Vectors

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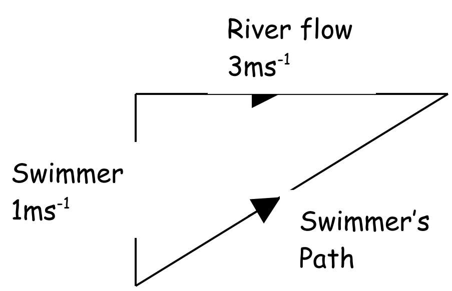
Vectors are first introduced at GCSE and students regularly ask how they apply to the real world. One simple example would be to consider a swimmer attempting to cross a river. If the swimmer can maintain a velocity of $1ms^{-1}$ and the river is flowing at $3ms^{-1}$ then the swimmer will obviously move on a diagonal path. The path that the swimmer tackles is said to be the resultant of the two forces (swimmer and current). This idea is explained in the diagram below.

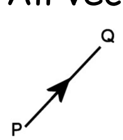


The swimmer will be moving with a speed of $\sqrt{10}ms^{-1}$

Recap of GCSE content

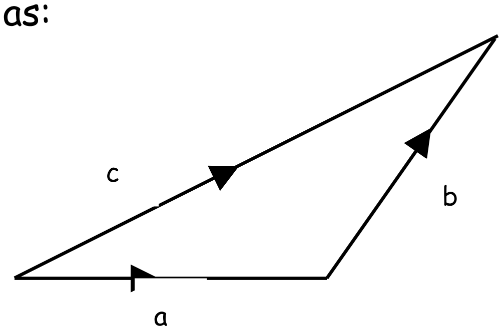
Vector quantities require direction and magnitude to be truly defined. **Scalar** quantities are completely specified by their magnitude. Examples of each would be a car moving with a velocity of $25ms^{-1}$ on a bearing of 045°, and a second car traveling along a road with a speed of $15ms^{-1}$.

All vectors are represented by a directional line segment



PQ

Vectors can also be represented in bold type and the triangle law is written as:

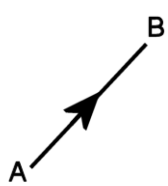


a + b = c

The magnitude (length) of a vector is given as $|PQ|$.

Vectors can only be equal if they have the same magnitude and direction.

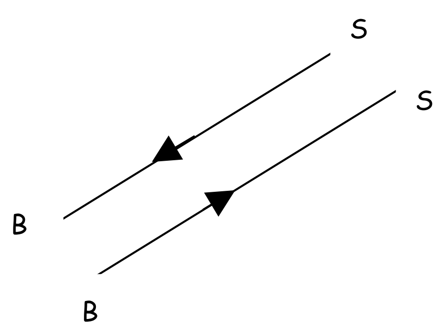
e.g.



Vectors parallel and same magnitude $\therefore$ $AB = MN$

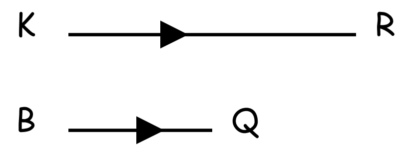
Remember that direction is important

$BS \neq SB$



Vectors may have different magnitudes but still be parallel.
e.g.

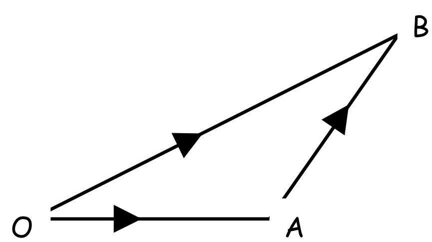
$KR = xBQ$



(where x is a scalar quantity).

Adding Vectors

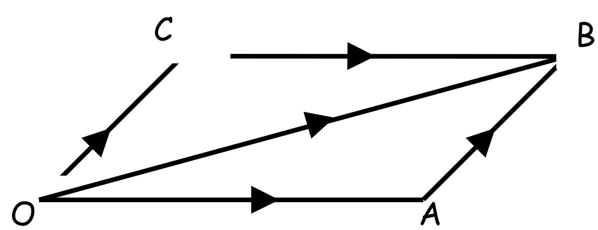
The earlier real world example introduced the triangle law by default. In more formal terms it can be said that for two vectors OA and OB:



$$OA + AB = OB$$

This can be shown quite easily by using column vectors.

