PARTICLE KINEMATICS & VECTOR MOTION

Take g as 9.8 ms^{-2} when required

1. A particle's position at time t is given by:

$$\mathbf{r} = (t^2 - kt)\mathbf{i} + t^3\mathbf{j}$$

a) When t = 2, the particle's velocity is parallel to $\mathbf{i} + 6\mathbf{j}$. Find the value of k.

[5]

b) Show that the particle's acceleration is never zero.

[3]

c) Another particle has position vector **s**, where $\mathbf{s} = (2t - 3)\mathbf{i} + (4t^2 - 9)\mathbf{j}$
Show that the two particles collide, and state the time at which this occurs.

[6]

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2. At noon, ship A is at point P, and ship B is 9 km due west of A. A then sets off with velocity vector **v**. B sets off with speed 15 kmh^{-1} in the direction $3\mathbf{i} + 4\mathbf{j}$, where **i** and **j** are unit vectors East and North respectively.

a) Find the velocity vector of B.

[2]

b) Write down expressions for the displacement of A and B from point P at time t.

[3]

c) Given that the ships meet in 6 hours, find **v**.

[3]

Ship C is 8 km due East of P at noon, and thereafter moves with velocity vector $2\mathbf{i} + 24\mathbf{j}$.

d) Find an expression for the distance between B and C at time t, and hence find the minimum distance between them, giving your answer to 3 significant figures.

[10]

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3. The distance, x, of a particle from the origin at time t is given by $x = (t - 2)^2(t + 1)$ $t \geq 0$

a) Show that x may be written as $x = t^3 - 3t^2 + 4$

[3]

b) Find expressions for the velocity and acceleration of the particle at time t

[4]

c) Find the distance of the particle from the origin at the time(s) when it is at rest.

[5]

d) Find the distance covered by the particle in the first two seconds of its motion.

[1]

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4. The distance of a particle from the origin at time t is given by $x = \frac{4 - t}{t^2}$ $t > 0$

a) Find an expression for the velocity of the particle at time t

[3]

b) State the time at which the particle passes through the origin, and find its velocity at this time.

[3]

c) Show that the particle changes direction exactly once, and state the time at which this occurs.

[3]

d) Find the distance travelled by the particle between t = 7 and t = 10.

[7]

e) Explain what happens to the particle for very large values of t

[2]

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5. Particles P and Q move along the x-axis. Their positions at time t are given by x_P and x_Q respectively, where $x_P = (2t - 3)^2$ and $x_Q = t^2 - 2t + 17$ $t \geq 0$

a) Find the minimum distance of particle Q from the origin.

[3]

b) Find the time at which the two particles collide.

[4]

c) Find the time at which the particles are moving with the same speed, but in opposite directions

[4]

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6. A particle of mass 2kg is moving in a straight line under the influence of a force F.
At time t, its distance (x) from a fixed point O is given by: $x = 3t^2 - 11t - 4$ $t \geq 0$

a) Show that F is a constant force, and find its magnitude.

[4]

The point A is at distance 16 from point O in the positive direction.

b) Find the velocity of the particle as it passes through point A.

[4]

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7. A ship S is 6km North of a point A on the coast at 12 noon. It then sets off at a constant velocity of $6\mathbf{i}+3\mathbf{j}$ kmh^{-1}, where **i** and **j** are unit vectors in the East and North directions respectively. Simultaneously, boat B sets off from point A with velocity **v** to intercept the ship S. B successfully intercepts S five hours later.

a) Write down an expression for the displacement of ship S from point A when the ship has been travelling for 5 hours.

[3]

b) Hence find the velocity **v** of boat B.

