before coming to rest. The ground is assumed to exert a constant resistive force of magnitude F newtons. Find:

a) the value of t to 3 sig fig;
b) the value of F to 3 sig fig.

a) Using constant acceleration equations to find t:

$s = 22, \quad a = 9.8, \quad u = 0, \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$22 = 0 + 0.5 \times 9.8 \times t^2$$

$$t = 2.12 \text{sec}$$

b) The ball is brought to rest in 1.9cm so by using constant acceleration equations we can find the deceleration and hence the force:

Firstly we need to find the speed with which the ball hits the surface

$u = 0, \quad v = ?, \quad t = 2.12, \quad a = 9.8$

$$v = u + at$$

$$v = 9.8 \times 2.12$$

$$v = 20.776 \text{ms}^{-1}$$

And now for the deceleration:

$s = 0.019,\quad a = ?,\qquad v = 0,\qquad u = 20.776$

$$v^2 = u^2 + 2as$$

$$0 = 20.776^2 + 2 \times a \times 0.019$$

$$a = -11359ms^{-2}$$

And finally using an equation of motion for the particle:

$$F = ma$$

$$F = 0.9 \times 11359$$

$$F = 10.2KN$$

Note that we were asked for the magnitude hence the positive answer. How realistic is this answer? Ask a physics teacher for some other examples.

Example 5

A ball of mass 5kg falls from a height of 6m into a jar containing a viscous liquid. The upward force exerted by the liquid is of magnitude 75N. How far will the ball sink into the liquid? Calculate the total time that the ball is in motion.

Firstly we need to calculate the speed with which the ball hits the liquid.

Assuming that down is positive:

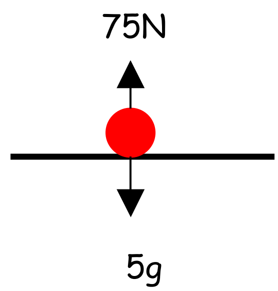
$s = 6$, $a = 9.8$, $u = 0$, $v = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.8 \times 6$$

$$v = 10.84\text{ms}^{-1}$$

Secondly we can set up an equation of motion for the ball to work out the deceleration.



$$F = ma$$

$$5g - 75 = 5a$$

$$a = -5.2\text{ms}^{-2}$$

Thirdly we need to calculate the distance that the ball travels through the liquid before coming to rest.

$u = 10.84$, $v = 0$, $a = -5.2$ $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 117.6 - 2 \times 5.2 \times s$$

$$s = 11.31\text{m}$$

Finally the total time taken by the ball in motion must be done in two parts seeing as before the ball hits the liquid it has an acceleration of $9.8ms^{-2}$ whereas, whilst falling through the liquid, it has an acceleration of $-5.2ms^{-2}$.

Time to meet the surface of the liquid:

$a = 9.8, \quad u = 0, \quad v = 10.84, \quad t = ?$

$$v = u + at$$

$$10.84 = 0 + 9.8t$$

$$t = 1.1 \text{ sec}$$

Time to come to rest:

$a = -5.2, \quad u = 10.84, \quad v = 0, \quad t = ?$

$$v = u + at$$

$$0 = 10.84 - 5.2t$$

$$t = 2.1 \text{ sec}$$

Therefore the total time in motion is 3.3 sec. The liquid is rather viscous, how realistic is the resistance force? What about a ball falling in to a jar of syrup?

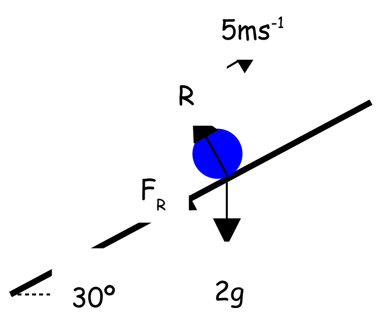
The next example introduces friction on a slope. The questions are beginning to get more challenging but a good diagram is always the best place to start. Examiners regularly report that the most successful candidates in mechanics M1 and M2 always draw diagrams.