The triangle law can be applied to prove the parallelogram law.

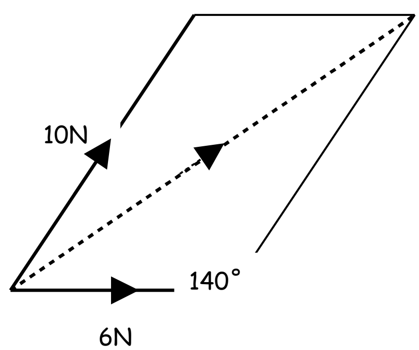


The vector OB is said to be the resultant and is the diagonal of the parallelogram OABC

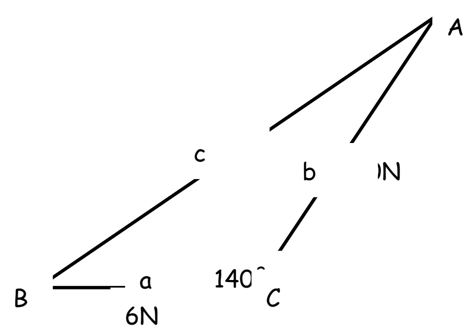
This idea is used in the following example:

Example 1

A particle P is acted upon by two forces P and Q of magnitude 6N and 10N respectively. The angle between the two vectors is 140°. Find the magnitude of the resultant and the angle it makes with the force P.



This problem is solved by applying sine and cosine rule since the diagonal of the parallelogram is the resultant force.



$$c^2 = a^2 + b^2 - 2abCosC$$

$$c^2 = 36 + 100 - 2 \times 6 \times 10 \times Cos\ 140$$

$$c = 15.87N$$

The resultant has a magnitude of 15.9N.

The resultant makes an angle with force P of:

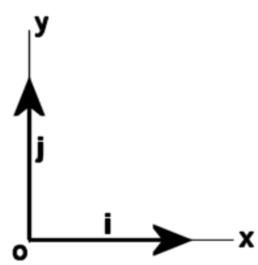
$$\frac{Sin B}{10} = \frac{Sin 140}{15.87}$$

$$B = 23.9^{\circ}$$

Cartesian unit vectors and components

A **unit vector** is a vector with magnitude of 1 unit.

i, j notation



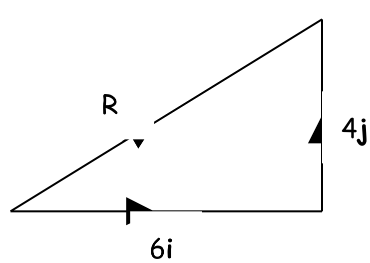
The diagram above shows the two unit vectors **i** and **j**. By definition they are vectors of magnitude one unit along the x and y coordinate axis respectively.

Example 2

A vector R = (6**i**+4**j**)
represents a displacement of:

- 6 units in the direction of the unit vector **i**,
- 4 units in the direction of the unit vector **j**.

And is displayed diagramatically as:



Adding vectors in **i**, **j** notation

When vectors are given in terms of unit vectors **i** and **j**, you can add them together by adding their terms involving **i** and **j** separately.

Example 3

Consider the two vectors a and b where: $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j}$

$$\mathbf{a} + \mathbf{b} = (2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - 2\mathbf{j})$$

$$=6\mathbf{i} + \mathbf{j}$$

Subtracting vectors in **i**, **j** notation

Using the same vectors a and b as in the example above

$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j}$

$$a - b = (2\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - 2\mathbf{j})$$

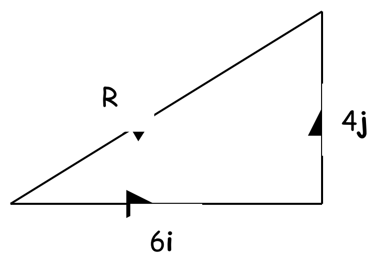
$$= (-2\mathbf{i} + 5\mathbf{j})$$

Please take care with signs as these questions are not mathematically difficult but students are sometimes prone to make silly mistakes.

Magnitude of a vector in **i**, **j** notation

When a vector R is given in terms of the unit vectors **i** and **j** you can find its magnitude by using Pythagoras' Theorem.

