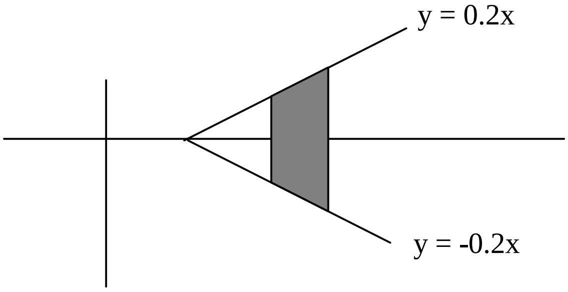
CENTRE OF MASS 2

1. a) Let m = mass of unit area,



Consider a strip as shown — M1 A1

length is 2y, thickness is $\delta x \Rightarrow$ mass of strip = $2my\delta x$.

Moment of strip about y-axis = $2my\delta x \times x$.

For whole triangle, area = $\frac{1}{2} \times 0.4H \times H = 0.2H^2$

Let $\bar{x}$ be distance of centre of mass from y-axis.

Then moment of whole = $0.2H^2 m\bar{x}$. — B1

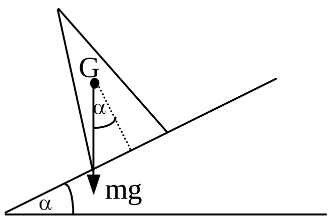
$$\text{So: } 0.2H^2 m\bar{x} = \int_0^H 2myx\,dx = 2m\int_0^H 0.2x^2\,dx = 2m\left[\frac{0.2x^3}{3}\right]_0^H \quad \text{M1}$$

$$0.2H^2 m\bar{x} = \frac{0.4mH^3}{3} \quad \text{A1}$$

$$\bar{x} = \frac{0.4H}{0.6} = \frac{2H}{3} \quad \text{A1}$$

[6]

b) Consider the limiting case when triangle is about to topple: — M1 (topples if line of action of weight outside area of contact)



From a), we have : $\tan\alpha = \frac{0.2H}{\frac{H}{3}}$ — M1 A1

$= \frac{3}{5}$ — A1

If $\alpha > \tan^{-1}\left(\frac{3}{5}\right)$, it will topple — M1

$\tan^{-1}\left(\frac{3}{5}\right)$, = 31°$\Rightarrow$ topples — A1

[6]

CENTRE OF MASS 2

2. a) $V = \pi\int y^2dx$ M1

$= \pi\int_0^4 x^3dx$

$= \pi\left[\frac{x^4}{4}\right]_0^4$ A1

$= 64\pi$ A1

[3]

b) Let m be mass of unit volume
Consider the solid to be composed of discs.
Each disc has radius y and thickness δx.
Each disc has mass $\pi y^2 m\delta x$.
$\Rightarrow$ moment of disc about y-axis = $\pi m y^2 x\delta x$ M1 A1

Let $\bar{x}$ be distance of centre of mass from y-axis.
So moment of whole body = $64\pi m\bar{x}$ B1

So $64\pi m\bar{x} = \pi m\int_0^4 y^2xdx$ M1

$= \pi m\int_0^4 x^4dx$ A1

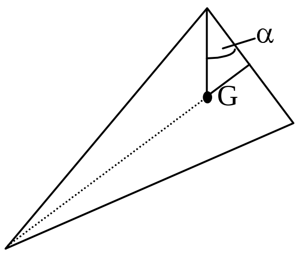
$= \pi m\left[\frac{x^5}{5}\right]_0^4$

$64\pi m\bar{x} = \frac{1024\pi m}{5}$ A1

$\bar{x} = \frac{16}{5}$ A1

[7]

c)



M1 (Centre of mass directly below point of suspension)

$\tan\alpha = \frac{4 - \frac{16}{5}}{8}$ M1 ($\tan\alpha$) A1 f.t. ($4 - \frac{16}{5}$)

A1 c.a.o. (8)

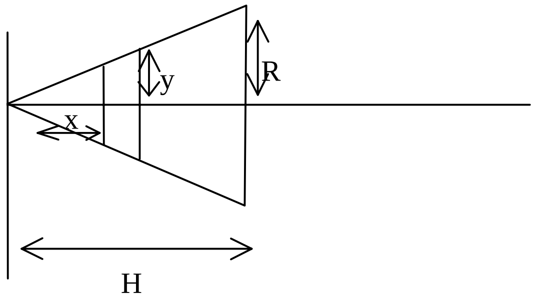
$= 0.1$

$\alpha = 5.71°$ A1

[5]

CENTRE OF MASS 2

3. a) Let m = mass of unit volume.



