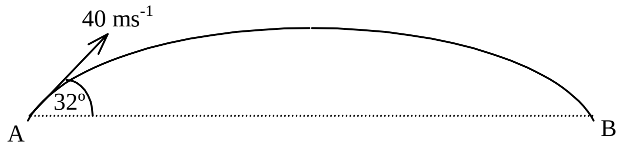
PROJECTILES

1.



Vertically from A to B using $s = ut + \frac{1}{2}at^2 \Rightarrow 0 = (40\sin 32)t - \frac{1}{2}gt^2$	M1 A1
$t = \dfrac{2 \times 40 \sin 32°}{9.8} = 4.33$ seconds.	A1
Range horizontally = (40 cos 32°) (time of flight)	M1
= 147 m (nearest metre)	A1
	[5]

---

2. For time of flight: $0 = v\sin\alpha t - \frac{1}{2}gt^2$	M1
$\Rightarrow t = \dfrac{2v\sin\alpha}{g} = 4$	A1
So $v\sin\alpha = 19.6$ ①	
For range: $v\cos\alpha \times \dfrac{2v\sin\alpha}{g} = 4v\cos\alpha = 160$	M1 A1
So $v\cos\alpha = 40$ ②	
① and ② $\Rightarrow$ $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{19.6}{40} \Rightarrow \alpha = 26°$	M1 A1
$v = \dfrac{19.6}{\sin\alpha} = 44.5\text{ms}^{-1}$	M1 A1
Greatest height: $-2gh = -v^2\sin^2\alpha$	M1
$\Rightarrow h = 19.6$m	A1
	[10]

---

3. a) Using s = ut horizontally as no acceleration	M1
$960 = 30t \Rightarrow t = 32$ seconds	A1
	[2]
b) Greatest height: $-2gh = 0 - 40^2$	M1
$\Rightarrow h = \dfrac{1600}{2g} = 81.6$ m	A1
	[2]

PROJECTILES

4.	a) Using $s = ut + \frac{1}{2}at^2$, $s = -14$; $u = 0$; $a = -g$	M1
	$-14 = -4.9t^2$ $t = 1.69$	A1
		[2]
	b) Horizontally no acceleration $\Rightarrow s = ut$	M1
	Distance $= 12 \times 1.69 = 20.3$ m	A1 f.t
		[2]
	c) Vase modelled as a particle	B1
	Air resistance neglected.	B1
		[2]

5.	a) Using $s = ut + \frac{1}{2}at^2$ vertically	M1
	$s = 0$, $u = 50\sin 60°$	B1 (both)
	$a = -g$	
	$0 = 50t\sin 60^o - 4.9t^2$	
	$0 = t(50\sin 60^o - 4.9t)$	M1 (solving)
	$t = 8.84$ seconds	A1
		[4]
	b) No acceleration $\Rightarrow s = ut$	M1
	$u = 50\cos 60°$,	
	$t = 8.84$	
	$s = 50\cos 60^o \times 8.84 = 221$m	A1
		[2]

PROJECTILES

6. a) Let speed = U; angle = α
In air for 6 seconds $\Rightarrow$ vertical displacement 0 after this time.
$\Rightarrow$ Using $s = ut + \frac{1}{2}at^2$ M1
$s = 0$; $a = -g$; $t = 6$ $u = U\sin\alpha$ A1
$0 = U\sin\alpha \times 6 - 4.9 \times 36$
$U\sin\alpha = 29.4$ A1

Horizontal range 120 m
$\Rightarrow$ using $s = ut + \frac{1}{2}at^2$ horizontally M1
$s = 120$; $a = 0$; $u = U\cos\alpha$, $t = 6$ A1
$120 = 6U\cos\alpha$

So: $U\sin\alpha = 29.4$
$U\cos\alpha = 20$
$\Rightarrow \tan\alpha = 1.47$ M1 A1
$\Rightarrow \alpha = 56^o$ A1
[8]

b) $U\sin\alpha = 29.4$
$\Rightarrow U = \dfrac{29.4}{\sin\alpha}$ M1 A1
$= 35.6\ ms^{-1}$ A1