Using the vector R from the earlier example:



$R^2 = \sqrt{6^2 + 4^2}$

$R = 10$

The following example combines unit vectors and **i,j** notation.

Example 4

Find the unit vector in the direction of the vector $a = 4i + 7j$

The magnitude of a is:

$$|a| = \sqrt{4^2 + 7^2}$$

$$|a| = \sqrt{65}$$

So the unit vector in the same direction as a is

$$\frac{a}{|a|} = \frac{1}{\sqrt{65}}(4i + 7j))$$

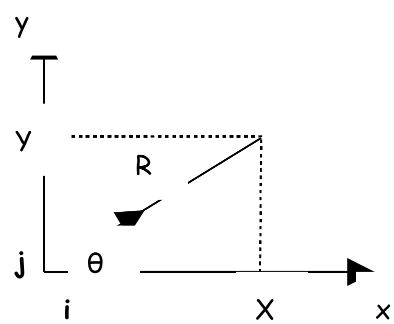
In more simple terms the vector a is $\sqrt{65}$ in length and the unit vector in the same direction of a must be $\frac{1}{\sqrt{65}}$ times the vector a.

Equal Vectors

Two vectors are equal if and only if the **i** components are equal and the **j** components are equal.

Components of a vector

Any two vectors can be written as a single vector (triangle law). However, sometimes it is useful to reverse this process. This process is called **resolving a vector into Cartesian components**. This idea is used extensively in mechanics when dealing with a number of forces (see Statics chapter).



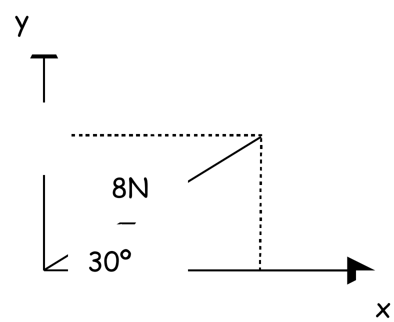
The diagram shows the vector R, which makes an angle θ with the x axis. The X and Y components can be given as:

$$X = R \cos \theta$$

$$Y = R \sin \theta$$

Example 5

A force of magnitude 8N acts on a particle at an angle of 30° to the horizontal. Find the x and y components of the force.



X = 8 Cos 30 = 6.93N

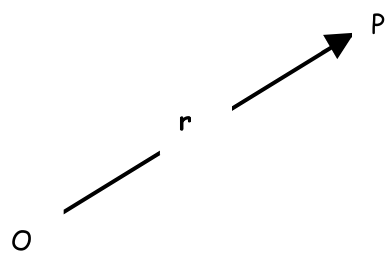
Y = 8 Sin 30 = 4N

Vectors and Mechanics

So far very little in the way of mechanics has been discussed. The positions of particles and their motion can be described by the use of vectors.

<u>Position Vectors</u>

If a particle is moving in a plane, where O is a fixed point, then the position of P is defined by $OP = r$

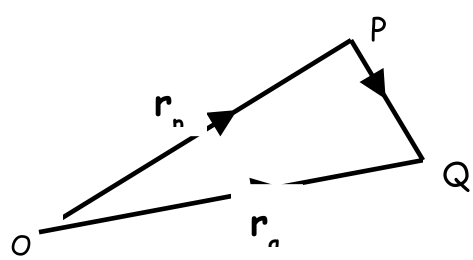


The vector r is known as the **position vector** of P relative to O.

<u>Relative position vectors</u>

As the name suggest we aim to find the vector of one particle relative to another.

Imagine two particles P and Q with position vectors OP and OQ respectively.



The vector PQ gives the position vector of **Q relative to P**. It's called the **relative position vector** (ie how do you get from P to Q).

Velocity as a Vector

We discussed earlier that velocity is a vector quantity but defined more formally:

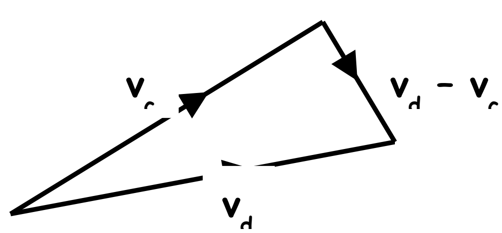
The velocity of a particle is a vector in the direction of motion whose magnitude is equal to the speed of the particle (usually denoted by v)

Relative Velocity

We have discussed relative position vectors, and seeing as velocity is a vector we should be able to consider relative velocity vectors. But what do they mean?