Consider cone to be composed of discs of radius y, thickness δx.
Then moment of each disc about y-axis = $\pi m y^2 x \delta x$. M1 A1

Let distance of centre of mass from y-axis be $\bar{x}$.
Then moment of whole thing = $\frac{1}{3}\pi R^2 H m \bar{x}$ B1

Similar triangles $\Rightarrow$ $\frac{y}{x} = \frac{R}{H}$ M1

$y = \frac{R}{H}x$ A1

So $\frac{\pi R^2 Hm}{3}\bar{x} = \int_0^H \pi m\left(\frac{R}{H}x\right)^2 x\,dx$ M1

$= \frac{\pi m R^2}{H^2}\int_0^H x^3\,dx$

$= \frac{\pi m R^2}{H^2}\left[\frac{x^4}{4}\right]_0^H$

$\frac{\pi R^2 Hm}{3}\bar{x} = \frac{\pi m R^2 H^4}{4H^2}$ A1

so $\bar{x} = \frac{3H}{4}$ A1 c.a.o.

[8]

b)

Body	Mass	Distance of centre of mass from common plane face	
Cone	$\frac{1}{3}\pi r^2 hm$	$\frac{h}{4}$	M1 A1
Hemisphere	$\frac{2}{3}\pi r^3 m$	$-\frac{3r}{8}$	A1
Combined body	$\frac{\pi r^2}{3}m(h + 2r)$	d	

CENTRE OF MASS 2

QUESTION 3 b) CONTINUED

So $\frac{1}{3}\pi r^2 hm \times \frac{h}{4} + \frac{2}{3}\pi r^3 m \times -\frac{3r}{8} = \frac{\pi r^2 m}{3}(h + 2r) \times d$ M1

$\frac{h^2}{4} + \frac{-3r^2}{4} = (h + 2r)d$ M1

$\frac{h^2 - 3r^2}{4(h + 2r)} = d$ A1

[6]

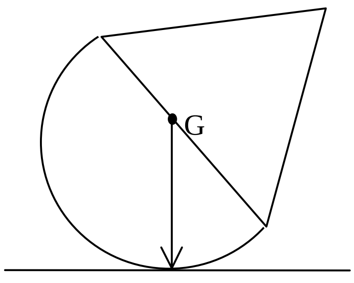
c) $d = 0$

$h^2 - 3r^2 = 0$ M1

$h = \sqrt{3}r$ A1

[2]

d)



The radius GP will be perpendicular to the ground B1

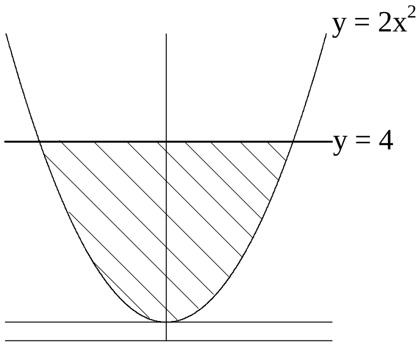
since a radius at a point is perpendicular to the tangent at that point. B1

So G is directly above the point of contact. B1

[3]

CENTRE OF MASS 2

4. a) Line intersects curve when $x = \pm 2$ B1



Area of lamina = area of rectangle – area under curve M1

Area of rectangle = 16 B1

Area under curve $= \displaystyle\int_{-2}^{2} x^2 dx$ M1

$= \left[\dfrac{x^3}{3}\right]_2^2$

$= \frac{16}{3}$ A1

Area of lamina $= \frac{32}{3}$ A1

[6]

b) Because the y-axis is a line of symmetry of A B1

[1]

c) Let m be the mass of unit area.
Consider the lamina to be composed of strips
Each has length 2x and thickness δy.
Then moment of each strip about x-axis is $m \times 2x\delta y \times y$ M1A1

Let $\bar{y}$ = distance of centre of mass from x-axis.

Then moment of lamina $= \dfrac{32m}{3}\bar{y}$ B1

So $\dfrac{32m}{3}\bar{y} = 2m\displaystyle\int_0^4 xy dy$ M1

$x = \sqrt{y}$ B1

So $\dfrac{32m}{3}\bar{y} = 2m\displaystyle\int_0^4 y^{\frac{1}{2}} y dy = 2m\int_0^4 y^{\frac{3}{2}} dy$

$= 2m\left[\dfrac{y^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4 = 2m \times \dfrac{64}{5}$ A1

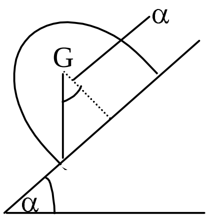
So $\bar{y} = \frac{12}{5}$ A1

[7]

CENTRE OF MASS 2

QUESTION 4 CONTINUED

d) Limiting case:



M1

$$\tan\alpha = \frac{2}{4 - \frac{12}{5}} \qquad \text{M1 A1}$$

$$= \tfrac{5}{4}$$

$$\alpha = 51° \qquad \text{A1}$$

[4]

