## HORIZONTAL CIRCULAR MOTION

**1.** $\omega = \frac{2\pi}{T} = \frac{2\pi}{5}$ rad s$^{-1}$ B1

$F = mr\omega^2$ M1

$= 0.2 \times 0.2 \times \frac{4\pi^2}{25}$ A1

$= 0.0632$N A1

**[4]**

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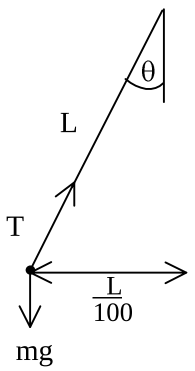
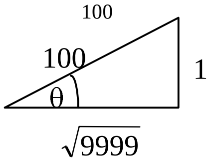
**2.** a)




$\sin\theta = \frac{1}{100}$ B1




Resolving M1

Vertically : $T\cos\theta = mg$ A1

Horizontally : $T\sin\theta = m\frac{v^2}{\frac{L}{100}}$ A1

so $\frac{mv^2}{\frac{L}{100}\sin\theta} = \frac{mg}{\cos\theta}$ M1 (combining)

$v^2 = g\frac{\sin\theta}{\cos\theta}\frac{L}{100}$ A1

$\tan\theta = \frac{1}{\sqrt{9999}}$ B1

$v^2 = \frac{g}{\sqrt{9999}}\frac{L}{100} = \frac{gL}{100\sqrt{9999}}$ A1

**[8]**

b) $T = \frac{mv^2}{\frac{L}{100}\sin\theta}$ M1

$= \frac{m}{\frac{L}{100} \times 0.01}\frac{gL}{100\sqrt{9999}} = \frac{100mg}{\sqrt{9999}}$ A1

**[2]**

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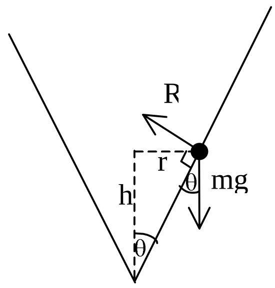
**HORIZONTAL CIRCULAR MOTION**

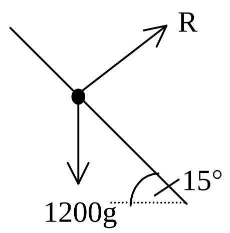
**3.**




| | | |
|---|---|---|
| Resolving | | M1 |
| Vertically: $R\sin\theta = mg$ | ① | A1 |
| Horizontally : $R\cos\theta = \dfrac{mu^2}{r}$ | ② | A1 |
| $\dfrac{①}{②}$ : $\tan\theta = \dfrac{gr}{u^2}$ | | M1 A1 |
| But $\dfrac{r}{h} = \tan\theta$ | | B1 |
| So $\dfrac{r}{h} = \dfrac{gr}{u^2}$ | | M1 |
| $\dfrac{u^2}{g} = h$ | | A1 |
| | | **[8]** |

## HORIZONTAL CIRCULAR MOTION

**4.** a)




Resolving — M1

Vertically : $R\cos 15 = 1200g$ ① — A1

Horizontally : $R\sin 15 = \dfrac{1200v^2}{80}$ ② — A1 A1

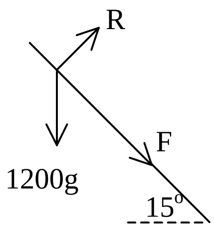
$\dfrac{②}{①}$ $\tan 15 = \dfrac{v^2}{80g}$ — M1

$v^2 = 210.07$

$v = 14.5\,\text{ms}^{-1}$ — A1

**[6]**

b)




Resolving: — M1

Vertically : $1200g + F\sin 15 = R\cos 15$ ①

Horizontally : $R\sin 15 + F\cos 15 = \dfrac{1200v^2}{80}$ ② — A1

About to slip $\Rightarrow F = \mu R = 0.6R$ — B1

① $\Rightarrow 1200g = R(\cos 15 - 0.6\sin 15)$

② $\Rightarrow \dfrac{1200v^2}{80} = R(\sin 15 + 0.6\cos 15)$ — M1 A1

Dividing : $\dfrac{v^2}{80g} = \dfrac{\sin 15 + 0.6\cos 15}{\cos 15 - 0.6\sin 15}$ — M1

