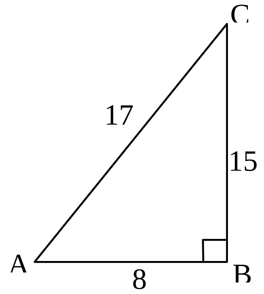
CENTRE OF MASS I

1. a)	Using Σ mass $\times$ position = total mass $\times$ position of centre of mass	M1
	$3(0, 2) + 4(1, 6) + 5(-3, 2) = 12(x, y)$	M1
	$(-11, 40) = 12(x, y)$	
	$\left(-\frac{11}{12}, \frac{10}{3}\right) = (x, y)$	A1 A1
		[4]
b)	$3(0,2) + 4(1,6) + 5(-3,2) + 10(x,y) = (0,0)$	M1
	$10x - 11 = 0 \quad \Rightarrow x = 1\frac{1}{10}$	A1
	$40 + 10y = 0 \quad \Rightarrow y = -4$	A1
		[3]

CENTRE OF MASS I

2. a)



Let m = mass of unit length

Body	Mass	Distance of centre of mass from AB	Distance of centre of mass from BC
AC	17m	$7\frac{1}{2}$	4
AB	8m	0	4
BC	15m	$7\frac{1}{2}$	0
Whole thing	40m	$\overline{Y}$	$\overline{X}$

M1 A1 A1

i) $17m \times 7\frac{1}{2} + 15m \times 7\frac{1}{2} = 40m\,\overline{Y}$ M1

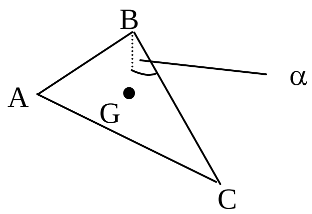
$6 = \overline{Y}$ A1

ii) $17m \times 4 + 8m \times 4 = 40m\,\overline{X}$

$2.5 = \overline{X}$ A1

[6]

b)



M1 (C of M below point of suspension)

$\tan \alpha = \dfrac{\overline{X}}{\overline{Y}}$ M1

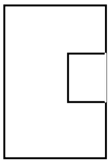
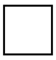
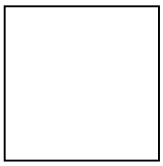
$= \dfrac{5}{12}$ A1 f.t.

$\alpha = 22.6°$ A1

[4]

CENTRE OF MASS I

3. (a) Let m = mass of unit area.

Body	Mass	Distance of centre of mass from AD	Distance of centre of mass from AB
	11m	$\overline{X}$	$\overline{Y}$
	m	$3\frac{1}{2}$	$1\frac{1}{2}$
	12m	2	$1\frac{1}{2}$

M1 A1 A1

a) $11m\overline{X} + 3\frac{1}{2}m = 2 \times 12\text{ m}$ — M1

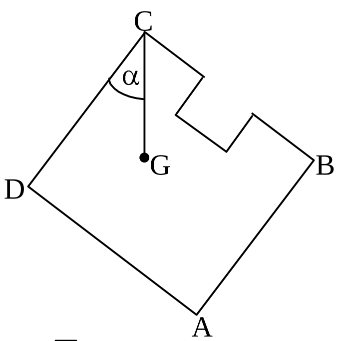
$\overline{X} = \dfrac{41}{22}$ — A1

b) $11m\overline{Y} + 1\frac{1}{2}m = 1\frac{1}{2} \times 12m$

$\overline{Y} = \dfrac{3}{2}$ — A1

[6]

b)



M1 (C of M below point of suspension)

$\tan\alpha = \dfrac{3-\overline{Y}}{4-\overline{X}}$ — M1

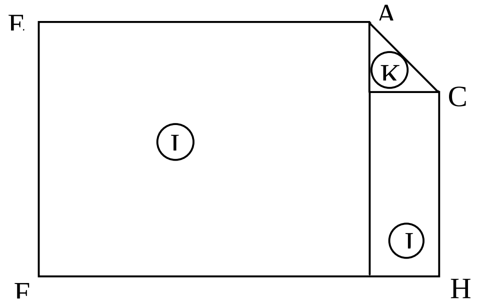
$= \dfrac{33}{47}$ — A1

$\alpha = 35.1°$ — A1

[4]

CENTRE OF MASS I

4. a) Let m be mass of unit area.