OR $U^2\sin^2\alpha + U^2\cos^2\alpha = 29.4^2 + 20^2$ M1 A1
$U^2 = 1264 \Rightarrow U = 35.6$ A1
[3]

---

7. a) Let speed = U
Using $s = ut + \frac{1}{2}at^2$ M1
Horizontally: $150 = Ut\cos 50^o$ ① A1
Vertically: $0 = Ut\sin 50^o - \frac{1}{2}gt^2$ ② A1
② $\Rightarrow t = \dfrac{2U\sin 50}{g}$
$150 = \dfrac{2U^2\sin 50\cos 50}{g}$ M1 A1
$U^2 = \dfrac{150g}{2\sin 50\cos 50}$ A1
$\Rightarrow U = 38.6\ ms^{-1}$ A1
[7]

b) No air resistance or wind B1
Ball modelled as particle. B1
[2]

PROJECTILES

8. a) Let angle of projection be α

Vertical displacement = 0

$\Rightarrow$ use $s = ut + \frac{1}{2}at^2$ M1

$0 = 30\sin\alpha t - \frac{1}{2}gt^2$ A1

$0 = t(30\sin\alpha - \frac{1}{2}gt)$ M1 (solving)

$t = \frac{60\sin\alpha}{g}$ A1

Now use horizontal displacement:

$s = 30\cos\alpha \times \frac{60\sin\alpha}{g}$ M1

$= \frac{1800\sin\alpha\cos\alpha}{g}$ A1

$= \frac{900\sin 2\alpha}{g}$

Maximum when $\sin 2\alpha = 1$

$\Rightarrow \frac{900}{g} = 91.8$ is max range B1

[7]

b) $\sin 2\alpha = 1 \Rightarrow 2\alpha = 90°$ B1

$\alpha = 45°$

[1]

---

9. a) Maximum height is achieved when projecting vertically. B1

Using $2as = v^2 - u^2$ M1

$2 \times -gs = -45^2 \Rightarrow s = \frac{45^2}{2g}$ A1

$s = 103$ m. A1

[4]

b) Using $v = u + at$ M1

$0 = 45 - gt$

$t = \frac{45}{g} = 4.59$ seconds A1

[2]

c) Using $s = ut + \frac{1}{2}at^2$ vertically M1

$s = 45\sin 60°\left(\frac{45}{g}\right) - \frac{1}{2}g\left(\frac{45}{g}\right)^2$ A1

$= 75.6$ m A1

[3]

PROJECTILES

10.a) Let speed with which it is projected be U

Using $s = ut + \frac{1}{2}at^2$	M1
Vertically: $0 = U\sin 30t - \frac{1}{2}gt^2$ ①	A1
Horizontally: $175 = U\cos 30t$ ②	A1
Substituting ② into ① :	M1
$0 = \dfrac{U\sin 30° \times 175}{U\cos 30°} - \dfrac{g}{2} \times \dfrac{175^2}{U^2\cos^2 30°}$	
$0 = 175\tan 30° - \dfrac{175^2 g}{2U^2\cos^2 30°}$	A1
$U = 44.5\ ms^{-1}$	M1 A1
	[7]

b) To find maximum height:

$2as = v^2 - u^2$ vertically	
$-2gs = -(44.5\sin 30)^2$	M1
$s = 25.3$ m.	A1
	[2]

11.a) Suppose he fires at an angle α.

Using $s = ut + \frac{1}{2}at^2$	M1
Vertically: $10 = 150\sin\alpha t - \frac{1}{2}gt^2$ ①	A1
Horizontally: $500 = 150\cos\alpha t$ ②	A1
Substitute ② into ①: $10 = 150\sin\alpha \times \dfrac{500}{150\cos\alpha} - \dfrac{g}{2} \times \dfrac{500^2}{150^2\cos^2\alpha}$	M1
$10 = 500\tan\alpha - \dfrac{g}{2} \times \dfrac{100}{9\cos^2\alpha}$	A1

Need to solve for α:

$10 = 500\tan\alpha - \dfrac{50g}{9}\sec^2\alpha$	M1
$10 = 500\tan\alpha - \dfrac{50g}{9}(1 + \tan^2\alpha)$	A1
$\dfrac{50g}{9}\tan^2\alpha - 500\tan\alpha + 10 + \dfrac{50g}{9} = 0$	A1
Solving: $\tan\alpha = 9.053$ or 0.1308	M1 A1 A1
$\alpha = 7.5°$ or $83.7°$	A1 f.t
	[12]

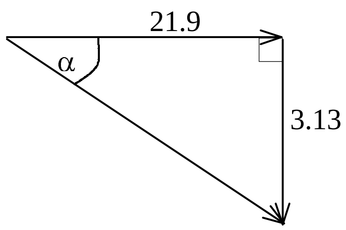
PROJECTILES

12. a) Let speed be U

Using $s = ut + \frac{1}{2}at^2$		M1
Horizontally:	$7 = Ut$ ①	A1
Vertically:	$0.5 = \frac{1}{2}gt^2$ ②	A1
② $\Rightarrow t = 0.319$ seconds		A1
Substitute into ①: $7 = U(0.319)$		M1
$U = 21.9\ ms^{-1}$		A1
		[6]

b) Using $v = u + at$

Vertically:	$v = -g(0.319) = -3.13\ ms^{-1}$	M1
Horizontally:	$v = 21.9\ ms^{-1}$	A1



So speed $= \sqrt{21.9^2 + 3.13^2}$ M1

$= 22.1\ ms^{-1}$ A1

At $\tan^{-1}\dfrac{3.13}{21.9} = 8.1°$ B1

Below horizontal B1

[6]

---

13. a)

Using $s = ut + \frac{1}{2}at^2$		M1
Vertically:	$-40 = -\frac{1}{2}gt^2$ ①	A1
Horizontally:	$D = 30t$ ②	A1
① $\Rightarrow t = \sqrt{8.16} = 2.86$ seconds		M1 (solving)
Substitute into ②: $D = 30 \times 2.86m = 86m$ (2SF)		A1
		[5]

b) Air resistance neglected/ Wind neglected/ Stone modelled as particle B2 (any 2 from 3)

Air resistance: decrease range

Wind: could increase or decrease range B1

[3]

PROJECTILES

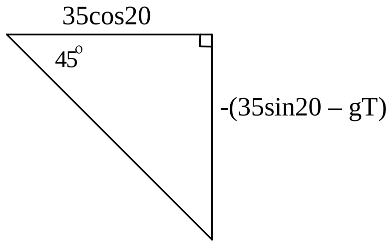
14. a)	$s = ut + \frac{1}{2}at^2$	M1
	$-65 = 15\sin 35t - \frac{1}{2}gt^2$	A1 A1
	$0 = 4.9t^2 - 15\sin 35t - 65$	M1 (Solving) A1
	$t = 4.62$s (Negative root not applicable)	A1
		[6]
b)	First stone: distance from cliff $= 15\cos 35° \times 4.62 = 56.8$m	M1 A1
	Second stone: use $s = ut + \frac{1}{2}at^2$	M1
	$65 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\dfrac{2 \times 65}{g}} = 3.64$ seconds	
	Distance $= 15 \times 3.64 = 54.6$m	A1
	$56.8\text{m} - 54.6\text{m} = 2.2\text{m}$	B1 f.t.
		[5]

---

15. a)	Using $s = ut + \frac{1}{2}at^2$	M1
	Horizontally: $2000 = 300\cos 15t$	A1
	$t = 6.90$ seconds	A1
	Vertically downwards: $s = 300\sin 15t + \frac{1}{2}gt^2$	M1 A1
	$= 300\sin 15(6.90) + 4.9(6.90)^2 = 769$ m	A1
		[6]

---

16. a)	Using $v = u + at$	M1
	Vertically: $v = 35\sin 20 - gT$	A1
	Horizontally: $v = 35\cos 20$	B1
	45° below vertical $\Rightarrow 35\sin 20 - gT < 0$	B1