Imagine that two trains are traveling at equal velocities in the same direction at $90kmh^{-1}$. As far as passengers on the trains are concerned they will appear to be stationary. In this case we can say that their relative velocity is zero. If on the other hand the two trains were traveling in the same direction but at velocities of $90kmh^{-1}$ and $50kmh^{-1}$ then the relative velocity would be $40kmh^{-1}$. This principle can be applied to vectors:

Consider two particles C and D. If their velocities are v_c and v_d respectively then the velocity of D relative to C is $v_d - v_c$.



In real terms the vector $v_d - v_c$ is the velocity vector required to get from C to D. Unfortunately as time passes the particles C and D will get further apart and the vector between them will be a multiple of $v_d - v_c$.

The only other property of the vector $v_d - v_c$ is that if we imagine that C and D are aeroplanes that set off at the same time from the same point, then it is the direction that a passenger on a plane C would look to see plane D.

Acceleration as a Vector

Considering that velocity can be a vector, and that acceleration is the rate of change velocity. It follows that acceleration can be a vector. Using the constant acceleration equations find final velocities.

Example 6

A particle has is moving with velocity $(8\mathbf{i} - 12\mathbf{j})\text{ms}^{-1}$ and it experiences an acceleration of $(3\mathbf{i} + 6\mathbf{j})\text{ms}^{-2}$ for 4 seconds. Find the final velocity.

The constant acceleration equation needed is $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, however I find that it is best to write the equation in words:

$$\text{Final velocity} = \text{initial velocity} + (\text{acceleration} \times \text{time})$$

The following example uses the equation of motion. A number of vector exam questions will involve ideas from other areas of the M1 course and therefore you need to have a decent grasp of the whole course.

Example 7

A particle Q, of mass 7.5Kg is moving under the action of a constant force F. Initially the velocity of Q is $(12\mathbf{i} - 16\mathbf{j})ms^{-1}$ and 8 seconds later it is $(32\mathbf{i} + 16\mathbf{j})ms^{-1}$.

a) find, in vector form, the acceleration of Q.

b) calculate the magnitude of F.

a) Given that acceleration is change in velocity over time:

$$\Delta \text{Velocity} = (32\mathbf{i} + 16\mathbf{j}) - (12\mathbf{i} - 16\mathbf{j})$$

$$= (20\mathbf{i} + 32\mathbf{j})$$

Therefore:

$$\text{Acceleration} = (20\mathbf{i} + 32\mathbf{j})/8 = (2.5\mathbf{i} + 4\mathbf{j})ms^{-2}$$

b) Using the equation of motion:

$$F = ma$$

$$F = 7.5 \times (2.5\mathbf{i} + 4\mathbf{j})$$

$$F = (18.75\mathbf{i} + 30\mathbf{j})N$$

The question asks for the magnitude of the force:

$$|F| = 35.4N$$

The next example uses more concepts but shouldn't cause any problems!

Example 8

A particle A, of mass 3.5Kg is acted upon by two constant forces (6**i** – 3**j**)N and (8**i** + 10**j**)N.

a) Find, in vector form, the resultant force F acting on A.

b) Find, in degrees to 3 sig fig, the angle between F and **i**.

c) Find the magnitude of the acceleration of A.

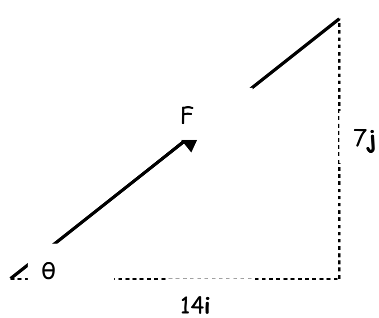
d) Given that the initial velocity is (-6**i** + 7**j**)ms^{-1}, find the speed of A after 6 seconds.

a) The resultant force is simply the sum of the forces (as they are in **i**,**j** form):

$$F = (6\mathbf{i} - 3\mathbf{j}) + (8\mathbf{i} + 10\mathbf{j})$$

$$F = (14\mathbf{i} + 7\mathbf{j})N$$

b) The diagram below shows the force F and angle θ:



$$\tan\theta = \frac{7}{14}$$

$$\theta = 26.6^\circ$$

c) The acceleration can be found by using the equation of motion for the particle.

