

GCSE 9 -1 Mathematics Higher Tier Grade 9 'Tough Paper' Paper 1



Total marks 80
1 Hour 30 minutes

PLEASE NOTE:

This paper does not claim the questions included are 'Grade 9 questions'.
This paper was designed for pupils aiming for Grade 9s who are looking for
challenging questions within the GCSE 9-1 syllabus.

**CLICK ON THE
QUESTION NUMBER
FOR THE WORKED
VIDEO SOLUTION**

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[illegible]

(Total for Question 1 is 4 marks)

(2) Freda plays the lottery.

There are 49 balls to choose from.

The balls are numbered 1 – 49.

Freda chooses the 6 numbers shown below in the order in which they appear.



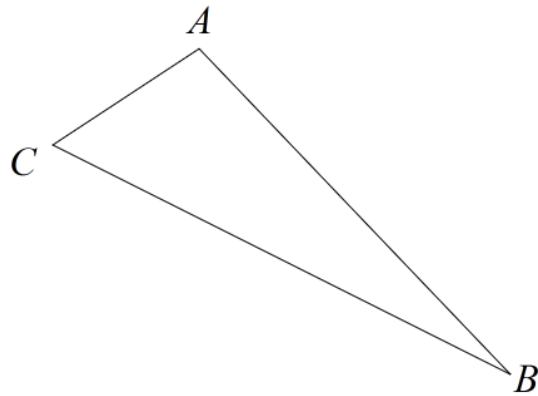
John believes the numbers were chosen randomly.

Show that John could be wrong stating a reason for your choice.

[illegible]

(Total for Question 2 is 4 marks)

(3) Triangle ABC is shown in the diagram below.



$$AC = x$$

$$BC = 3x$$

Angle $ACB = 60^\circ$

Show that the perimeter of the triangle is $(4 + \sqrt{7})x$.

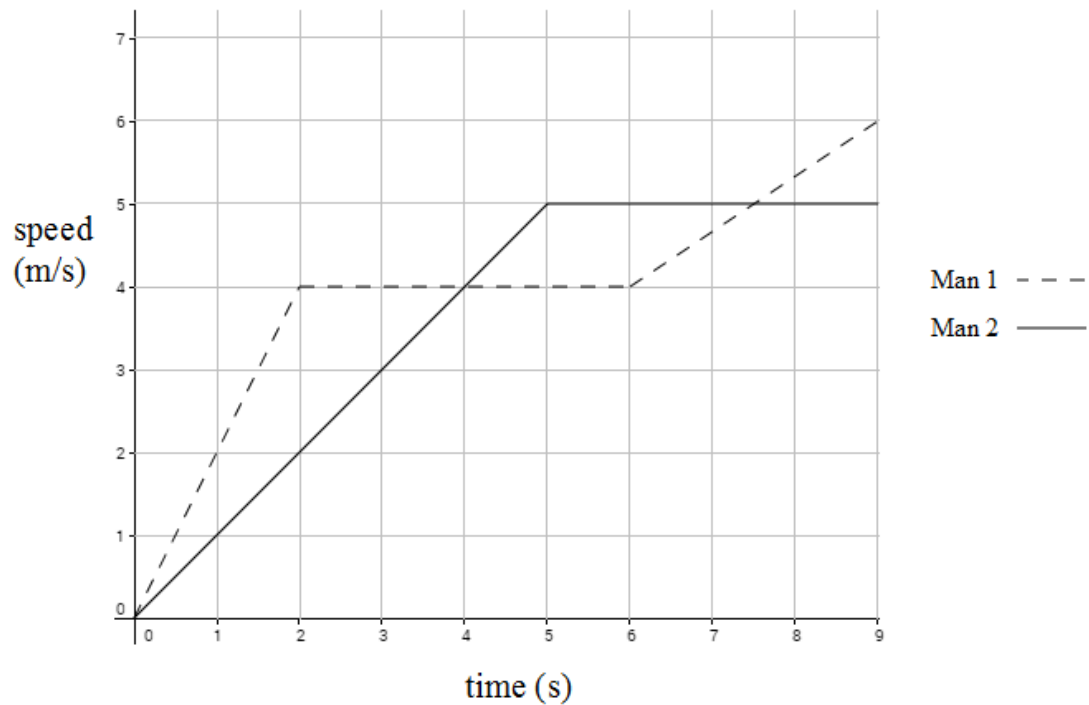
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(Total for Question 3 is 5 marks)

[illegible]

(Total for Question 5 is 2 marks)

(5) Two men walk together along a road, starting at the same time. The speed-time graph below shows the first 9 seconds of the walk.