Example 6

A ball of mass 2kg is projected up a line of greatest slope inclined at an angle of 30° to the horizontal. The coefficient of friction between the plane and the ball is 0.4. The initial speed of the ball is 5 ms^{-1}. Find:

a) the frictional force acting whilst the ball moves up the plane.

b) the distance moved up the plane by the ball before it comes to instantaneous rest.



a) Since the ball is moving then Friction must be at its maximum value.

Resolving perpendicular to the plane gives:

$$R = 2g \cos 30°$$

$$R = 16.97N$$

Using $F_R = \mu R$:

$$F_R = 0.4 \times 16.97$$

$$F_R = 6.79N$$

b) The frictional force and the weight component of the ball are trying to slow the ball down. By setting up an equation of motion for the ball we can calculate the deceleration.
Assuming that uphill is positive:

$$F = ma$$

$$-F_R - \text{weight comp down the plane} = ma$$

$$-6.79 - 2g\cos 30^\circ = 2a$$

$$a = -8.295ms^{-2}$$

At the point the ball comes to instantaneous rest it will have zero velocity and we can use constant acceleration equations to calculate the distance traveled.

$u = 5$, $v = 0$, $a = -8.295$, $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 25 - 2 \times 8.295 \times s$$

$$s = 1.51m$$

Motion of Two Connected Particles

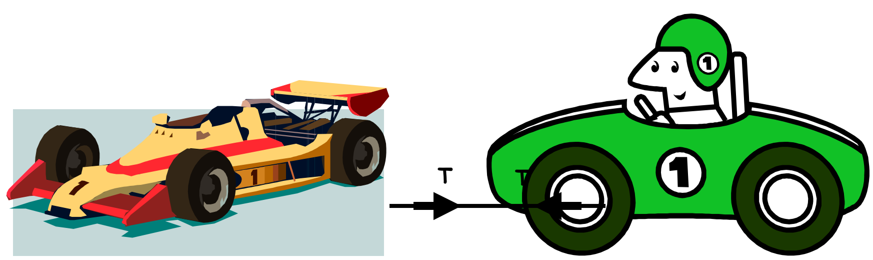
Newton's Third Law

Before we can consider the motion of two connected particles we need to discuss **Newton's Third Law**. This law states that action and reaction are equal and opposite.

If two bodies A and B are in contact and exert forces on each other, then the force exerted by A on B is equal in magnitude and opposite in direction to the force exerted by B on A.

This principle will be applied to tow truck problems and pulleys to name but two.

Consider the situation below where the cartoon car is towing a racing car.



The racing car is pulled forward by tension in the tow bar. The racing car will exert an equal but opposite force on the car. If the car is slowing down and there are no breaks on the racing car then some force must be acting in the opposite direction to the direction of motion of the two cars. In this case the tow bar will exert a thrust on both cars (arrows change directions).

Example 7

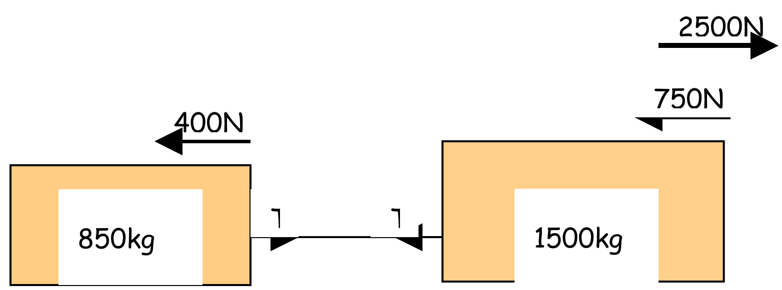
The AA man is towing a car along a straight horizontal road. The truck has a mass of 1500kg and the car has a mass of 850kg. The truck is connected to the car by a bar which is to be modelled as a light inextensible string. The truck's engine produces a constant driving force of 2500N. The resistance to motion of the truck and the car are constant and of magnitude 750N and 400N respectively. Find:

a) the acceleration of the truck and the car;

b) the tension in the rope.