$v^2 = 810.83 \Rightarrow v = 28.5\text{ms}^{-1}$ — A1

**[7]**

**HORIZONTAL CIRCULAR MOTION**

**5.** Hooke's Law : $T = \dfrac{\lambda x}{L}$ — M1

$= \dfrac{6x}{0.8}$

Resolving inwards : $\dfrac{6x}{8} = \dfrac{1 \times v^2}{r}$

$\dfrac{6x}{8} = \dfrac{1 \times 1.5^2}{r}$ — M1 A1

But r = 0.8 + x — M1

$\dfrac{6x}{8} = \dfrac{2.25}{0.8 + x}$

$\Rightarrow 6x(0.8 + x) = 2.25 \times 8$ — M1 (reasonable attempt to solve)

$4.8x + 6x^2 = 18$

$x = 1.38$ or $-2.18$ — M1 (solving) A1 (1.38)

so r = 2.18m — B1

**[8]**

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**6.** a) Particle about to slip $\Rightarrow F = \mu R$ — M1

$F = 0.4 \times 0.5g$

$= 0.2g$ — A1

Resolving inwards : $0.2g = \dfrac{0.5v^2}{r}$ — M1

$v^2 = 0.4gr$ — A1

i) $v = 0.885\text{ms}^{-1}$ — B1

ii) $v = 1.25\text{ms}^{-1}$ — B1

**[6]**

b) none — B1

**[1]**

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**HORIZONTAL CIRCULAR MOTION**

**7.** a) At surface, $mg = \dfrac{kmM}{R^2}$ M1 A1 A1

$k = \dfrac{gR^2}{M}$ A1

**[4]**

b) $D = \dfrac{3R}{2}$ B1

Gravitational force = $\dfrac{gR^2}{M}\dfrac{mM}{\left(\frac{3R}{2}\right)^2}$ M1

$= \dfrac{4mg}{9}$ A1

Resolving inwards :

$\dfrac{4mg}{9} = m\omega^2\dfrac{3R}{2}$ M1

$\omega^2 = \dfrac{8g}{27R}$ A1

$\omega = \dfrac{2}{3}\sqrt{\dfrac{2g}{3R}}$ A1

**[7]**

c) $\omega = \dfrac{2\pi}{24 \times 60 \times 60} = \dfrac{\pi}{43200}$ M1 (give for $\frac{2\pi}{24}$) A1

$\dfrac{gR^2}{M}\dfrac{mM}{D^2} = mD\omega^2$ M1 A1

$\dfrac{gR^2}{D^2} = D\dfrac{\pi^2}{43200^2}$

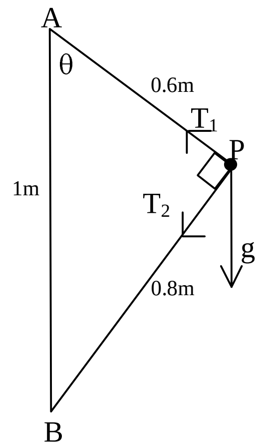
$D^3 = \dfrac{43200^2 gR^2}{\pi^2}$ M1

$D = \sqrt[3]{\dfrac{43200^2 gR^2}{\pi^2}}$ $\left( = 720\sqrt[3]{\dfrac{5gR^2}{\pi^2}} \right)$ A1

**[6]**

## HORIZONTAL CIRCULAR MOTION

**8.** a)




M1 (correct diagram)

