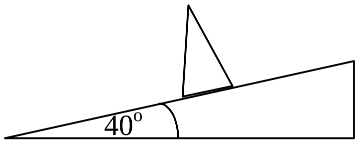
CENTRE OF MASS 2

1. An isosceles triangle is defined by the area enclosed between the lines y=0.2x, y= -0.2x and x=H.

a) Use integration to show that the centre of mass of the triangle has coordinates ($\frac{2}{3}$H, 0)

[6]

This triangle is placed on an inclined plane of angle 40^o, as shown.



Assuming the plane is sufficiently rough to prevent slipping,

b) Find whether or not the triangle will topple.

[6]

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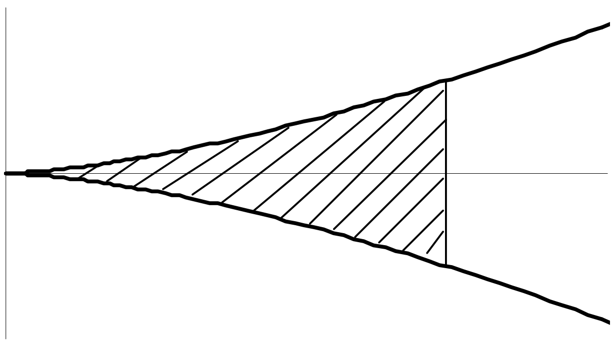
2. The area enclosed between the curve $y^2=x^3$ and the line x=4 is shown below. This area is rotated by π radians about the x axis to produce a solid.



a) Show that the volume of this solid is 64π.

[3]

b) Find the distance of the centre of mass of this solid from the y-axis.

[7]

The solid is suspended from a point on the rim of its circular face.

c) Find the angle which its circular face makes with the vertical.

[5]

CENTRE OF MASS 2

3. a) Use integration to show that the centre of mass of a solid cone of height H is at a distance of $\frac{3h}{4}$ from its vertex.
[8]

The plane face of a cone of radius r and height h is joined to the plane face of a hemisphere of radius r. Assuming the two bodies are of the same density,

b) Show that the centre of mass of the combined body is a distance of $\frac{h^2 - 3r^2}{4(2r + h)}$ from the common plane face.
[6]

c) Given that the centre of mass is on this plane face, deduce the value of h in terms of r.
[2]

d) Explain why the combined body can rest in equilibrium on a flat plane if it rests on any part of its hemispherical surface.
[3]

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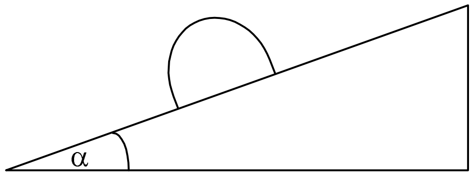
4. The lamina A is defined by the region enclosed between the curve $y = x^2$ and the line $y = 4$.

a) Show that the area of A is $\frac{32}{3}$
[6]

b) Explain why the centre of mass of A must lie on the y-axis
[1]

c) Find the distance of the centre of mass of A from the x-axis.
[7]

The lamina rests on a rough inclined plane as shown in the diagram below.

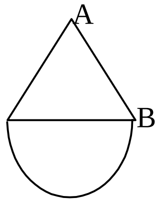


The angle α which the plane makes to the horizontal is such that the lamina is just about to topple.

d) Find the angle α, giving your answer correct to the nearest degree.
[4]

CENTRE OF MASS 2

5. The area enclosed between the curves $y=x^2$ and $y=2-x$, and the **y-axis** is rotated through 2π radians about the y-axis to make the body shown below



a) Show that the two curves intersect at the point (1,1)

[4]

b) Find the volume of the body formed

[6]

c) Explain why the x coordinate of the centre of mass of this body must be zero, and find its y coordinate

[13]

d) Show that this body can rest without toppling on the side AB

[4]

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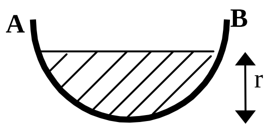
6. A light, thin hemispherical bowl of radius 2r is filled with cement to a depth of r as shown.



a) Use integration to show that the volume of the cement is $\dfrac{5\pi r^3}{3}$

[7]

b) Show that the centre of mass of the cement is at a distance of 1.35 r from the top of the bowl

[7]

The bowl is suspended from the point A.

c) Find the angle that AB makes with the vertical, giving your answer to the nearest degree.

[4]

CENTRE OF MASS 2

7. The area enclosed by the curve $y^2 = x$, the x axis and the line $x = 4$ is rotated through π radians about the x-axis.

a) Find the volume of the body formed, giving your answer in terms of π

[3]

b) Show that the centre of mass of the solid formed is at the point $(\frac{8}{3}, 0)$.

[8]

The base of a cone of radius 2 and height 2 is joined to the flat face of this body.
The centre of mass of the whole object is at the centre of the base of the cone.

c) Find the ratio of the masses of the two bodies which have been joined.