When the truck and the car are traveling at $22ms^{-1}$ the tow bar breaks. If the magnitude of the resistance to motion of the truck remains at 750N calculate:

c) the time difference in achieving a speed of $30ms^{-1}$ with and without the car in tow.



a) Setting up equations of motion for the car and truck separately gives:

Car $T - 400 = 850a$ Truck $2500 - 750 - T = 1500a$

Adding the two equations gives:

$$1350 = 2350a$$
$$a = 0.574ms^{-2}$$

b) Finding the tension:

Substituting the value into the car's equation of motion gives:

$$T - 400 = 850 \times 0.574$$

$$T = 888N$$

c) If we assume that the bar doesn't break then the time required to reach $30ms^{-1}$ is calculated by using the constant acceleration equations.

$u = 22$, $v = 30$, $a = 0.574$, $t = ?$

$$v = u + at$$

$$30 = 22 + 0.574t$$

$$t = 13.9 \text{ sec}$$

At the point that the tow bar breaks, the tension in the bar is no longer acting against the truck. Therefore the equation of motion of the truck becomes:

$$2500 - 750 = 1500a$$

$$a = 1.167ms^{-2}$$

$u = 22$, $v = 30$, $a = 1.167$, $t = ?$

$$v = u + at$$

$$30 = 22 + 1.167t$$

$$t = 6.9 \text{ sec}$$

Therefore there is a time difference of 7 seconds.

Pulleys

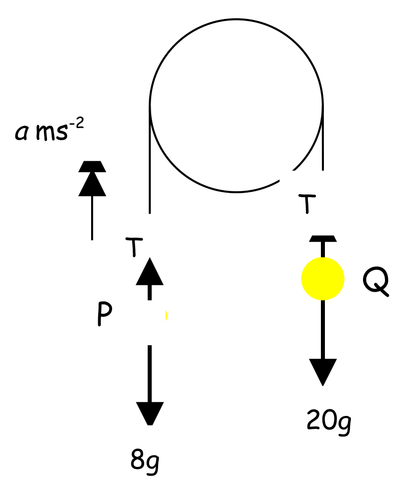
In all questions in M1 the pulley system will be smooth. This implies that the motion of the particles at the end of the string are unaffected by the string passing over the pulley. A further assumption is that the string is light and inextensible. These modelling assumptions make the problem simpler but we can still get pretty realistic answers. If two particles are connected by a string where the string passes over a smooth pulley, then we can assume that the particles will have equal accelerations but in opposite directions.

<u>Example 8</u>

Two particles P and Q are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest. Find:

a) the magnitude of the acceleration;

b) find the tension in the string.



To start the problem set up an equation of motion for particle P. The particle will accelerate upwards hence:

$$F = ma$$

$$T - 8g = 8a \qquad (1)$$

When considering particle Q, its weight will cause it to accelerate down hence the equation of motion is:

$$F = ma$$

$$20g - T = 20a \qquad (2)$$

By adding the two equations the tension will be eliminated:

$$12g = 28a$$

$$a = 4.2\text{ms}^{-2}$$

Substituting the value of the acceleration into equation (1) will give the tension.

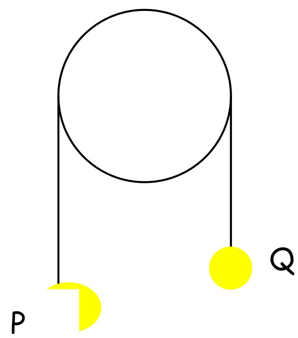
$$T - 8g = 8 \times 4.2$$

$$T = 112\text{N}$$

The following example is more algebraic but this does not mean that it is more complicated.

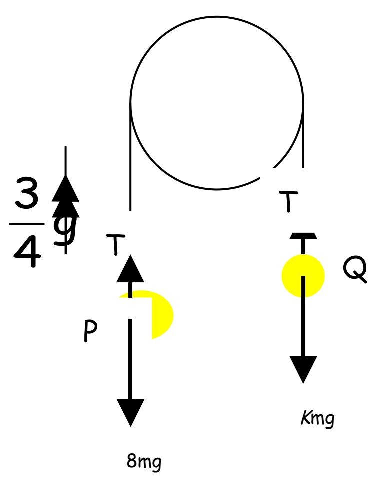
Example 9

Two particles P and Q of masses have masses 8m and *K*m, where *K* > 8. They are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut and the hanging parts of the string vertical, as shown below. Initially P has an acceleration of magnitude of $\frac{3}{4}g$.



