PARTICLE KINEMATICS AND VECTOR MOTION

1. a) $\mathbf{v} = (2t - k)\mathbf{i} + 3t^2\mathbf{j}$	M1 A1
$t = 2 \Rightarrow \mathbf{v} = (4 - k)\mathbf{i} + 12\mathbf{j}$	B1
Parallel to $\mathbf{i} + 6\mathbf{j} \Rightarrow 4 - k = 2$	M1
$\Rightarrow k = 2$	A1
	[5]
b) $\mathbf{a} = 2\mathbf{i} + 6t\mathbf{j}$	M1 A1
So **i** component never 0.	A1
	[3]
c) Collide if $(t^2 - 2t)\mathbf{i} + t^3\mathbf{j} = (2t - 3)\mathbf{i} + (4t - 9)\mathbf{j}$	M1
i: $t^2 - 2t = 2t - 3$	M1
$t^2 - 4t + 3 = 0$	
$(t - 3)(t - 1) = 0$	
$t = 1, 3$	A1 A1
If $t = 1$: **j** component: $t^3\mathbf{j} = \mathbf{j}$ $(4t^2 - 9) = -5\mathbf{j} \Rightarrow$ no collision	M1
If $t = 3$: **j** component: $t^3\mathbf{j} = 27\mathbf{j}$ $(4t^2 - 9) = 27\mathbf{j} \Rightarrow$ collide	A1
	[6]

PARTICLE KINEMATICS AND VECTOR MOTION

2. a) $\mathbf{v} = \frac{15}{5}(3\mathbf{i} + 4\mathbf{j})$ M1

$= 9\mathbf{i} + 12\mathbf{j}$ A1

[2]

b) $\mathbf{r}_A = \mathbf{v}t$ B1

$\mathbf{r}_B = -9\mathbf{i} + (9\mathbf{i} + 12\mathbf{j})t$ M1 A1

$= (9t - 9)\mathbf{i} + 12t\mathbf{j}$

[3]

c) $t = 6$, $\mathbf{r}_A = \mathbf{r}_B$: M1

$6\mathbf{v} = 45\mathbf{i} + 72\mathbf{j}$ A1

$\mathbf{v} = 7\frac{1}{2}\mathbf{i} + 12\mathbf{j}$ A1

[3]

d) $\mathbf{r}_C = 8\mathbf{i} + t(2\mathbf{i} + 24\mathbf{j})$ M1 A1

$= (8 + 2t)\mathbf{i} + 24t\mathbf{j}$

So distance $= \sqrt{(9t - 9 - 8 - 2t)^2 + (12t - 24t)^2}$ M1 A1

$= \sqrt{(7t - 17)^2 + (-12t)^2}$

$= \sqrt{49t^2 - 238t + 289 + 144t^2}$ M1

$= \sqrt{193t^2 - 238t + 289}$ A1

$D^2 = 193t^2 - 238t + 289$

$\frac{d(D^2)}{dt} = 386t - 238 = 0$ for minimum M1 A1

$t = \frac{238}{386}$ A1

$D_{min} = 14.7$ km (3 S.F.) A1 f.t.

[10]

PARTICLE KINEMATICS AND VECTOR MOTION

3. a)	$(t-2)^2(t+1) \equiv (t^2-4t+4)(t+1)$	M1 A1
	$\equiv t^3-4t^2+4t-t^2-4t+4$	
	$\equiv t^3-3t^2+4$	A1
		[3]
b)	$v = \dfrac{dx}{dt}$	M1
	$= 3t^2-6t$	A1
	$a = \dfrac{dv}{dt}$	M1
	$= 6t-6$	A1 f.t.
		[4]
c)	At rest $\Rightarrow v = 0$	M1
	$3t^2-6t=0$	
	$3t(t-2)=0$	
	$t = 0, 2$	A1 A1
	$t = 0 \Rightarrow x = 4$	M1
	$t = 2 \Rightarrow x = 0$	A1 (both)
		[5]
d)	4 metres	B1
		[1]