CENTRE OF MASS 2

5. a) $y = x^2$; $y = 2 - x$

$\Rightarrow x^2 = 2 - x$ M1

$x^2 + x - 2 = 0$

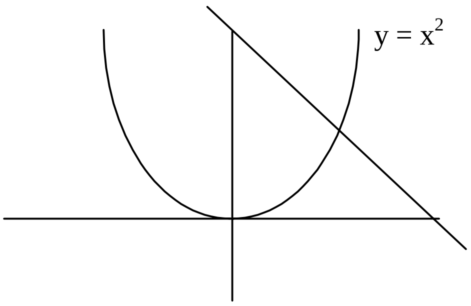
$(x + 2)(x - 1) = 0$ M1

$x = 1, -2$ A1 ($x = 1$)

$y = 1, 4$ A1 ($y = 1$)

[4]

b)



Cone has height 1, radius 1 B1

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{\pi}{3}$ B1

For curve:

volume $= \pi\int_0^1 x^2 dy$ M1

$= \pi\int_0^1 y dy$ A1

$= \pi\left[\frac{y^2}{2}\right]_0^1 = \frac{\pi}{2}$ A1

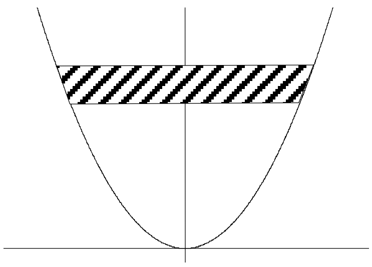
total volume $= \frac{5\pi}{6}$ B1

[6]

c) The y-axis is a line of symmetry, so the centre of mass must lie on it B1

Let m be mass of unit volume.

For the "$y = x^2$" portion : M1 (splitting)



Consider a disc of radius x, thickness δy.

Then moment of disc about x-axis is $\pi m x^2 y \delta y$ } M1 A1

Let distance of centre of mass of body from x-axis be $\bar{y}$.

Then moment of body is $\frac{\pi m}{2}\bar{y}$ B1

CENTRE OF MASS 2

QUESTION 5 c) CONTINUED

So $\frac{\pi m}{2}\bar{y} = \pi m\int_0^1 x^2 y\,dy$ M1

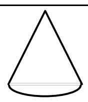
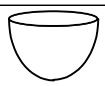
$= \pi m\int_0^1 y^2\,dy$ A1

$= \pi m\left[\frac{y^3}{3}\right]_0^1$

$= \frac{\pi m}{3}$

$\bar{y} = \frac{2}{3}$ A1

Whole thing :

Body	Mass	Distance of centre of mass from x-axis
	$\frac{1}{3}\pi m$	$1+\frac{1}{4}$
	$\frac{1}{2}\pi m$	$\frac{2}{3}$
Whole thing	$\frac{5\pi m}{6}$	d

M1 A1

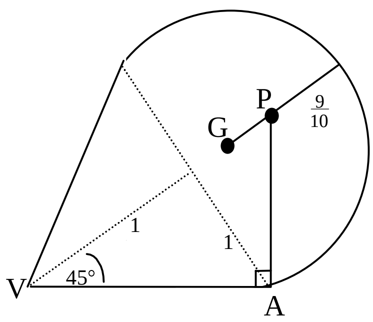
A1

So $\frac{1}{3}\pi m \times \frac{5}{4} + \frac{1}{2}\pi m \times \frac{2}{3} = \frac{5\pi m}{6} \times d$ M1

$\frac{9}{10} = d$ A1

[13]

d)



Require centre of mass to be vertically above area of contact. M1

The furthest point from the cone's vertex for which this is true is point P, as shown, which is vertically above point A. M1 A1

$VA = \sqrt{2}$

so $VP = 2$

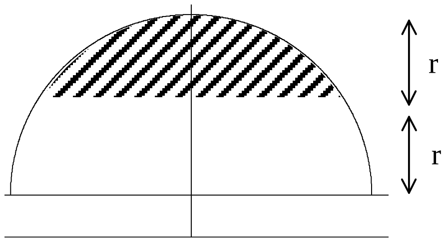
But distance from V to centre of mass is $1\frac{1}{10}$

$\Rightarrow$ can rest in equilibrium. A1

[4]

CENTRE OF MASS 2

6. a)



Equation of semicircle is $x^2 + y^2 = 4r^2$ — M1 A1

Require volume between this and the line $y = r$ — M1

Volume $= \pi\int x^2 dy$ — M1

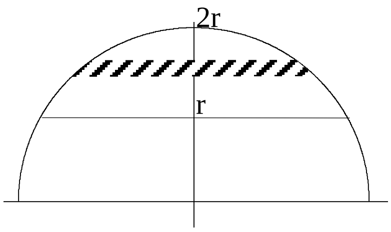
$$= \pi\int_r^{2r} 4r^2 - y^2 dy$$ — A1

$$= \pi\left[4r^2 y - \frac{y^3}{3}\right]_r^{2r}$$ — A1

$$= \pi\left(8r^3 - \frac{8r^3}{3} - 4r^3 + \frac{r^3}{3}\right) = \frac{5\pi r^3}{3}$$ — A1

[7]

b) Let m be mass of unit area.