Divide sheet as above.

Body	Mass	Distance of centre of mass from FH	Distance of centre of mass from FE
I	16m	2	2
J	4m	1	5
K	4m	$2\frac{2}{3}$	$4\frac{2}{3}$
Whole thing	24m	$\overline{Y}$	$\overline{X}$

M1 A1

M1 A1 (triangle)

so $16m \times 2 + 4m \times 1 + 4m \times 2\frac{2}{3} = 24m\overline{Y}$ M1

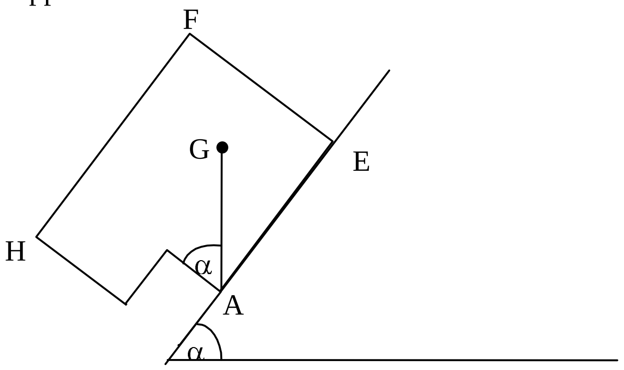
$\frac{35}{18} = \overline{Y}$ A1

$16m \times 2 + 4m \times 5 + 4m \times 4\frac{2}{3} = 24m\overline{X}$

$\frac{53}{18} = \overline{X}$ A1

[7]

b) If about to topple



M1 (centre of mass vertically above A)

$\tan \alpha = \frac{4 - \overline{X}}{4 - \overline{Y}}$ M1

$= \frac{19}{37}$ A1

$\alpha = 27.2°$ A1

[4]

CENTRE OF MASS I

5. a) Let m be mass of unit volume.

Body	Mass	Distance of centre of mass from AB
cylinder	$64\pi m$	8
"upper cone"	$\frac{32\pi m}{3}$	16 + 2
"lower cone"	$\frac{32\pi m}{3}$	2
toy	$64\pi m$	d

M1 A1 A1

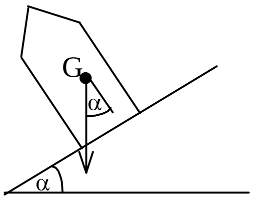
$$64\pi m \times 8 + \frac{32\pi m}{3} \times 18 = \frac{32\pi m}{3} \times 2 + 64\pi m \times d$$ M1 A1

$$704 = 21\tfrac{1}{3} + 64d$$

$$10\tfrac{2}{3} = d$$ A1

[6]

b) If toy is about to topple:



M1

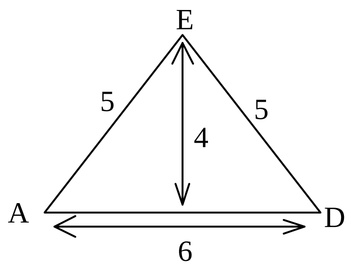
$$\tan\alpha = \frac{2}{10\frac{2}{3}}$$ M1 A1 f.t

$$\alpha = 10.6°$$ A1

[4]

CENTRE OF MASS I

6. a)



Body	Mass	Distance of centre of mass from AD
AB	60	-3
BC	60	-6
CD	60	-3
DE	50	2
EA	50	2
Whole thing	280	d

M1 A1

A1

so: 60(-3) + 60(-6) + 60(-3) + 50(2) + 50(2) = 280(d) M1

$$-\frac{13}{7} = d$$ A1

[5]

b) Centre of mass of combined body now on AD M1

Body	Mass	Distance of centre of mass from AD
Wire	280	$-\frac{13}{7}$
M	M	4
Combined body	280+M	0

M1 A1

so: $280 \times -\frac{13}{7} + 4m = 0$ M1

m = 130g A1

[5]