	So: $\dfrac{gT - 35\sin 20}{35\cos 20} = 1$	B1
	$T = 4.58$ seconds	B1
		[6]
b)	Using $s = ut + \frac{1}{2}at^2$	M1
	$= 35\sin 20(4.58) - \frac{1}{2}g(4.58)^2$	
	$= -48.0$m (- indicates below roof of building)	A1
		[2]

PROJECTILES

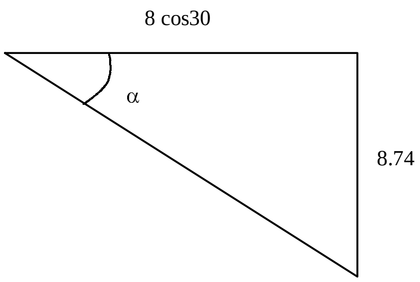
17. a)	Using $s = ut + \frac{1}{2}at^2$ vertically	M1
	$0 = 5\sin 20t - \frac{1}{2}gt^2$	A1
	$0 = t(5\sin 20 - \frac{1}{2}gt)$	M1 (Solving)
	$t = 0.349$ seconds	A1
		[4]
b)	$-3 = 5\sin 20t - \frac{1}{2}gt^2$	B1
	$0 = \frac{1}{2}gt^2 - 5\sin 20t - 3$	M1 (Solving)
	$t = 0.976$ seconds	A1
		[3]
c)	Using $v = u + at$	M1
	Vertically: $v = 5\sin 20 - 0.976g$	
	$v = -7.85\ ms^{-1}$	A1
	Horizontally: $v = 5\cos 20$	B1
	Speed $= \sqrt{(5\cos 20)^2 + (-7.85)^2}$	M1
	$= 9.15\ ms^{-1}$	A1
OR	Use conservation of energy	
	$\frac{1}{2}v^2 = 3g + \frac{1}{2} \times 25$	M1 A1 A1
	$v^2 = 6g + 25$	A1
	$v = 9.15\ ms^{-1}$	A1
		[5]

---

18. a)	$s = ut + \frac{1}{2}at^2$ vertically	M1
	$-3 = 8\sin 30t - \frac{1}{2}gt^2$	A1
	$4.9t^2 - 4t - 3 = 0$	M1
	$t = 1.29$ seconds	A1
		[4]
b)	Horizontally: $x = ut$	M1
	$\Rightarrow x = 8\cos 30(1.29)$	
	$x = 8.94$ m	A1
	So lands in basket	B1
		[3]
c)	No wind/ No air resistance/ Egg behaves as a particle.	B1 (Any one)
		[1]

PROJECTILES

19. a) Using $s = ut + \frac{1}{2}at^2$	M1
Horizontally: $9 = 8\cos 30t$ ①	A1
Vertically: $s = 8\sin 30t - \frac{1}{2}gt^2$ ②	A1
① $\Rightarrow$ time = 1.30 seconds	M1
So vertical distance = -3.07 m	A1
Height of ball above street level = 15.5 – 3.07 = 12.43 m	M1
$\Rightarrow$Ball goes through window.	B1
	[7]
b) Using v =u + at	M1
Vertically: $v = 8\sin 30° - g(1.30)$	
$v = -8.74\text{ms}^{-1}$	A1
Horizontally: $v = 8\cos 30°$	



Speed $= \sqrt{(8\cos 30°)^2 + (8.74)^2}$	M1
$= 11.2\text{ ms}^{-1}$	A1
At $\arctan \dfrac{8.74}{8\cos 30°} = 51.6°$	B1
Below horizontal	B1
	[6]
c) 11.2 ms^{-1} at 51.6° above horizontal	B1 f.t B1 f.t.
	[2]
d) Neglecting air- resistance may mean a higher speed would be necessary/ Neglecting wind may increase or decrease necessary speed.	B1 (either)
	[1]

