MOTION UNDER VARIABLE ACCELERATION

Take g as 9.8ms^{-2} when required.

1. A particle is moving along a straight line. Its acceleration a ms^{-2} is always inversely proportional to its velocity v ms^{-1}. Initially the particle is at a fixed point O, and it has a velocity of 1 ms^{-1} and an acceleration of 4 ms^{-2}.

a) Show that $a = \frac{4}{v}$

[2]

b) Find:

i) the speed of the particle when it is 4m from O, giving your answer to 3 significant figures;

[6]

ii) its distance from O at any time t.

[8]

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2. A small toy parachute of mass 0.25kg is thrown vertically upwards with a speed 7 ms^{-1} from a point 1m above the ground. Whilst the toy is moving upwards, there is no resistance to motion but when it begins to descend the parachute opens and offers a resistance to motion of $0.25\,v^2$ N where v is the speed of the parachute.

a) Show that the greatest height reached by the parachute is 3.5m above the ground.

[5]

b) Find its speed when it returns to the ground, giving your answer to 3 significant figures.

[11]

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3. A particle of mass 2 kg is falling from rest in a medium that offers a resistance to motion of kv N where k is a constant and v is the speed of the particle. The limiting speed of the particle is g ms^{-1}.

a) Show that k = 2.

[4]

b) Given that the particle is released from rest in the medium, find its speed and the distance it has travelled in 3 seconds, giving your answers in terms of g and e.

[11]

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4. A particle of mass 1 kg moves in a straight line along the x-axis with a velocity given by $v = 2e^{-t} + 3$ Initially, the particle is at the point x = -2.

a) Find expressions for the displacement and acceleration of the particle at time t.

[4]

When t = ln 5, the force acting on the particle is replaced by a constant force of magnitude 6N acting towards the origin.

b) Find the <u>total</u> distance moved by the particle from t = 0 to t = 2, giving your answer to 3 significant figures.

[11]

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5. A particle P of mass m kg is moving along in a straight line OP. Initially it is at O and has a speed of u ms^{-1} in the direction $\overrightarrow{OP}$. At any time t seconds it is subject to a retarding force of $\frac{kmvu}{(x+k)^2}$ where k is a positive constant, x m is its displacement from O and v is its velocity.

Find its distance from O when its speed has been halved, and the time taken to reach this point.

[15]

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6. A particle P is moving on the x axis. Initially it is at the point where x = 2 and is moving with speed 4 ms^{-1} in the direction $\overrightarrow{OP}$. At any time t seconds it is subject to a retarding force of 2v ms^{-2} where v ms^{-1} is the velocity of the particle.

a) Prove that $v = 4e^{-2t}$.

[7]

b) Describe the motion of P, and show that x can never exceed 4.

[8]

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7. A particle of mass 1 kg is moving vertically downwards in a medium which offers a resistance to motion which is proportional to the square of the speed of the particle.

a) If the terminal speed of the particle if 2 ms^{-1} show that at any time t seconds the acceleration a ms^{-2} of the particle is given by $a = g - \frac{gv^2}{4}$, where v ms^{-1} is the velocity of the particle.

[3]

b) Given that the particle falls from rest, find its speed after 1 second, giving your answer to 3 significant figures.

[9]

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8. In a game of hopscotch a small disc of mass 0.5 kg is given an impulse of magnitude 3 Ns which causes it to begin to move across a rough horizontal playground. The coefficient of friction between the disc and the ground is $\frac{4}{g}$, where g is the acceleration due to gravity. In addition to the frictional force the hopscotch is subject to a retarding force of 2x where x m is the distance it has travelled.

a) Show that the acceleration, a ms^{-2} of the hopscotch is given by $a = -4(1 + x)$

[6]

b) Find the distance travelled by the hopscotch before it comes to rest.

