SIMPLE HARMONIC MOTION

Take g = 9.8ms^{-2} where a numerical value is required

1. A particle is moving along the x axis. Its displacement from 0 at time t is x m where $x = 4 - 3\sin\frac{\pi t}{2}$

a) Find its initial distance from 0 and the times when it is instantaneously at rest.

[5]

b) Show that the particle performs simple harmonic motion about (4, 0) and find the period and the amplitude of the motion.

[6]

---

2. A light elastic string AB of natural length 2 m is fastened to a peg at A. A mass of 6 kg is attached to B and the system is in equilibrium with AB vertical. The modulus of elasticity of the string is 10g N.

a) Find the stretched length of the string.

[4]

b) The mass is pulled down by a distance a.

i) Show that the mass will perform simple harmonic motion with amplitude a, provided $a \leq d$, where d is a constant whose value should be stated.

[6]

ii) Given a = 1 m, find, in terms of g and π, the mass's maximum speed during the motion and the time required to complete one full oscillation.

[8]

---

3. A light elastic spring AB of natural length 5m is resting on a smooth horizontal table. It is fixed at the end A and has a particle of mass 1 kg attached at B. A force of magnitude 4 N in the direction $\overrightarrow{BA}$ is required to maintain the particle in equilibrium. The modulus of elasticity of the spring is 10 N.

a) Find the compressed length of the spring AB.

[5]

b) The force is then removed. Show that the particle performs simple harmonic motion, and find the time taken for the spring to first reach its natural length.

[9]

SIMPLE HARMONIC MOTION

4. Two light elastic strings AB and BC lying on a smooth, horizontal table are joined together at B. The natural lengths of AB and BC are 2L m and L m respectively and their moduli of elasticity are λ_A N and λ_C N respectively. A particle of mass 1 kg is attached to point B, and points A and C are fixed in a horizontal line so that AC=6L m. The particle rests in equilibrium with AB = BC = 3L m.

a) Find the ratio $\lambda_A : \lambda_C$

[6]

The particle is displaced $\frac{L}{2}$ m towards A.

b) Show that it performs simple harmonic motion.

[6]

c) Find its speed when it is $\frac{11L}{4}$ m from B, giving your answer in terms of L and λ_C.

[11]

---

5. A particle of mass m hangs on a light, inextensible string of length 1 m.
The particle is displaced through an angle α radians from its equilibrium position, then released.

a) Show that the particle performs simple harmonic motion with period $\frac{2\pi}{\sqrt{g}}$. (You may assume $\alpha < 0.1$)

[6]

b) Find the time during 1 oscillation for which the particle is within $\frac{\alpha}{2}$ radians of its equilibrium position.

[6]

c) Find the angular speed of the particle as it passes through the equilibrium position.

[2]

d) As the particle move through the equilibrium position, it collides and coalesces with a stationary particle of mass 2 m. Find the greatest vertical height to which the combined particles rise.

[6]

SIMPLE HARMONIC MOTION

6. A particle of mass 6m is suspended on a light elastic string of modulus 12 mg and natural length 2L. The string hangs vertically with its other end attached to a fixed point.

a) Find the length of the string when the particle hangs in equilibrium

[4]

The particle is pulled down to a distance 2L

b) Show that the particle initially performs simple harmonic motion, and explain why it does not perform complete oscillations.

[5]

c) Find, in terms of g and L, the speed of the particle when the length of the string is 2L, and the time taken for the particle to reach this point.

[7]

d) Find the highest point reached by the particle.

[3]

e) Find the time taken for the particle to complete one full "oscillation" – i.e. to return to the point from which it was released.

[5]

---

7. A particle of mass m is suspended on a light elastic spring of modulus Kmg and natural length L which is fixed at the other end.

a) Find the extension of the spring when the mass hangs in equilibrium.

[3]

b) Given that when the mass is pulled down by $\frac{L}{2}$ from its equilibrium position then released, it performs simple harmonic motion of period $\frac{10\pi}{7}$,

i) Find, in terms of L, the value of K.

[7]

ii) Write down the maximum speed and maximum acceleration of the particle.

[5]

SIMPLE HARMONIC MOTION

8. A particle of mass 2 kg moves along the x-axis under a force directed towards the origin proportional to the particle's distance from the origin, when the particle is 2 m from the origin, the force on it is 36 N.

a) Show that the motion of the particle satisfies the differential equation $\frac{d^2x}{dt^2} + 9x = 0$

[5]

The maximum speed of the particle is 12 ms^{-1}. Initially it is 2 m from the origin, moving towards the origin. Find:

b) the amplitude of the motion.

[4]

c) the time at which the particle passes through the origin for the second time, giving your answer in terms of π.

[10]

---

9. A particle is performing simple harmonic motion of period $\frac{\pi}{8}$ and amplitude 0.6 m about point O.

a) Find the maximum speed of the particle

[4]

Initially the particle is at O, moving towards point A (OA = 0.6 m).
Point B is 0.4 m away from O, on the opposite side to point A

b) Find to 3 significant figures, the first time at which the particle passes through point B moving <u>towards</u> O, and find its speed at this point.

[7]

---

10. A particle is performing simple harmonic motion along the x-axis with centre the origin. When $t = \frac{\pi}{3}$, the particle is at x = 3, moving with speed - $6\sqrt{3}$ ms^{-1} towards the origin. Its maximum speed is 12 ms^{-1}.

a) Find the period and amplitude of the motion.

[8]

b) Find, in terms of π, the time at which the particle first passes through the origin moving in the positive direction.

[12]

SIMPLE HARMONIC MOTION

11. Dan is swimming on the surface of a rectangular swimming pool of length 20m and width 10m. His friend Andy is standing at the centre of the pool. Taking **i** and **j** to be unit vectors parallel to the length and width of the swimming pool respectively, Dan's position vector **r** relative to Andy at time t is given by $\mathbf{r} = 2(\cos 2t\,\mathbf{i} + \sin Bt\,\mathbf{j})$, where B is a positive constant.

a) Given that Dan's motion satisfies the differential equation $\ddot{\mathbf{r}} = -k^2\mathbf{r}$, where k is a constant, show that B = 2. [6]

Dan's little brother Craig is running along the length of the swimming pool, so that he stays level with Dan at all times.

c) Show that Craig is performing simple harmonic motion, and state its amplitude and period. [5]

d) Find Dan's distance from Craig at the time that Craig's speed first takes half its maximum value. [6]

---

12. A particle P of mass m is attached to the midpoint of a light spring of length 4L and modulus of elasticity 2mg. One end of the spring is attached to point A, on the ceiling of a room. The other end of the spring is attached to point B, which is on the floor, a distance 6L vertically below A.

a) Find the distance of the particle above B when it is in equilibrium [5]

b) The particle is displaced by a distance 2L upwards. Show that it performs simple harmonic motion and state the period and amplitude of the motion. [7]

c) Find the speed of the particle when AP = 3L. [3]