a) Find, in terms of m and g, the tension, T, in the string.
b) Find the value of *K*.

a) Adding forces to the system:



Setting up an equation of motion for particle P:

$$F = ma$$

$$T - 8mg = 8m \times \frac{3}{4}g$$

$$T = 14mg \quad (1)$$

b) Considering particle Q:

$$Kmg - T = km \times \frac{3}{4}g$$

Using (1):

$$\frac{1}{4}Kmg = 14mg$$

$$K = 56$$

The next problem involves a pulley system where one of the particles is being dragged across a horizontal table as the other particle is falling. The problem will be made more complex when the horizontal table is considered to be rough. The questions are increasing in complexity but the same basic principles apply.

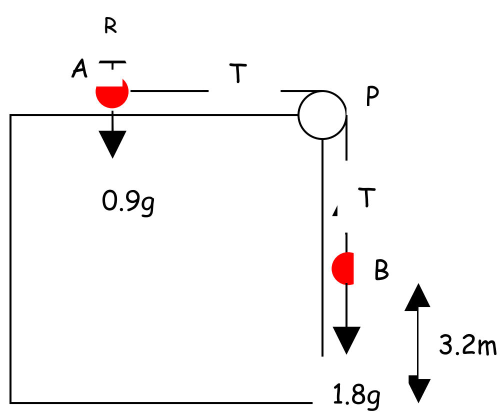
Example 10

A particle A, of mass 0.9kg, rests on smooth horizontal table and is attached to one end of a light inextensible string. The string passes over a smooth pulley P fixed at the edge of the table. The other end of the string is attached to a particle B of mass 1.8kg which hangs freely below the pulley. The system is released from rest with the string taut and B at a height of 3.2m above the ground. In the subsequent motion A does not reach the pulley before B reaches the ground. Find:

a) the tension in the string before B reaches the ground.

b) the time taken by B to reach the ground.

Then, to make the model more realistic, assume that the coefficient of friction between the particle and the table is 0.3. Using this modification find the time taken by B to reach the ground.



a) Setting up equations of motion for the two particles gives:

A $F = ma$ B $F = ma$

$T = 0.9a$ $1.8g - T = 1.8a$

Adding the two equations gives:

$$1.8g = 2.7a$$

$$a = 6.53ms^{-2}$$

Therefore:

$$T = 0.9 \times 6.53$$

$$T = 5.88N$$

b) The particle B is falling with acceleration $6.53ms^{-2}$. So by using constant acceleration equation we can find the time it takes to reach the floor.

$u = 0$, $a = 6.53$, $s = 3.2$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$3.2 = \frac{1}{2} \times 6.53 \times t^2$$

$$t = 1.01 sec$$

Seeing as the particle is moving friction must be at its maximum value and hence $F_R = \mu R$.
Setting up equations of motion for the two particles with friction included gives:

A $F = ma$

$T - F_R = 0.9a$ (1)

B $F = ma$

$1.8g - T = 1.8a$ (2)

Resolving vertically for A:

$$R = 0.9g$$

Using $\quad F_R = \mu R \quad F_R = 0.27g$

Adding equations (1) and (2) gives:

$$1.8g - 0.27g = 2.7a$$

$$a = 5.553\text{ms}^{-2}$$

The new value of acceleration can now be used to calculate the new time:

$u = 0, \quad a = 5.553, \; s = 3.2, \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

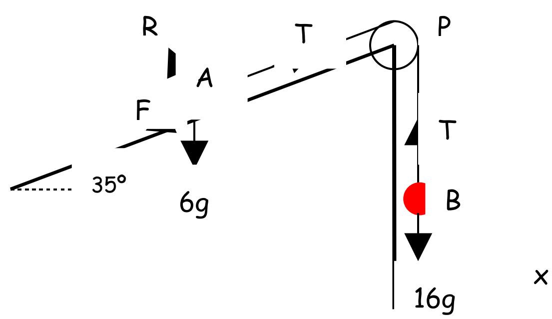
$$3.2 = \frac{1}{2} \times 5.553 \times t^2$$

$$t = 1.07\,sec$$

In the next problem a particle is being pulled up a rough inclined plane by the motion of another particle falling towards a floor. This is very similar to an exam question and would be worth in excess of 10 marks on an M1 paper.