Resolving — M1

Vertically : $T_1\cos\theta = g + T_2\sin\theta$ — A1

Horizontally : $T_1\sin\theta + T_2\cos\theta = r\omega^2$ — A1

$\sin\theta = 0.8$; $\cos\theta = 0.6$ — B1

$r = 0.6\sin\theta = 0.48$ — M1 A1

So $0.6T_1 = g + 0.8T_2$

$0.8T_1 + 0.6T_2 = 0.48\omega^2$ — B1

Solving simultaneously — M1

$T_1 = \frac{3g}{5} + \frac{48}{125}\omega^2$ — A1

$T_2 = \frac{36\omega^2}{125} - \frac{4g}{5}$ — A1

**[10]**

b) $T_2 \geq 0$ — M1

$\frac{36\omega^2}{125} - \frac{4g}{5} \geq 0$

$\omega^2 \geq \frac{4g}{5} \times \frac{125}{36}$ — A1

$\omega \geq \frac{5\sqrt{g}}{3}$ (5.22 rad $s^{-1}$) — A1

**[3]**

c) $T_1 > T_2$, so $T_1 \leq 40$ — B1

$\frac{3g}{5} + \frac{48}{125}\omega^2 \leq 40$ — M1

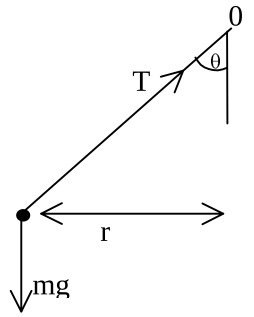
$\omega^2 \leq \frac{125}{48}\left(40 - \frac{3g}{5}\right)$

$\omega^2 \leq 88.854 \Rightarrow \omega \leq 9.43$ rad $s^{-1}$ — A1

**[3]**

**HORIZONTAL CIRCULAR MOTION**

**9.** a)




Resolving : M1

Vertically : $T\cos\theta = mg$ ① A1

Horizontally : $T\sin\theta = mr\omega^2$ ② A1

Hooke's law : $T = 4mgx$ ③ M1 A1

$r = (1 + x)\sin\theta$ B1

② $\Rightarrow T\sin\theta = m(1 + x)\sin\theta\omega^2 \Rightarrow T = m(1 + x)\omega^2$ ④ B1

so $4mgx = m(1 + x)\omega^2$ M1

giving $4gx = \omega^2 + x\omega^2 \Rightarrow x = \dfrac{\omega^2}{4g - \omega^2}$ A1

**[9]**

b) Using ① and ④

$\dfrac{mg}{\cos\theta} = m(1 + x)\omega^2$ M1 A1

But $x = \dfrac{\omega^2}{4g - \omega^2}$

$\dfrac{mg}{\cos\theta} = m\omega^2\left(1 + \dfrac{\omega^2}{4g - \omega^2}\right)$ M1

$\dfrac{g}{\cos\theta} = \omega^2\left(\dfrac{4g - \omega^2 + \omega^2}{4g - \omega^2}\right)$ M1

$\dfrac{g}{\cos\theta} = \dfrac{4g\omega^2}{4g - \omega^2} \Rightarrow \cos\theta = \dfrac{4g - \omega^2}{4\omega^2}$ A1

**[5]**

c) $0 < \cos\theta < 1$ M1

$4g - \omega^2 > 0$

$4g > \omega^2 \Rightarrow 2\sqrt{g} > \omega$ A1

$\dfrac{4g - \omega^2}{4\omega^2} < 1 \Rightarrow 4g - \omega^2 < 4\omega^2$ A1

Hence $4g < 5\omega^2$, giving $2\sqrt{\dfrac{g}{5}} < \omega$ A1

Hence $2\sqrt{\dfrac{g}{5}} < \omega < 2\sqrt{g}$

**[4]**

**HORIZONTAL CIRCULAR MOTION**

| | | |
|---|---|---|
| **10.**a) | $v = \omega r = 0.9\text{ms}^{-1}$ | B1 |
| | k.e. $= \frac{1}{2}mv^2$ | M1 |
| | $= 0.81\text{J}$ | A1 |
| | | **[3]** |
| b) | Distance moved $= \dfrac{0.6\pi}{4}$ | B1 |
| | $F = \mu R = 0.1g$ | B1 |
| | Work done $= 0.15\pi \times 0.1g$ | M1 |
| | $= 0.015\pi g$ | A1 |
| | | **[4]** |
| c) | Work done against friction = loss in energy | M1 |
| | New kinetic energy $= \frac{1}{2} \times 2v^2 = 0.81 - 0.015\pi g$ | A1 |
| | $v^2 = 0.348$ | |
| | $T = \dfrac{mv^2}{r} = \dfrac{1 \times 0.348}{0.3} = 1.16\text{N}$ | A1 |
| | | **[3]** |
| d) | Require kinetic energy to decrease to zero | M1 |
| | $0.81 = 0.1g \times \text{distance}$ | M1 |
| | $0.827 = \text{distance}$ | |
| | angle $= \dfrac{0.827}{0.6\pi} \times 360$ | M1 |
| | $= 158°$ | A1 |
| | | **[4]** |

