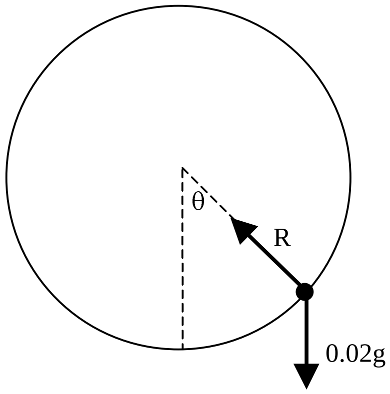
VERTICAL CIRCULAR MOTION

1.



$\omega = \dfrac{2\pi \times 1000}{60} = \dfrac{100\pi}{3}$ rad s^{-1} — B1

At all times, $R - 0.02g\cos\theta = 0.02 \times 0.5\left(\dfrac{100\pi}{3}\right)^2$ — M1 (resolving) M1($mr\omega^2$) A1

$R = 0.01 \times \dfrac{10000\pi^2}{9} + 0.02g\cos\theta$

So $R_{max} = 0.01 \times \dfrac{10000\pi^2}{9} + 0.02g$ — M1

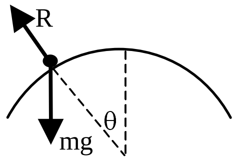
$= 109.86$ N

$\Rightarrow$ 110 N — A1

[6]

VERTICAL CIRCULAR MOTION

2. a)

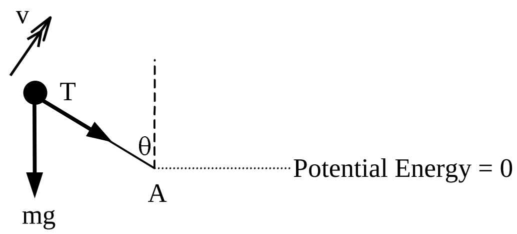


At any time, $mg\cos\theta - R = \dfrac{mv^2}{r}$	M1 A1
$= \dfrac{mv^2}{50}$	
We require $R \geq 0$	M1
So $mg\cos\theta - \dfrac{mv^2}{50} \geq 0$	A1
$50g\cos\theta \geq v^2$	
minimum value of $\cos\theta$ is when $\theta = 60°$	B1
So $\dfrac{50g}{2} \geq v^2$	
$V_{max} = 15.7\ ms^{-1}$	B1
	[6]

b) Car is modelled as a particle	B1
	[1]

VERTICAL CIRCULAR MOTION

3. a)



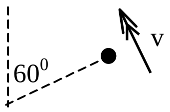
Using conservation of energy	M1
Initially k.e. = $\frac{1}{2}mu^2$	
p.e. = $-mgL$	A1 (both)
When string makes angle θ with upward vertical	
k.e. = $\frac{1}{2}mv^2$	
p.e. = $mgL\cos\theta$	A1 (both)
So $\frac{1}{2}mv^2 + mgL\cos\theta = \frac{1}{2}mu^2 - mgL$	A1
Resolving inwards:	M1
$T + mg\cos\theta = \dfrac{mv^2}{L}$	A1
Combining:	
$T = \dfrac{mu^2}{L} - 2mg - 2mg\cos\theta - mg\cos\theta$	M1
When $\theta = \alpha$, $T = 0$	M1
$0 = \dfrac{mu^2}{L} - 2mg - 3mg\cos\alpha$	
$u^2 = gL(2 + 3\cos\alpha)$	A1
	[9]

b) $v^2 = u^2 - 2gL - 2gL\cos\theta$	
$= 3gL\cos\alpha + 2gL - 2gL - 2gL\cos\alpha$	M1 (substituting)
$v = \sqrt{gL\cos\alpha}$	A1
	[2]