Consider a disc of radius x, thickness δy

Then moment of disc about x-axis = $\pi m x^2 y \delta y$ — M1

Let centre of mass of whole body be at distance $\bar{y}$ from x-axis.

Then moment of cement = $\frac{5\pi r^3}{3} m\bar{y}$ — B1

So $\frac{5\pi r^3}{3} m\bar{y} = \pi m\int_r^{2r} x^2 y dy$ — M1

$$= \pi m\int_r^{2r} 4r^2 y - y^3 dy$$ — A1

$$= \pi m\left[2r^2y^2 - \frac{y^4}{4}\right]_r^{2r}$$

$$\frac{5\pi r^3}{3} m\bar{y} = \pi m\left(8r^4 - 4r^4 - 2r^4 + \frac{r^4}{4}\right) = \frac{9\pi m r^4}{4}$$ — A1

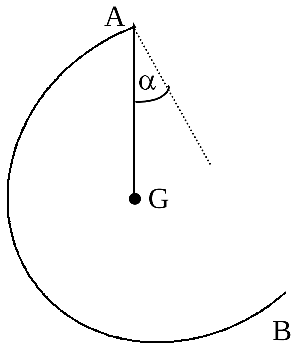
$$\bar{y} = \frac{27r}{20}$$ — A1

[7]

CENTRE OF MASS 2

QUESTION 6 CONTINUED

c)



Centre of mass vertically below point of suspension.	M1
$\tan\alpha = \dfrac{\frac{27r}{20}}{2r}$	M1 A1
$\alpha = 34°$	A1
	[4]

CENTRE OF MASS 2

7. a) $V = \pi\int_0^4 y^2 dx$ M1

$= \pi\int_0^4 x dx$

$= \pi\left[\frac{x^2}{2}\right]_0^4$ A1

$= 8\pi$ A1

[3]

b) Centre of mass must be on x-axis, by symmetry B1

Let m = mass of unit volume
Consider the solid to be composed of discs.
Each disc has radius y, thickness δx.
each disc has mass $m\pi y^2\delta x$.
So moment of disc about y-axis is $\pi m y^2 x\delta x$. M1 A1

Let $\bar{x}$ be distance of centre of mass of body from y-axis.
So moment of whole body $= 8\pi m\bar{x}$ B1

So $8\pi m\bar{x} = \pi m\int_0^4 y^2 x dx$ M1

$= \pi m\int_0^4 x^2 dx$ A1

$= \pi m\left[\frac{x^3}{3}\right]_0^4$

$8\pi m\bar{x} = \frac{64\pi m}{3}$ A1

$\bar{x} = \frac{8}{3}$ A1

[8]

c)

Body	Mass	Distance of centre of mass from base of cone
Cone	M_1	$\frac{1}{2}$
Solid of rotation	M_2	$-\frac{4}{3}$
Combined body	$M_1 + M_2$	0

M1 A1

So $\frac{1}{2}M_1 - \frac{4}{3}M_2 = 0$ M1

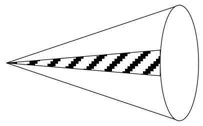
$M_1 = \frac{8}{3}M_2$ A1

(or ratio 8 : 3)

[4]

CENTRE OF MASS 2

8. a) Consider the cone to be composed of triangular strips, as shown: M1



The centre of mass of each triangle is at two thirds of
the distance from the plane face to the vertex. B1

So the centre of mass of entire cone is at distance $\frac{2}{3}$H from vertex, A1

and by symmetry, is on the axis of the cone. B1

[4]

b) Masses are in ratio 1 : 3 B1

Body	Mass	Distance of centre of mass from base of cone
Small Cone	m	$3R + \frac{R}{3}$
Large Cone	3m	2R
Combined body	4m	d

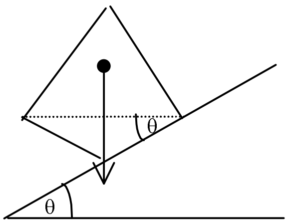
M1 A1

So $m \times \frac{10R}{3} + 3m \times 2R = 4md$ M1

$\frac{7}{3}R = d$ A1

[5]

c) The limiting case is shown in the diagram.



