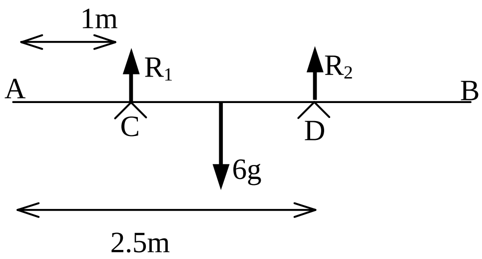
STATICS – EQUILIBRIUM OF RIGID BODIES

1.



Moments about A : M1

$R_1 + 2.5R_2 = 12g$ ① A1 A1

Resolving vertically: M1

$R_1 + R_2 = 6g$ ② A1

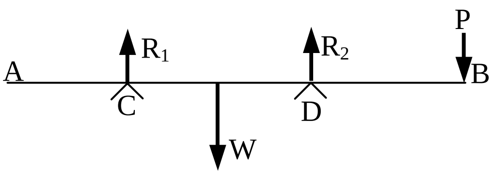
① – ② : $1.5R_2 = 6g$ M1 (solving)

$R_2 = 4g$ A1

$R_1 = 2g$ A1

[8]

2.



Losing contact at C $\Rightarrow R_1 = 0$ B1

Taking moments about D : M1

$LW = LP$

$W = P$ A1

[3]

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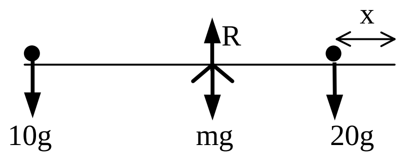
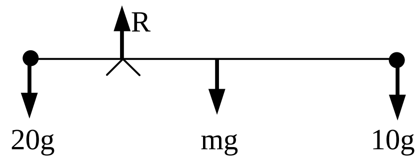
3. a)



Moments about pivot	M1
$10g \times 1.5 = 20g \times (1.5 - x)$	A1
$x = 0.75$m	A1
	[3]

b)

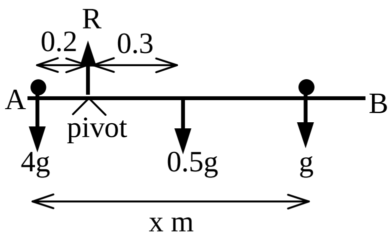


Moments about pivot	M1
$20g \times 1.2 = mg \times 0.3 + 10g \times 1.8$	A1 A1
$20\text{kg} = m$	A1
	[4]

c) Children can be modelled as particles
Plank can be modelled as a rod — B1 (either)

[1]

4.



Taking moments about pivot:	M1
$0.2 \times 4g = 0.3 \times 0.5g + (x - 0.2) \times g$	A1 A1
$x = 0.85$m	A1
	[4]

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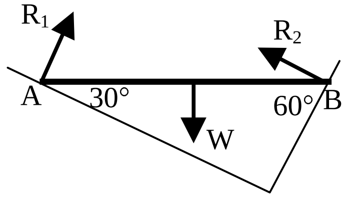
5.



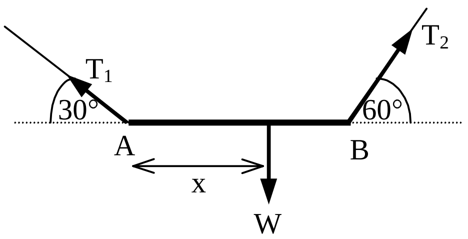
Moments about A :

$WL = R_2 2L\sin 30$	M1
$W = R_2$	A1

Moments about B :

$WL = R_1 2L\sin 60$	M1
$\frac{1}{\sqrt{3}} W = R_1$	A1
	[4]

6.



Resolving horizontally :	M1
$T_1\cos 30 = T_2\cos 60$	A1
$T_1\frac{\sqrt{3}}{2} = T_2\frac{1}{2}$	
$T_1\sqrt{3} = T_2$ ①	
Resolving vertically :	
$T_1\sin 30 + T_2\sin 60 = W$	A1
$\frac{1}{2}T_1 + \frac{\sqrt{3}}{2}T_2 = W$ ②	
① and ② $\Rightarrow \frac{1}{2}T_1 + \frac{\sqrt{3}}{2}T_1\sqrt{3} = W$	M1
$2T_1 = W$	
$T_1 = \frac{1}{2}W$	A1
Moments about B :	M1
$(L - x)W = LT_1\sin 30$	A1
$(L - x)W = L\frac{1}{4}W$	M1 (substituting T_1)
$L - x = \frac{1}{4}L$	
$x = \frac{3}{4}L$	A1 c.a.o.
	[9]

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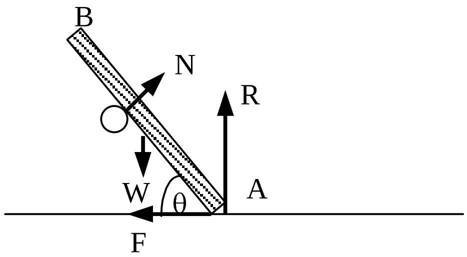
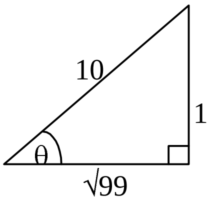
7.