PARTICLE KINEMATICS AND VECTOR MOTION

4. a) $x = \frac{4}{t^2} - \frac{1}{t}$ M1

$v = \frac{dx}{dt} = -\frac{8}{t^3} + \frac{1}{t^2}$ M1 A1

[3]

b) $x = 0 \Rightarrow t = 4$ B1

$v = -\frac{8}{4^3} + \frac{1}{4^2} = -\frac{1}{16}$ B1 ($\frac{1}{16}$)

B1 (– or direction given)

[3]

c) Changes direction if v changes sign M1

$v = -\frac{8}{t^3} + \frac{1}{t^2}$

$= \frac{-8+t}{t^3}$ M1

$v = 0$ when $t = 8$ only A1

[3]

d) Must split into $t = 7$ to 8 and $t = 8$ to 10 M1

$t = 7 : x = -\frac{3}{49}$ $t = 8 : x = -\frac{4}{64}$ A1 A1

so distance covered between $t = 7$ and $t = 8$ is $\frac{1}{784}$ (= 0.00128) B1

$t = 10$, $x = -\frac{6}{100}$. A1

so distance covered between $t = 8$ and $t = 10$ is $\frac{1}{400}$ (= 0.0025) B1

so total distance is $\frac{37}{9800}$ (= 0.00378) B1

[7]

e) Particle's velocity tends to zero B1

and its displacement tends to zero B1

[2]

PARTICLE KINEMATICS AND VECTOR MOTION

5. a)	Minimum distance occurs when $\frac{dx_Q}{dt} = 0$	M1
	$\Rightarrow 2t - 2 = 0$	
	$t = 1$	A1 (or comp sq)
	$t = 1,\ x_Q = 16$	A1
		[3]
b)	$x_P = x_Q$	M1
	$(2t-3)^2 = t^2 - 2t + 17$	
	$4t^2 - 12t + 9 = t^2 - 2t + 17$	
	$3t^2 - 10t - 8 = 0$	A1
	$(3t + 2)(t - 4) = 0$	M1 (solving)
	$t = \frac{2}{3},\ t = 4$	
	$t \geq 0 \Rightarrow t = 4$	A1 (t = 4 only)
		[4]
c)	$v = \frac{dx}{dt}$	M1
	$\left.\begin{array}{l} v_P = 8t - 12 \\ v_Q = 2t - 2 \end{array}\right\}$	A1
	Require $v_P = -v_Q$	M1
	$8t - 12 = 2 - 2t$	
	$10t = 14 \Rightarrow t = 1.4$	A1
		[4]

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6. a)	$a = \frac{d^2x}{dt^2}$	M1
	$\frac{dx}{dt} = 6t - 11$	A1
	$\frac{d^2x}{dt^2} = 6$	A1
	$F = ma \Rightarrow F = 2 \times 6 = 12N$	A1
		[4]
b)	Particle at A $\Rightarrow$ $16 = 3t^2 - 11t - 4$	M1
	$0 = 3t^2 - 11t - 20$	
	$0 = (3t + 4)(t - 5)$	M1 (solving)
	t must be positive $\Rightarrow t = 5$	A1 (t = 5 only)
	When $t = 5$, $v = 19$	B1
		[4]

PARTICLE KINEMATICS AND VECTOR MOTION

7. a) $\mathbf{r}_s = 6\mathbf{j} + 5(6\mathbf{i} + 3\mathbf{j})$	B1 (6**j**) M1 (**v**t)
$= 30\mathbf{i} + 21\mathbf{j}$	A1
	[3]
b) Must have $5\mathbf{v} = 30\mathbf{i} + 21\mathbf{j}$	M1
$\mathbf{v} = 6\mathbf{i} + 4.2\mathbf{j}$	A1
	[2]
c) Speed $= \sqrt{6^2 + 4.2^2}$	M1
$= 7.32$ kmh^{-1}	A1
Distance $= 5 \times 7.32 = 36.6$ km	M1 A1
	[4]