M1 A1

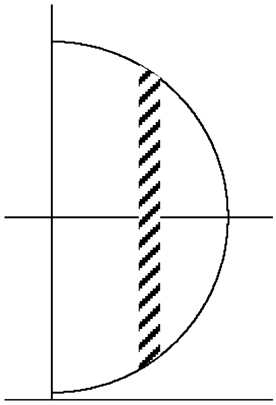
$\tan\theta = \frac{R}{R} = 1$ M1

$\theta = 45°$ A1

[4]

CENTRE OF MASS 2

9. a) Let m = mass of unit area



Consider the lamina to be composed of strips of length 2y, thickness δx.
mass of each strip = $2my\delta x$
moment of each strip above y-axis = $2mxy\delta x$. M1 A1

Let $\bar{x}$ be distance of centre of mass of lamina from y-axis
Then moment of whole body = $\frac{1}{2}\pi r^2 m\bar{x}$ B1

So $\frac{1}{2}\pi r^2 m\bar{x} = 2m\int_0^r xy\,dx$ M1

But $x^2 + y^2 = r^2$ B1

So $\frac{1}{2}\pi r^2 m\bar{x} = 2m\int_0^r x\sqrt{r^2 - x^2}\,dx$

Using the substitution $u^2 = r^2 - x^2$
so $2u\,du = -2x\,dx$
$x = 0 \Rightarrow u = r$
$x = r \Rightarrow u = 0$ M1A1

So $\frac{1}{2}\pi r^2 m\bar{x} = 2m\int_r^0 -u^2\,du$ A1

$= 2m\left[-\frac{u^3}{3}\right]_r^0$

$= \frac{2mr^3}{3}$

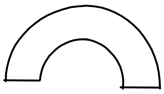
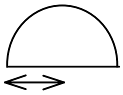
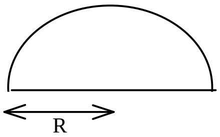
$\bar{x} = \frac{4r}{3\pi}$ A1

[9]

CENTRE OF MASS 2

QUESTION 9 CONTINUED

b)

Body	Mass	Distance of centre of mass from AB
	$\dfrac{\pi m\left(R^2 - r^2\right)}{2}$	d
	$\dfrac{\pi m r^2}{2}$	$\dfrac{4r}{3\pi}$
	$\dfrac{\pi m R^2}{2}$	$\dfrac{4R}{3\pi}$

M1 A1

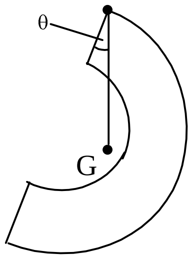
So $\pi m\left(R^2 - r^2\right)d + \pi m r^2 \dfrac{4r}{3\pi} = \pi m R^2 \dfrac{4R}{3\pi}$ M1

$$d = \frac{4\left(R^3 - r^3\right)}{3\pi\left(R^2 - r^2\right)}$$ A1

$$\left(= \frac{4}{3\pi}\left(\frac{R^2 + Rr + r^2}{R + r}\right)\right)$$

[4]

c)



M1 (centre of mass vertically below point of suspension)

$\tan\theta = \dfrac{d}{R}$ M1

$$d = \frac{4}{3\pi}\frac{\left(27r^3 - r^3\right)}{\left(9r^2 - r^2\right)}$$

$$= \frac{4}{3\pi} \times \frac{26r}{8} = \frac{13r}{3\pi}$$ B1

$$\tan\theta = \frac{\frac{13r}{3\pi}}{3r} = \frac{13}{9\pi}$$ A1

$\theta = 24.7°$ A1

[5]

CENTRE OF MASS 2

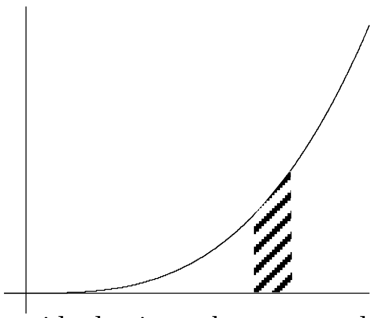
10.a) Area of A $= \int_0^2 x^3 dx$ M1

$= \left[\frac{x^4}{4}\right]_0^2 = 4$ A1

Mass = 0.1 × 4 = 0.4kg A1

[3]

b)



Consider lamina to be composed of strips of length y, thickness δx.
Mass of each strip = 0.1yδx. M1

Centre of mass of each strip has coordinates (x, $\frac{y}{2}$) B1 B1

For whole body, centre of mass has coordinates $(\bar{x}, \bar{y})$

Moments about y-axis :

$0.4\bar{x} = \int_0^2 0.1yx dx$ M1

$= \int_0^2 0.1x^4 dx$

$0.1\left[\frac{x^5}{5}\right]_0^2$

$0.4\bar{x} = 0.1 \times \frac{32}{5}$ A1

$\bar{x} = \frac{8}{5}$ A1

Moments about x-axis :