Resolving		M1
vertically : $R + N\cos\theta = W$	①	A1
horizontally : $F = N\sin\theta$	②	A1
Moments about A :		M1
$W \times 3L\cos\theta = N \times 4L$	③	A1



$\cos\theta = \frac{\sqrt{99}}{10}$

③ $\Rightarrow N = \frac{3}{4}W\frac{\sqrt{99}}{10} = \frac{3\sqrt{99}W}{40}$ B1

Substituting in ① : $R = W - \frac{3\sqrt{99}W}{40}\frac{\sqrt{99}}{10}$ M1

$R = \frac{103W}{400}$ A1

② $\Rightarrow F = \frac{1}{10}\frac{3\sqrt{99}W}{40} = \frac{3\sqrt{99}W}{400}$ B1

Using $F = \mu R$: M1

$\frac{3\sqrt{99}W}{400} = \mu\frac{103W}{400}$

$\frac{3\sqrt{99}}{103} = \mu$

$\mu = 0.290$ (3 S.F.) A1

[11]

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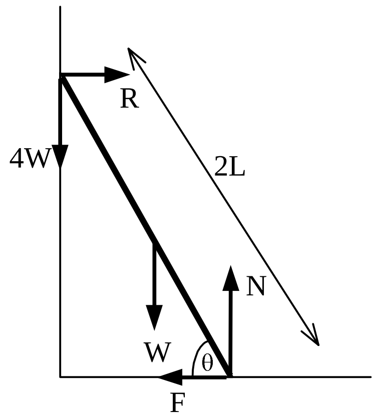
8. a)



Taking moments about the foot of the ladder:	M1
$WL\cos\theta + 4W \times 2L\cos\theta = R \times 2L\sin\theta$	A1
$\sin\theta = \frac{4}{5}$	
$\therefore\ R = \dfrac{27}{8}\ W$	A1 c.a.o.
Resolving horizontally and vertically:	M1
$F = R$ and $N = 5W$	
$\therefore\ F = \dfrac{27}{8} W$	A1
In order for ladder not to slip $F \le \mu N$	M1
so we need $F \le 4W$	A1
$\therefore$ Since $F = 3\frac{3}{8}W < 4W$ builder can reach top	A1 f.t.
	[8]
b) Assuming he reaches the top with the bricks	M1
moments about foot give $R = \dfrac{33}{8} W$	M1 A1
$\therefore\ F = \dfrac{33}{8} W > 4W$	M1
$\therefore$ Ladder would slip $\therefore$ cannot reach the top safely	A1
	[5]

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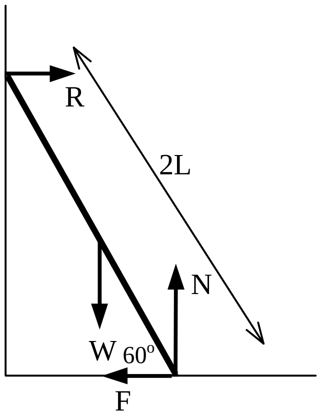
9. a)



Taking moments about the base of ladder	M1
$W \times L \cos 60^0 - R \times 2L \sin 60^0 = 0$	A1
$R = \dfrac{W}{2\sqrt{3}}$	A1
	[3]
b) Resolving gives	M1
$N = W$	A1
$F = R = \dfrac{W}{2\sqrt{3}}$	A1 f.t
$F \le \mu N$	M1
Min $\mu = \dfrac{1}{2\sqrt{3}}$	A1
	[5]
c) Ladder can be modelled as a rod.	B1
	[1]

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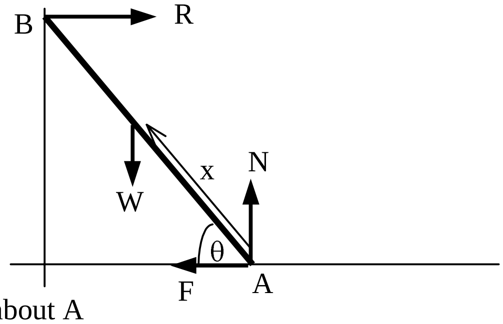
10.