Example 11

A particle, A of mass 6kg, rests on a rough plane inclined at an angle of 35° to the horizontal. The particle is attached to one end of a light inextensible string which lies in a line of greatest slope of the plane and passes over a light smooth pulley P fixed at the top of the plane. The other end of the string is attached to a particle B of mass 16kg. The particles are released from rest with the string taut. The particle B moves down with an acceleration of $\frac{2}{5}g$.



Find:

a) the tension, T, in the string.

b) the coefficient of friction between the plane and A.

a) Setting up equations of motion for A and B gives:

A F = ma B F = ma

$T - 6g \sin 35° - F_R = 6 \times \frac{2}{5}g$ (1) $16g - T = 16 \times \frac{2}{5}g$ (2)

Using (2) to find the tension:

$$T = \frac{48}{5}g = 94.08\text{N}$$

Resolving vertically for A:

$$R = 6g\cos 35° = 48.166$$

Using $\qquad F_R = \mu R \qquad F_R = 48.166\mu$

Therefore equation (1) becomes:

$$94.08 - 33.726 - 48.166\mu = \frac{12}{5}g$$

Rearranging for μ:

$$\mu = \frac{60.35 - \frac{12}{5}g}{48.166}$$

$$\mu = 0.765$$

Extension

Assume that in the example above the particle is 5m above a level surface and that after 0.5sec the string breaks. Calculate the total time that the particle B is in flight and the distance that A moves up the plane before it comes to instantaneous rest (assume that it does not reach the pulley).

This is a rather large value for μ and this obviously accounts for the low value for the acceleration.
What is the resultant force on the pulley?

Momentum and Impulse

Momentum

The momentum of a body of mass m, having a velocity v is mv. The units of momentum are Newton seconds (Ns).

Momentum = mv

The momentum of a body is dependent upon its velocity therefore momentum is a vector quantity. This implies that direction is very important and great care must be taken with signs.

Example 12

Find the momentum of a hockey ball of mass 0.9kg hit at $18ms^{-1}$.

Momentum = mass × velocity

Momentum = 0.9 × 18 = 16.2Ns

Change in Momentum

If a particle experiences a change in velocity then, by definition, its momentum must change. Let the initial velocity be u and the final velocity be v then the change in momentum is given by:

Change in Momentum = mv – mu = m(v - u)

Example 13

If the hockey ball from example 12 hits a wall directly and returns with a velocity of $12ms^{-1}$. Find the change in momentum.

Always draw a diagram.

Taking left to right as positive, therefore u = 18, v = -12

Change in Momentum = m(v - u)

= 0.9 × (-12 – 18)

= -27Ns

The wall in the above example has experienced a force as the hockey ball hits it. This action takes place in a very short time period and is called the impulse.

<u>Impulse</u>

When a force F, is applied to a particle for a period of time t, then this quantity is defined as the impulse of the force. Obviously an impulse will bring about a change in velocity and therefore momentum will change.
Therefore:

Impulse = F × t = m(v- u)

The derivation of the formula comes from the equation of motion and constant acceleration equations:

$$F = ma \qquad\qquad v = u + at$$

$$a = \frac{v - u}{t}$$

$$F = m\left(\frac{v - u}{t}\right)$$

Therefore $\quad Ft = m(v - u)$

Example 14

A particle of mass 7.5kg is acted on by a force for 6 seconds and in the process its velocity increases from $6ms^{-1}$ to $15ms^{-1}$. Find the magnitude of the force.

Impulse = change in momentum

$$F \times t = m(v - u)$$

$$F \times 6 = 7.5\,(15 - 6)$$

$$F = 11.25N$$

Example 15

A ball of mass 1.2kg is moving vertically with a speed of 14 ms^{-1} when it hits a smooth horizontal floor. It rebounds with a speed 8 ms^{-1}. Find the magnitude of the impulse exerted by the floor on the ball.

