NEWTON'S LAW OF RESTITUTION

1. a) Taking direction of $8ms^{-1}$ as positive:

$-e \times 8 = -5$	M1
$e = \frac{5}{8}$	A1
	[2]

b)

Initial kinetic energy = $0.5 \times 10 \times 8^2$	M1
Final kinetic energy = $0.5 \times 10 \times 5^2$	A1
Loss in kinetic energy = 195J	A1 cao
	[3]

c)

Impulse = change in momentum	M1
$= 50 - -80 = 130$ Ns	A1
	[2]

d)

Initial momentum = 50	
Final momentum = 15v	M1 A1
$\Rightarrow v = \frac{10}{3}\ ms^{-1}$	A1
	[3]

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2. a)

Initial momentum = $2m \times 4 + m \times 2 = 10m$	B1
Conservation of momentum	M1
$10m = 2mv_A + mv_B$	A1
$e = \frac{\text{separation speed}}{\text{approach speed}} = \frac{v_B - v_A}{4 - 2}$	M1
$1 = v_B - v_A$	A1
Solving simultaneously	M1
$v_A = 3\ ms^{-1}$ $\quad v_B = 4\ ms^{-1}$	A2
	[8]

b)

impulse = change in momentum	M1
$= 6m - 8m$	
$= -2m$ Ns	A1
	[2]

c)

k.e. before = $0.5 \times 2m \times 4^2 + 0.5 \times m \times 2^2 = 18m$	M1 A1
k.e. before = $0.5 \times 2m \times 3^2 + 0.5 \times m \times 4^2 = 17m$	A1
loss = m	A1 f.t.
	[4]

NEWTON'S LAW OF RESTITUTION

3. $m \times ku + 2m \times u = 2mv$	M1 A1
$\Rightarrow v = \frac{1}{2}(k + 2)u$	A1
$v = -e(u - ku)$	M1 A1
$\frac{1}{2}(k + 2)u = -e(u - ku)$	
$e = \frac{k+2}{2(k-1)}$	M1 A1
	[7]
b) $e \leq 1$, so $\frac{k+2}{2(k-1)} \leq 1$	M1
$k + 2 \leq 2k - 2$ (since $k > 1$)	
$4 \leq k$	A1
	[2]
c) $k = 6 \Rightarrow v = 4u$ and $e = \frac{4}{5}$	M1 A1
So B is moving with speed $\frac{1}{2} \times 4u = 2u$ after hitting wall	M1 A1
For second collision:	
$m \times 0 + 2m \times 2u = mv_A + 2mv_B$	M1 A1 ft
so $4u = v_A + 2v_B$	
and $v_B - v_A = -\frac{4}{5}(2u - 0)$	M1 A1 ft
so $5v_A - 5v_B = 8u$	
Solving simultaneously $v_A = \frac{12}{5}u$ and $v_B = \frac{4}{5}u$	M1 A1 A1cao
	[11]

NEWTON'S LAW OF RESTITUTION

4. a)	Taking initial direction of A as positive	
	Impulse = change in momentum	B1
	= -mu – 2mu	M1
	Magnitude = 3mu; direction opposite to original direction of A	A1 A1
		[4]
b)	Impulse on B = 3mu in original direction of A	B1
	For B: 3mu = mv – (-mu)	M1
	v = 2u	
	Speed of B is 2u, its direction is reversed	A1 A1
		[4]
c)	$v_A - v_B = -e(u_A - u_B)$	M1
	$-u - 2u = -e(2u - - u)$	
	e = 1	A1
	Spheres are perfectly elastic	B1
		[3]

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5. a)	Conservation of momentum	M1
	$mu = mv_A + 2mv_B$	A1
	Restitution: $v_A - v_B = -\frac{1}{2}u$	M1 A1
	Solving: $v_A = 0$ and $v_B = \frac{1}{2}u$	M1 A1 cao
		[6]
b)	Next collision is B on C	B1
	Conservation of momentum $2m\times\frac{1}{2}u = 2mv^1_B + 4mv_C$	M1
	Restitution: $v^1_B - v_C = -\frac{1}{2}\left(\frac{1}{2}u\right)$	M1
	Solving, obtain $v^1_B = 0$ $v_C = \frac{1}{4}u$	A1
	There are no further collisions since A and B are now at rest	B1 B1
	and C is moving with speed $\frac{1}{4}u$ in direction of original motion of A	B1
		[7]

NEWTON'S LAW OF RESTITUTION

6. a) First collision A with B

Conservation of momentum	M1
$mu + \frac{1}{2}mu = mv_A + mv_B$	A1
Restitution: $v_A - v_B = -\frac{1}{2}\left(u - \frac{1}{2}u\right)$	M1 A1
Solving: $v_A = \frac{5}{8}u$ and $v_B = \frac{7}{8}u$	M1 A1
	[6]

b)

B then hits the wall. After this, its velocity is $-\frac{1}{7} \times \frac{7}{8}u = -\frac{1}{8}u$	M1 A1 ft
Second collision between A and B:	
Conservation of momentum: $\frac{5}{8}mu - \frac{1}{8}mu = mw_A + mw_B$	M1 A1 ft
Restitution: $w_A - w_B = -\frac{1}{2}\left(\frac{5}{8}u + \frac{1}{8}u\right)$	M1 A1 ft
Solving: $w_A = \frac{1}{16}u$ $w_B = \frac{7}{16}u$	A1 cao
B then hits the wall again. After this, its velocity is $-\frac{1}{7} \times \frac{7}{16}u = -\frac{1}{16}u$	B1 ft
	[8]

c)