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8. a) Length of $12\mathbf{i} - 5\mathbf{j} = \sqrt{12^2 + 5^2} = 13$	M1 A1
so $\mathbf{v} = \dfrac{26}{13}(12\mathbf{i} - 5\mathbf{j})$	M1
$= 24\mathbf{i} - 10\mathbf{j}$	A1
	[4]
b) After 1 second, position vector of ball is $4\mathbf{i} + 6\mathbf{j} + 1(24\mathbf{i} - 10\mathbf{j}) = 28\mathbf{i} - 4\mathbf{j}$	M1 A1
Position of 2nd player $= 7\mathbf{i} - 2\mathbf{j} + 1(a\mathbf{i} + b\mathbf{j})$	
$= (7 + a)\mathbf{i} + (-2 + b)\mathbf{j}$	A1
Intercepts ball $\Rightarrow$ these are the same	M1
$(7 + a)\mathbf{i} + (-2 + b)\mathbf{j} = 28\mathbf{i} - 4\mathbf{j}$	
So $a = 21$ $b = -2$	A1 A1
	[6]
c) Speed required $= \sqrt{21^2 + 2^2}$	M1
$= 21.1$	A1
$\Rightarrow$ not possible	B1
	[3]

PARTICLE KINEMATICS AND VECTOR MOTION

9. a)	$t = 0$, $\quad \mathbf{r} = \mathbf{i} + \mathbf{j}$	
	so $\mathbf{i} + \mathbf{j} = \mathbf{i} + B\mathbf{j}$	M1
	$B = 1$	A1
	velocity $= \dfrac{d\mathbf{r}}{dt} = 2t\mathbf{i} + A\mathbf{j}$	M1 A1
	$t = 0$, $\mathbf{v} = -2\mathbf{j}$	B1
	$A = -2$	A1
		[6]
b)	$\mathbf{v} = 2t\mathbf{i} - 2\mathbf{j}$	
	Speed $= \sqrt{4t^2 + 4}$	M1 A1
	Minimum speed $= \sqrt{4} = 2$	B1
		[3]
c)	$\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = 2\mathbf{i}$	M1 A1
	$\mathbf{F} = m\mathbf{a}$	M1
	$\mathbf{F} = 4\mathbf{i}$	
	So magnitude is 4	A1
		[4]

10.a)	$\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 3t^2\mathbf{i} + 4\mathbf{j}$	M1 A1
	$\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = 6t\mathbf{i}$	M1 A1
		[4]
b)	Require $3t^2 : 4$ in same ratio as $3 : 1$	M1
	so $\dfrac{3t^2}{4} = 3$	M1
	$t^2 = 4 \Rightarrow t = 2$ (since $t \geq 0$)	A1
	$t = 2$, $\mathbf{v} = 12\mathbf{i} + 4\mathbf{j}$	
	speed $= \sqrt{12^2 + 4^2} = 12.6$	M1 A1
		[5]
c)	$t = 2 \Rightarrow a = 12\mathbf{i}$	B1
	so magnitude = 12	B1
	in positive **i** direction	B1
		[3]

PARTICLE KINEMATICS AND VECTOR MOTION

11.a)	$v = \int a\,dt \quad \therefore\ v = t^2 - 3t + C$	M1 A1
	Initially $v = -4$	B1
	$\therefore\ v = t^2 - 3t - 4$	A1 f.t
		[4]
b)	$s = \int v\ dt \qquad \therefore\ s = \frac{t^3}{3} - \frac{3t^2}{2} - 4t + K$	M1 A1 f.t.
	Initially $s = 1 \quad \therefore\ s = \frac{t^3}{3} - \frac{3t^2}{2} - 4t + 1$	A1 f.t.
		[3]
c)	$v = 0 \quad \Rightarrow \quad t^2 - 3t - 4 = 0$	M1
	$(t-4)(t+1) = 0 \quad \therefore\ t = 4 \quad \text{as } t \geq 0$	A1
	$s = \frac{4^3}{3} - \frac{3 \times 4^2}{2} - 4 \times 4 + 1 = -17\frac{2}{3}$ m	M1 A1 c.a.o
	$\therefore$ distance is $17\frac{2}{3}$ m	A1 f.t.
		[5]