Take care with signs

Assuming down to be positive

Impulse = Change in Momentum

$= m(v - u)$

$= 1.2(-8 - 14)$

$= 1.2(- 22)$

$= -26.4$ Ns

The Principle of the Conservation of Momentum

When a collision occurs between two bodies, A and B, then the force exerted on A by B will be equal and opposite to the force exerted on B by A (by application of Newton's third law). If no other forces are present then the change in momentum in one particle will equate to the loss of momentum in the other particle. Momentum is conserved and therefore the sum of the momentum of the particles before collision must equal the sum of momentum after the collision. This is referred to as the **Principle of Conservation of Momentum**.

If two particles of masses, m_1 and m_2, with initial velocities u_1 and u_2, collide then, given that their final velocities are v_1 and v_2 we can say that:

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

The following examples illustrate the principle and once again it is always best to draw a diagram as this will help to avoid mistakes with signs and direction.

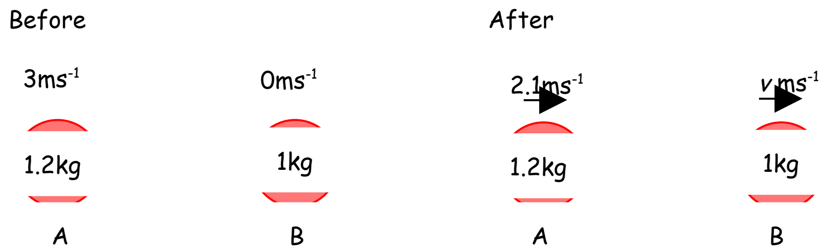
<u>Example 16</u>

Two particles A and B have masses of 1.2kg and 1kg respectively. Particle A is moving towards a stationary particle B with a velocity of 3 ms^{-1}. Immediately after the collision the speed of A is 2.1ms^{-1} and its direction is unchanged. Find:

a) the speed of B after the collision;

b) the magnitude of the impulse exerted on A in the collision.

a)



By conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$3 \times 1.2 = 2.1 \times 1.2 + v$$

$$3.6 - 2.52 = v$$

$$v = 1.08 \text{ ms}^{-1}.$$

b) Impulse = Change in Momentum

$= m(v - u)$

$= 1.2(2.1 - 3)$

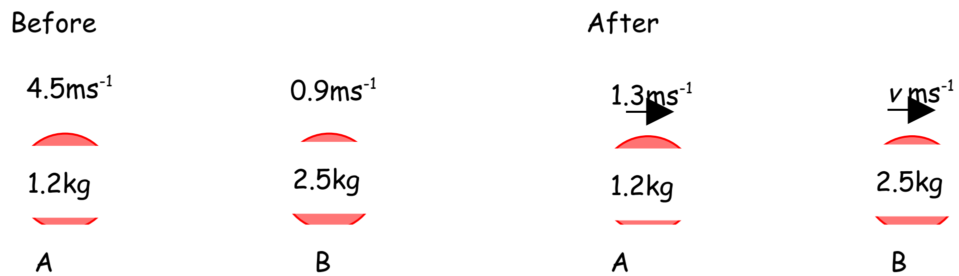
$= -1.08$ Ns

Example 17

Two small balls A and B have masses 1.2kg and 2.5kg respectively. They are moving in opposite directions on a smooth horizontal surface when they collide directly. Immediately before the collision, the speed of A is $4.5ms^{-1}$ and the speed of B is $0.9ms^{-1}$. The speed of A immediately after the collision is $1.3ms^{-1}$. The direction of A remains unchanged after the collision. Find:

a) the speed of B immediately after the collision;

b) the magnitude of the impulse exerted on B in the collision.



a) By conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1.2 \times 4.5 - 2.5 \times 0.9 = 1.2 \times 1.3 + 2.5v$$

$$v = 0.636ms^{-1}$$

b) Impulse = Change in Momentum

$= m(v - u)$

$= 2.5 (0.636 - - 0.9)$

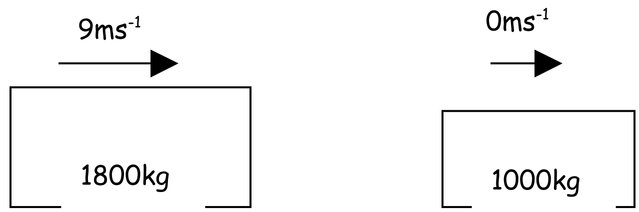
$= 3.84Ns$

Example 18

A locomotive A, of mass 1800kg is moving along a straight horizontal track with a speed of $9ms^{-1}$. It collides directly with a stationary coal truck, B, of mass 1000kg. In the collision, A and B are coupled and move off together.