VERTICAL CIRCULAR MOTION

QUESTION 3 CONTINUED

c) $\alpha = 60^\circ \;\Rightarrow\; v = \sqrt{\dfrac{gL}{2}}$ B1



Upward component of v is $v\cos 30^\circ = \sqrt{\dfrac{3gL}{8}}$ M1 A1

Using "$2as = v^2 - u^2$" M1

$a = -g$; $v = 0$; $u = \sqrt{\dfrac{3gL}{8}}$ B1 (all)

$-2gs = 0 - \dfrac{3gL}{8}$

$s = \dfrac{3L}{16}$ A1

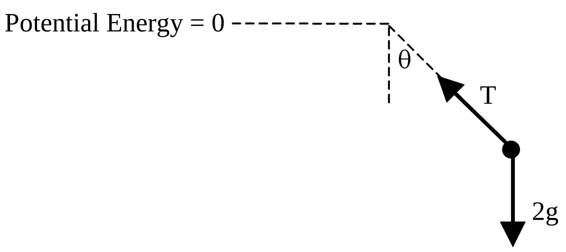
So maximum height above A $= L\cos 60^\circ + \dfrac{3L}{16}$ M1

$= \dfrac{11L}{16}$ A1

[8]

VERTICAL CIRCULAR MOTION

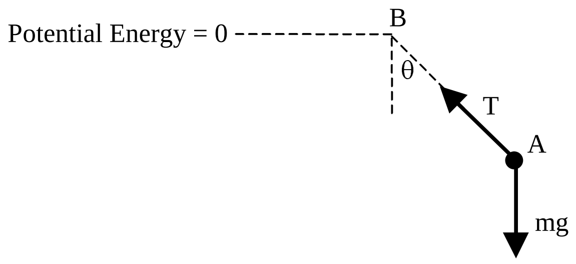
4. a)



Conservation of energy:	M1
When $\theta = 60^\circ$ p.e. $= -2g\cos 60^\circ$	A1
$= -g$	
k.e. $= 0$	B1
When $\theta = 0$ p.e. $= -2g$	
k.e. $= \frac{1}{2} \times 2u^2$	A1 (both)
So: $-g = u^2 - 2g$	
$g = u^2$	
$u = \sqrt{g}$ (= 3.13 ms^{-1})	A1
	[5]
b) When $\theta = 10^\circ$, p.e. $= -2g\cos 10^\circ$	
k.e. $= \frac{1}{2} \times 2v^2$	B1 (both)
$v^2 - 2g\cos 10^\circ = -g$	
$v^2 = 2g\cos 10^\circ - g$	B1
Resolving inwards: $T - 2g\cos 10^\circ = 2v^2$	M1 A1
$T = g\left(4\cos 10^\circ - 2 + 2\cos 10^\circ\right)$	
$= 38.3$ N	A1
	[5]
c) This occurs when $\theta = 0^\circ$	M1
Resolving:	
$T - 2g = 2g \Rightarrow T = 4g = 39.2$N	A1
	[2]

VERTICAL CIRCULAR MOTION

5. a)



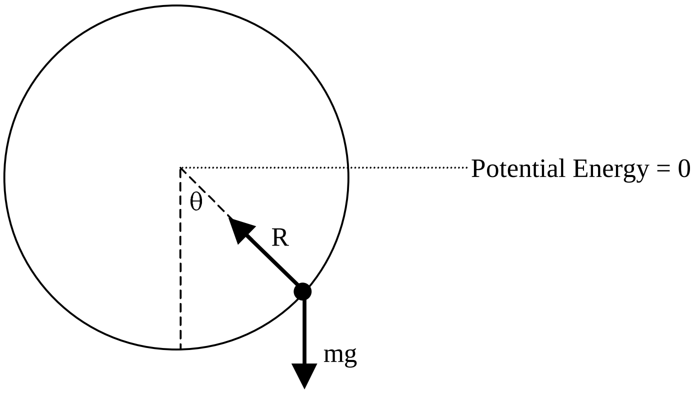
Conservation of energy	M1
Initially, p.e. = -mgL	
k.e. = $\frac{1}{2}mu^2$	A1 (both)
When AB makes an angle θ:	
p.e. = $-mgL\cos\theta$	
k.e. = $\frac{1}{2}mv^2$	A1 (or using $\theta = 180^\circ$)
So $\frac{1}{2}mu^2 - mgL = \frac{1}{2}mv^2 - mgL\cos\theta$	A1
Since rigid rod, just require $v^2 \geq 0$, when $\theta = 180^\circ$	M1
For U_{min}: $\frac{1}{2}mu^2 - mgL = mgL$	
$u = 2\sqrt{gL}$	A1
	[6]

