Kinematics

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Introduction and examples

Kinematics is the study of the motion of particles. In M1 all motion will have constant acceleration. This leads to the development of the constant acceleration equations.

Definitions

a = acceleration (ms^{-2})
u = initial velocity (ms^{-1})
v = final velocity (ms^{-1})
t = time (second)
s = displacement (metres)

If we know any of the three we can find the other two.

From GCSE you should remember that:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$\therefore \quad a = \frac{v-u}{t}$$

$$\text{so } v = u + at \quad (1)$$

As mentioned above the acceleration is constant hence the average velocity is simply the average of u and v.

$$\text{average velocity} = \frac{u+v}{2}$$

Another definition is that:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{s}{t}$$

Therefore:

$$\frac{u+v}{2} = \frac{s}{t}$$

$$\therefore \quad s = \left(\frac{u+v}{2}\right)t \quad (2)$$

By using equation (1) we can eliminate v.

$$\therefore \quad s = \left(\frac{u+u+at}{2}\right)t$$

$$s = ut + \frac{1}{2}at^2 \quad (3)$$

If equation (1) is rearranged to make t the subject:

$$t = \frac{v-u}{a}$$

Then by substituting into equation (2):

$$\therefore \quad s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \quad (4)$$

You must learn all four equations above and remember that they only apply to constant acceleration problems. Since velocity is a vector quantity, getting the direction right in these problems is vital.

Example 1

A particle is moving in a straight line from O to P with a constant acceleration of $4ms^{-2}$. Its velocity at P is $48ms^{-1}$ and it takes 12 seconds to travel from O to P. Find (a) the particle's velocity at O and (b) the distance OP.

With all problems of this nature, write down what you are given and the one required. It should then be obvious as to which equation you need to use.

a) $a = 4$, $u = ?$, $v = 48$, $t = 12$

Obviously we need to use equation (1):

$$v = u + at$$

$$48 = u + 4 \times 12$$

Therefore $u = 0ms^{-1}$

b) $a = 4$, $u = o$, $v = 48$, $t = 12$, $s = ?$

Using equation (3):

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times 4 \times 12^2$$

$$s = 288m$$

Example 2

Percy starts from rest at a station S and moves with constant acceleration. He passes a signal box B 14 seconds later with a speed of $84kmh^{-1}$. Modelling the train as a particle, find the acceleration of the train in ms^{-2} and the distance in metres between the station and the signal box.

$u = 0, \quad t = 14, \quad v = 84kmh^{-1} \quad a = ?, \quad s = ?$

Using equation (1) :

$$v = u + at$$

$$\mathbf{84000/60/60} = 0 + 14a$$

$$a = 1.67ms^{-2}$$

Once again using equation (3)

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times \frac{5}{3} \times 14^2$$

$$s = 163.33m = 163m$$

Example 3

ET is travelling up a hill on his BMX. He experiences a constant retardation of magnitude $3ms^{-2}$. Given that his speed at the bottom of the hill was $15ms^{-1}$ determine how far he will travel before he comes to rest.

Retardation is simply deceleration.

$a = -3, \quad u = 15, \quad v = 0, \quad s = ?$

There is no mention of time therefore we use equation (4):

$$v^2 = u^2 + 2as$$

$$0 = 225 - 2 \times 3 \times s$$

$$s = 37.5m$$

Vertical Motion Under Gravity

A few assumptions need to be stated before continuing.

1. Objects will be treated as particles.
2. Motion will only be in a straight line.
3. No evidence of spinning or turning of objects.
4. Particles will have constant acceleration of g ($9.8ms^{-2}$).

If an object is projected vertically upwards and it falls 3m below the point of release then the time taken can be calculated by setting s = -3m and a = -9.8 and substitute the values into equation (3). A lot of students work out the time the object takes to reach the top, then the time to return to point of release and finally the -3m part, but this much more complicated than what you need to do!

It is also worth noting at this point that time taken to reach maximum height and fall back down to the point of release are the same. This type of question regularly appears on exam papers and it is far quicker to use the first method.