**HORIZONTAL CIRCULAR MOTION**

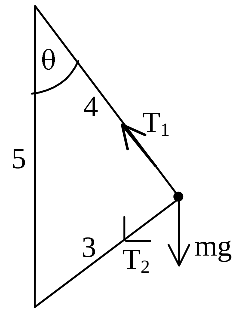
**11.** a) Hooke's law $\quad T = \dfrac{\lambda x}{L}$ M1

$AP: \ T = \dfrac{kmg3}{1} = 3kmg$ A1

$PB: \ T = \dfrac{kmg2}{1} = 2kmg$ A1

**[3]**

b)




Resolving M1

Vertically : $\quad T_1\cos\theta = mg + T_2\sin\theta \quad ①$ A1

Horizontally : $\quad T_1\sin\theta + T_2\cos\theta = mr\omega^2 \quad ②$ A1

Using values of $T_1$ and $T_2$ and $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$ M1 A1

① : $\quad \dfrac{12kmg}{5} = mg + \dfrac{6kmg}{5}$ A1

$\dfrac{6kmg}{5} = mg \Rightarrow k = \frac{5}{6}$ A1

**[7]**

c) Using ② :

$\frac{5}{2}mg \times \frac{3}{5} + \frac{5}{3}mg \times \frac{4}{5} = mr\omega^2$ M1 A1

But $r = 4\sin\theta = \frac{12}{5}$ M1 A1

$$\frac{3mg}{2} + \frac{4mg}{3} = \frac{12m\omega^2}{5}$$

$\omega^2 = \dfrac{85}{72}g$ A1

$\omega = 3.40 \text{ rad s}^{-1}$ A1

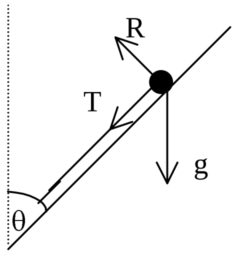
**[6]**

**HORIZONTAL CIRCULAR MOTION**

| | |
|---|---|
| **12.** $T = \frac{\lambda x}{L} = 2gx$ | M1 A1 |
| Radius = $\frac{1}{2}(1 + x)$ | B1 |
| Resolving inward for either particle : | M1 |
| $2gx = 4 \times \frac{1}{2}(1 + x)$ | A1 A1 |
| $\Rightarrow 2gx = 2 + 2x$ | |
| $\Rightarrow x(2g - 2) = 2$ | M1 (solving) |
| $x = \frac{1}{g-1}$ | A1 |
| $T = \frac{2g}{g-1} = 2.23\text{N}$ | A1 |
| | **[9]** |

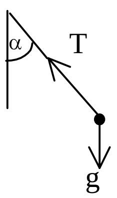
**HORIZONTAL CIRCULAR MOTION**

**13.**a)




| | |
|---|---|
| Resolving | M1 |
| Vertically : $R\sin\theta = T\cos\theta + g$ ① | A1 |
| Horizontally : $R\cos\theta + T\sin\theta = \frac{25}{r}$ ② | A1 |
| $r = 1\sin\theta = 0.6$ | B1 |
| $\cos\theta = 0.8$ | B1 |
| so ① : $0.6R = 0.8T + g$ | |
| ② : $0.8R + 0.6T = \frac{25}{0.6}$ | |
| Solving : T=17.16N | M1 A1 |
| | **[7]** |

b)



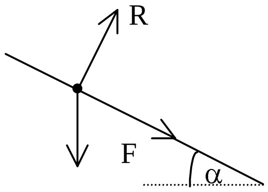

| | |
|---|---|
| Resolving vertically : $T\cos\alpha = g$ | B1 |
| $\cos\alpha = \frac{g}{17.16}$ | |
| $\alpha = 55°$ | B1 f.t. |
| | **[2]** |

c) Resolving horizontally :

| | |
|---|---|
| $T\sin\alpha = \frac{25}{r}$ | M1 |
| $r = L\sin\alpha$ | B1 |
| $L = \frac{25}{T\sin^2\alpha}$ | |
| $= 2.16\text{m}$ | A1 |
| | **[3]** |