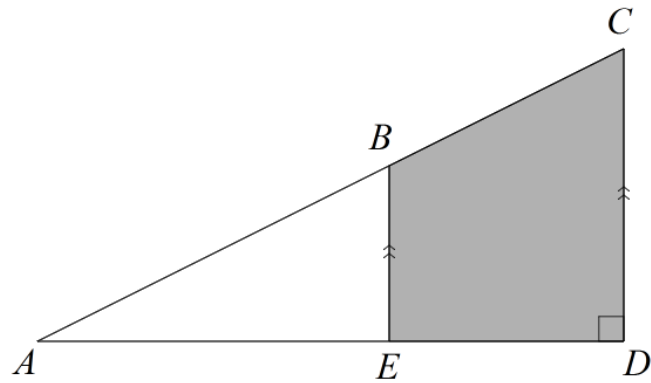
The ratio of the distance covered by Man 1 to the distance covered by Man 2 in the first 9 seconds of the walk can be written in the form $m:n$ where m and n are double digit integers.

Find the value of m and the value of n .

[illegible]

(Total for Question 5 is 5 marks)

(6) Triangle ACD is shown in the diagram below.



AED is a straight line.

$$AB = 3\sqrt{5}$$

$$AE = 2BE$$

$$3AD = 5AE$$

Find the area of the shaded quadrilateral $BCDE$.

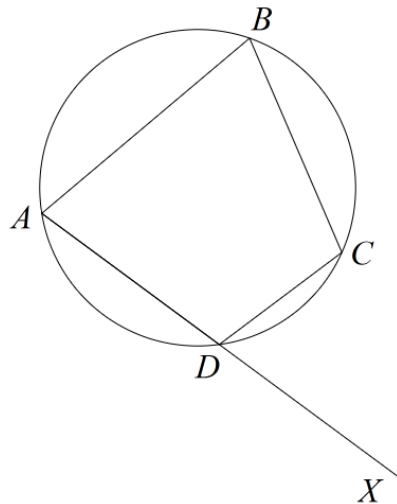
[illegible]

(Total for Question 6 is 5 marks)

[illegible]

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(8) A, B, C and D are all points on the circumference of a circle as shown in the diagram below.



$$\text{Angle } DAB = x^2 - 5x - 8$$

$$\text{Angle } BCD = x^2 + 4x - 88$$

$$\text{Angle } CDA = y^2 - 15y + 90$$

$$\text{Angle } ABC = 5y - 6$$

A line is drawn from D to X .

$$\text{Angle } CDX = x^2 - 70$$

Prove that ADX is a straight line.

[illegible]

(Total for Question 8 is 6 marks)

(9) The first five terms of an arithmetic sequence are:

$$x+1, \quad 2x, \quad \frac{2(2x+3)}{6-x}, \quad x^2-2, \quad 5x-3$$

Show that the term $4x^2 - 3$ is not in the sequence.

[illegible]

(Total for Question 9 is 6 marks)

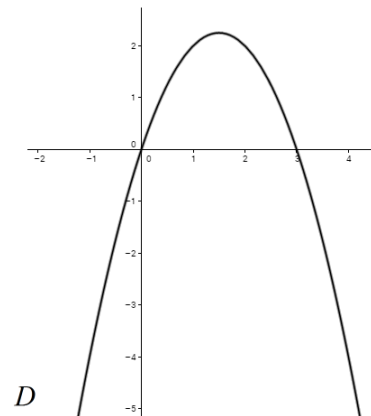
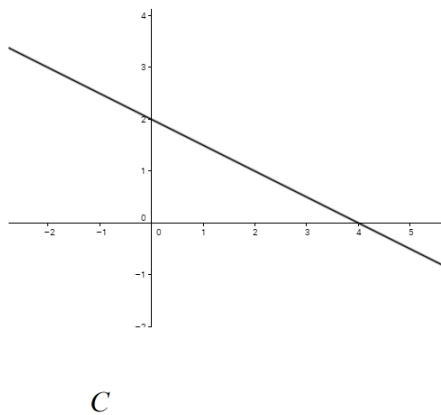
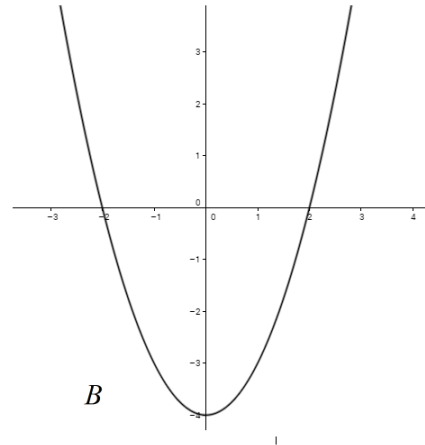
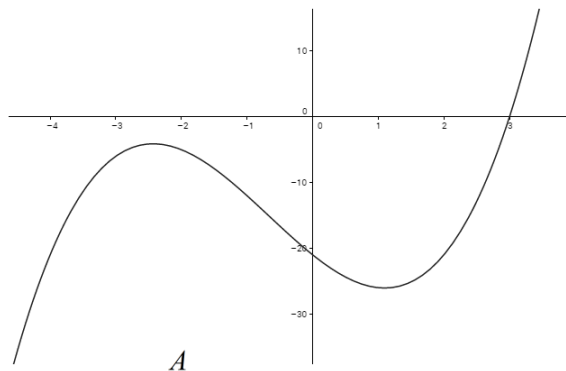
(11) Two functions are given below:

$$f(x) = (x + p)(x + q)$$

$$g(x) = \frac{r}{x}, \quad x \neq 0$$

p, q and r are constants.

State which of the following graphs could be used to solve the equation $f(x) = g(x)$
You must give a reason for your choice.



(Total for Question 11 is 3 marks)

When $A = 0.5$, $B = 64$.

Express C in terms of A .

(Total for Question 12 is 5 marks)

The two games are Game A and Game B.

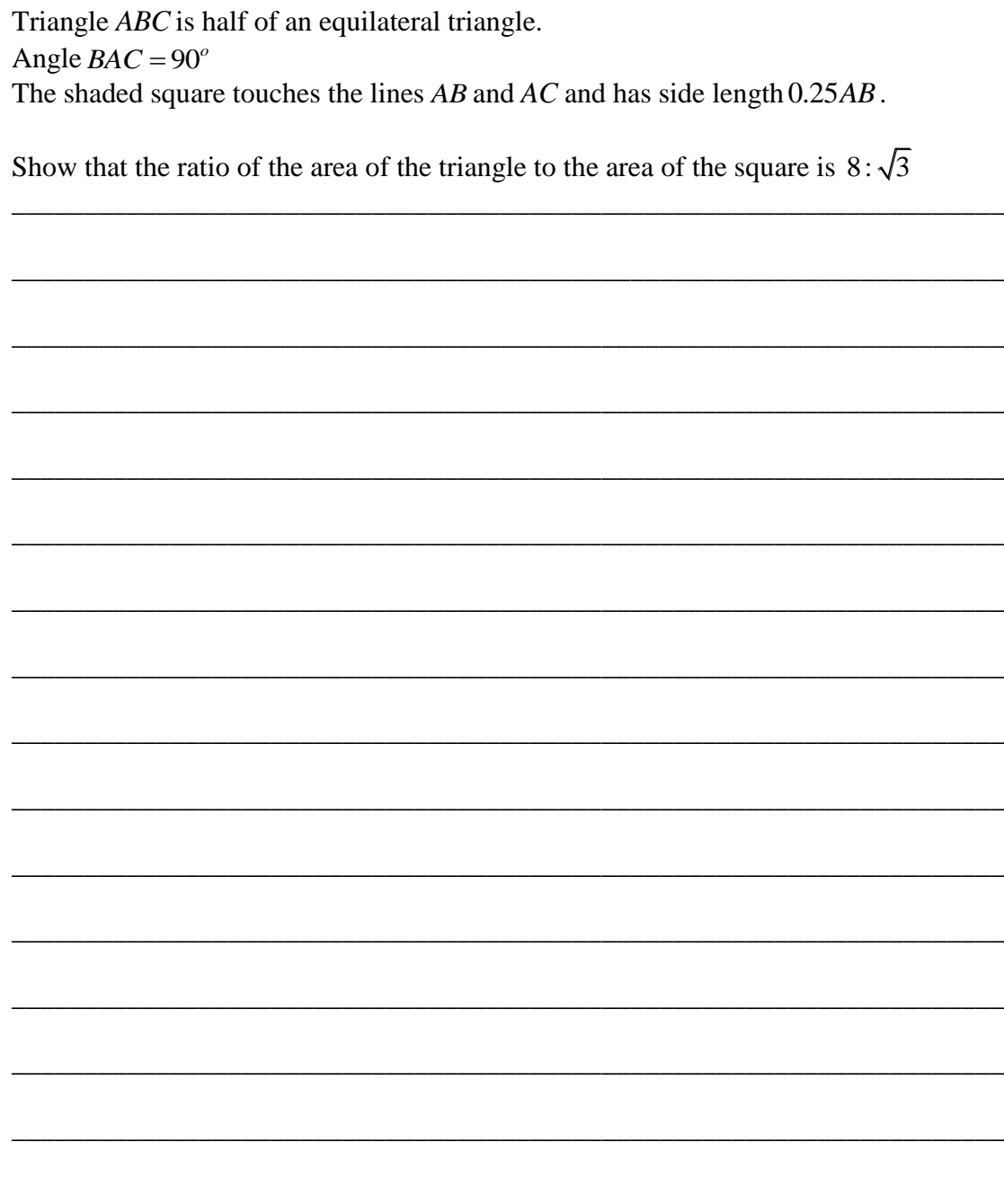
The probability that Mr Lucky wins both games is $\frac{9}{25}$.