[8]

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9. A particle of mass 1 kg moves on the x-axis under the influence of a force directed towards the origin of magnitude e^{-x}. Initially, the particle is at the point where x = ln2, moving with speed 1 ms^{-1} away from the origin.

a) Show that the velocity of the particle is given by $v = \sqrt{2}e^{-\frac{x}{2}}$

[6]

b) Hence show that the particle's position at time t is given by $x = \ln\left(\frac{(t+2)^2}{2}\right)$

[8]

c) Find an expression for the particle's velocity at time t.

[3]

d) State the limiting velocity of the particle as $t \to \infty$.

[1]

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10. A particle moves in a straight line under the action of a resistive force proportional to its speed and inversely proportional to time.

a) Express this information as a differential equation.

[2]

b) Show that $v = At^{-L}$, where A and L are constants.

[6]

c) Given that L < -1 and that initially the particle is at the origin, show that the particle's position (x) at time t is given by $x = \frac{At^{1-L}}{1-L}$

[4]

d) Given that when t = 3, x = v = 18, find the values of the constants A and L

[7]

e) Hence find the distance travelled by the particle between t = 2 and t = 4.

[3]

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11. A particle P moves so that its position vector **r** at time t seconds is given by **r** = 3cos 2t **i** + 3sin 2t **j**

a) Show that $|\mathbf{r}|$ is constant and hence describe the locus of P.

[4]

b) Find **v** ms^{-1}, the velocity vector of P. State what can be deduced about the speed of P.

[4]

c) P is initially at the point A. Find the time when it first passes again through A, giving your answer in terms of π.

[3]

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12. Relative to a fixed origin O, **r** m, **v** ms^{-1} and **a** ms^{-2} are the position, velocity and acceleration vectors at time t seconds of a particle P. $\mathbf{r} = 3\cos 2t\,\mathbf{i} - 2\sin 2t\,\mathbf{j}$

a) Find **v** and **a**.

[4]

b) Show that, at all times, $\mathbf{a} = k\mathbf{r}$, where k is a constant. State the value of k.

[2]

c) Find the cartesian (i.e. x, y) equation of the path of the particle.

[3]

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13. A particle moves in a straight line so that its velocity v ms^{-1} at any time t is given by $v = \cos t - \sqrt{3}\sin t$ Initially it is at a fixed point O. In the subsequent motion, find, in terms of π:

a) the time when the particle first comes instantaneously to rest, and its acceleration at this time;

[7]

b) the distance from O at any time t.

[4]

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14. A particle of mass 3 kg moves in the x, y plane. Its position vector **r** at any time t is given by $\mathbf{r} = 3e^{2t}\mathbf{i} - 2e^{-2t}\mathbf{j}$ where **i** and **j** are unit vectors parallel to the x and y axes respectively.

a) Find the velocity of the particle at time t.

[2]

b) Show that the force acting on the particle is always parallel to $\overrightarrow{OP}$ and find its initial magnitude, giving your answer to 1 decimal place.

[8]

c) Show that the co-ordinates (x,y) of the particle at time t satisfy xy = -6.

[5]

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15. The position vector **r** m of a particle P, relative to the origin O, at any time t s is given by
$\mathbf{r} = \left(2 + \sqrt{3}\cos t\right)\mathbf{i} + \sin t\,\mathbf{j}$

a) Find the initial distance of P from O.

[3]

b) i) Find **v** ms^{-1} and **a** ms^{-2}, the velocity and acceleration vectors of P at time t.

[3]

ii) The velocity and acceleration vectors are at angles θ and ϕ to the horizontal respectively.
Show that $\tan\theta = \frac{-1}{\sqrt{3}}\cot t$ and $\tan\phi = \frac{1}{\sqrt{3}}\tan t$.

[3]

iii) Show that if α is the angle between the velocity and acceleration vectors, $\tan\alpha = \frac{\sqrt{3}}{2}\left(\tan t + \cot t\right)$.

[5]

iv) Using the identity $\tan t + \cot t \equiv \frac{2}{\sin 2t}$ or otherwise, show that the velocity and acceleration are perpendicular initially, and find, in terms of π, the next value of t for which this occurs.

[7]

c) Find the maximum and the minimum speed of the particle.

[4]