## HORIZONTAL CIRCULAR MOTION

**14.** a)




$F = \mu R$, as about to slip — B1

Resolving — M1

Vertically : $R\cos\alpha = F\sin\alpha + 1200g$ ① — A1

Horizontally : $R\sin\alpha + F\cos\alpha = \dfrac{1200 \times 40^2}{200}$ ② — A1

$\cos\alpha = \dfrac{\sqrt{99}}{10}$ — B1

① : $R\dfrac{\sqrt{99}}{10} = \dfrac{\mu R}{10} + 1200g$

② : $\dfrac{R}{10} + \dfrac{\mu R\sqrt{99}}{10} = 9600$

M1 A1

So, from ① : $R(\sqrt{99} - \mu) = 12000g$

② : $R(1 + \mu\sqrt{99}) = 96000$

Dividing : $\dfrac{\sqrt{99} - \mu}{1 + \mu\sqrt{99}} = \dfrac{12000g}{96000}$ — M1 A1

$8\sqrt{99} - 8\mu = g + g\mu\sqrt{99}$ — M1

$\mu = \dfrac{8\sqrt{99} - g}{8 + g\sqrt{99}} = 0.662$ — A1

**[11]**

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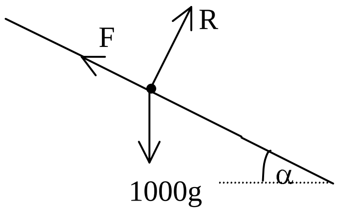
## HORIZONTAL CIRCULAR MOTION

**15.** a)




Resolving — M1

Vertically : $R\cos\alpha + F\sin\alpha = 1000g$ — A1

Horizontally : $R\sin\alpha - F\cos\alpha = \dfrac{1000v^2}{100}$ — A1

About to slip $\Rightarrow F = \mu R$ — M1

So : $R(\cos\alpha + \mu\sin\alpha) = 1000g$ ①
$R(\sin\alpha - \mu\cos\alpha) = 10v^2$ ② — A1

$\dfrac{②}{①}$ : $\dfrac{\sin\alpha - \mu\cos\alpha}{\cos\alpha + \mu\sin\alpha} = \dfrac{v^2}{100g}$ — M1 A1

Multiplying top and bottom of left-hand side by $\dfrac{1}{\cos\alpha}$ : — M1

$$\frac{\tan\alpha - \mu}{1 + \mu\tan\alpha} = \frac{v^2}{100g}$$

$$v^2 = \frac{100g(\tan\alpha - \mu)}{1 + \mu\tan\alpha}$$ A1

**[9]**

b) $$v^2 = \frac{100g(\tan\alpha - 0.2)}{1 + 0.2\tan\alpha}$$

$v^2 \geq 0 \Rightarrow \tan\alpha \geq 0.2$ — M1

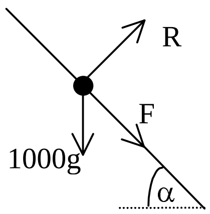
$\alpha \geq 11.3°$ — A1

**[2]**

**HORIZONTAL CIRCULAR MOTION**

**QUESTION 15 CONTINUED**

c)




Resolving — M1

Vertically : $R\cos\alpha = 1000g + F\sin\alpha$

Horizontally : $R\sin\alpha + F\cos\alpha = \dfrac{1000u^2}{100}$ — A1

About to slip out $\Rightarrow F = 0.2R$ — M1

So : $R(\cos\alpha - 0.2\sin\alpha) = 1000g$ ①

$R(\sin\alpha + 0.2\cos\alpha) = 10u^2$ ② — A1

$\dfrac{②}{①} \quad \Rightarrow u^2 = \dfrac{(\sin\alpha + 0.2\cos\alpha)100g}{\cos\alpha - 0.2\sin\alpha}$ — M1 A1

$$\left( = \frac{100g(\tan\alpha + 0.2)}{1 - 0.2\tan\alpha} \right)$$

**[6]**

d) $u^2 \geq 0$ so $1 > 0.2\tan\alpha$ — M1

$5 > \tan\alpha$ — A1

$78.7° > \alpha$

so $78.7 > \alpha \geq 11.3$ — B1

**[3]**

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