VERTICAL CIRCULAR MOTION

Wherever a numerical value is required, take $g = 9.8ms^{-2}$

1. A spin drier rotates at 1000 revolutions per minute about a horizontal axis and has a drum of diameter 50cm. It contains a small item of clothing of mass 20g which remains in contact with the drum and at rest relative to it, at all times, so that it moves in a vertical circle.
By modelling the item of cloth as a particle, find the maximum magnitude of the contact force between the item of clothing and the drum.

[6]

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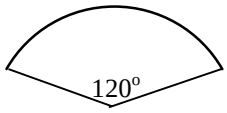
2. A car travelling at constant speed passes over a humpbacked bridge which is in the form of an arc of angle 120^0 from a circle with radius 50m, as shown.



a) Find the maximum speed at which the car can cross the bridge without losing contact with the road.

[6]

b) State one assumption you have made in your mathematical modelling

[1]

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3. A particle of mass m kg is attached to a fixed point A by a light inelastic string of length L. The particle hangs vertically below A and is given a horizontal velocity of u ms^{-1}. It moves in a vertical circle until the string makes an angle α with the upward vertical through A.

a) Show that $u^2 = gL(3\cos\alpha + 2)$

[9]

b) Find the speed of the particle at the point when its motion ceases to be circular

[2]

c) Given that $\alpha = 60^0$, find the maximum height above A reached by the particle.

[8]

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4. A particle of mass 2kg is attached by a light, inextensible string of length 1m to a fixed point. The mass is made to swing in a vertical arc through an angle of 60^0 either side of the vertical through the fixed point. Find:

a) the velocity of the particle at the lowest point of the arc

[5]

b) the tension in the string when it makes an angle of 10^o with the vertical

[5]

c) the maximum tension in the string.

[2]

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5. A particle of mass m kg is attached to end A of a light rigid rod AB of length L metres. B is attached to a fixed point O, and the rod hangs with A below B. A horizontal impulse is applied to the particle, which causes it to start moving with speed u ms^{-1}.

a) Given that the particle moves in complete vertical circles, find the minimum possible value of u, giving your answer in terms of g and L

[6]

When the particle first returns to its starting point, it collides and coalesces with an identical particle. Given that u takes the minimum value found in a),

b) Find the initial speed of the combined mass.

[2]

c) Find the maximum height to which the combined mass rises.

[4]

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6. A hollow smooth cylinder of radius 1m is placed with its axis horizontal. A small ball of mass m is placed inside the cylinder, and projected with speed u so that it starts to move in a vertical circle.

a) Given that the motion of the ball is oscillatory, find the maximum possible value of u.

[5]

Given that u takes half this maximum value

b) Find the maximum height of the ball above the lowest point of the cylinder.

[6]

c) Find the maximum force exerted by the cylinder on the ball

[3]

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7. A smooth hemisphere of radius 0.6 m and centre O is placed flat-side down on a flat horizontal table. A particle P is placed on the highest point of the hemisphere, and slightly disturbed, so that it begins to move down the hemisphere.

a) Show that the particle loses contact with the hemisphere when it is a height of 0.4m above the table.

[10]

b) Find the time taken for the particle to hit the table from the instant when it leaves the hemisphere.

[8]

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8. A particle of mass 2kg is attached to a light, inextensible string of length 2m. The other end of the string is attached to a fixed point, O. The particle is held vertically above O, and projected horizontally with speed 5 ms^{-1}.

a) Find expressions for the speed of the particle and the tension in the string when the string makes an angle of θ with the upward vertical.

[8]

b) State the maximum and minimum values of the tension in the string.

[2]

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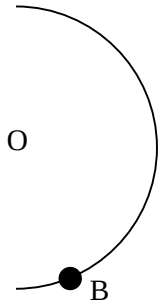
9. A bead B of mass 0.1kg is threaded on a light, rigid semicircular piece of wire of radius 0.3m and centre O which is positioned vertically, as shown below.



The bead is projected from the lowest point of the wire with speed u so that it moves up the wire.

a) Given that the bead reaches the highest point of the wire, find the minimum value of u.

[5]

Given that u takes this value,

b) find the angle that OB makes with the downward vertical at the instant when the normal contact force between the bead and the wire is zero.

[8]

c) Find the time taken for the bead to pass O if it falls off the wire after reaching the highest point.

[3]

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10. A bowl is in the form of a smooth hemisphere with centre O and radius a. The bowl is fixed with its rim horizontal and uppermost. A particle P of mass m is held in contact with the inside of the bowl, level with O, then released.

a) Find an expression for the speed of the particle when OP makes an angle of α with the downward vertical.
[4]

When OP makes an angle of 60^o to the downward vertical, P collides and coalesces with a stationary particle of twice its mass.

b) Find the maximum height to which the combined particle will rise in the subsequent motion
[10]

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11. The diagram below shows a section of track for a toy car. The curved section BC is a quarter of the circumference of a circle of radius 0.2m, and the straight section AB is of length 0.6m. The section is fixed in a vertical plane.

A child projects a toy car of mass 0.03kg from A along the track with a speed of 1.5 ms^{-1}.

a) Given that the coefficient of friction between section AB of the track and the toy car is 0.1, find the speed of the car when it reaches B.
[6]

b) Assuming section BC of the track is smooth
i) Find the maximum height the toy car reaches above the level of AB
[3]

ii) Find the magnitude of the instantaneous change in the normal reaction force exerted by the track on the car as it passes point B.
[5]

iii) State what effect it would have on your answer to b) a) if it was not assumed that the track was smooth, and explain your answer
[2]

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12. A particle P of mass 1kg is attached to one end of a light, inextensible string of length $\sqrt{2}$ metres. The other end of the string is attached to a fixed point O, with P vertically below O.

P is projected horizontally with speed U ms^{-1}. It initially moves in a vertical circle, but leaves its circular path when OP makes an angle of 45^o with the upward vertical.

a) Show that $U^2 = g(2\sqrt{2} + 3)$ [9]

b) Find the speed of the particle as it leaves the circular path. [3]

c) Find the maximum height above O reached by the particle in its subsequent motion [3]

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13. A particle P of mass 1kg is placed at the midpoint of a light inextensible string AB of length 2m. The ends of the string are fastened in a horizontal line so that AB =$\sqrt{2}$ m, and the particle hangs in equilibrium.

a) Find the tension in the string, giving your answer in terms of g and surds [4]

The particle is set in motion horizontally with speed U. It performs complete vertical circles.

b) Find the minimum possible value of U [11]

c) Given U takes this value, find the tension in the strings when the particle is level with AB. [3]
