Moments

Introduction and examples

The moment of a force is the measure of its capacity to turn the body on which it is acting.

$$\text{Moment} = \text{Force} \times \text{Perpendicular Distance}$$

Example 1

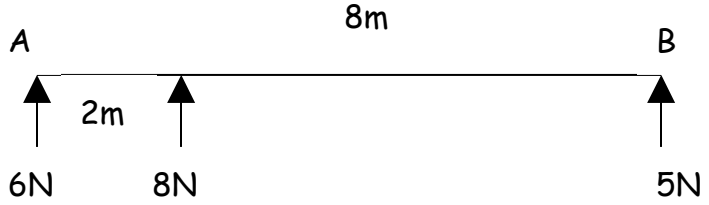
A door of width 1.2m is being pushed by a force of 25N. Find the moment about the hinge.

Moment = Force × Perpendicular Distance

= 25 × 1.2 = 30Nm

Example 2

Forces of magnitude 6N, 8N and 5N are applied to a light rod AB, of length 8m, as outlined in the diagram below.

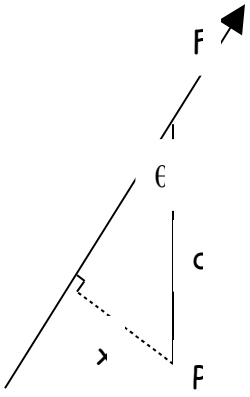


Calculate the sum of the moments about the midpoint of the rod.

Moment of 6N force is given by: 6 × 4 = 24Nm clockwise
Moment of 8N force is given by: 8 × 2 = 16Nm clockwise
Moment of 5N force is given by: 5 × 4 = 20Nm anti-clockwise

Therefore the sum of the moments is 20Nm clockwise.

It is more likely however that the forces are not being applied at right angles to the object.



The diagram above shows a force F, acting on an object P at a given angle θ and given distance d. The force brings about a turning effect at P. The dotted line is the perpendicular distance. $x = d \times \sin\theta$.

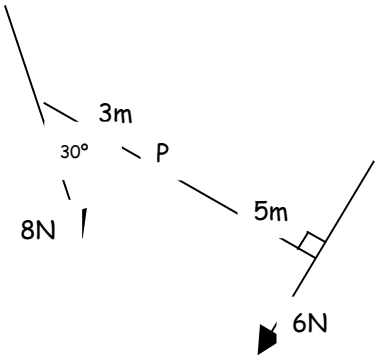
Therefore the moment about P is given by:

Moment = Force × Perpendicular Distance from line of action of force to pivot point

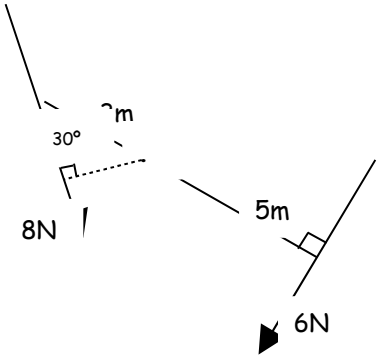
$$\text{Moment} = FdSin\theta$$

Example 3

Two forces are applied to a light rod as outlined in the diagram below. Find the sum of the moment about P.



The 6N force gives a 30Nm clockwise moment.



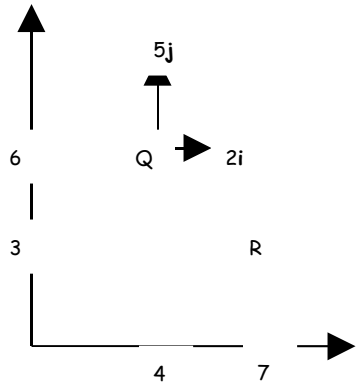
The dotted line has been added to show the perpendicular distance. The 8N force gives an anticlockwise moment of magnitude:

$$8 \times 3 \times Sin30 = 12Nm.$$

Therefore the system is experiencing a clockwise moment of magnitude 18Nm.

Example 4

A point Q(4,6) is acted upon by a force (2**i** + 5**j**)N. Calculate the sum of the moment about the origin.



The 2**i** force gives a clockwise moment of: 2 × 6 = 12Nm
The 5**j** force gives a anticlockwise moment of: 5 × 4 = 20Nm
Therefore the sum of the moments is 8Nm anticlockwise.

Find the sum of the moments about the point R(7,3).
The 2**i** force gives a clockwise moment of: 2 × 3 = 6Nm
The 5**j** force gives a clockwise moment of: 5 × 3 = 15Nm
Therefore the sum of the moments is 21Nm clockwise.