Find the probability that Mr Lucky wins only one of the two games he plays.

[illegible]

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A right-angled triangle ABC is shown, with the right angle at vertex A . A square is inscribed within the triangle, with one vertex at A , one vertex on side AB , and one vertex on side AC . The hypotenuse of the triangle is BC .



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A graph of a shaded region R in the xy -plane. The region is a triangle with vertices at $(0, 12)$, $(0.6, 8)$, and $(0.4, 4)$. The x -axis ranges from -0.2 to 0.6 , and the y -axis ranges from 0 to 12 . The region is labeled with a bold R .

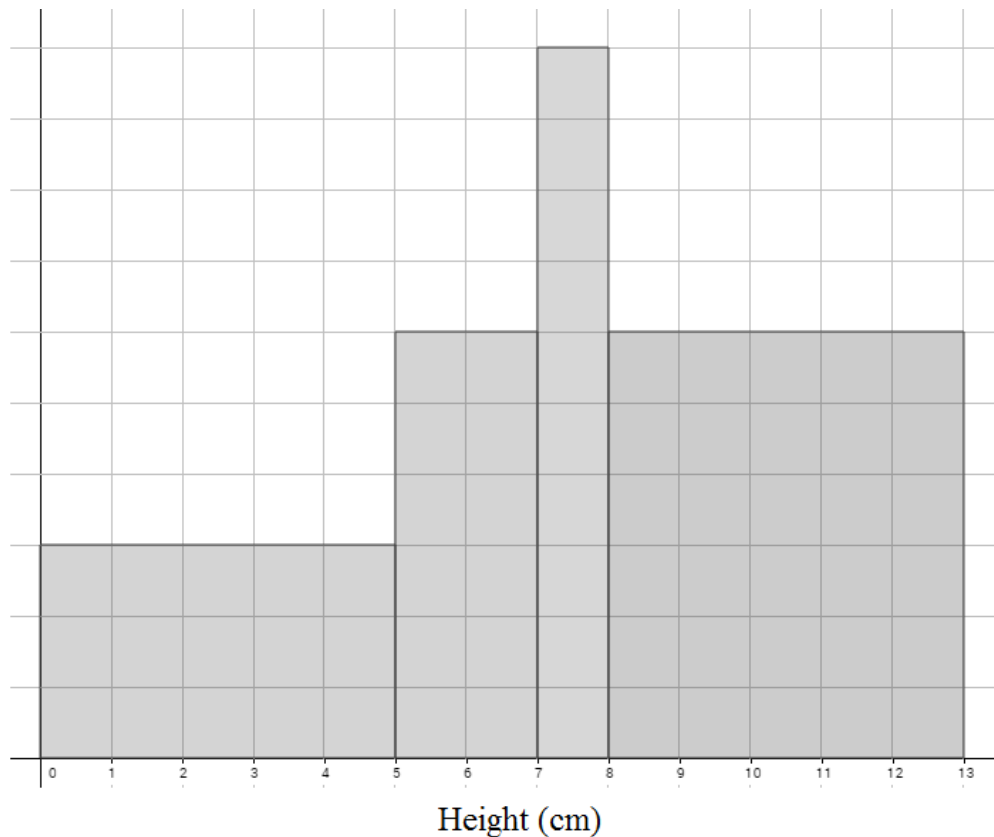
[illegible]

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[illegible]

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(17) The histogram below shows information about the height (cm) of a number of plants.



There were 40 plants between 7 and 8cm tall.

Michael takes two plants at random from the sample and doesn't replace them. He writes down his calculations for the probability and its answer as:

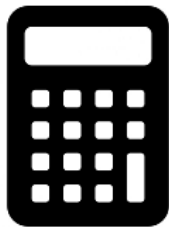
$$\frac{30}{67} \times \frac{16}{89} = \frac{480}{5963}.$$

Write down the minimum height of each of the plants Michael chooses.