12.a)	$\mathbf{v} = \int \mathbf{a}\,dt \quad \therefore\ \mathbf{v} = (t^2 - t)\mathbf{i} + 3t\mathbf{j} + \mathbf{C}$	M1 A1
	$\mathbf{v} = 0$ when $t = 0 \Rightarrow \mathbf{C} = 0$	
	$\therefore\ \mathbf{v} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$	A1 f.t.
		[3]
b)	$\mathbf{s} = \int \mathbf{v}\ dt \quad \therefore \quad \mathbf{s} = \left(\frac{t^3}{3} - \frac{t^2}{2}\right)\mathbf{i} + \frac{3t^2}{2}\mathbf{j} + \mathbf{K}$	M1 A1
	$\mathbf{s} = 0$ when $t = 0 \Rightarrow \mathbf{K} = 0$	
	$\therefore\ \mathbf{s} = \left(\frac{t^3}{3} - \frac{t^2}{2}\right)\mathbf{i} + \frac{3t^2}{2}\mathbf{j}$	A1 f.t.
	$\therefore$ when $t = 2 \quad \mathbf{s} = \frac{2}{3}\mathbf{i} + 6\mathbf{j}$	M1 A1 f.t.
	So distance $= \sqrt{\left(\frac{2}{3}\right)^2 + 6^2} = \sqrt{36\frac{4}{9}} = 6.04$m	M1 A1 f.t.
		[7]
c)	When $t = 2 \quad \mathbf{a} = 3\mathbf{i} + 3\mathbf{j}$	B1
	$\therefore$ Magnitude of acceleration $= \sqrt{3^2 + 3^2} = 3\sqrt{2}$	M1 A1
	$\therefore$ Force = mass $\times$ acceleration	B1
	$\therefore$ Magnitude of force $= 2 \times 3\sqrt{2} = 6\sqrt{2}$	A1 f.t.
		[5]

PARTICLE KINEMATICS AND VECTOR MOTION

13.a)	$v = 0 \Rightarrow t^2 - 6t + 5 = 0$	M1
	$\Rightarrow t = 1$ or $t = 5$	A1
	$a = \dfrac{dv}{dt} \quad \therefore \quad a = 2t - 6$	M1 A1
	$t = 1 \Rightarrow a = -4 \qquad t = 5 \Rightarrow a = 4$	M1 A1 f.t.
		[6]
b) i)	$s = \int v\, dt \quad \therefore \quad s = \dfrac{t^3}{3} - 3t^2 + 5t + C$	M1 A1
	$s = 0$ when $t = 0 \quad \therefore \quad C = 0$	
	$s = \dfrac{t^3}{3} - 3t^2 + 5t$	A1 f.t.
	When $t = 2$, $\quad s = \frac{8}{3} - 12 + 10 = \frac{2}{3}$	A1
		[4]
ii)	Particle reverses direction at $t = 1$, since $v = 0$ then.	M1
	When $t = 1$, $s = 2\frac{1}{3}$.	B1
	So distance between $t = 0$ and $t = 1$ is $2\frac{1}{3}$	M1 (separating)
	Distance between $t = 1$ and $t = 2$ is $2\frac{1}{3} - \frac{2}{3} = 1\frac{2}{3}$	M1 A1
	Total $= 2\frac{1}{3} + 1\frac{2}{3} = 4$ m	B1
		[6]
c)	For the first three seconds the particle has a positive acceleration, then the acceleration is negative.	B1
	After one second it changes direction, then it does so again after 5 seconds and then continues in that direction	B1
	Speed reduces to zero during first second, remains negative until $t = 5$, then becomes positive and remains so, tending to infinity as t tends to infinity. .	B1
		[3]

PARTICLE KINEMATICS AND VECTOR MOTION

14. a) $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = (6t - 1)\mathbf{i} + \mathbf{j}$ — M1 A1

$\mathbf{F} = m\mathbf{a}$. When $t = 2 \Rightarrow \mathbf{F} = 22\mathbf{i} + 2\mathbf{j}$ — M1 A1 f.t.