20.a) Use $s = ut + \frac{1}{2}at^2$	M1
Vertically: $0.3 = U\sin 60t - \frac{1}{2}gt^2$ ①	A1
Horizontally: $15 = U\cos 60t$ ②	A1
Substituting ② into ① :	M1
$0.3 = U\sin 60° \dfrac{15}{U\cos 60°} - \dfrac{g}{2} \times \dfrac{15^2}{U^2\cos^2 60°}$	
$0.3 = 15\tan 60° - \dfrac{225g}{\frac{2u^2}{4}}$	A1
$u = 13.1\text{ ms}^{-1}$	M1 A1
	[7]
b) Use $2as = v^2 - u^2$ vertically	M1
$2 \times -gs = 0 - (13.1\sin 60°)^2$	A1
$s = 6.57$ m	A1
	[3]

PROJECTILES

21.a)	Use $s = ut + \frac{1}{2}at^2$	M1
	Horizontally: $10 = 15\cos 30t$ ①	A1
	Vertically: $s = 15\sin 30t - \frac{1}{2}gt^2$ ②	A1
	① $\Rightarrow t = 0.770$	M1
	Substitute into ② $\Rightarrow s = 2.87$ m	A1
	$\Rightarrow$ Height of 4.47 m	B1
		[6]
b)	Let rebound speed be u.	
	Using $s = ut + \frac{1}{2}at^2$	M1
	Horizontally: $8 = ut$ ①	A1
	Vertically: $-4.47 = -\frac{1}{2}gt^2$ ②	A1
	② $\Rightarrow t = 0.955$	M1 (Combining)
	$\Rightarrow u = \frac{8}{t} = 8.38\ \text{ms}^{-1}$	A1
		[5]

PROJECTILES

22.a) When it hits wall, vertical velocity = 0. B1

Use v = u + at M1

$0 = U\sin\alpha - gt$

$t = \dfrac{U\sin\alpha}{g}$ A1

So distance horizontally: $X = U\cos\alpha \times t$ M1

$$X = \frac{U\cos\alpha \times U\sin\alpha}{g} = \frac{U^2}{g}\sin\alpha\cos\alpha$$ A1

[5]

b) When ball rebounds:

Initial speed = $\dfrac{U\cos\alpha}{2}$ horizontally

Using $s = ut + \frac{1}{2}at^2$ M1

Horizontally: $X - 2 = \dfrac{U\cos\alpha}{2} \times t$ A1

Vertically, it will take the same time to fall as it took to reach the wall. B1

So $t = \dfrac{U\sin\alpha}{g}$

So $X - 2 = \dfrac{U\cos\alpha}{2} \times \dfrac{U\sin\alpha}{g}$ B1

But $X = \dfrac{U\cos\alpha U\sin\alpha}{g}$

$\Rightarrow \dfrac{U\cos\alpha U\sin\alpha}{g} - 2 = \dfrac{U\cos\alpha U\sin\alpha}{2g}$ M1

So: $\dfrac{U^2\cos\alpha\sin\alpha}{2g} = 2$ A1

$U^2 = \dfrac{4g}{\cos\alpha\sin\alpha}$

$U = 2\sqrt{\dfrac{g}{\cos\alpha\sin\alpha}}$ A1

[7]

PROJECTILES

23.a) Use $s = ut + \frac{1}{2}at^2$ where $a = -g$ $t = 1$ $u = 30$ M1

Ball at B: $S_B = 30 - \frac{1}{2}g$ A1

Ball at A: Horizontally: $S_{A(H)} = U\cos\alpha$ A1

Vertically: $S_{A(V)} = U\sin\alpha - \frac{1}{2}g$ A1

When they collide:

$$\left.\begin{aligned} S_{A(H)} &= 50 \\ S_{A(V)} &= S_B \end{aligned}\right\}$$ B1

$$\left.\begin{aligned} 50 &= U\cos\alpha \quad ① \\ 30 - \tfrac{1}{2}g &= U\sin\alpha - \tfrac{1}{2}g \quad ② \end{aligned}\right\}$$ B1

② ⇒ $30 = u\sin\alpha$ ③

③ ÷ ① ⇒ $\frac{30}{50} = \tan\alpha$ M1

$\tan\alpha = \frac{3}{5}$ A1

[8]

b) $U = \frac{30}{\sin\alpha}$ M1

$= 58.3 \text{ ms}^{-1}$ A1

[2]