[2]

c) Find the speed of boat B, and hence find the distance it has travelled from point A when it intercepts the ship, giving your answers to three significant figures

[4]

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8. A hockey player standing at the point A, which has position vector $4\mathbf{i} + 6\mathbf{j}$, hits the ball, which moves off with a speed of 26 ms^{-1} in the direction of the vector $12\mathbf{i} - 5\mathbf{j}$ ms^{-1}. A second player, standing at the point B, which has position vector $7\mathbf{i} - 2\mathbf{j}$, needs to run with velocity $a\mathbf{i} + b\mathbf{j}$ ms^{-1} to intercept the ball after one second.

a) Find the velocity vector of the ball after the first player has hit it.

[4]

b) By considering the positions of the ball and the second player, find the value of the constants a and b.

[6]

The second player cannot run faster than 10 ms^{-1}.

c) Find whether it is possible for the second player to intercept the ball after one second.

[3]

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9. The position vector **r** of particle P of mass 2 kg at time t is given by $\mathbf{r} = (t^2+1)\mathbf{i} + (At + B)\mathbf{j} \quad t \geq 0$ Initially the particle is at the point with position vector $\mathbf{i} + \mathbf{j}$, and is moving with speed 2 ms^{-1} in the negative **j** direction.

a) Find the values of the constants A and B. [6]

b) Find an expression for the speed of the particle at time t, and hence state the minimum speed of the particle

[3]

c) Find the magnitude of the force acting on the particle.

[4]

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10. Particle P, which is of mass 1kg, has position vector $\mathbf{r}$ at time t, where $\mathbf{r} = t^3\mathbf{i} + (4t - 3)\mathbf{j}$ $t \geq 0$

a) Find the velocity and acceleration vectors of P at time t.

[4]

b) Find the time at which the velocity of P is parallel to the vector $3\mathbf{i} + \mathbf{j}$, and the speed of the particle at this time, giving your answer correct to three significant figures.

[5]

c) State the magnitude and direction of the force acting on P when t = 2.

[3]

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11. A particle moves in a straight line with an acceleration of a ms^{-2} at time t seconds where $a = 2t - 3$ Initially it is a distance of 1m from a fixed point O on this line, and has a velocity of 4 ms^{-1} directed towards O. Find:

a) its velocity at any time t;

[4]

b) its distance from O at any time t;

[3]

c) its distance from O when the velocity is zero.

[5]

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12. The acceleration $\mathbf{a}$ ms^{-2} at time t seconds of a particle of mass 2 kg is given by

$$\mathbf{a} = (2t - 1)\mathbf{i} + 3\mathbf{j}$$

Initially the particle is at rest at the origin. Find:

a) the velocity of the particle at time t seconds;

[3]

b) the distance in metres of the particle from the origin when t = 2;

[7]

c) the magnitude of the force acting on the particle when t = 2, giving your answer in surd form.

[5]

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13. A particle which is moving in a straight line has a velocity v ms^{-1} at time t seconds where $v = t^2 - 6t + 5$. Initially it is at a fixed point O on the straight line.

a) Find the acceleration of the particle at the time when the velocity is zero.

[6]

b) Find:

i) the displacement of the particle after 2 seconds;

[4]

ii) the distance travelled by the particle in the first 2 seconds of the motion.

[6]

c) Describe the motion of the particle

[3]

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14. A particle of mass 2 kg has a velocity $\mathbf{v}$ ms^{-1} at time t seconds where $\mathbf{v} = (3t^2 - t)\mathbf{i} + t\mathbf{j}$
The initial position vector of the particle is $3\mathbf{i}$. Find:

a) the magnitude of the force acting on the particle at time t = 2, giving your answer in terms of the simplest possible surds.

[6]

b) the position vector $\mathbf{r}$ of the particle at any time t seconds.

[4]

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15. A particle is moving along a straight line with its acceleration a ms^{-2} at time t s being given by $a = 2 - 2t$. Initially it is at a point A and has a speed of 6 ms^{-1} .

a) Find, in terms of t, expressions for v ms^{-1} (the velocity of the particle) and x m (its displacement from A) at time t s.

[6]

b) Find the time taken for the particle to first return to A, and its speed at that instant.

[6]

c) Describe the subsequent motion of the particle.