b) Conservation of momentum:	
$m \times 2\sqrt{gL} = 2mv$	M1
$\sqrt{gL} = v$	A1
	[2]

c) Using energy equation;	M1
$\frac{1}{2} \times 2mgL - 2mgL = \frac{1}{2} \times 2mv^2 - 2mgL\cos\theta$	
Rises until $v = 0$	M1
$mgL - 2mgL = -2mgL\cos\theta$	
$\cos\theta = \frac{1}{2}$	A1
So height is $\frac{1}{2}L$ below B.	B1
	[4]

VERTICAL CIRCULAR MOTION

6. a)



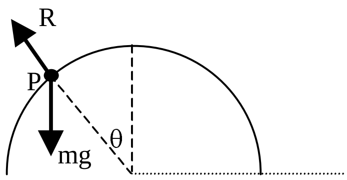
Conservation of energy:	M1
Initially: p.e. = -mg	
k.e. = $\frac{1}{2}mu^2$	A1 (both)
At angle θ	
p.e. = $-mg\cos\theta$	
k.e. = $\frac{1}{2}mv^2$	A1 (both)
So $\frac{1}{2}mu^2 - mg = \frac{1}{2}mv^2 - mg\cos\theta$	
Oscillations $\Rightarrow$ v becomes 0 when $\theta \le 90^\circ$	M1
$v = 0 \Rightarrow u^2 - 2g = -2g\cos\theta$	
$\theta = 90^\circ \Rightarrow u^2 - 2g = 0$ so $u_{max} = \sqrt{2g}$	A1
	[5]

b) $u = \frac{1}{2}\sqrt{2g}$	
So using energy equation:	M1
$\frac{1}{2}m \times \frac{1}{4} \times 2g - mg = \frac{1}{2}mv^2 - mg\cos\theta$	A1
At maximum height, $v = 0$	M1
$\frac{g}{4} - g = -g\cos\theta$	
$\cos\theta = \frac{3}{4}$	A1
Maximum height $= L - L\cos\theta = \frac{1}{4}L$	M1 A1
	[6]

c) This is when $\theta = 0^\circ$	M1
Resolving inwards	M1
$R - mg = m\left(\frac{1}{2}\sqrt{2g}\right)^2 = \dfrac{3mg}{2}$	A1
	[3]

VERTICAL CIRCULAR MOTION

7. a)

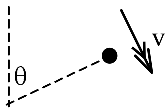


Using conservation of energy:	M1
Initially k.e. = 0	
p.e. = 0.6mg	A1 (both)
When OP makes an angle θ with vertical	
k.e. = $\frac{1}{2}mv^2$	
p.e. = $0.6mg\cos\theta$	A1 (both)
So $\frac{1}{2}mv^2 + 0.6mg\cos\theta = 0.6mg$	
$\Rightarrow v^2 = 1.2g(1-\cos\theta)$	B1
Resolving inwards:	
$mg\cos\theta - R = \dfrac{mv^2}{0.6}$	M1 A1
$mg\cos\theta - R = 2mg(1-\cos\theta)$	A1 (f.t.)
Loses contact $\Rightarrow$ R = 0	M1
$mg\cos\theta = 2mg - 2mg\cos\theta$	
$\cos\theta = \frac{2}{3}$	A1
Height = $L\cos\theta = 0.4$	A1
	[10]