Moments about A	M1
$\Rightarrow W \times x\cos\theta = R \times 3\sin\theta$	A1
$\cos\theta = \frac{3}{5}$	
$R = \dfrac{Wx}{4}$	A1
Resolving horizontally and vertically	
$F = R$	M1
$N = W$	A1
Limiting equilibrium $\therefore$ $F = \mu N$	M1
$\therefore \dfrac{Wx}{4} = 0.6W$	
$x = 2.4$	A1
Centre of mass 2.4 m from A.	
	[7]

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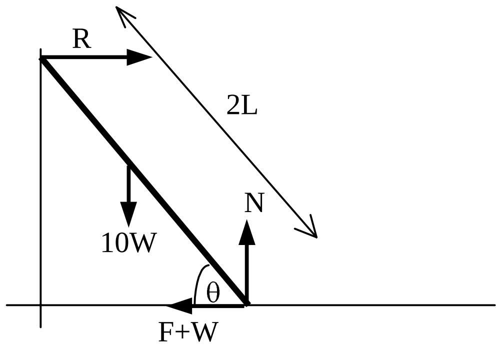
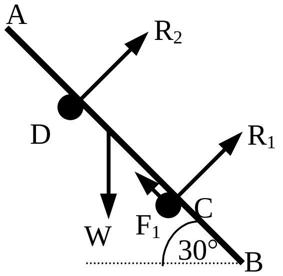
11.



Taking moments about the foot of the ladder	M1
$10\,W \times L\cos\theta = R \times 2L\sin\theta$ ①	A1
Resolving vertically and horizontally	M1
$F + W = R$ ② and $N = 10\,W$ ③	A1 A1
Limiting equilibrium $\therefore F = \mu N \Rightarrow F = 0.2 \times 10W = 2W$	B1
③ $\Rightarrow R = 3W$	
Substituting into ① : $10WL\cos\theta = 6\,WL\sin\theta$	M1
$\frac{10}{6} = \tan\theta$	A1 (solving)
$59.0° = \theta$	A1 c.a.o.
	[9]

12.



Resolving parallel to plank : $F_1 = W\sin30$	M1 A1
Moments about D :	M1
$LW\cos30 = 2LR_1$	A1
$F_1 = \mu R_1$	M1
$W\sin30 = \mu\frac{W\cos30}{2}$	M1
$2\tan30 = \mu$	A1
$\frac{2}{\sqrt{3}} = \mu$	A1
	[8]

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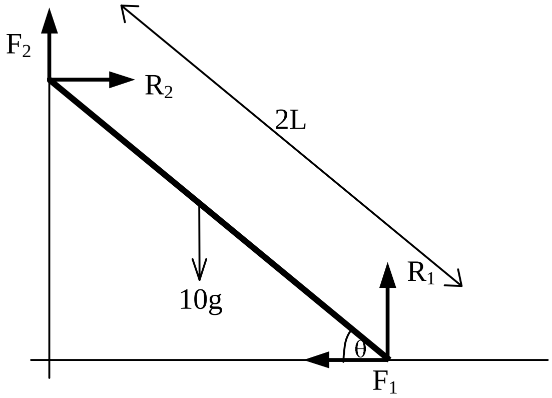
13.



Moments about foot of ladder	M1
$L10g\cos\theta = 2LF_2\cos\theta + 2LR_2\sin\theta$	A1
$5g\cos\theta = F_2\cos\theta + R_2\sin\theta$ ①	
Resolving	M1
horizontally : $R_2 = F_1$ ②	A1
vertically : $F_2 + R_1 = 10g$ ③	A1
Friction limiting $\Rightarrow$ $F_1 = \mu_1 R_1$ ④; $F_2 = \mu_2 R_2$ ⑤	B1
Eliminating F_1, F_2 :	M1
① $\Rightarrow 5g\cos\theta = \mu_2 R_2\cos\theta + R_2\sin\theta$ (I)	A1
② $\Rightarrow R_2 = \mu_1 R_1$ (II)	A1
③ $\Rightarrow \mu_2 R_2 + R_1 = 10g$ (III)	A1
(II) and (III) $\Rightarrow \mu_2\mu_1 R_1 + R_1 = 10g$; $R_1 = \dfrac{10g}{1+\mu_1\mu_2}$; hence $R_2 = \dfrac{10g\mu_1}{1+\mu_1\mu_2}$	M1 (R_1 or R_2)
Substituting in (I) :	
$5g\cos\theta = \dfrac{10g\mu_1\mu_2}{1+\mu_1\mu_2}\cos\theta + \dfrac{10g\mu_1}{1+\mu_1\mu_2}\sin\theta$	M1
$\div 5g\cos\theta$: $1 = \dfrac{2\mu_1\mu_2}{1+\mu_1\mu_2} + \dfrac{2\mu_1\tan\theta}{1+\mu_1\mu_2}$	M1 (simplifying)
$\times (1+\mu_1\mu_2)$: $1 + \mu_1\mu_2 = 2\mu_1\mu_2 + 2\mu_1\tan\theta$	
$\dfrac{1-\mu_1\mu_2}{2\mu_1} = \tan\theta$	A1 c.a.o.
	[14]