Equilibrium of a lamina under parallel forces

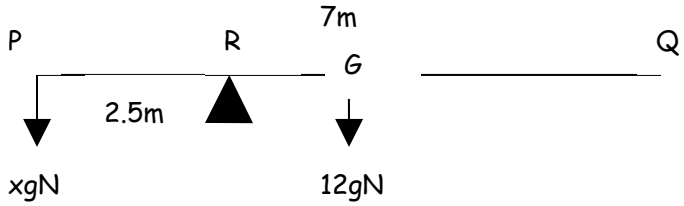
If a system is in equilibrium then the following must hold:

1. The component of the resultant force in any direction must be zero.
2. The algebraic sum of the moments about any point must be zero.

For uniform non light systems the weight acts through the centre of mass. A rod is said to be uniform if it has even weight distribution and therefore the centre of mass acts at the centre of the rod.

Example 5

A uniform rod PQ of length 7m and mass 12kg is pivoted at the point R where PR is 2.5m. Calculate the mass of the particle that must be attached at P to maintain the rod in a horizontal position.



The moment at P about R must equate to the moment at G about R.

The mass of the rod gives a clockwise moment of:

$$12g \times 1 = 12gNm$$

The mass added at P gives an anticlockwise moment of:

$$xg \times 2.5 = 2.5xNm.$$

Therefore:

$$12gNm = 2.5xNm$$

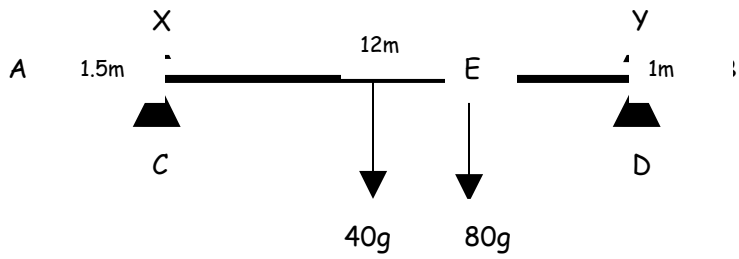
$$x = 4.8Kg$$

Uniform Beams

Example 6

A uniform beam AB of mass 40kg and length 12m is supported in a horizontal position at C and D, where AC = 1.5m and DB = 1m. A man of mass 80Kg stands on the beam at the point E where EB = 2.5m. Find the reactions at C and D.

Examiners always suggest that a diagram is **VITAL**.



Resolving vertically gives:

$$X + Y = 120g \qquad (1)$$

Taking Moments about C gives:

The mass of the beam gives a clockwise moment of: $40g \times 4.5 = 180gNm$
The mass of the man gives a clockwise moment of: $80g \times 8 = 640gNm$
The reaction at D gives an anticlockwise moment of: $9.5YNm$

Therefore:

$$9.5Y = 820g$$

$$Y = \frac{1640}{19}g = 845.9N$$

Using equation (**1**) from above

$$X + Y = 120g$$

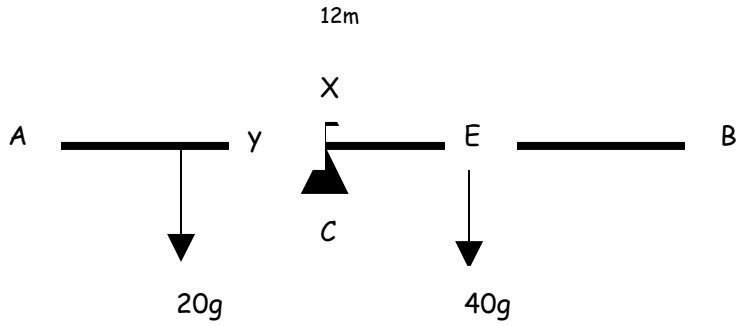
$$X + 845.9 = 120g$$

$$X = 330N$$

Non uniform beams

Example **7**

A non uniform beam AB of mass 20kg and length 12m has an object of mass 40kg placed at a point 8m from A. The beam is in equilibrium in a horizontal position resting on a support C at the midpoint of AB. Find the position of the centre of mass.



