A LEVEL MATHEMATICS QUESTIONBANKS

HORIZONTAL CIRCULAR MOTION

All answers should be given to three significant figures unless otherwise stated ***in the question***
Where a numerical value is required, take $g = 9.8ms^{-2}$

1. A marble of mass 0.2 kg is placed on a horizontal smooth, circular tray of radius 20cm with a rim. The marble is placed in contact with the rim of the tray, then set in motion so that it remains in contact with the rim and performs horizontal circular motion. Given that the marble makes one revolution of the tray every 5 seconds, find the force exerted by the rim of the tray on the marble.

[4]

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2. A particle of mass m grams hangs from a light inelastic string of length L metres. The particle is made to rotate in a horizontal circle of radius L *centimetres*.

a) Find, in terms of L and g, an expression for the square of the velocity of the particle.

[8]

b) Find the tension in the string in terms of m and g.

[2]

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3. A hollow cone with a smooth inner surface is placed with its axis vertical and its vertex downwards. A small particle moves with constant speed u in a horizontal circle on the inner surface of the cone.

Show that this circle will be a height of $\frac{u^2}{g}$ above the vertex

[8]

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4. A car of mass 1200kg travels around a bend, which is part of a circular arc of radius 80m. The track is banked at an angle of 15^o to the horizontal.

a) Calculate the speed at which the car must travel in order not to slip sideways if there is no friction between the car and the track.

[6]

b) Calculate the maximum speed at which the car must travel if it is not to slip outwards, given that the coefficient of friction between the car and the track is 0.6

[7]

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5. A particle of mass 1kg is attached to a light elastic string of natural length 0.8m and modulus of elasticity 6N. The other end of the string is fixed to a smooth horizontal table. The particle is made to move in a horizontal circle with constant speed 1.5 ms^{-1}. Find the radius of the circle.

[8]

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6. A particle of mass 0.5kg is placed r cm from the centre of a turntable. The turntable starts to rotate, and its angular speed increases steadily. The coefficient of friction between the particle and the turntable is 0.4.

a) Find the angular speed at which the particle begins to slip if
i) r = 20cm ii) r = 40cm

[6]

The 0.5kg is replaced by a 1kg particle.

b) State what effect this will have on the angular speeds calculated.

[1]

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7. The gravitational attraction between two bodies of masses m_1 and m_2 which are distance D apart is given by $\frac{km_1m_2}{D^2}$, where k is a constant.

a) Given that the radius of the earth is R, and its mass is M, express the constant k in terms of M, R and g.

[4]

A satellite of mass m is orbiting the earth at a height of ½ R *above the earth's surface*.

b) Calculate the angular speed with which the satellite is moving, giving your answer in terms of M, R and g

[7]

Another satellite is in *geo-stationary orbit* – i.e. it orbits the earth once every twenty-four hours, so that it is always above the same point on the earth's surface.

c) Find, in terms of M, R and g, the radius of the satellite's orbit.

[6]

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8. A particle P of mass 1kg is connected to a light, inextensible string AB so that AP=0.6 metres and PB=0.8 metres. The string is then fastened so that A and B are in a vertical line, with A 1 metre above B.
The particle is set into motion with angular speed ω so that it completes horizontal circles, with both strings taut.

a) Find an expression for the tension in each string, leaving your answer in terms of ω and g

[12]

b) Deduce the minimum possible angular speed of the particle

[3]

Given, further, that each string will break if the tension in it exceeds 40N

c) Find the maximum possible angular speed of the particle

[3]

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9. A particle P of mass m is attached to a light, elastic string of natural length 1 metre and modulus of elasticity 4mg. The other end of the string is attached to a fixed point O, and the particle is set in motion so that it completes horizontal circles with angular speed ω.

a) Show that the extension of the string is $\frac{\omega^2}{4g - \omega^2}$.

[9]

b) The angle between OP and the vertical is θ. Show that $\cos\theta = \frac{4g - \omega^2}{4\omega^2}$

[5]

c) Deduce the range of values that ω may take.

[4]

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10. A particle of mass 2kg is attached to a light, inextensible string of length 30cm. The other end of the string is attached to a fixed point on a rough, horizontal table, and the particle is placed so that the string is taut. The particle is set in motion with an angular velocity of 3 rad s^{-1}.

a) Calculate the initial kinetic energy of the particle

[3]

b) Given that the coefficient of friction between the particle and the table is 0.05, find, in terms of π and g, the total work done against friction when the particle has made a quarter of one revolution.

[4]

c) Hence find, to three significant figures, the tension in the string at this time

[3]

d) Find the total angle through which the particle rotates before it stops moving, giving your answer to the nearest degree

[4]

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11. A particle P of mass m is joined to the midpoint of a light, elastic string AB of length 2 metres and modulus of elasticity kmg. The string is fastened vertically, with A a distance 5 metres above B. The particle is then set in motion so that it moves in a horizontal circle with angular speed ω. While this motion takes place, AP = 4 metres and PB = 3 metres.

a) Find the tension in each string in terms of m, k and g

[3]

b) Show that $k = \frac{5}{6}$.

[7]

c) Find the value of ω.

[6]

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12. Two identical particles of mass 1 kg are attached to the ends of a light elastic spring of natural length 1metre and modulus 2g. The string and particles are placed on a smooth horizontal table with the spring at its natural length, and the system is set in motion so that the particles perform horizontal circular motion with angular speed 2 rad s^{-1} about their centre of mass. Find the tension in the spring.

[9]

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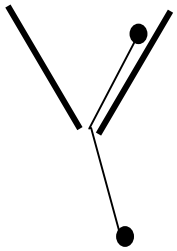
13. A smooth cone of semi-vertical angle $\sin^{-1}0.6$ has a small, smooth hole at its vertex. The cone is fixed with its vertex downwards, and a light, inextensible string is threaded through the hole. A particle of mass 1kg is attached to each end of the string, and both particles perform circular motion with speed $5ms^{-1}$. The length of string inside the cone is 1 metre.



a) By considering the motion of the particle inside the cone, find the tension in the string

[7]

b) The portion of the string outside the cone makes an angle α with the downwards vertical. Find the value of α, giving your answer to the nearest degree

[2]

c) Find the length of the portion of the string outside the cone

[3]

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14. A circular race-track has a radius of 200m. It is banked at an angle of $\sin^{-1}(0.1)$ to the horizontal. A car of mass 1200kg can travel at a maximum speed of $40ms^{-1}$ without slipping outwards. Find the coefficient of friction between the car and the track

[11]

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15. A track of radius 100m is banked at an angle α. A car of mass 1000kg travels around the track; the coefficient of friction between the car and the track is μ.

a) The minimum speed at which the car can travel without slipping inward is V.

Show that $V^2 = \dfrac{100g(\tan\alpha - \mu)}{1 + \mu\tan\alpha}$

[9]

b) Given that $\mu = 0.2$, deduce the minimum possible value of α, giving your answer in degrees to one decimal place.

[2]

There is a maximum possible speed of travel for the car, beyond which it will start to slip outwards.

c) Find this maximum speed, giving your answer in terms of α.

[6]

d) State the range of possible values of α.

[3]