Magnitude of $\mathbf{F} = \sqrt{22^2 + 2^2} = \sqrt{488} = 2\sqrt{122}$ — M1 A1

[6]

b) $\mathbf{r} = \int \mathbf{v}\, dt = \left(t^3 - \dfrac{t^2}{2}\right)\mathbf{i} + \dfrac{t^2}{2}\mathbf{j} + \mathbf{C}$ — M1 A1

$t = 0$ $\mathbf{r} = 3\mathbf{i} \Rightarrow 3\mathbf{i} = \mathbf{C}$ — A1 f.t

$\mathbf{r} = \left(t^3 - \dfrac{t^2}{2} + 3\right)\mathbf{i} + \frac{1}{2}t^2\mathbf{j}$ — A1

[4]

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15. a) $a = 2 - t \Rightarrow \dfrac{dv}{dt} = 2 - 2t$ $v = 2t - t^2 + C$ — M1 A1

$v = 6$ $t = 0 \Rightarrow v = 2t - t^2 + 6$ — A1 f.t

$\dfrac{dx}{dt} = 2t - t^2 + 6$ — M1

$x = t^2 - \dfrac{t^3}{3} + 6t + K$ — A1 f.t

$x = 0$ $t = 0 \Rightarrow K = 0 \therefore x = t^2 - \dfrac{t^3}{3} + 6t$ — A1 f.t.

[6]

b) When $x = 0$ $t^2 - \dfrac{t^3}{3} + 6t = 0$ — M1

$t^3 - 3t^2 - 18t = 0$

$t(t - 6)(t + 3) = 0$ — A1 ca.o.

$\therefore$ $t = 6$ when it returns — A1 ca.o

$t = 6 \Rightarrow v = 12 - 36 + 6 = -18$ — M1 A1 f.t.

$\therefore$ speed is 18 ms^{-1} — B1

[6]

c) Particle will not return to A. — B1

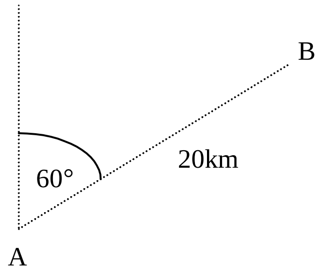
It will continue to move in direction opposite to original one, — B1

with increasing speed. — B1

[3]

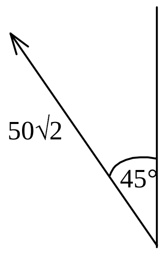
PARTICLE KINEMATICS AND VECTOR MOTION

16. a) i)



$\mathbf{r}_B = 20\sin 60\mathbf{i} + 20\cos 60\mathbf{j}$	M1 A1
$\mathbf{r}_B = 10\sqrt{3}\,\mathbf{i} + 10\mathbf{j}$	A1
	[3]

ii)



$\mathbf{v} = -50\sqrt{2}\sin 45\mathbf{i} + 50\sqrt{2}\cos 45\mathbf{j}$	M1
$= -50\mathbf{i} + 50\mathbf{j}$	A1
	[2]
iii) $\mathbf{r}_D = -50t\mathbf{i} + 50\mathbf{j}$	B1 f.t.
	[1]
b) $\mathbf{r}_2 = 10\sqrt{3}\,\mathbf{i} + 10\mathbf{j} + t(-60\mathbf{i} + 15\mathbf{j})$	B1 ($\mathbf{r}_B$) B1 ($t\mathbf{v}$)
	[2]
c) Require two position vectors equal	M1
$-50t\mathbf{i} + 50t\mathbf{j} = 10\sqrt{3}\,\mathbf{i} + 10\mathbf{j} + t(-60\mathbf{i} + 15\mathbf{j})$	
$\mathbf{i}$ component : $-50t = 10\sqrt{3} - 60t$	M1 (cmpts) A1
$\Rightarrow \quad t = \sqrt{3}$	M1 (finding t) A1
$\mathbf{j}$ component : $50t = 10 + 15t$	A1
$t = \frac{2}{7}$	A1
Since t-values different, do not meet	B1
	[8]