Taking moments about C gives:

$$40g \times 2 = 20g \times y$$

Y = 4m

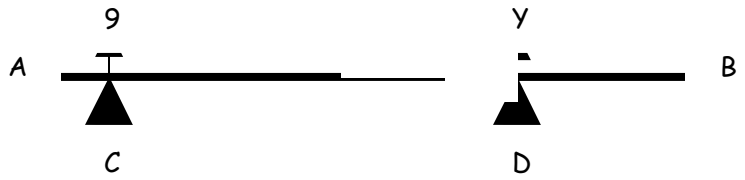
Questions

(some diagrams have forces missing, you need to figure out which ones!)

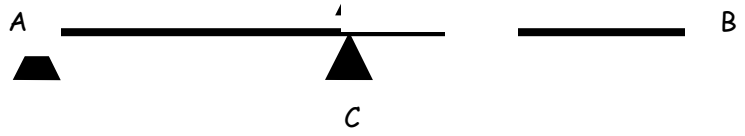
1 A uniform rod AB of weight 70N and length 5m. It rests in a horizontal position supported at point C and D, where AC = 0.4m. the reaction on the rod at C has magnitude 9N. Find

a) the magnitude of the reaction on the rod at D

b) the distance AD.



2 A uniform rod AB of length 6m and mass 40Kg. It is supported by two smooth pivots in a horizontal position at A and C where AC = 3m. A woman of mass 75Kg stands on the rod which remains in equilibrium. The magnitudes of the reactions at the two pivots are equal to R Newtons.

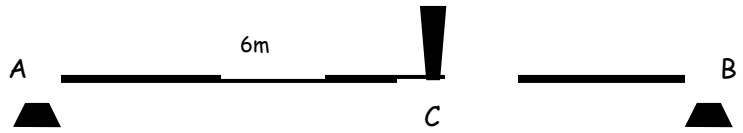


Find

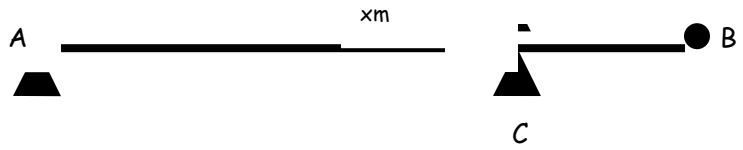
a) the value of R.

b) the distance of the woman from A.

3 A non uniform plank of wood AB of length 10m and mass 100Kg is smoothly supported in a horizontal position at A and B. An object of mass 90Kg is put on the plank at C, where AC = 6m. The plank is in equilibrium and the magnitudes of the reactions at A and B are equal. Find:
a) the magnitude of the reaction R, on the plank at B.
b) the distance, x, of the centre of mass of the plank from A.



4 A uniform plank AB has weight 80N and length xm. The plank rests in equilibrium on two supports at A and C, where AC = 3m. A rock of weight 20N is placed at B and the plank remains in equilibrium. The reaction on the plank at C has magnitude 70N.
a) find the value of x

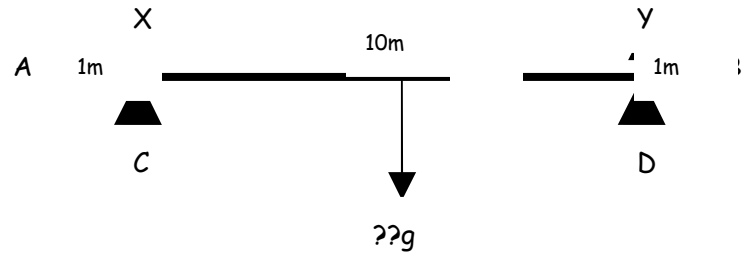


The support at A is now moved to a point D on the plank and the plank remains in equilibrium with the rock at B. The reaction on the plank at C is now two times the reaction at D.
b) find the distance AD.

5 A non uniform rod, AB, of length 7m and mass 10Kg is suspended in equilibrium in a horizontal position by ropes attached to the points E and F of the rod, where AE = 2m and AF = 6m. The tensions in the ropes are equal. Find the distance of the centre of mass from A.

Extension

1 A large log AB is 10m long. It rests in a horizontal on two supports C and D, where AC = 1m and BD = 1m. An estimate of the weight of the log is required, but the log is too heavy to lift off the supports. When a force of magnitude 1100N is applied vertically to the log at A, the log is about to tilt about D.
a) state the value of the reaction on the log at C for this case.
b) by modeling the log as a uniform rod, estimate the weight of the log.



The force at A is removed and a force vertically upwards is applied at B. The log is about to tilt about C when the force has a magnitude of 1600N. By modeling the log as a non uniform rod, with the distance of the centre of mass of the log being x metres from A, find:
c) a new estimate for the weight of the log.
d) the value of x.