CENTRE OF MASS I

7. a) Let m = mass of unit area

Body	Mass	Distance of centre of mass from A
small circle	$\pi r^2 m$	r
lamina	$\pi m(R^2 - r^2)$	d
big circle	$\pi R^2 m$	R

M1 A1

$\pi r^2 mr + \pi m(R^2 - r^2)d = \pi R^2 mR$ M1

$$d = \frac{R^3 - r^3}{R^2 - r^2}$$ A1

$$\left(= \frac{R^2 + Rr + r^2}{R + r} \right)$$

[4]

b) $\dfrac{7r}{3} = \dfrac{R^3 - r^3}{R^2 - r^2}$

$\dfrac{7r}{3} = \dfrac{k^3 r^3 - r^3}{k^2 r^2 - r^2}$

M1 A1

$7r^3(k^2 - 1) = 3r^3(k^3 - 1)$ M1

$7k^2 - 7 = 3k^3 - 3$

$3k^3 - 7k^2 + 4 = 0$ A1

[4]

b) Try k = 1 : 3 – 7 + 4 = 0 M1

so: $3k^3 - 7k^2 + 4 \equiv (k - 1)(3k^2 - 4k - 4)$ M1 A1

$\equiv (k - 1)(3k + 2)(k - 2)$ A1

so $k = 1,\ -\frac{2}{3},\ 2$

k = 1 not possible, since lamina would not exist

$k = -\frac{2}{3}$ not possible, since negative

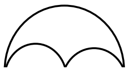
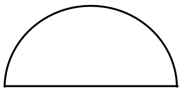
so k = 2 A1

[5]

CENTRE OF MASS I

8. a) For a semicircle, distance of centre of mass from straight edge is $\dfrac{2r\sin\frac{\pi}{2}}{\frac{3\pi}{2}} = \dfrac{4\pi}{3\pi}$ M1 A1

Let m be mass of unit area

Body	Mass	Distance of centre of mass from AB
	$\frac{1}{2}\pi R^2 m$	$\dfrac{4R}{3\pi}$
	$\frac{1}{2}\pi R^2 m$	$\dfrac{4R}{3\pi}$
	$\pi R^2 m$	d
	$\frac{1}{2}\pi 4R^2 m$	$\dfrac{4(2R)}{3\pi}$

M1 A1 A1

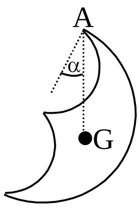
$$\frac{1}{2}\pi R^2 m\frac{4R}{3\pi} + \frac{1}{2}\pi R^2 m\frac{4R}{3\pi} + \pi R^2 md = 2\pi R^2 m\frac{8R}{3\pi}$$ M1

$$\frac{4R}{3\pi} + d = \frac{16R}{3\pi}$$

$$d = \frac{12R}{3\pi} = \frac{4R}{\pi}$$ A1

[7]

b)



M1 (C of M below point of suspension)

$$\tan\alpha = \frac{d}{2r}$$ M1

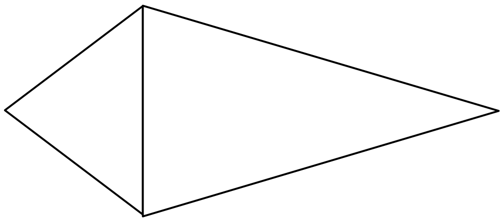
$$= \frac{2}{\pi}$$ A1 f.t.

$$\alpha = 32.5°$$ A1

[4]

CENTRE OF MASS I

9. a)



Let m be mass of unit area of large triangle.