[illegible]

(Total for Question 17 is 5 marks)
TOTAL FOR PAPER IS 80 MARKS

GCSE 9 -1 Mathematics Higher Tier Grade 9 'Tough Paper' Paper 2



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(1) Island X is a small island.

In the winter of 1986 the ratio of natives to tourists on the island was 7:1.

In the summer of 1987 the ratio of natives to tourists on the island was 155:69.

The number of natives on the island decreased by 100 from the winter of 1986 to summer of 1987.

The number of tourists on the island increased by 220 from the winter of 1986 to summer of 1987.

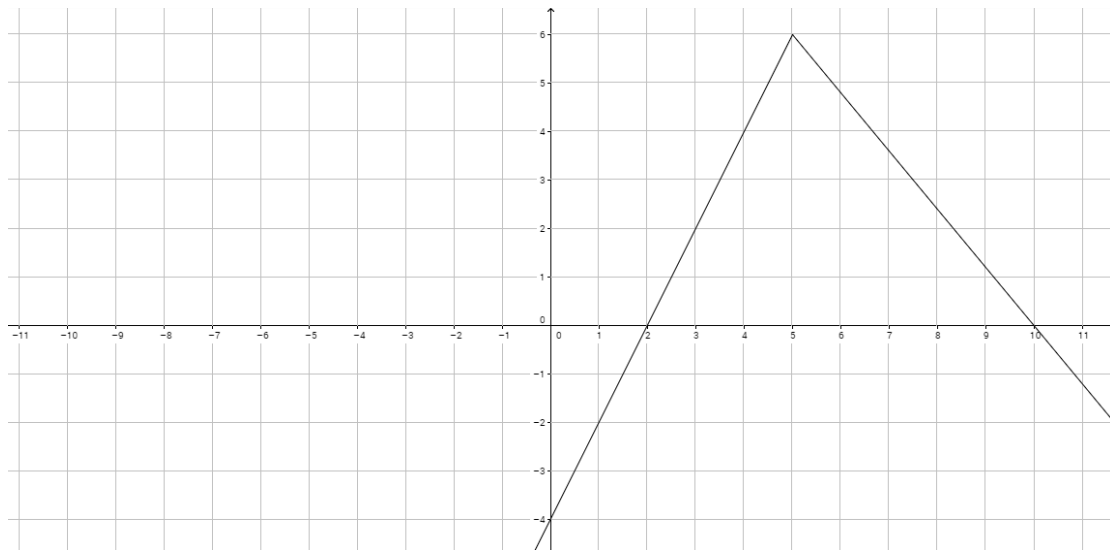
Find the number of tourists that were there on Island X in the winter of 1986.

(Total for Question 1 is 4 marks)

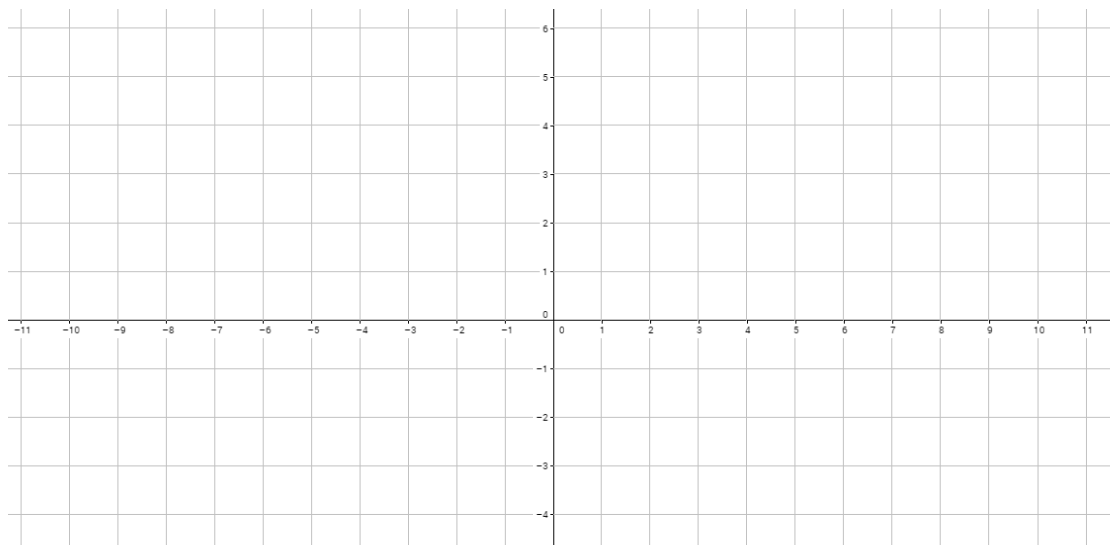
[illegible]