PARTICLE KINEMATICS AND VECTOR MOTION

17. a)	$\mathbf{r} = \left(\frac{3}{t}+1\right)\mathbf{i} + \left(\frac{4}{t}-2t\right)\mathbf{j}$	M1
	$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -\frac{3}{t^2}\mathbf{i} + \left(-\frac{4}{t^2}-2\right)\mathbf{j}$	M1 A1 A1
	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{6}{t^3}\mathbf{i} + \frac{8}{t^3}\mathbf{j}$	M1 A1
		[6]
b)	$\mathbf{F} = m\mathbf{a}$	M1
	$\mathbf{F} = 0.5\left(\frac{6}{t^3}\mathbf{i}+\frac{8}{t^3}\mathbf{j}\right)$	
	$= \frac{3}{t^3}\mathbf{i}+\frac{4}{t^3}\mathbf{j}$	A1
	$= \frac{1}{t^3}(3\mathbf{i} + 4\mathbf{j})$	B1
	$\Rightarrow$ parallel to $3\mathbf{i} + 4\mathbf{j}$	
	$\lvert\mathbf{F}\rvert = \sqrt{\frac{9}{t^6}+\frac{16}{t^6}} = \frac{5}{t^3}$	M1 A1
		[5]
c)	acceleration tends to zero	B1
	velocity tends to $-2\mathbf{j}$	B1
	i component of displacement tends to 1	B1
	j component of displacement becomes large and negative	B1 B1
		[5]

PARTICLE KINEMATICS AND VECTOR MOTION

18. a)	$q = 0$	M1
	$2t - t^2 = 0$	
	$t(2 - t) = 0$	
	$t = 0, 2$	A1 A1
	$\Rightarrow$ on positive x-axis for 2 seconds	B1 **[4]**
b)	$v = \dfrac{dp}{dt}$	M1
	$= \dfrac{2}{\sqrt{t}} - 2t$	A1 A1
	$v = 0$	M1
	$\dfrac{2}{\sqrt{t}} = 2t$	
	$1 = t\sqrt{t} = t^{\frac{3}{2}}$	A1
	$1^2 = t^3$	
	$1 = t$	A1 **[6]**
c)	$p = q$	M1
	$4\sqrt{t} - t^2 = 2t - t^2$	
	$4\sqrt{t} = 2t$	
	$16t = 4t^2$	A1
	$4t(t - 4) = 0$	M1
	$t = 0, 4$	
	But cannot be 0, since $t > 0$ for p	
	so $t = 4$	A1
	Position : substitute back into p or q	M1
	$2 \times 4 - 4^2 = -8$	A1 **[6]**

PARTICLE KINEMATICS AND VECTOR MOTION

19.a)	Collide if $\mathbf{r}_P = \mathbf{r}_Q$	M1
	$(3 + 2t)\mathbf{i} + (3 - 5t)\mathbf{j} = (5 - t)\mathbf{i} + (2 - 6t)\mathbf{j}$	
	i component : $3 + 2t = 5 - t$ ①	M1 (coefficients)
	j component : $3 - 5t = 2 - 6t$ ②	A1
	① $\Rightarrow t = \frac{2}{3}$	A1 f.t.
	② $\Rightarrow t = -1$	A1 f.t.
	$\Rightarrow$ do not collide	
		[5]
b)	$D^2 = [(3 + 2t) - (5 - t)]^2 + [(3 - 5t) - (2 - 6t)]^2$	M1 A1
	$= (-2 + 3t)^2 + (1 + t)^2$	A1
	$= 9t^2 - 12t + 4 + 1 + 2t + t^2$	M1
	$= 10t^2 - 10t + 5$	A1
		[5]
c)	$\frac{dD^2}{dt} = 20t - 10$	B1
	$\frac{dD^2}{dt} = 0$ for minimum	M1
	$\Rightarrow t = \frac{1}{2}$	A1
	$D^2 = \frac{10}{4} - \frac{10}{2} + 5 = \frac{10}{4}$	M1 (substituting)
	$D = \frac{\sqrt{10}}{2}$	A1
		[5]