VERTICAL CIRCULAR MOTION

QUESTION 7 CONTINUED

b)



When particle loses contact,

$v^2 = 1.2g\left(1 - \frac{2}{3}\right) = 0.4g \Rightarrow v = \sqrt{0.4g}$ B1

Vertical component $= v\sin\theta$ M1

$= \sqrt{0.4g}\sqrt{1-\left(\frac{2}{3}\right)^2}$ A1

$= \frac{\sqrt{5}}{3}\sqrt{0.4g}$

$= \frac{\sqrt{2g}}{3}$ A1 (or decimal)

Time to hit table:

$s = ut + \frac{1}{2}at^2$ M1

$s = 0.4 \quad u = \frac{\sqrt{2g}}{3} \quad a = g$ B1

$0.4 = \frac{\sqrt{2g}}{3}t + \frac{g}{2}t^2$ M1 (solving)

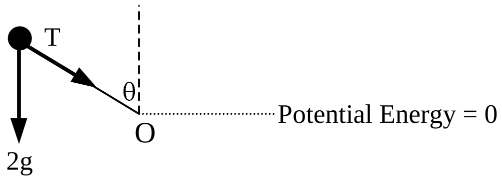
$t = 0.172$ or -0.474

$\Rightarrow t = 0.172$ seconds A1

[8]

VERTICAL CIRCULAR MOTION

8. a)

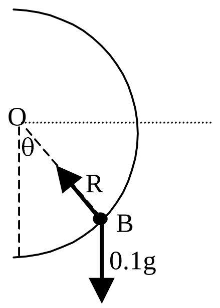


Conservation of energy	M1
Initially p.e. = 4g	
k.e. = 25	A1 (both)
When string makes an angle θ:	
p.e. = $4g\cos\theta$	
k.e. = v^2	A1 (both)
So $25 + 4g = v^2 + 4g\cos\theta$	
$v^2 = 25 + 4g(1-\cos\theta)$	B1
$v = \sqrt{25+4g(1-\cos\theta)}$	
Resolving inwards:	
$T + 2g\cos\theta = \dfrac{2v^2}{2}$	A1
$T + 2g\cos\theta = 25 + 4g(1-\cos\theta)$	M1 (combining)
$T = 25 + 2g(2-3\cos\theta)$	A1
	[8]

b) $T_{max} = 25 + 2g(5)$	
$= 123$ N	B1
$T_{min} = 25 + 2g(-1)$	
$= 5.4$ N	B1
	[2]

VERTICAL CIRCULAR MOTION

9. a)



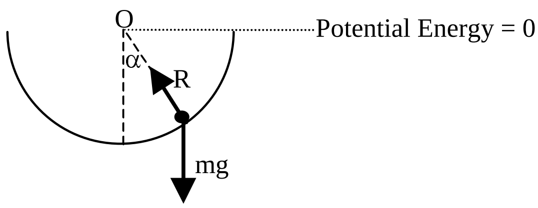
Conservation of energy	M1
Initially: p.e. = $-0.1 \times 0.3g$	
k.e. = $\frac{1}{2} \times 0.1u^2$	A1 (both)
When OB makes angle θ with downward vertical:	
p.e. = $-0.03g\cos\theta$	
k.e. = $\frac{1}{2} \times 0.1v^2$	A1 (both)
So $-0.03g + 0.05u^2 = -0.03g\cos\theta + 0.05v^2$	
Require $v^2 \geq 0$ when $\theta = 180^\circ$	M1
For u_{min}: $-0.03g + 0.05u^2 = 0.03g$	
$u^2 = \frac{6g}{5} \Rightarrow u_{min} = \sqrt{\frac{6g}{5}}$	A1
	[5]

b) Resolving inwards:	
$R - 0.1g\cos\theta = \frac{0.1v^2}{0.3}$	M1 A1
Substituting from energy equation:	M1
$R - 0.1g\cos\theta = \frac{1}{3}\left(\frac{6g}{5} - \frac{3g}{5} + \frac{3g\cos\theta}{5}\right)$	A1
$R = \frac{g}{5} + \frac{3g}{10}\cos\theta$	A1
$R = 0 \Rightarrow \cos\theta = -\frac{2}{3}$	M1 A1
$\theta = 132^\circ$	A1
	[8]

c) Initially no vertical velocity	B1
$s = ut + \frac{1}{2}at^2 \Rightarrow 0.3 = \frac{1}{2}gt^2$	M1
t = 0.247 seconds	A1
	[3]