$$F = ma$$

$$(14\mathbf{i} + 7\mathbf{j}) = 3.5 \times a$$

$$a = (4\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$$

d) Seeing as the acceleration is constant we can use constant acceleration equations:

$u = (-6\mathbf{i} + 7\mathbf{j})$, $a = (4\mathbf{i} + 2\mathbf{j})$, $t = 6$

$$v = u + at$$

$$v = (-6\mathbf{i} + 7\mathbf{j}) + 6 \times (4\mathbf{i} + 2\mathbf{j})$$

$$v = (18\mathbf{i} + 19\mathbf{j})\text{ms}^{-1}$$

Speed is the magnitude of the velocity:

$$|v| = \sqrt{(18^2 + 19^2)}$$

$$\text{Speed} = 26.2\text{ms}^{-1}$$

AS Exam Questions

A few basic principles are used over and again in M1 exam questions. Hopefully the solutions given below should give a few ideas as to what might appear. Examiner Reports usually state that vectors is the worst answered question on the paper, but with a little thought half marks is easily attainable.

Example 9

A particle B moves with constant acceleration $(3\mathbf{i} + 7\mathbf{j})\text{ms}^{-2}$. At time t its velocity is v ms^{-1}. When $t = 0$, $v = (12\mathbf{i} - 14\mathbf{j})\text{ms}^{-1}$.

a) find the time when B is moving parallel to the vector **i**.

b) find the speed of B when t = 8.

c) find the angle between the direction of motion of B and the vector **i** when t = 8.

a) The particle will be moving parallel to the vector **i** when it has no **j** component.

The velocity at any time is given by:

Velocity = initial velocity + (acceleration × time)

$$= (12\mathbf{i} - 14\mathbf{j}) + t(3\mathbf{i} + 7\mathbf{j}) \qquad (1)$$

We are only interested in the **j** component and particularly when it is zero:

$$-14\mathbf{j} + 7t\mathbf{j} = 0$$

Therefore: $t = 2$

b) By substituting t = 8 into equation (1) we can find the velocity of the particle. Speed is the magnitude of the velocity.

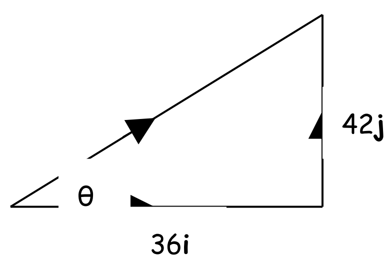
$$\text{Velocity} = (12\mathbf{i} - 14\mathbf{j}) + t(3\mathbf{i} + 7\mathbf{j})$$

When t = 8 $\quad \text{Velocity} = (12\mathbf{i} - 14\mathbf{j}) + 8 \times (3\mathbf{i} + 7\mathbf{j})$

$$= (36\mathbf{i} + 42\mathbf{j})\ \text{ms}^{-1}$$

$$\text{Speed} = \sqrt{36^2 + 42^2} = 55.3\text{ms}^{-1}$$

c) The velocity at any time gives the direction of motion.



Therefore the direction is given by:

$$\theta = \text{Tan}^{-1}\left(\frac{42}{36}\right)$$

$$\theta = 49.4^\circ$$

Example 10

A particle has position vector (3**i** + 7**j**) and is moving with speed 39ms^{-1} in the direction (12**i** – 5**j**). Find the position vector at time t = 5 and the distance from the point (3,4) at this time.

The hint in the question is that the velocity of the particle must be a multiple of (12i – 5j)ms^{-1}. The magnitude of this vector is 13, but the speed of the particle is 39ms^{-1}, therefore the velocity of the particle must be (36i – 15j)ms^{-1}. This one little trick is often used but if you don't spot it the question is almost impossible.