Then mass of large triangle = $\frac{1}{2} \times 30 \times 20m = 300m$ B1

mass of small triangle = $\frac{1}{2} \times 30 \times 8km = 120km$ B1

Body	Mass	Distance of centre of mass from base
small triangle	120km	$-\frac{8}{3}$
big triangle	300m	$\frac{20}{3}$
whole thing	60m(5 +2k)	d

M1 A1

$120km(-\frac{8}{3}) + 300m(\frac{20}{3}) = 60m(5 + 2k)d$ M1

$-16k + 100 = 3(5 + 2k)d$

$\frac{100 - 16k}{3(5 + 2k)} = d$ A1

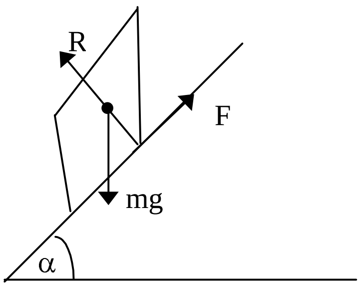
$\left(\frac{4(25 - 4k)}{3(5 + 2k)} = d\right)$

[6]

b) $\frac{100 - 16k}{3(5 + 2k)} = 0 \Rightarrow k = \frac{100}{16} = \frac{25}{4}$ M1 A1 f.t.

[2]

c)



Resolving: parallel to plane: $mg\sin\alpha = F$

perpendicular to plane: $mg\cos\alpha = R$ M1 A1

$F \le \mu R$ M1

$mg\sin\alpha \le \mu mg\cos\alpha$

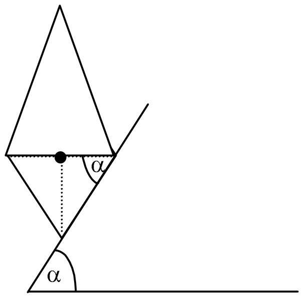
$\tan\alpha \le \mu$ A1

[4]

CENTRE OF MASS I

QUESTION 9 CONTINUED

d)



If about to topple, centre of mass is directly above vertex of small triangle M1

$\tan\alpha = \frac{8}{15}$ M1 A1

$\alpha = 28.1°$ A1

[4]

CENTRE OF MASS I

10.a) Let m = mass of unit volume

Body	Mass	Distance of centre of mass from vertex
small cone	$\frac{1}{3}\pi R^2Hm$	$\frac{3H}{4}$
large cone	$\frac{1}{3}\pi R^2kHm$	$H + \frac{kH}{4}$
whole thing	$\frac{1}{3}\pi R^2Hm(1 + k)$	d

M1 A1 A1

$$\tfrac{1}{3}\pi R^2Hm\left(\frac{3H}{4}\right) + \tfrac{1}{3}\pi R^2kHm\left(H + \frac{kH}{4}\right) = \tfrac{1}{3}\pi R^2Hm(1 + k)d$$ M1

$$\frac{3H}{4} + k\left(H + \frac{kH}{4}\right) = (1 + k)d$$

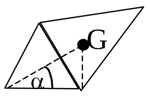
$$\frac{H(3 + 4k + k^2)}{4(1 + k)} = d$$ A1

$$\frac{H(k + 3)(k + 1)}{4(1 + k)} = d$$

$$\frac{H(k + 3)}{4} = d$$ A1

[6]

b) If body about to topple, centre of mass is directly above last point of contact M1



Semi-vertical angle of smaller cone is 45^o B1

So $\dfrac{\text{slant height}}{d} = \cos 45^o = \dfrac{1}{\sqrt{2}}$ M1

slant height = $\sqrt{R^2 + R^2} = \sqrt{2}R$ B1

$$\frac{\sqrt{2}R}{d} = \frac{1}{\sqrt{2}}$$ A1

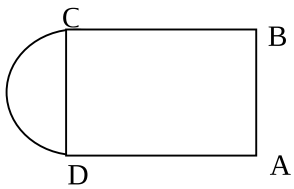
d = 2R A1

k = 5 A1

[7]

CENTRE OF MASS I

11.a)



Let m = mass of unit length

Body	Mass	Distance of centre of mass from AB
AB	2Rm	0
AD	Lm	$\frac{L}{2}$
BC	Lm	$\frac{L}{2}$
CD	2Rm	L
Arc	πRm	$L + \frac{2R}{\pi}$
Whole thing	$m(2L + 4R + \pi R)$	d