Example 4

A Kinder Surprise falls off a shelf which is 0.9 m above the floor. Find:

(a) the time it takes to reach the floor

(b) the speed with which it will reach the floor.

a) $s = 0.9$, $a = 9.8$, $u = 0$, $t = ?$

Using equation (3):

$$s = ut + \frac{1}{2}at^2$$

$$0.9 = \frac{1}{2} \times 9.8 \times t^2$$

$$s = 0.429sec$$

b) $s = 0.9$, $a = 9.8$, $u = 0$, $t = 0.429$, $v = ?$

Using equation (1):

$$v = u + at$$
$$v = 0 + 9.8 \times 0.429$$
$$v = 4.20 ms^{-1}$$

Example 5

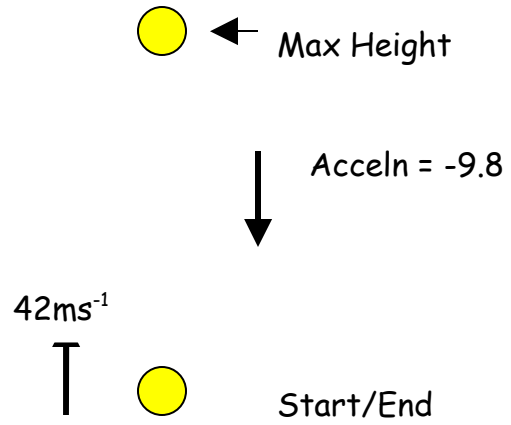
A bouncy ball B is projected vertically upwards from a point O with a speed of $42ms^{-1}$. Find:

(a) the greatest height h above O reached by B

(b) the total time before B returns to O

(c) the total distance travelled by the particle.

With questions of this nature one has to be careful as to which direction is positive.



a) At the greatest height, $v = 0$,

$u = 42$, $a = -9.8$, $s = ?$

Using equation (4):

$v^2 = u^2 + 2as$

$0 = 42^2 - 2 \times 9.8 \times s$

$$s = \frac{42^2}{19.6}$$

$s = 90m$

b) When the ball returns to B its displacement will be zero!

$s = 0, \quad a = -9.8, \quad u = 42, \quad t = ?$

Using equation (3):

$$s = ut + \frac{1}{2}at^2$$

$$0 = 42t - 4.9t^2$$

Factorising gives:

$$7t(0.7t - 6) = 0$$

Therefore $t = 0, t = 8.57$sec

c) In part (a) we found the distance to the top therefore we only need to double the answer.

Total distance = 180m

Example 6

A cricket ball is thrown vertically upwards with a velocity of $15ms^{-1}$. Modelling the ball as a particle moving under gravity alone, find for how long its height exceeds 10 m.

We need to find the times at which the ball has a displacement of 10m.

$a = -9.8, \quad u = 15, \quad s = 10, \quad t = ?$

Using equation (3):

$$s = ut + \frac{1}{2}at^2$$

$$10 = 15t - 4.9t^2$$

$$4.9t^2 - 15t + 10 = 0$$

By using the quadratic formula we find the two values of t.

$$t = \frac{15 \pm \sqrt{225 - 4 \times 10 \times 4.9}}{9.8}$$

$t = 2.08$ or 0.98.

Therefore the ball is above 10m for 1.1 second.

Questions A

1 A car moves with constant acceleration along a straight horizontal road. The car passes the point A with speed $7ms^{-1}$ and 5 seconds later it passes the point B, where AB = 53m.
a) Find the acceleration of the car.
When the car passes the point C, it has a speed of $26ms^{-1}$.
b) Find the distance AC.

2 A competition diver makes a dive from a highboard into a pool. She leaves the board vertically with a speed of $4ms^{-1}$ upwards. When she leaves the board, she is 6m above the surface of the pool. The diver is modelled as a particle moving vertically under gravity alone and it is assumed that she does not hit the springboard as she descends.
a) Find her speed as she reaches the surface of the pool.
b) Find the time taken to reach the surface of the pool.
c) State two physical characteristics that have been ignored in the model.