The position vector of the particle at any time is given by:

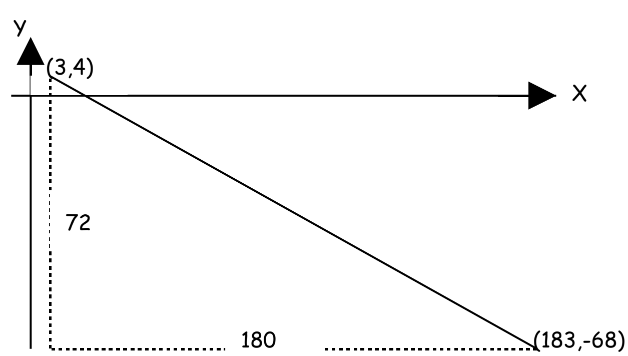
Position = initial position + (velocity × time)

r = (3**i** + 7**j**) + t(36**i** – 15**j**)

When t = 5

r = (183**i** – 68**j**)

The distance between the point and the particle can be found by Pythagoras theorem.



Distance = $\sqrt{72^2 + 180^2}$ = 193.9m

The final two examples are of the more challenging type. Setting the equations for the position vectors is pretty straight forward as long as you follow the rule:

$$r = \text{initial position} + (\text{velocity} \times \text{time})$$

In most cases the ships or aeroplanes are on a collision course and therefore you must set the equations equal to each other and equate coefficients. These questions may carry in excess of 10 marks but you should be able to make a start.

Example 11

A command post O monitors the movement of two of its ships in the Gulf. At 1200 hrs a battleship (B) has position ($-2\mathbf{i} + 10\mathbf{j}$) km relative to O and has constant velocity of ($3\mathbf{i} + 2\mathbf{j}$) kmh^{-1}. A frigate (F) is at the point with position vector ($4\mathbf{i} + 5\mathbf{j}$) km and has constant velocity ($-3\mathbf{i} + 7\mathbf{j}$) kmh^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors directed due east and due north respectively.

a) The captain of one ship has been taken ill, show that the two ships will collide.

The command post contacts the battleship and orders it to reduce its speed to move with velocity ($2\mathbf{i} + 2\mathbf{j}$) kmh^{-1}.

b) Find an expression for the vector BF at time t hours after noon.

c) Find the distance between B and F at 1400 hrs.

d) Find the time at which F will be due north of B.

a) The position of each ship is given by it's position vector:

$$\text{position} = \text{initial position} + (\text{velocity} \times \text{time})$$

So for the battleship:

$$r_b = (-2\mathbf{i} + 10\mathbf{j}) + t(3\mathbf{i} + 2\mathbf{j})$$

And for the frigate:

$$r_f = (4\mathbf{i} + 5\mathbf{j}) + t(-3\mathbf{i} + 7\mathbf{j})$$

If the two ships are to collide then for some value of t their respective **i** and **j** components must be equal.

Therefore by equating **i**'s:

$$-2 + 3t = 4 - 3t$$

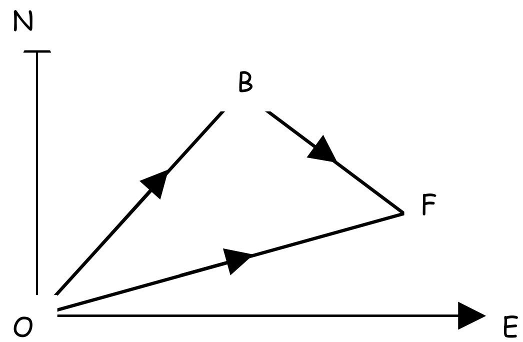
$$t = 1$$

Substituting the value of $t = 1$ into r_b and r_b gives the same position vector of (**i** + 12**j**). Therefore the two ships will collide after one hour at the point with position vector (**i** + 12**j**).

b) The position vector for the battleship must change to take account of the new velocity:

$$r_b = (-2\mathbf{i} + 10\mathbf{j}) + t(2\mathbf{i} + 2\mathbf{j})$$

We have been asked to find the vector BF as shown in the diagram below.



