

CENTRE OF MASS I

Unless otherwise stated in the question, all answers should be given to three significant figures

1. Three particles of mass 3kg, 4kg and 5kg lie at the points with coordinates (0, 2), (1, 6) and (-3, 2) respectively.

a) Determine the coordinates of their centre of mass.

[4]

b) A mass of 10kg is added to the system so that the centre of mass of all four masses is at (0,0). Determine the coordinates of the 10kg mass.

[3]

2. A uniform wire of length 40cm is bent into a right angled triangle ABC, such that $BC = 15\text{cm}$ and $AC = 17\text{ cm}$.

a) Calculate the distance of the centre of mass of the triangle from
(i) AB (ii) BC

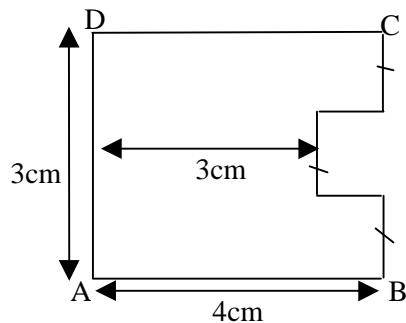
[6]

The wire triangle is suspended from B.

b) Calculate the angle that BC makes with the vertical.

[4]

3. The diagram below shows a uniform lamina



a) Calculate the distance of the centre of mass of the lamina from
i) AD ii) AB

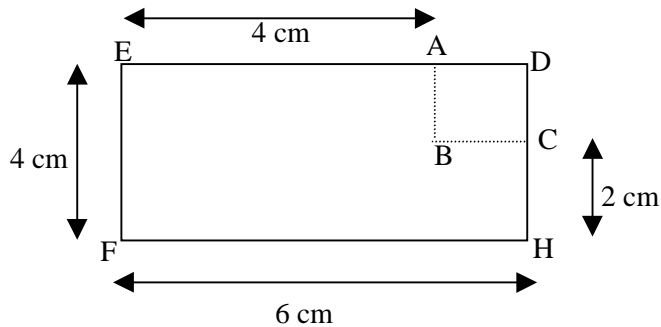
[6]

b) The lamina is suspended from C. Calculate the angle that DC makes with the vertical.

[4]

CENTRE OF MASS I

4. A uniform rectangular sheet of metal is 4cm by 6cm. It is folded along AC so that D is in contact with B.



- a) Calculate the distance of the centre of mass of the folded shape from FH and from FE

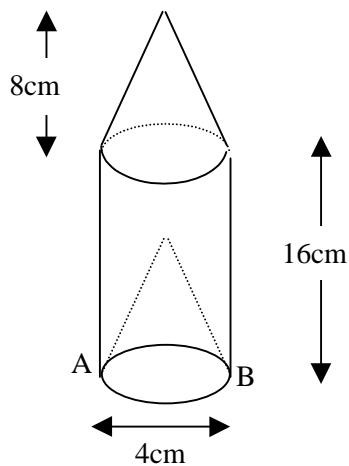
[7]

The shape is placed on a rough inclined plane with EA in contact with the plane and with A below E. The angle of the plane is gradually increased.

- b) Assuming the plane is sufficiently rough to prevent slipping, find the angle of inclination of the plane to the horizontal when the shape is about to topple.

[4]

5. The stacking toy shown below consists of a solid cylinder of length 16cm and diameter 4cm with a solid cone of height 8cm added to one end of the cylinder. At the other end of the cylinder an identical cone has been removed.



- a) Calculate the distance of the centre of mass of the toy from AB.

[6]

The shape is placed on a rough inclined plane with AB in contact with the plane. The angle of the plane is then gradually increased. Assuming that the plane is sufficiently rough to prevent slipping,

- b) Find the angle of inclination of the plane to the horizontal when the toy is about to topple.

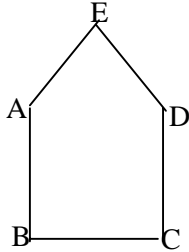
[4]

CENTRE OF MASS I

6. A uniform piece of wire of mass 280 grammes is bent into the pentagon shown.
 $AB = BC = CD = 6\text{cm}$, $AE = ED = 5\text{cm}$. Angles at B and C are right angles

a) Calculate the distance of the centre of mass from AD.

[5]

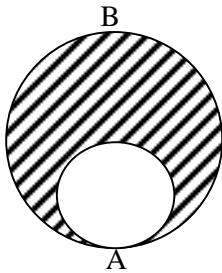


A mass M grammes is tied to the frame at E so that when the pentagon is suspended from A, AD is vertical.

b) Find the value of M .