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(3) The diagram below shows part of the graph of $y = 2f(x-1)$



On the grid below draw the graph of $y = -f(-x)$



(Total for Question 3 is 4 marks)

[illegible]

(Total for Question 4 is 6 marks)

The flight path of the toy rocket can be modelled by the equation $h = -2t^2 + 6t + 1$.
 h is the height in metres the rocket reaches above the ground.
 t is the time in seconds after the rocket is launched.

[illegible]

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(6) Shape A is a regular polygon.

The ratio of the size of the interior angle to the size of the exterior angle is 7:2.

The ratio of the side **length** to the **number** of sides is 5:3.

Find the perimeter of Shape A .

[illegible]

(Total for Question 6 is 4 marks)

Jamal will start from Point A and paddle on a bearing of 050° until he reaches Point B. Once at Point B, Jamal will then paddle 1.6km on a bearing of 142° to reach Point C. Finally Jamal will paddle directly back to Point A from Point C where he will finish. The bearing of Point C from Point A is 098° .

Give your answer to the nearest minute.

[illegible]

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(8) Triangle ABC is an isosceles triangle.

$$AB = BC$$

The points X and Y lie on the line AC .

$$AY = 3AX$$

$$AC = 4AX$$

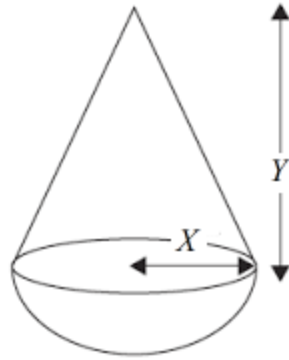
Prove that triangle ABX and triangle CBY are congruent.

[illegible]

(Total for Question 8 is 3 marks)

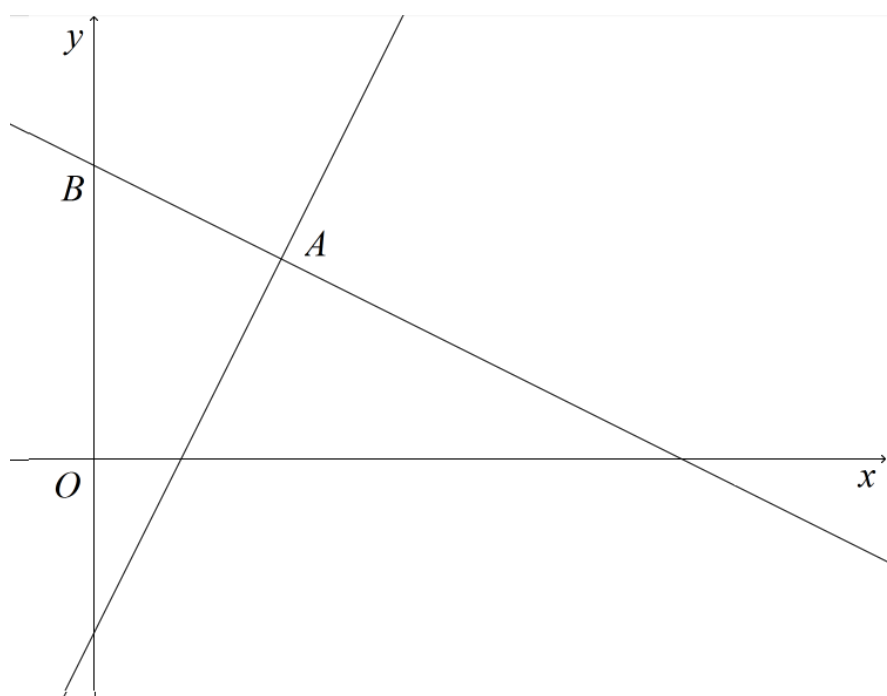
The cone will be attached to the top of the hemisphere as shown below.

Give your answer in its simplest form.



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(11) The diagram below shows Line 1 and Line 2.



Line 1 has gradient 2.

Line 2 is perpendicular to Line 1.