VERTICAL CIRCULAR MOTION

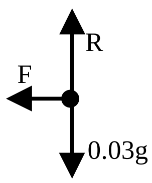
10. a)



Conservation of energy:	M1
Initially k.e. = p.e. = 0	B1
When OP makes angle α :	
k.e. = $\frac{1}{2}mv^2$ p.e. = $-mga\cos\alpha$	A1
So $\frac{1}{2}mv^2 = mga\cos\alpha$	
$v = \sqrt{2ga\cos\alpha}$	A1
	[4]
b) $\alpha = 60^\circ \Rightarrow v = \sqrt{ga}$	B1
Conservation of momentum	M1
$m\sqrt{ga} = 3mV$	
$V = \frac{1}{3}\sqrt{ga}$	A1
Conservation of energy for combined particle:	M1
Initially (at angle 60°)	
k.e. = $\frac{1}{2}\times 3m \times \left(\frac{1}{3}\sqrt{ga}\right)^2$	A1
p.e. = $\frac{-3mga}{2}$	A1
When OP makes angle θ with downward vertical:	
k.e. = $\frac{1}{2}\times 3mv^2$	
p.e. = $-3mga\cos\theta$	B1 (both)
So $\frac{mga}{6} - \frac{3mga}{2} = \frac{3mv^2}{2} - 3mga\cos\theta$	
Rises until v = 0	M1
$\frac{-4mga}{3} = -3mga\cos\theta$	
$\frac{4}{9} = \cos\theta$	A1
height is $\frac{4}{9}a$ below 0	A1
	[10]

VERTICAL CIRCULAR MOTION

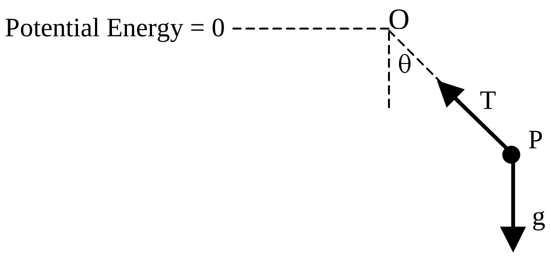
11. a)



$F = 0.1 \times 0.03g$	M1
$= 0.003g$	A1
Work done against friction = loss in energy	M1
Initial energy = $\frac{1}{2} \times 0.03 \times 1.5^2$	B1
Work done by friction = force $\times$ distance = $0.003g \times 0.6$	M1
So final energy = $\frac{1}{2} \times 0.03v^2 = \frac{1}{2} \times 0.03 \times 1.5^2 - 0.003g \times 0.6$	
$v = 1.036\,\text{ms}^{-1}$	A1
	[6]
b) i) Rises until k.e. = 0	M1
Conservation of energy:	M1
$0.03gh = \frac{1}{2} \times 0.03v^2$	
$h = 0.0548\,\text{m}$	A1
	[3]
ii) Before point B, R = 0.03g	B1
After: resolving inwards: $R - 0.03g = \dfrac{0.03v^2}{0.2}$	M1 A1
$R = 0.455$ N	A1
Difference = 0.161 N	B1
	[5]
iii) It would decrease it	B1
Because friction would reduce v to 0 before this height was reached.	B1
	[2]