Third collision between A and B	
Conservation of momentum $\Rightarrow \frac{1}{16}mu - \frac{1}{16}mu = x_A + x_B$	M1
Restitution: $x_A - x_B = -\frac{1}{2}\left(\frac{1}{16}u + \frac{1}{16}u\right)$	M1
$x_A = -\frac{1}{32}u$ and $x_B = \frac{1}{32}u$	A1 cao
B hits wall again; its velocity then becomes $-\frac{1}{7} \times \frac{1}{32}u = -\frac{1}{224}u$	A1 ft
No further collisions, since B is now moving in the same direction as A but slower and cannot catch A up	B1
	[5]

NEWTON'S LAW OF RESTITUTION

7. a)	Initial momentum = $2m \times 3u + m \times u$	B1
	Final momentum = $2m \times 2u + m \times v$	B1
	Conservation of momentum $7mu = 4mu + mv$	M1
	$v = 3u$	A1 cao
		[4]
b)	$v_A - v_B = -e(u_A - u_B)$	M1
	$2u - 3u = -e(3u - u)$	A1 ft
	$e = \frac{1}{2}$	A1 ft (if < 1)
		[3]
c)	kinetic energy before impact = $\frac{1}{2}(2m)(3u)^2 + \frac{1}{2}mu^2 = 9\frac{1}{2}mu^2$	M1 A1
	kinetic energy after impact = $\frac{1}{2}(2m)(2u)^2 + \frac{1}{2}m(3u)^2 = 8\frac{1}{2}mu^2$	A1 ft
	Loss in kinetic energy = mu^2	M1 A1 ft
		[5]

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8. a)	$0.1 \times 2.5 = 0.25$ kg ms^{-1}	M1 A1
		[2]
b)	$F \times 0.05 = 0.25$	M1
	$F = 5N$	A1
		[2]
c)	$0.1 \times 2.5 = 0.1 \times V_W + 0.1 \times V_B$	M1 A1
	$\frac{3}{4} \times 2.5 = V_B - V_W$	B1
	$\frac{15}{8} + \frac{5}{2} = 2V_B$	M1 (solving)
	$\frac{35}{16} = V_B$	A1
	$\frac{5}{16} = V_W$	
		[5]
d)	Initial k.e. = $\frac{1}{2} \times 0.1 \times 2.5^2$	M1 A1
	Final k.e. = $\frac{1}{2} \times 0.1 \times \left(\frac{5}{16}\right)^2 + \frac{1}{2} \times 0.1 \times \left(\frac{35}{16}\right)^2$	
	Loss = 0.0684 J	A1
		[3]
e)	$\frac{35}{16} \times 0.8 - \frac{5}{16} \times 0.8$	M1
	=1.5m	A1
		[2]

NEWTON'S LAW OF RESTITUTION

9. a)	$6m = mv_A + mv_B$	M1 A1
	$6e = v_B - v_A$	B1
	$\Rightarrow 6 - 6e = 2v_A$	M1
	$v_A = 3(1 - e)$	A1
	$v_B = 6e + v_A$	
	$= 3(1 + e)$	A1
		[6]
b)	$3m(1 + e) = mB + mC$	B1
	$3e(1 + e) = C - B$	B1
	$3(1 + 2e + e^2) = 2C$	M1
	$\frac{3}{2}(1+e)^2 = C$	A1
	$\frac{3}{2}\left(1-e^2\right) = B$	
		[4]

10.a)	$mu = mv_A + 3mku$	M1
	$v_A = u(1 - 3k)$	A1
		[2]
b)	$eu = ku - u(1 - 3k)$	M1 A1 f.t.
	$e = k - 1 + 3k$	
	$= 4k - 1$	
	$0 \le e \le 1$	M1
	$0 \le 4k - 1 \le 1$	
	$\frac{1}{4} \le k \le \frac{1}{2}$	A1
		[4]

11.a)	$3mu - 2mu = mv_B \quad \Rightarrow u = v_B$	M1 A1
	$e(u + 3u) = v_B \Rightarrow e = \frac{1}{4}$	M1 A1
		[4]
b)	$3mu$	B1
		[1]
c)	Initial k.e.= $\frac{1}{2}(9mu^2 + 2mu^2) = \frac{11}{2}mu^2$	M1 A1
	Final k.e. = $\frac{1}{2}mu^2$	A1
	Loss = $5mu^2$	A1
		[4]

NEWTON'S LAW OF RESTITUTION

12. a)	$mu - kmku = mv_A + kmv_B$	M1 A1
	(so, $u(1 - k^2) = v_A + kv_B$)	
	$e(u + ku) = v_B - v_A$	M1 A1
	$u(1 - k^2 + e + ke) = v_B(1 + k)$	M1
	$u\left(\dfrac{1-k^2}{1+k} + \dfrac{e(1+k)}{1+k}\right) = v_B$	
	$u(1 - k + e) = v_B$	A1
	$v_A = u(1 - k^2) - kv_B$	M1
	$= u(1 - k^2 - k + k^2 - ke)$	
	$= u(1 - k - ke)$	A1
		[8]
b)	$v_B = 0$	M1
	$1 - k + e = 0$	
	$k = 1 + e$	A1
		[2]
c)	After collision with wall,	
	$v_A = -\frac{1}{2}u(1 - 1.4 - 0.56)$	
	$= 0.48u$	B1
	In second collision:	
	$0.48\ mu = mA + 1.4\ mB$	M1
	$0.4 \times 0.48u = B - A$	M1
	so	
	$B = 0.28u$	M1
	$A = 0.088u$	A1
		[5]
	d) If no collisions occur, both are moving away from the wall and $v_B > v_A$	M1
	v_B, v_A both positive and $v_B - v_A = 0.28u - 0.088u = 0.192u > 0$	A1
		[2]