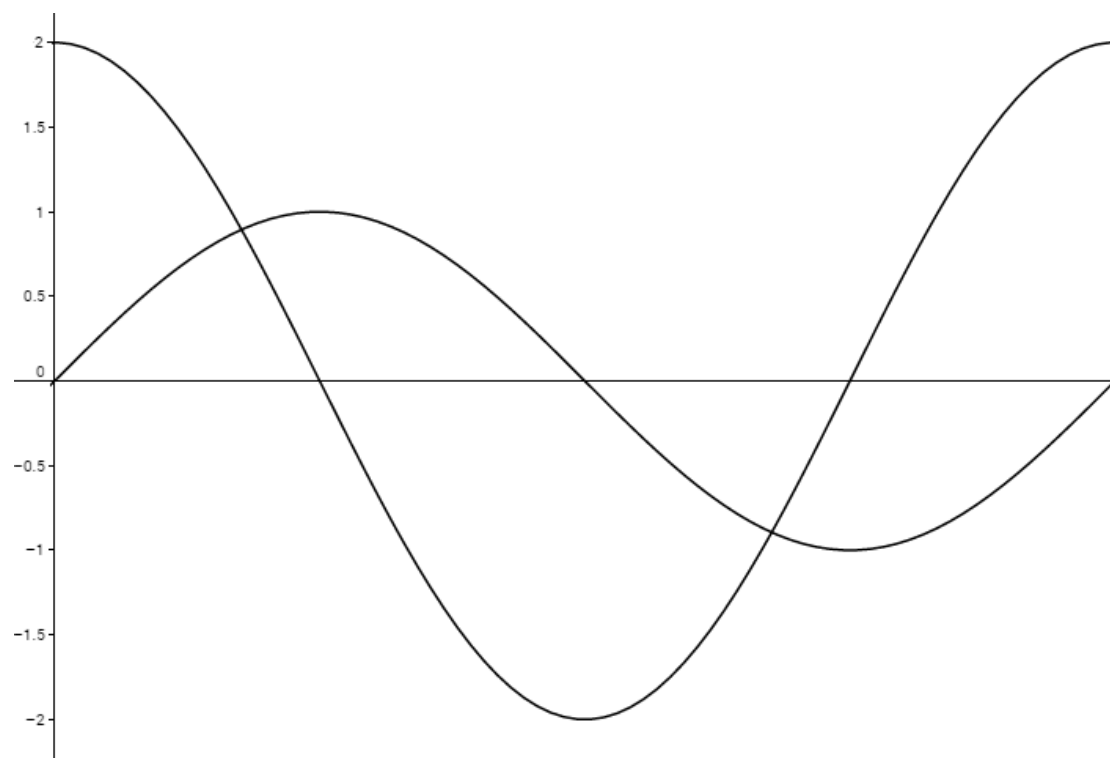
Line 1 and Line 2 intersect at the point $A(p, q)$.

Line 2 crosses the y axis at the point B .

Show that the coordinates of point B can be written as $\left(0, \frac{p}{2} + q\right)$.

(Total for Question 11 is 4 marks)

(12) The graphs of $y = 2\cos(x)$ and $y = \sin(x)$ are shown in the diagram below for $0 \leq x \leq 360^\circ$.



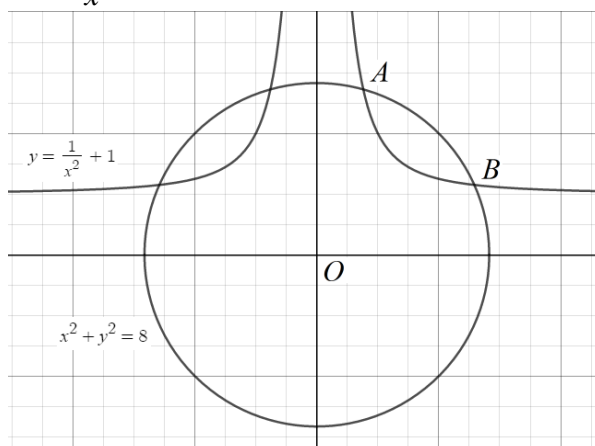
Use the graphs to find estimates for the solutions of the equation:

$$\sin(x) - 2\cos(x) = 0 \text{ for } 0 \leq x \leq 360^\circ.$$

You must show all of your working.

(Total for Question 12 is 4 marks)

(13) The diagram below shows the graph of the equation $x^2 + y^2 = 8$ and part of the graph of the equation $y = \frac{1}{x^2} + 1$.



The points $A(p, q)$ and $B(r, s)$ are two of the points where the graphs intersect.

(a) Using the graphs above, find the solutions to the simultaneous equations:

$$y = \frac{1}{x^2} + 1$$

$$x^2 + y^2 = 8$$

giving your answers in terms of p, q, r and s .