a) Find the speed of the combined train.

b) After collision a constant breaking force of magnitude R Newtons is applied. The train comes to rest after 15 seconds. Find the value of R.



a) By conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$9 \times 1800 = 2800 \times v$$

$$v = 5.79ms^{-1}$$

b) The combined train comes to rest in 15 seconds therefore we need to calculate the acceleration for use in an equation of motion.

v = 0, u = 5.79, a = ?, t = 15

Using:

$$v = u + at$$
$$0 = 5.79 + 15a$$
$$a = -0.386ms^{-2}$$

Assuming that the train acts as one body:

Equation of motion:

$$F = ma$$

$$R = -2800 \times 0.386$$

$$R = 1080.8N = 1.10KN$$

Jerk in a String

If two particles are connected by a light inextensible string and one of the particles is projected away from the other then at some point there will be a jerk in the string. At the instant before the jerk one of the particles will have momentum. As soon as the string becomes taut the particles will move onwards with the same velocity. The overall momentum must be conserved so therefore the velocity after the jerk must be lower than the initial velocity. This idea is best explained through an example.

Example 19

Two particles P and Q of masses 4kg and 7.5kg respectively are connected by a light inextensible string which is initially slack. Q is projected away from P with velocity $5ms^{-1}$. When the string becomes taught the two particles move on together with a common speed. Find the common speed and the impulse exerted on P by the string.

Using the conservation of momentum where $v_1 = v_2$:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_1$$

$$4 \times 0 + 7.5 \times 5 = 4 \times v_1 + 7.5 \times v_1$$

$$37.5 = 11.5v_1$$

$$v_1 = 3.26ms^{-1}$$

So the common speed is $3.26ms^{-1}$

Impulse is the change in momentum so considering particle P:

$$\text{Impulse} = m(v - u)$$

$$= 4(3.26 - 0)$$

$$= 13.0\text{Ns}$$

Questions B

1 A hockey ball of mass 0.125kg, is moving horizontally at $22ms^{-1}$ when it hits a vertical kick board at right angles. The ball rebounds horizontally at $12ms^{-1}$.

a) Find, in Ns, the impulse of the force exerted by the ball on the kick board.

Given that the ball is in contact with the kick board for 0.15s:

b) Find, in N, the force, assumed constant exerted by the ball on the kick board.

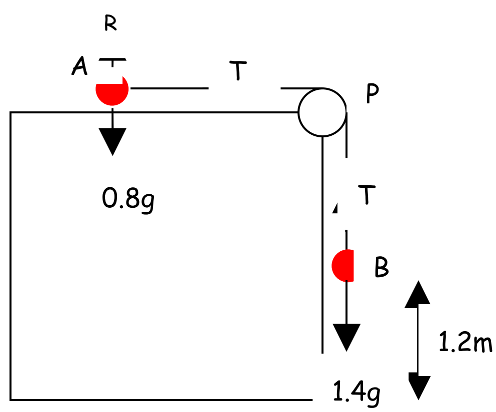
2 Two particles of mass 5kg and 10kg are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut. Find the acceleration of the system and the tension in the string.

3 A particle of mass 0.8kg rests on rough horizontal table and is attached to one end of a light inextensible string. The string passes over a smooth pulley P fixed at the edge of the table. The other end of the string is attached to a particle P of mass 1.4kg which hangs freely below the pulley. The coefficient of friction between the particle and the table is 0.45. The system is released from rest with the string taut and B at a height of 1.2m above the ground. At the point of release A is 1.8m from P. Find:

a) the acceleration of the particles;

b) the time taken by B to reach the ground.

c) the speed with which A hits P.