By triangle law:

$$OB + BF = OF$$

$$\overrightarrow{BF} = \overrightarrow{OF} - \overrightarrow{OB}$$

Where $OB = r_b$ and $OF = r_f$

Therefore: $BF = r_f - r_b$

$$= (4\mathbf{i} + 5\mathbf{j}) + t(-3\mathbf{i} + 7\mathbf{j}) - ((-2\mathbf{i} + 10\mathbf{j}) + t(2\mathbf{i} + 2\mathbf{j}))$$

$$BF = (6\mathbf{i} - 5\mathbf{j}) + t(-5\mathbf{i} + 5\mathbf{j})$$

c) The magnitude of BF gives the distance between the two ships, at 1400 hrs, t = 2

$$= (6\mathbf{i} - 5\mathbf{j}) + 2(-5\mathbf{i} + 5\mathbf{j})$$

$$= (-4\mathbf{i} + 5\mathbf{j})$$

Magnitude = $\sqrt{41}Km$

d) If F is due north of B, then $\overrightarrow{BF}$ will have no **i** component.

$$\overrightarrow{BF} = (6\mathbf{i} - 5\mathbf{j}) + t(-5\mathbf{i} + 5\mathbf{j})$$

$$6\mathbf{i} - 5t\mathbf{i} = 0$$

$$t = 1\text{hr } 12\text{mins}$$

<u>Example 12</u>

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is 25ms^{-1} due east and the velocity of Q is $(10\mathbf{i} + 8\mathbf{j})$ms^{-1}. Initially P is at rest at the origin and Q has the position vector 230**i**m relative to the origin. At time t seconds the position vectors of P and Q are *r* metres and *s* metres respectively.

a) Find expressions for *r* and *s* in terms of t.
b) Write an expression for $\overrightarrow{PQ}$.
c) Find the time when the bearing of Q from P is 045°
d) Find the time when the cars are 200m apart.

a) The position vector for car P is given by:

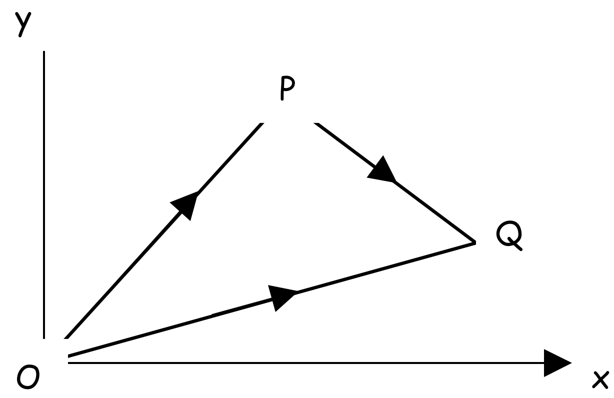
$$r = \text{initial position} + (\text{velocity} \times \text{time})$$

$$r = 0 + 25t\mathbf{i}$$

Therefore for Q:

$$s = 230\mathbf{i} + t(10\mathbf{i} + 8\mathbf{j})$$

b) Using the triangle law:



$OP + PQ = OQ$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

Where $OP = \mathbf{r}$ and $OQ = \mathbf{s}$

Therefore: $PQ = \mathbf{s} - \mathbf{r}$

$= 230\mathbf{i} + t(10\mathbf{i} + 8\mathbf{j}) - 25t\mathbf{i}$

$= 230\mathbf{i} + t(8\mathbf{j} - 15\mathbf{i})$

c) If the bearing of Q from P is 045°, then the vector PQ must be parallel to the vector $(\mathbf{i} + \mathbf{j})$.

Therefore

$$230\mathbf{i} + t(8\mathbf{j} - 15\mathbf{i}) = m(\mathbf{i} + \mathbf{j})$$

Equating coefficients:

$$230 - 15t = m \quad (1) \qquad 8t = m \quad (2)$$

Substituting gives:

$$230 - 15t = 8t$$

$$t = 10$$

The bearing of Q from P is 045° after 10 seconds.

d) The magnitude of PQ will give the distance that the two cars are apart. We need the value of t where $|PQ| = 200$
Rewriting PQ gives:

$$PQ = (230 - 15t)\mathbf{i} + 8t\mathbf{j}$$

$$|PQ| = \sqrt{(230 - 15t)^2 + 64t^2}$$

$$|\overrightarrow{PQ}| = \sqrt{52900 - 6900t - 225t^2 + 64t^2}$$

$$200^2 = 52900 - 6900t - 161t^2$$

$$161t^2 + 6900t - 12900 = 0$$

$$t = 1.79$$

Example 13

At 1200 hrs the position vectors of two helicopters A and B are r_A and r_B as outlined below. Use the velocity vectors v_A and v_B to give the position vectors of A and B at a time t hours after noon.