3 A car moves from rest at a point O and moves in a straight line. The car moves with constant acceleration $5ms^{-2}$ until it passes the point A when it is moving with speed 14 ms^{-1}. It then moves with constant acceleration 2 ms^{-2} for 8 seconds until it reaches the point B.
a) Find the speed of the car at B.
b) Find the distance OB.

4 An aircraft moves along a straight horizontal runway with constant acceleration. It passes a point A on the runway with speed $15ms^{-1}$. It then passes the point B on the runway with speed $39ms^{-1}$. The distance AB is 190m.
a) Find the acceleration of the aircraft.
b) Find the time taken by the aircraft in moving from A to B.
c) Find the speed of the aircraft when it passes the mid point of A and B.

5 A racing car moves along a straight horizontal road with constant acceleration. It passes the point O with speed $11ms^{-1}$. It passes the point A, 4 seconds later with speed $55ms^{-1}$.
a) Find the acceleration of the car.
b) Find the distance OA.
c) Find the speed of the car as it passes the midpoint of OA.

6 A body is projected vertically upwards from ground level at a speed of $49ms^{-1}$. Find the length of time for which the body is at least 78.4m above the ground.

7 A body is projected vertically upwards from ground level at a speed of $14ms^{-1}$. Find the height of the body above the level of projection after:
a) 1 second of motion
b) 2 seconds of motion
Find the distance travelled by the body in the 2^{nd} second of motion.

8 A balloon is moving vertically upwards with a steady speed $3ms^{-1}$. When it reaches a height of 36m above the ground an object is released from the balloon. The balloon then accelerates upwards at a rate of $2ms^{-2}$. Find

a) the greatest height of the object above the ground.

b) the speed of the object as it hits the ground.

c) the time taken by the object from leaving the balloon to striking the ground.

d) the speed of the balloon as the object hits the ground.

9 A cyclist travels on a straight road with a constant acceleration of $0.6\ ms^{-2}$. P and Q are 120m apart. Given that the cyclist increases his speed by $6ms^{-1}$ as he travels from P to Q. Find:

a) the speed of the cyclist at P

b) the time taken to travel from P to Q

10 Oblivion at Alton Towers has a vertical drop of 60m. Assuming that the ride starts from rest, calculate the speed of the ride at the bottom of the pit.

Speed Time Graphs

Definitions

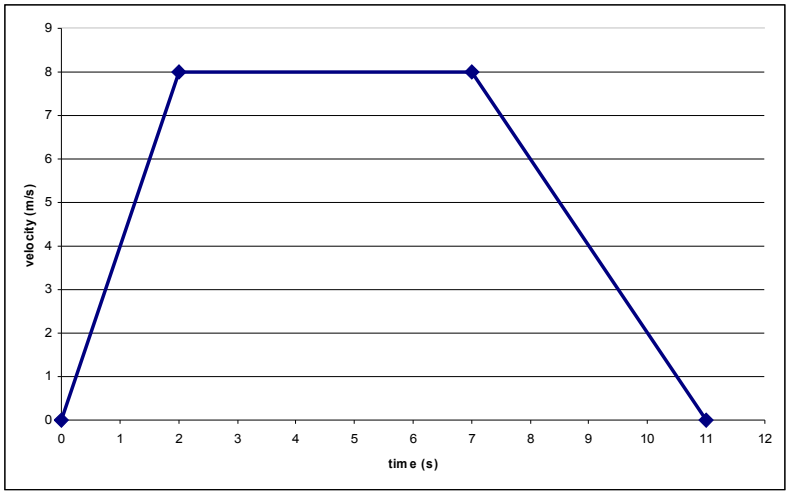
For constant acceleration problems the speed time graph will be a straight line.
The gradient of the graph is the acceleration.
The area under the graph represents the distance traveled.