[3]

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16. Towns A and B lie on a flat plane. Point B is 20km from point A, at a bearing of 060^o.
A driver sets off at a speed of $50\sqrt{2}$ kmh^{-1} from point A, on a bearing of 315^o.

a) Taking **i** and **j** as unit vectors East and North respectively, find vector expressions for the following, leaving your answers in surd form where appropriate:

i) the displacement of town B from town A

[3]

ii) the velocity of the driver

[2]

iii) the displacement of the driver from town A at time t

[1]

A second driver sets off simultaneously from town B, with velocity vector $\mathbf{v} = -60\mathbf{i} + 15\mathbf{j}$.

b) Write down an expression for the displacement of the second driver from town A at time t.

[2]

c) Hence determine whether the drivers meet.

[8]

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17. Particle P is of mass 0.5kg, and has position vector **r** at time t, where

$\mathbf{r} = \left(\frac{3+t}{t}\right)\mathbf{i} + \left(\frac{4-2t^2}{t}\right)\mathbf{j} \quad t > 0$

a) Write down expressions for the velocity and acceleration vectors of P at time t

[6]

b) Show that the force acting on P is always parallel to the vector $3\mathbf{i} + 4\mathbf{j}$, and find an expression for its magnitude at time t

[5]

c) Describe the behaviour of P when t is very large.

[5]

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18. Particles P and Q move along the x axis. Their displacements from the origin at time t are given by p and q respectively, where $p = 4\sqrt{t} - t^2 \quad t > 0$ and $q = 2t - t^2 \quad t \geq 0$

a) Find the times at which Q passes through the origin, and hence find the time for which Q is on the positive part of the x axis.

[4]

b) Show that particle P is stationary only when t = 1

[6]

c) Find the position at which P and Q collide

[6]

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19. The position of particle P at time t is given by $\mathbf{r_P} = (3 + 2t)\mathbf{i} + (3 - 5t)\mathbf{j}$.
The position of particle Q at time t is given by $\mathbf{r_Q} = (5 - t)\mathbf{i} + (2 - 6t)\mathbf{j}$

a) Show that P and Q never collide.

[5]

b) If D is the distance between P and Q at time t, show that $D^2 = 10t^2 - 10t + 5$

[5]

c) By differentiating D^2, or otherwise, show that the minimum distance between the particles is $\frac{1}{2}\sqrt{10}$.

[5]

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20. The displacement of particle P from a fixed point O at time t is given by $\mathbf{r} = 15t\mathbf{i} + (20t - 5t^2)\mathbf{j}$ where **i** and **j** are unit vectors in the horizontal and vertical directions respectively.

a) Find the time at which particle P returns to the same horizontal level as point O, and find its distance from O at this time.

[4]

b) Find the maximum height attained by the particle P.

[6]

c) Show that the particle cannot pass through the point with position vector $45\mathbf{i} + 10\mathbf{j}$.

[6]

d) Find the time at which the velocity of the particle makes an angle of 45^0 to the upward vertical.

[3]

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21. A particle of mass 1.5kg moves under the action of a constant force of magnitude 15N, parallel to the vector $3\mathbf{i} - 4\mathbf{j}$.

a) Find the acceleration vector of the particle.

[3]

Given that initially, the particle was at rest at the point with position vector $6\mathbf{i} + 36\mathbf{j}$,

b) Find the velocity and displacement vectors of the particle at time t.

[6]

c) Show that the particle can never pass through the origin

[3]

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22. A particle of mass 1kg moves in a straight line under the action of a force F, where $F = 6 + 6t$ for $t \geq 0$
Initially, the particle is at rest at the origin.

a) Find expressions for the velocity and displacement of the particle at time t

[6]

b) Hence obtain its velocity and displacement when $t = 2$

[2]

After 2 seconds, the force acting on the particle instantaneously changes to $F = -4$

c) Find the velocity of the particle when $t = 5$

[3]