There are 2 real solutions to the simultaneous equations:

$$x^2 + y^2 = 8$$

$$x = a$$

(b) Find the set of values of a giving your answer in simplified surd form.

(Total for Question 13 is 5 marks)

(14) The students in Class X and Class Y sat the same maths exam.
Information is given about the performance of each class in the table below.

	X	Y
Lowest Score	$x - 1$	$y + 1$
Lower Quartile	$x + 2$	$2(y + 1)$
Median	$x^2 - 3$	$y(y - 1)$
Upper Quartile	$4x + 2$	$3y + 1$
Highest Score	$2(x^2 + 2)$	$5y - 4$

The median score for Class X was half the median score for Class Y .
The interquartile range for Class X was three times the interquartile range for Class Y .

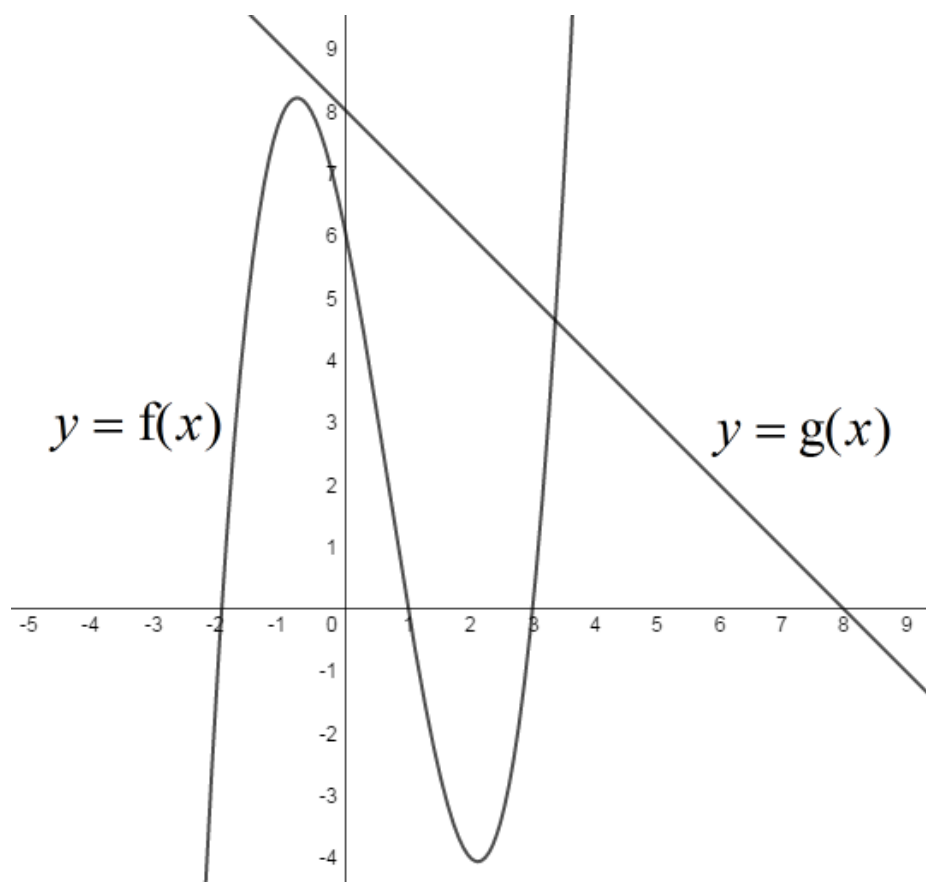
Michael scored 17 marks in his maths exam.
Complete the following sentence;

“Michael was in the top _____ % of performers in Class _____”

You must show all of your working.

(Total for Question 14 is 6 marks)

(15) The diagram below shows parts of the graphs of $y = f(x)$ and $y = g(x)$.



Find the integer value of $gff(-2)$.

(Total for Question 15 is 3 marks)

The diagram illustrates a geometric configuration involving two circles and several lines. A large circle on the left and a smaller circle on the right are tangent to each other at a single point. A vertical line passes through the center of the large circle, with points A and B marked on it. A line passing through point A is tangent to the large circle. A line passing through point B is tangent to the small circle. A line passing through point C , the center of the small circle, is also tangent to the large circle.

Circle 2 has radius 4cm.

The line AB lies on a diameter of Circle 1 .

AC and BC are tangents to Circle 1 and pass through C .

Give your answer to 3 significant figures.

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Find the largest possible size of the two acute angles in the parallelogram. Give your answer correct to 3 decimal places.

[illegible]

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