$0.4\bar{y} = \int_0^2 0.1y\frac{y}{2} dx$ M1

$= \frac{0.1}{2}\int_0^2 x^6 dx$ A1

$= \frac{0.1}{2}\left[\frac{x^7}{7}\right]_0^2$

$0.4\bar{y} = \frac{0.1}{2} \times \frac{128}{7}$

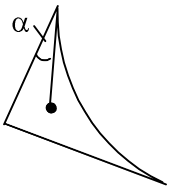
$\bar{y} = \frac{16}{7}$ A1

[9]

CENTRE OF MASS 2

QUESTION 10 CONTINUED

c)



M1 (Centre of mass below point of suspension)

$\tan\alpha = \dfrac{\frac{16}{7}}{\frac{8}{5}}$ M1

$\alpha = 55.0°$ A1

[3]

CENTRE OF MASS 2

11.a) Volume $= \pi\int y^2 dx$ M1

$= \pi\int_2^3 e^{4x}dx = \pi\left[\frac{e^{4x}}{4}\right]_2^3$ A1

$= \pi\left(\frac{e^{12}-e^8}{4}\right)$ A1

[3]

b) Let m = mass of unit volume
Consider body to be composed of discs of radius y, thickness δx.
mass of each disc = $\pi m y^2 \delta x$
Moment of each disc about y-axis = $\pi m y^2 x\delta x$ M1 A1

If $\bar{x}$ = distance of centre of mass of body from y-axis,
moment of whole body about y-axis = $\pi\left(\frac{e^{12}-e^8}{4}\right)m\bar{x}$ B1

So $\frac{\pi m}{4}\left(e^{12}-e^8\right)\bar{x} = \pi m\int_2^3 y^2 x dx$ M1

$= \pi m\int_2^3 xe^{4x}dx$ A1

Using integration by parts :

$\frac{\pi m}{4}\left(e^{12}-e^8\right)\bar{x} = \pi m\left(\left[\frac{xe^{4x}}{4}\right]_2^3 - \int_2^3 \frac{e^{4x}}{4}dx\right)$ M1 A1

$= \pi m\left[\frac{xe^{4x}}{4} - \frac{e^{4x}}{16}\right]_2^3$ A1

$= \pi m\left(\frac{11e^{12}}{16} - \frac{7e^8}{16}\right)$ A1

$\bar{x} = \frac{4}{e^{12}-e^8}\left(\frac{11e^{12}-7e^8}{16}\right)$ M1

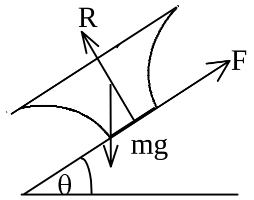
$= \frac{11e^4-7}{4\left(e^4-1\right)}$ A1

[11]

CENTRE OF MASS 2

QUESTION 11 CONTINUED

c)



If body is about to slide :

Resolving :	parallel to plane:	$mg\sin\theta = F$	
	perpendicular to plane:	$mg\cos\theta = R$	M1 A1
$F = \mu R \Rightarrow \mu = \tan\theta$			A1
since $\mu < 1$, $\theta < 45°$			B1
If body is about to topple :			M1
$\tan\theta = \dfrac{e^4}{d}$			A1
where d = distance of centre of mass from smaller face			
$d = \dfrac{11e^4 - 7}{4(e^4-1)} - 2$			B1
$\left(= \dfrac{11e^4 - 7 - 8e^4 + 8}{4(e^4-1)} = \dfrac{3e^4+1}{4(e^4-1)} \right)$			
This gives $\theta = 89.2°$			A1
So it will slide before it topples.			
			[8]

CENTRE OF MASS 2

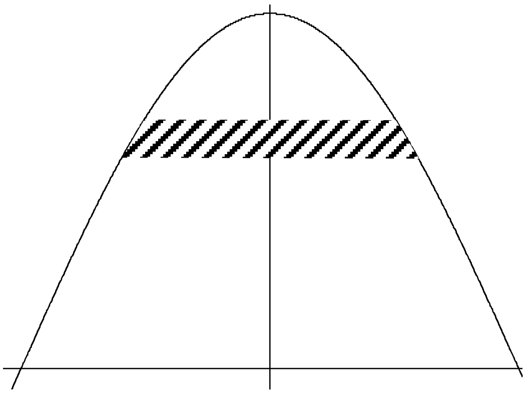
12. a) Area $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x$ M1

$= [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$ A1

[2]

b) 0 (by symmetry) B1
[1]

c) Let m = mass of unit area



Consider the lamina to be composed of strips of length 2x, thickness δy.
mass of each strip = $2mx\delta y$. M1 A1
Moment of each strip about x-axis = $2mxy\delta y$.