M1 A1 A1 A1

$Lm(\frac{L}{2}) + Lm(\frac{L}{2}) + 2Rm(L) + \pi Rm(L + \frac{2R}{\pi}) = m(2L + 4R + \pi R)d$ M1

$$\frac{L^2 + 2RL + \pi RL + 2R^2}{2L + 4R + \pi R} = d$$ A1

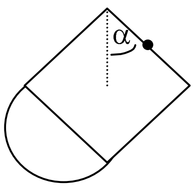
[6]

b) $L = R \Rightarrow d = R\frac{(5+\pi)}{(6+\pi)}$ B1

Body	Mass	Distance of centre of mass from AB
wire	m	$R\frac{(5+\pi)}{(6+\pi)}$
particle	2m	0
whole thing	3m	D

M1 A1

$Rm\frac{(5+\pi)}{(6+\pi)} = 3mD \Rightarrow R\frac{(5+\pi)}{3(6+\pi)} = D$ M1 A1



Centre of mass directly under point of suspension M1

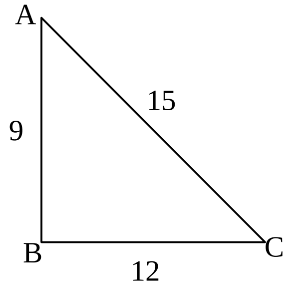
$\tan\alpha = \frac{D}{R} = \frac{5+\pi}{3(6+\pi)}$ M1

$\alpha = 16.5°$ A1

[8]

CENTRE OF MASS I

12.a)



Distance is $\frac{1}{3}$ height — M1

So from AB : 4cm — A1

from BC : 3cm — A1

[3]

b)

Body	Mass	Distance of centre of mass from AB	from BC
lamina	m	4	3
particle	km	12	0
whole thing	m(1 + k)	x	y

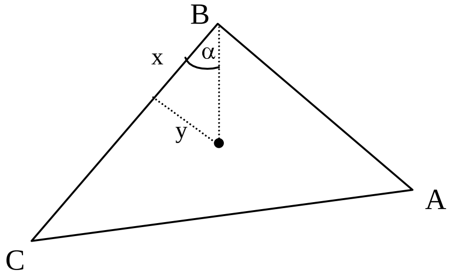
M1 A1

$4m + 12km = xm(1 + k)$ — M1

$\frac{4(1+3k)}{1+k} = x$ — A1

$3m = ym(1 + k)$ — M1

$\frac{3}{1+k} = y$ — A1



Centre of mass vertically below B — M1

$\tan\alpha = \frac{y}{x}$ — M1

$0.075 = \frac{\frac{3}{(1+k)}}{\frac{4(1+3k)}{(1+k)}}$ — M1

$0.075 = \frac{3}{4(1+3k)}$

$1 + 3k = \frac{3}{4(0.075)} = 10 \Rightarrow k = 3$ — A1

[10]

CENTRE OF MASS I

13.a) Let m = mass of unit volume.

Body	Mass	Distance of centre of mass from top of bowl
small hemisphere	$\frac{2}{3}\pi r^3 m$	$\frac{3}{8}r$
bowl	$\frac{2}{3}\pi m(R^3 - r^3)$	d
large hemisphere	$\frac{2}{3}\pi r R^3 m$	$\frac{3}{8}R$

M1 A1 A1

$$\frac{2}{3}\pi r^3 m\left(\frac{3r}{8}\right) + \frac{2}{3}\pi m\left(R^3 - r^3\right)d = \frac{2}{3}\pi R^3 m\left(\frac{3R}{8}\right)$$ M1

$$d = \frac{3(R^4 - r^4)}{8(R^3 - r^3)}$$ A1

[5]

b) $R = 2r \Rightarrow d = \dfrac{3(16r^4 - r^4)}{8(8r^3 - r^3)} = \dfrac{45r}{56}$ B1

Body	Mass	Distance of centre of mass from base of bowl of mass m
Bowl of mass m	m	$2r - \frac{45r}{56}$
Bowl of mass M	M	$2r + \frac{45r}{56}$
Whole thing	m + M	$\frac{10r}{7}$

M1 A1

$$m\left(\frac{67r}{56}\right) + M\left(\frac{157r}{56}\right) = (m + M)\left(\frac{10r}{7}\right)$$ M1

67m + 157M = 80m + 80M A1

77M = 13m A1

M : m is 13 : 77 **[7]**