PARTICLE KINEMATICS AND VECTOR MOTION

20.a)	**j** component must be zero	M1
	$20t - 5t^2 = 0$	
	$5t(4 - t) = 0$	M1
	$t = 4$	A1
	$t = 4$, $\mathbf{r} = 60\mathbf{i} \Rightarrow 60$	B1
		[4]
b)	Require vertical component of velocity is zero	M1
	$\mathbf{v} = 15\mathbf{i} + (20 - 10t)\mathbf{j}$	M1 A1
	$20 - 10t = 0$	
	$t = 2$	A1
	$t = 2$, $\mathbf{r} = 30\mathbf{i} + 20\mathbf{j}$	M1
	$\Rightarrow$ 20 is greatest height	A1
		[6]
c)	$45\mathbf{i} + 10\mathbf{j} = 15t\,\mathbf{i} + (20t - 5t^2)\mathbf{j}$	M1
	i component : $45 = 15t$ ①	M1 A1
	j component : $10 = 20t - 5t^2$ ②	
	① $\Rightarrow t = 3$	B1
	Substitute $t = 3$ in ② : $20t - 5t^2 = 15 \neq 10$	M1 A1
	$\Rightarrow$ no solution	
		[6]
d)	require **v** parallel to **i** + **j**	M1 (or equivalent)
	$15\mathbf{i} + (20 - 10t)\mathbf{j}$ parallel to $\mathbf{i} + \mathbf{j}$	
	so $20 - 10t = 15$	M1
	$t = \frac{1}{2}$	A1
		[3]

PARTICLE KINEMATICS AND VECTOR MOTION

21. a)	Magnitude of $3\mathbf{i} - 4\mathbf{j} = \sqrt{3^2 + 4^2} = 5$	B1
	$\Rightarrow \mathbf{F} = 3(3\mathbf{i} - 4\mathbf{j})$	M1
	$\mathbf{F} = 1.5\mathbf{a}$	
	$\Rightarrow \mathbf{a} = 2(3\mathbf{i} - 4\mathbf{j})$	A1
		[3]
b)	$\mathbf{v} = \int \mathbf{a}\, dt = 6t\mathbf{i} - 8t\mathbf{j} + \mathbf{c}$	M1 A1
	When $t = 0$, $\mathbf{v} = 0 \Rightarrow \mathbf{c} = 0$	
	$\mathbf{v} = 6t\mathbf{i} - 8t\mathbf{j}$	
	$\mathbf{r} = \int \mathbf{v}\, dt = 3t^2\mathbf{i} - 4t^2\mathbf{j} + \mathbf{d}$	M1 A1
	$t = 0$, $\mathbf{r} = 6\mathbf{i} + 36\mathbf{j} \Rightarrow \mathbf{d} = 6\mathbf{i} + 36\mathbf{j}$	
	so $\mathbf{r} = (3t^2 + 6)\mathbf{i} + (36 - 4t^2)\mathbf{j}$	A1 A1
		[6]
c)	$\mathbf{r} = 0 \Rightarrow 3t^2 + 6 = 0$	M1 A1
	and $36 - 4t^2 = 0$	
	$3t^2 + 6 = 0 \Rightarrow t^2 = -2$ impossible	B1
		[3]

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22. a)	$v = \int a\, dt = 6t + 3t^2 + c$	M1 A1
	$t = 0$, $v = 0 \Rightarrow c = 0$	B1
	so $v = 6t + 3t^2$	
	$x = \int v\, dt = 3t^2 + t^3 + k$	M1 A1
	$t = 0$, $x = 0 \Rightarrow k = 0$	
	$x = 3t^2 + t^3$	A1
		[6]
b)	$v = 6(2) + 3(2^2) = 24$	B1
	$x = 3(2^2) + 2^3 = 20$	B1
		[2]
c)	Now moving under constant acceleration of $-4\ \text{ms}^{-2}$	M1
	$\Rightarrow v = u + at$	M1
	$v = 24 + 3(-4) = 12$	A1
		[3]