Example 1

A body starts from rest, accelerates uniformly to a velocity of $8ms^{-1}$ in 2 seconds, maintains that velocity for a further 5 seconds, and then retards uniformly to rest. The entire journey takes 11 seconds.
Find:

a) the acceleration of the body during the initial part of the motion.
b) The retardation of the body in the final part of the motion.
c) The total distance traveled by the body

In these type of questions it is vital that you make a sketch of the motion.



a) As the definitions suggested earlier the acceleration is simply the gradient of the velocity time graph. This is:

grad = 8/2 = 4ms^{-2}.

b) Similarly for the retardation:

grad = 8/4 = 2ms^{-2}.

c) The total distance travelled by the body is equal to the area of under the graph.

Area = 0.5 × 8 × (11 + 5)

= 64m

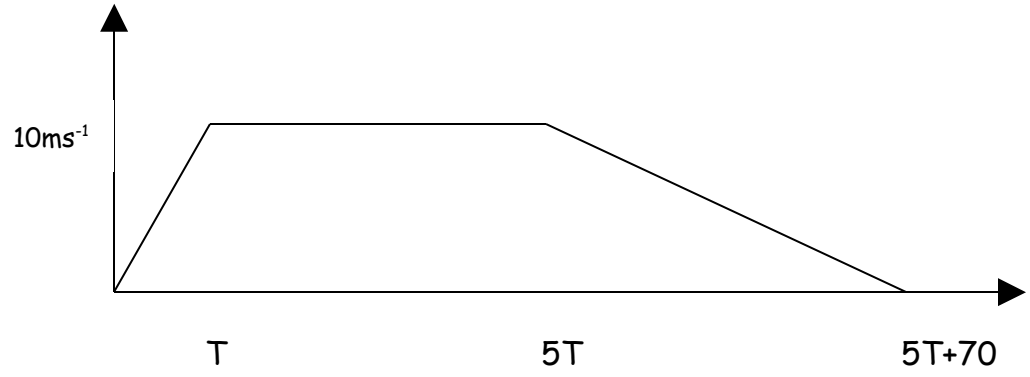
Example 2

A car accelerates uniformly from rest to a speed of 10ms^{-1} in T seconds. The car then travels for 4T seconds and finally decelerates uniformly to rest in a further 70s. The total distance traveled by the car is 1250m. Find:

a) the value of T.

b) the initial acceleration of the car.

Once again, sketch the journey.



a) the area under the graph must equal 1250.

1250 = 10T + 40T + 700/2

900 = 50T

T = 18sec.

b) The car accelerates to 10ms^{-1} in 18 seconds, therefore the acceleration is:

Acceln = 10/18 = 0.556ms^{-2}

Example 3

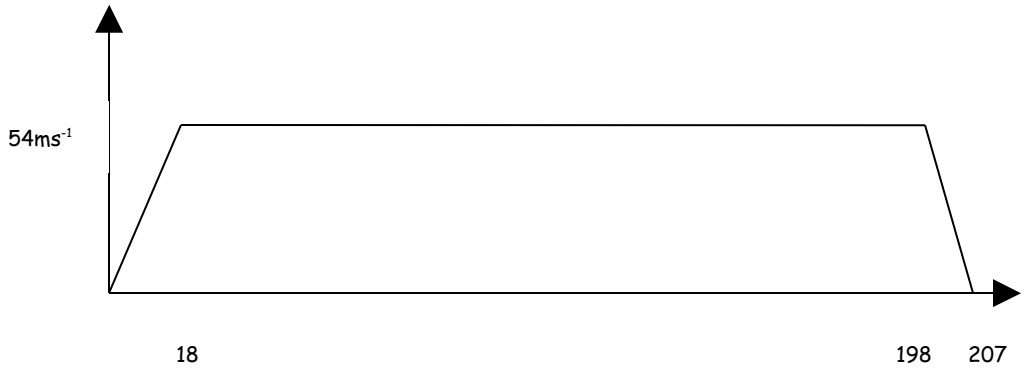
A train is traveling along a straight track between the points P and Q. It starts from rest at P and accelerates at $3ms^{-2}$ until it reaches a speed of $54ms^{-1}$. It continues at a constant speed of $54ms^{-1}$ for a further 180seconds and then decelerates at a constant deceleration of $6ms^{-2}$.

a) sketch a speed time graph for the train's journey.

b) calculate the total time for the journey from P to Q.

c) calculate the distance between P and Q.

a) The train will take 54/3 seconds to reach the constant speed (18seconds). It will take 54/6 seconds to decelerate to rest (9 seconds). Hence the graph will have the following shape.



b) the total time for the journey is 207 seconds.

c) the total distance is once again equal to the area under the graph.