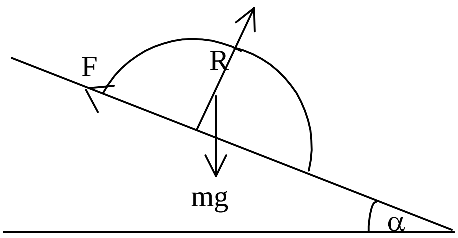
Moment of whole body about x-axis = $2m\bar{y}$ B1
where $\bar{y}$ is the y-coordinate of the centre of mass

so : $2m\int_0^1 xy\,dy = 2m\bar{y}$ M1

$x = \cos^{-1} y \Rightarrow \int_0^1 y\cos^{-1} y\,dy = \bar{y}$ A1

[5]

d)



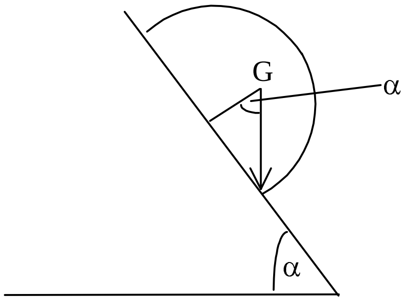
Resolving : parallel to plane: $F = mg\sin\alpha$
perpendicular to plane: $R = mg\cos\alpha$ M1 A1

Will not slip $\Rightarrow F \le \mu R$ M1
$mg\sin\alpha \le \mu mg\cos\alpha$
$\tan\alpha \le \mu$ A1
[4]

CENTRE OF MASS 2

QUESTION 12 CONTINUED

e) If about to topple :



M1

$\tan\alpha = \dfrac{\frac{\pi}{2}}{\frac{\pi}{8}}$ M1 A1

$= 4$ A1

$\Rightarrow$ topples if $\tan\alpha > 4$

[4]

f) Maximum value of μ is 1 B1
So once α reaches 45, it slips B1
[2]

CENTRE OF MASS 2

13. a) Volume $= \pi\int x^2 dy$ M1

$= \pi\int_0^{ka^2} \frac{y}{k} dy$ A1

$= \pi\left[\frac{y^2}{2k}\right]_0^{ka^2} = \frac{\pi ka^4}{2}$ A1

[3]

b) x-coordinate is 0, by symmetry B1

Let m = mass of unit volume
Consider solid to be composed of discs of radius x, thickness δy.
Moment of each disc about x-axis $= \pi m x^2 y \delta y$ } M1A1

Let $\bar{y}$ be coordinate of centre of mass of solid.

Moment of whole body about x-axis $= \frac{\pi ka^4}{2} m\bar{y}$ B1

So $\frac{\pi ka^4}{2} m\bar{y} = \int_0^{ka^2} \pi m x^2 y dy$ M1

$= \pi m \int_0^{ka^2} \frac{y^2}{k} dy$ A1

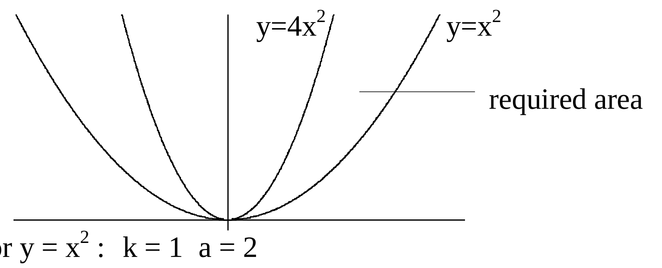
$= \pi m \left[\frac{y^3}{3k}\right]_0^{ka^2}$

$= \frac{\pi m k^2 a^6}{3}$

$\bar{y} = \frac{2ka^2}{3}$ A1

[7]

c) i)



For $y = x^2$: k = 1 a = 2 M1 A1

Volume $= \frac{\pi 2^4}{2} = 8\pi$ A1

For $y = 4x^2$ k = 4 a = 1 A1

Volume $= \frac{\pi 4}{2} = 2\pi$ A1

Total volume $= 8\pi - 2\pi = 6\pi$ M1 A1
[7]

CENTRE OF MASS 2

QUESTION 13 c) CONTINUED

ii)

Body	Mass	Distance of centre of mass from x-axis	
Solid from $y = 4x^2$	$2\pi m$	$\frac{2\times4\times1^2}{3} = \frac{8}{3}$	M1
A	$6\pi m$	d	M1 A1
Solid from $y = x^2$	$8\pi m$	$\frac{2\times1\times2^2}{3} = \frac{8}{3}$	

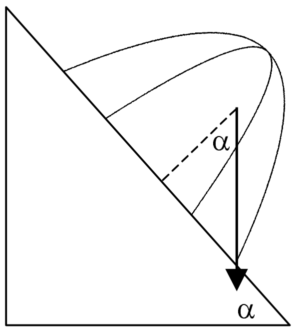
so $2\pi m \times \frac{8}{3} + 6\pi m \times d = 8\pi m \times \frac{8}{3}$ M1

$d = \frac{8}{3}$ A1

Coordinates are $(0, \frac{8}{3})$

[6]

d) If it is about to topple :



M1

$\tan\alpha = \dfrac{2}{\frac{8}{3}}$ M1 A1 A1 f.t.