[5]

7. The diagram below shows a uniform lamina consisting of a circle of radius R with a smaller circle of radius r removed. The centres of both circles lie on the line AB



a) Find an expression for the distance of the centre of mass of the lamina from A.

[4]

Given that the distance of the centre of mass from A is $\frac{7}{3}r$,

b) Show that, if $R=kr$, then k satisfies $3k^3 - 7k^2 + 4 = 0$

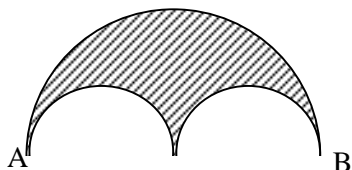
[4]

c) Find the value of k .

[5]

CENTRE OF MASS I

8. A semicircle of radius $2R$ has two smaller semicircles each of radius R removed from it, as shown in the diagram.



- a) Find the distance of the centre of mass of this lamina from AB , giving your answer in terms of π .
[7]
 - b) The lamina is suspended from point A and hangs in equilibrium. Find the angle which AB makes with the vertical.
[4]
-

9. A kite is formed by joining the bases of two isosceles triangles. Each triangle has a base of 30cm , and their heights are 8cm and 20cm . The ratio of the density of the smaller triangle to the larger triangle is $k:1$.

- a) Find an expression for the distance of the centre of mass of the kite from the base of the two triangles.
[6]

Given that this distance is zero,

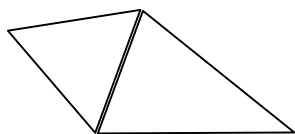
- b) Find the value of k
[2]

The kite is placed so that a shorter side is in contact with a rough plane inclined at angle α to the horizontal

- c) Given that the kite has not slipped, find in terms of α , the minimum possible value of μ , the coefficient of friction between the kite and the plane.
[4]
 - d) Given that the kite is about to topple, find the value of α .
[4]
-

CENTRE OF MASS I

10. The toy shown in the diagram is produced by joining two cones of equal density. Each cone is of radius R , but one has height H and the other height kH , where $k > 1$.



- a) Find, in terms of H , the distance of the centre of mass from the vertex of the cone of height H , giving your answer in as simple a form as possible.

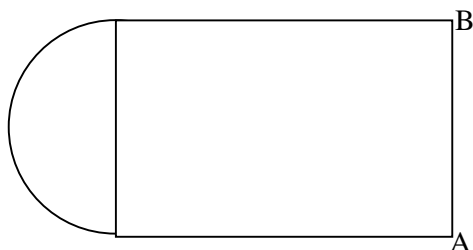
[6]

Given that $H=R$, and that the toy can rest in equilibrium on the surface of the cone of height H

- b) Find the maximum possible value of k .

[7]

11. The shape shown below is made of wire of uniform density. The curved portion of wire forms a semicircle of radius R , and the remainder of the wire forms a rectangle of sides $2R$ and L .



- a) Find the distance of the centre of mass of this body from AB

[6]

The total mass of the wire is M . A particle of mass $2M$ is attached at the midpoint of AB , and the combined body is suspended from A . Given that $L=R$,

- b) Find the angle which AB makes with the vertical.

[8]

CENTRE OF MASS I

12. A uniform lamina ABC is in the form of a right-angled triangle, where $AB=9\text{cm}$, $BC=12\text{cm}$ and $AC=15\text{cm}$.

a) Find the distance of the centre of mass of the lamina from AB and from BC

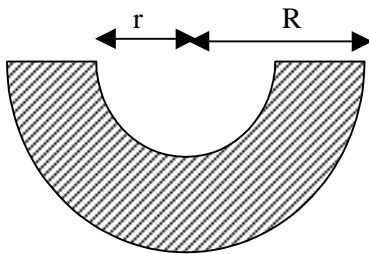
[3]

The lamina is of mass M . A particle of mass kM is attached to it at point C. The lamina is then suspended from point B.

b) Given that the angle between BC and the vertical is $\tan^{-1}0.075$, find the value of k .

[10]

13. A bowl is formed by removing a hemisphere of radius r from a hemisphere of radius R , as shown in the diagram.



a) Find an expression for the distance of the centre of mass of the bowl from its top

[5]

Two bowls, each of inner radius r and outer radius $2r$ but of masses m and M , are joined together to form a sphere. The centre of mass of this figure is at a distance $\frac{10}{7}r$ from the base of the bowl of mass m .

b) Find the ratio $M:m$

[7]