[4]

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8. A hollow cone has base radius R and height H.

a) Explain why the centre of mass of this cone is at a distance of $\frac{2H}{3}$ from its vertex

[4]

Two hollow cones made of identical material each have radius R but have heights R and 3R. Their bases are joined.

b) Find the distance of the centre of mass of the combined body from the vertex of the larger cone.

[5]

The combined body is placed on a rough inclined plane of angle θ. It rests on the surface of the smaller cone without toppling. Assuming that the plane is sufficiently rough to prevent sliding,

c) Find the maximum possible value of the angle θ.

[4]

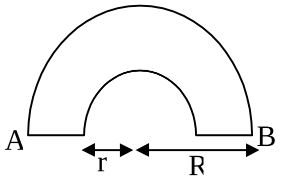
CENTRE OF MASS 2

9. a) Use integration to prove that the centre of mass of a semicircular lamina of radius r is at a distance of $\frac{4r}{3\pi}$ from its base.

[9]

b) Deduce the distance of the centre of mass from AB for the body shown below.

[4]



This body is suspended from point A. Given that R = 3r,

c) Find the angle which AB makes with the vertical.

[5]

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10. The curve C has equation $y = x^3$. The lamina A has area equal to that enclosed by C, the x-axis and the line x = 2. Given that A has a mass of 0.1kg per unit area,

a) find the mass of A

[3]

b) find the x coordinate of the centre of mass of A, and show that its y coordinate is $\frac{16}{7}$

[9]

c) The lamina is suspended from the point with coordinates (0,0). Find the angle which its x-axis makes with the vertical.

[3]

CENTRE OF MASS 2

11. The curve C has equation $y = e^{2x}$. The portion of this curve between x = 2 and x = 3 is rotated through 2π radians about the x-axis.

a) Find the volume of the solid generated, leaving your answer in terms of π and e.

[3]

b) Find the distance of the centre of mass of this body from the y-axis, giving your answer in terms of e

[11]

The body rests with its smaller plane face in contact with a rough piece of board. The inclination of the board is gradually increased from zero.

c) Show that the body will slide before it will topple.

[8]

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12. The lamina A is enclosed between the curve y=cosx, the x-axis and the lines $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

a) Find the area of A.

[2]

b) State the x coordinate of the centre of mass of A.

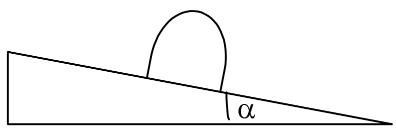
[1]

c) Show that the y coordinate of the centre of mass of A is given by the integral $\int_0^1 y\cos^{-1} y\,dy$

[5]

It is given that $\int_0^1 y\cos^{-1} y\,dy = \frac{\pi}{8}$

The lamina A is placed on an inclined plane which makes an angle α with the horizontal, as shown below. The coefficient of friction between the plane and the lamina is μ



d) Show that the lamina will not slip if $\mu \geq \tan\alpha$.

[4]

e) Show that the lamina will topple if $\tan\alpha > 4$.

[4]

f) Explain why the lamina will slip before it will topple if α is increased.

[2]

CENTRE OF MASS 2

13. A solid is formed by rotating the portion of the curve $y = kx^2$ between $y = 0$ and $y = ka^2$ through π radians about the y-axis.

a) Show that the volume of this solid is $\frac{1}{2}\pi ka^4$.

[3]

b) Find the coordinates of its centre of mass.

[7]

Solid A is produced by rotating the area enclosed between the curves $y = x^2$, $y = 4x^2$ and the line $y = 4$ through π radians about the y-axis.

c) Using your answers to a) and b)

i) Find the volume of A

[7]

ii) Find the coordinates of the centre of mass of A.

[6]

Solid A is placed with its plane face on a rough plane inclined at 40^o to the horizontal. The plane is sufficiently rough to prevent slipping.

d) Find whether A will topple.

[6]

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14. The curve C has equation $y = x(2-x)$. The portion of C between the lines $x=0$ and $x=1$ is rotated through 2π radians about the x-axis.

a) Find the volume of the solid formed.

[4]

b) State the y coordinate of the centre of mass of this solid, giving a reason for your answer

[2]

c) Find the x coordinate of the centre of mass of the solid.

[7]

The solid is placed with its plane face resting on a rough plane. The angle of inclination of the plane is gradually increased from zero.

d) Show that the solid will slide before it will topple.

[9]

CENTRE OF MASS 2

15. An arc of a circle is of angle 2α radians and radius r.

a) Use integration to show that the distance of the centre of mass of this arc from the centre of the circle is $\dfrac{r\sin\alpha}{\alpha}$

[7]

b) Hence use integration to find the position of the centre of mass of a circular sector of angle 2α and radius r.

[6]