$\alpha = 36.9^o$ A1

$\Rightarrow$ will topple if $\alpha = 40°$ B1

[6]

CENTRE OF MASS 2

14. a) $V = \pi\int y^2 dx$ M1

$= \pi\int_0^1 x^2(2-x)^2 dx$

$= \pi\int_0^1 4x^2 - 4x^3 + x^4 dx$ A1

$= \pi\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0^1$ A1

$= \frac{8\pi}{15}$ A1

[4]

b) 0, B1
since body is symmetrical about x-axis B1
[2]

c) Let m = mass of unit volume
Consider the body to be composed of discs of radius y, thickness δx.
mass of disc = $\pi m y^2 \delta x$
Moment of disc about y-axis = $\pi m x y^2 \delta x$ M1A1

Let $\bar{x}$ be x-coordinate of centre of mass of solid.

Moment of solid about y-axis = $\frac{8\pi}{15} m\bar{x}$ B1

So $\frac{8\pi}{15} m\bar{x} = \pi m\int_0^1 xy^2 dx$ M1

$= \pi m\int_0^1 4x^3 - 4x^4 + x^5 dx$ A1

$= \pi m\left[x^4 - \frac{4x^5}{5} + \frac{x^6}{6}\right]_0^1 = \frac{11\pi m}{30}$ A1

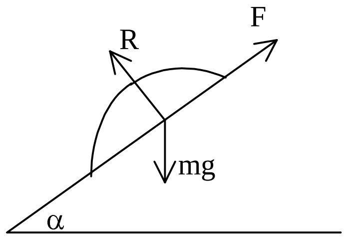
$\bar{x} = \frac{11}{16}$ A1

[7]

CENTRE OF MASS 2

QUESTION 14 CONTINUED

d)



If body is about to slide :

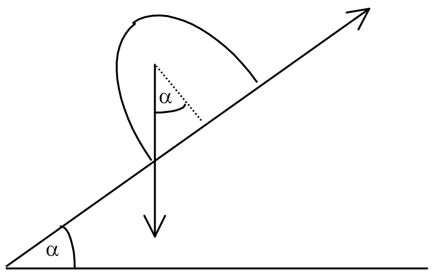
Resolving parallel to plane: $F = mg\sin\alpha$

perpendicular to plane $R = mg\cos\alpha$ M1 A1

$F = \mu R \Rightarrow \mu = \tan\alpha$ B1

Since $\mu < 1$, it will slide when $\alpha = 45$, or before B1

If body is about to topple:



M1

$\tan\alpha = \dfrac{1}{\frac{5}{16}} = \frac{16}{5}$ M1 A1

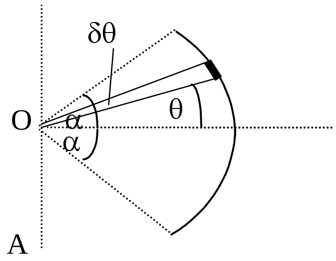
$\alpha > 45$ B1

so slides before it topples B1

[9]

CENTRE OF MASS 2

15. a)



Let m = mass of unit length
Consider an element subtending an angle $\delta\theta$, as shown
mass of element = $rm\delta\theta$. M1
Moment of element about OA = $rm\delta\theta \, r\cos\theta$ M1 A1

If d = distance of centre of mass of arc from OA,
Moment of whole body = $2mr\alpha d$ B1

So $2mr\alpha d = \int_{-\alpha}^{\alpha} mr^2 \cos\theta \, d\theta$ M1

$= mr^2[\sin\theta]_{-\alpha}^{\alpha}$

$= mr^2 2\sin\alpha$ A1

$d = \dfrac{r\sin\alpha}{\alpha}$ A1

[7]

b) Let m = mass of unit area
The sector can be considered to be composed of elemental arcs,
each of radius x, thickness δx
mass of each arc = $2m\alpha x\delta x$ M1

Moment of each arc about OA is $2m\alpha x\delta x \times \dfrac{x\sin\alpha}{\alpha}$ M1 A1

Let d = distance of centre of mass of lamina from OA.
Moment of whole body = $mr^2\alpha d$ B1

So $mr^2\alpha d = 2m\sin\alpha \int_0^r x^2 dx$ M1

$= 2m\sin\alpha \left[\dfrac{x^3}{3}\right]_0^r$

$= \dfrac{2mr^3}{3}\sin\alpha$

$d = \dfrac{2r\sin\alpha}{3\alpha}$ A1

[6]