Area = (54 × 18)/2 + 54 × 180 + (54 × 9)/2

Distance = 10449m = 10.4Km

Problems involving two vehicles

Example 4

Two particles A and B are traveling along a straight path PQ of length 20m. A leaves P, heading for Q, from rest with acceleration of $2ms^{-2}$ and at the same time B leaves Q, in the direction of P, from rest with a constant acceleration of $5ms^{-2}$. Find how far from A the two particles collide?

Let the point of collision be x metres from P and hence (20 – x)m from Q.

So for particle A: u = 0 a = 2 t = T s = x

Using:

$$s = ut + \frac{1}{2}at^2$$

$$x = t^2 \quad (i)$$

So for particle B: u = 0 a = 5 t = T s = 20 - x

Using

$$s = ut + \frac{1}{2}at^2$$

$$20 - x = 2.5t^2$$

Substituting (i) gives:

$$20 - t^2 = 2.5t^2$$

$$t = 2.39sec$$

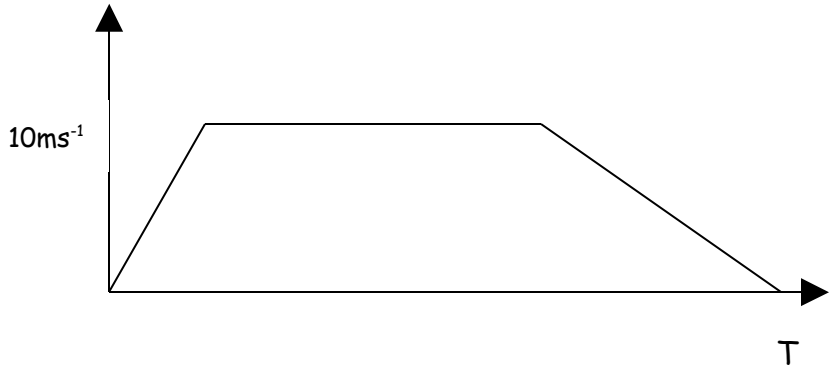
Using (i) again:

$$x = 2.39^2$$

$$x = 5.71m$$

Questions B

1 The diagram shows the sketch of a velocity time graph for a particle moving in a straight line. Find the value of T given that:



a) the distance covered is 100m.
b) the constant speed attained by the particle is $10ms^{-1}$
c) the rate of acceleration is twice the rate of retardation.
d) The time spent at constant speed is equal to the total time spent accelerating and decelerating.

2 Two trains, P and Q, run on parallel straight tracks. Initially both trains are at rest. At time t = 0, P moves off with constant acceleration for 15 seconds until it reaches a speed of $30ms^{-1}$. P then continues at a constant speed. At time t = 35seconds Q moves off with the same acceleration until it reaches a speed of $55ms^{-1}$. Q then continues at this constant speed. Train Q overtakes P whilst both trains are travelling at constant speeds after time T. Sketch a speed time graph for the journeys and find the value of T.

3 A particle Q starts from rest at a point O and accelerates at $3ms^{-2}$ until it reaches a speed V ms^{-2}. Q then continues at the constant speed for 90 seconds before decelerating at 1.5 ms^{-2} to come to rest at the point R. If the entire journey takes 150 seconds find the distance QR. Make sure that you draw a speed time graph.

4 A racing car moves in a straight line. The car accelerates at $8ms^{-2}$ for 5 seconds, maintains a steady speed for 15 seconds and then decelerates to rest in 9 seconds. Sketch a velocity time graph for the car and calculate the total distance travelled.