4 Thomas the tank engine has a mass of 12000Kg and is moving along horizontal rails at $1.2ms^{-1}$, strikes buffers and is brought to rest in 0.45s.

a) Calculate the impulse, in Ns, of the force exerted by the buffers on Thomas in bringing him to rest.

b) Calculate the magnitude of this force assuming it to be constant.

5 Two particles A and B, of masses 4m and 2m respectively are moving towards each other on a smooth horizontal surface with speeds 9v and 3v respectively. The particles collide directly and after the collision A continues to move in the same direction but its speed is halved. Find:

a) the speed of B after the impact.

b) the magnitude of the impulse exerted by A on B.

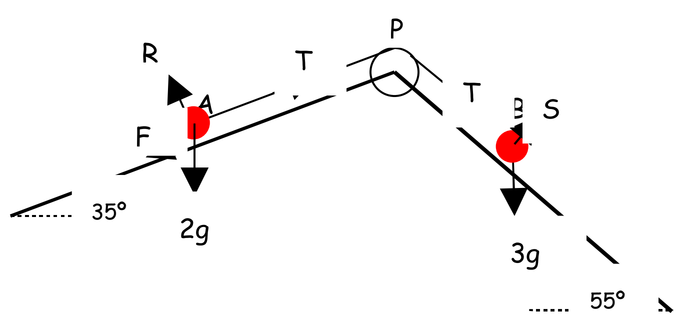
6 A bullet is fired horizontally with a speed of $450ms^{-1}$ into a block of wood of mass 0.15kg that is placed on a smooth horizontal surface. Given that the block begins to move with a velocity of $9ms^{-}1$, find, in kg the mass of the bullet.

7 Two particles P and Q of masses have masses 8kg and m, they are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut and the hanging parts of the string vertical. After 1.5 seconds P has fallen 2.5m. Assuming that Q does not reach the pulley, calculate the tension in the string and the value of m.

8 Two particles A and B of masses 2kg and 3kg are connected by a light inextensible string which passes over a smooth fixed pulley, as outlined in the diagram below. The system is released from rest with the string taut. Find the acceleration of the system and the tension in the string if:

a) both planes are smooth;

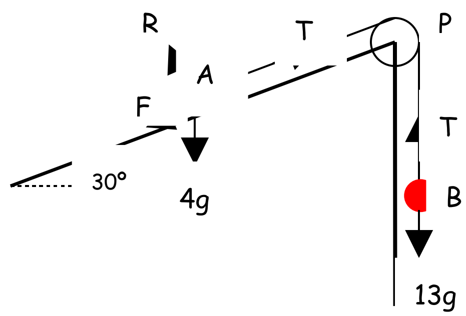
b) both planes are rough and the coefficient of friction between the particles and the plane is 0.1.



9 A particle, A of mass 4kg, rests on a rough plane inclined at an angle of 30° to the horizontal. The particle is attached to one end of a light inextensible string which lies in a line of greatest slope of the plane and passes over a light smooth pulley P fixed at the top of the plane. The other end of the string is attached to a particle B of mass 13kg. The particles are released from rest with the string taut. Given that the coefficient of friction between the particle A and the inclined plane is 0.35 calculate:

a) the tension in the string and the acceleration of the system.

b) the angle of inclination required to increase the acceleration by 50%.



10 A body of mass 6kg is moving with velocity (4**i** + 7**j**)ms^{-1} when an impulse is applied. The impulse causes its velocity to change to (-3**i** - 5**j**)ms^{-1}. Find the impulse.

11 A body of mass 7.5kg is initially at rest on a smooth horizontal surface, experiences a force (8**i** - 13**j**)N for 3 seconds. Find the final velocity of the body and its speed.

12 A pile driver of mass 250 Kg strikes a pile of mass 450kg and drives it into the ground. The pile driver strikes the pile directly with a velocity of $8.5ms^{-1}$. The driver does not rebound and in the subsequent motion the pile and the drive move as one.

a) Calculate the common speed of the pile and driver immediately after impact.

b) Calculate the impulse exerted by the driver on the pile.

c) The pile and the driver penetrate 0.55m into the ground before coming to rest. Assuming that the ground exerts a constant resistive force on the pile and driver, calculate the magnitude of the force.