$r_A = (3\mathbf{i} + 5\mathbf{j})$ $r_B = (5\mathbf{i} + 2\mathbf{j})$ $v_A = (\mathbf{i} + 2\mathbf{j})$ $v_B = (2\mathbf{i} + 3\mathbf{j})$

Find an expression at time t hours after noon for the position vector of B relative to A.

If d is the distance in Km between the two helicopters find the value of d^2 in terms of t.

Find the time at which the helicopters are closest together. Give the value of the minimum distance.

a) For Helicopters A and B the position vectors at time t hours after noon are:

$$r_A = (3\mathbf{i} + 5\mathbf{j}) + t(\mathbf{i} + 2\mathbf{j}) \qquad r_B = (5\mathbf{i} + 2\mathbf{j}) + t(2\mathbf{i} + 3\mathbf{j})$$

b) The position vector of B relative to A is given by:

$$r_B - r_A = 2\mathbf{i} - 3\mathbf{j} + t(\mathbf{i} + \mathbf{j})$$

$$= (2 + t)\mathbf{i} + (t - 3)\mathbf{j}$$

c) The magnitude of the vector in part (b) gives the distance between the two particles.
Therefore:

$$d^2 = (2 + t)^2 + (t - 3)^2$$

$$d^2 = 2t^2 - 2t + 13$$

By completing the square we can find the minimum value:

$$d^2 = 2[t^2 - t + 6.5]$$

Remembering to halve the t coefficient:

$(-0.5)^2 = 0.25$ we need 6.5

$$d^2 = 2[(t - 0.5)^2 + 6.25]$$

The minimum value occurs when $t = 0.5$. Therefore the minimum distance between the two helicopters is $\sqrt{12.5}$ Km

Questions

1 For each of the vectors below find the magnitude and the angle the vector makes with the positive x axis.
a) (6**i** + 3**j**) b) (**i** + 4**j**) c) (-4**i** + 3**j**) d) (-5**i** - 3**j**)
e) (3**i** - 2**j**)

2 The table below gives the magnitude and direction of a number of vectors. Express each in the form a**i** + b**j**, where **i** and **j** are unit vectors in the direction Ox and Oy respectively.

Vector	Magnitude	Direction(as measured from positive x axis)
P	6	30°
Q	9	60°
R	4.5	150°
S	12	240°

3 Find the speed of a body moving with velocity (6**i** - 8**j**)ms^{-1}

4 Find the speed of a body moving with velocity (-3**i** + 10**j**)ms^{-1}

5 A particle is moving with velocity (2**i** + a**j**)ms^{-1} has speed 5.2ms^{-1}. Find the two possible values of a.

6 A particle has an initial position vector of (6**i** - 8**j**)m. If the particle moves with constant velocity of (5**i** + 3**j**)ms^{-1}, find its position vector after:
a) 3 seconds b) 7 seconds.

7 How far is the particle in question 6 from the origin after 12 seconds?

8 A particle has initial position vector (7**i** + 5**j**)m. The particle moves with constant velocity of (2p**i** + q**j**)ms^{-1} and after three seconds it has position vector (-17**i** - **j**)ms^{-1}.

9 Given that a = 3**i** + 6**j** and b = -2**i** + 5**j**, find the magnitude of each of the following and angle that they make with the positive x axis

a) a + b b) a – b c) 2a – b d) 4a + 2b

10 Two joggers Chris (C) and Gwyneth (G) are moving with constant velocities across a level plane. At a certain instant Chris and Gwyneth have position vectors (-60**i** +170**j**) and (90**i** – 100**j**) respectively. Thirty seconds later they meet at the point with position vector (300**i** + 20**j**).

a) find in vector form the velocity of C and G.

b) Calculate the magnitude of the velocity of C relative to G.

11 The velocities of two particles P and Q are (a**i** – 7**j**)ms^{-1} and (5**i** + b**j**)ms^{-1} respectively. The velocity of Q relative to P is (2**i** – 3**j**)ms^{-1}. Find a and b.

12 The velocities of two particles P and Q are (14**i** + 7**j**)ms^{-1} and (5**i** + 12**j**)ms^{-1} respectively. Find:

a) the speed of Q.

b) the velocity of Q relative to P.

c) the angle between the velocity in part (b) and the vector **j**.