VERTICAL CIRCULAR MOTION

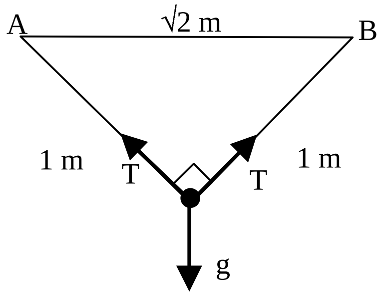
12. a)



Conservation of energy	M1
Initially k.e. = $\frac{1}{2}U^2$	
p.e. = $-\sqrt{2}g$	A1 (both)
At angle θ:	
k.e. = $\frac{1}{2}v^2$	
p.e. = $-\sqrt{2}g\cos\theta$	A1 (both)
So $\frac{1}{2}U^2 - \sqrt{2}g = \frac{1}{2}v^2 - \sqrt{2}g\cos\theta$	
Resolving inwards:	M1
$T - g\cos\theta = \frac{v^2}{\sqrt{2}}$	A1
So $T = g\cos\theta + \frac{u^2}{\sqrt{2}} - 2g + 2g\cos\theta$	M1 A1
When $\theta = 135^\circ$, $T = 0$	M1
$0 = \frac{-g}{\sqrt{2}} + \frac{u^2}{\sqrt{2}} - 2g - \frac{2g}{\sqrt{2}}$	
$u^2 = 3g + 2g\sqrt{2} = g(3 + 2\sqrt{2})$	A1
	[9]
b) $v^2 = g(3 + 2\sqrt{2}) - 2\sqrt{2}g - 2g$	M1
$= g$	A1
$v = \sqrt{g} = 3.13\text{ms}^{-1}$	A1
	[3]
c) Vertical component of velocity $= v\sin 45^\circ = \sqrt{\frac{g}{2}}$	B1
$2as = v^2 - u^2$	M1
$v = 0$; $a = -g$	
$-2gs = 0 - \frac{g}{2}$	
$s = \frac{1}{4}$	A1
	[3]

VERTICAL CIRCULAR MOTION

13.a)



Resolving vertically	M1
$2T\cos 45^\circ = g$	A1 A1
$T = \dfrac{\sqrt{2}g}{2}$	A1
	[4]

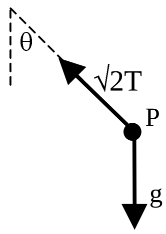
b) Vertical distance of P below AB = $1\sin 45^\circ$ — M1

$= \dfrac{1}{\sqrt{2}}$ — A1

Conservation of energy — A1

Initially: p.e. = $\dfrac{-g}{\sqrt{2}}$

k.e. = $\frac{1}{2}u^2$ — A1 f.t. (both)



When strings are at angle θ with vertical:

k.e. = $\frac{1}{2}v^2$

p.e. = $\dfrac{-g\cos\theta}{\sqrt{2}}$ — A1 (both)

So $\frac{1}{2}v^2 - \dfrac{g}{\sqrt{2}}\cos\theta = \frac{1}{2}u^2 - \dfrac{g}{\sqrt{2}}$

Resolving inwards: — M1

$\sqrt{2}T - g\cos\theta = \dfrac{v^2}{\sqrt{2}}$ — A1

$\sqrt{2}T - g\cos\theta = \dfrac{u^2}{\sqrt{2}} - g + g\cos\theta$ — M1 (combining) A1

VERTICAL CIRCULAR MOTION

QUESTION 13 b) CONTINUED

Require $T \geq 0$ when $\theta = 180^\circ$: M1

So minimum value of u is given by:

$$0 + g = \frac{u^2}{\sqrt{2}} - g - g$$

$$u^2 = 3\sqrt{2}g$$ A1

$$u = \sqrt{3\sqrt{2}g} \quad (= 6.45 \text{ ms}^{-1})$$

[11]

c) $\theta = 90^\circ$ B1

$$\sqrt{2}\,T = \frac{3\sqrt{2}g}{\sqrt{2}} - g = 2g \Rightarrow T = \sqrt{2}g$$ M1 A1

[3]