Secondary
Maths
Resources

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A LEVEL
TOPIC TESTS
PURE – EDEXCEL – 2004 – C1

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***Syllabus Reference**

Note All material in C1 to be done without a calculator!

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Teachers and Students Notes

These tests are designed to specifically show what students can do and so are great to encourage confidence and show progress. Each test looks to cover the **different types of problem** that students may encounter and so can be used to identify weaknesses. Additional difficult exam level questions have been added to test the more able. Each test follows the new specifications for **EDEXCEL C1** and a summary of what is tested in shown below. The tests numbers match the numbering in the specification. It is recommended that **students are given a copy of the specification** (or get one from the **internet**!) and are informed of the sections being tested. This should help students to become familiar with their specification. Any important notes about the test are summarised below and it's up to the teacher to decide where to pass this information on to students – possibly by supplying a copy Test Summary below.

Test Summary

*Please use this in conjunction with the specification. Relevant sections of the specification are shown but must **not** be relied upon. Note for example that **'Test 2' tests section 2 of the specification**!*

TEST

1a. **Algebra & Functions: Brackets, Indices, Surds**

Law of indices, fractional and negative indices including conversion from $\sqrt[m]{x^n}$, simplifying surds, rationalising surds.

Para 1/2 (8)

Laws of indices for all rational exponents.	The equivalence of $a^{m/n}$ and $\sqrt[n]{x^m}$ should be known
Use and manipulation of surds.	Candidates should be able to rationalise denominators.
Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.	Candidates should be able to use brackets. Factorisation of polynomials of degree n, $n \leq 3$, eg $x^3 + 4x^2 + 3x$.

1b. **Algebra & Functions: Quadratics, Simultaneous Equations and Factorisation**

Factorise quadratics and (cubics without a constant), Simultaneous equations, Completing the Square, Discriminant.

Para 3/4/5/6/8

Quadratic functions and their graphs.	
The discriminant of a quadratic function.	
Completing the square. Solution of quadratic equations.	Solution of quadratic equations by factorisation, use of the formula and completing the square.
Simultaneous equations: analytical solution by substitution.	For example, where one equation is linear and one equation is quadratic
Factorisation	Factorisation of polynomials of degree n, $n \leq 3$, eg $x^3 + 4x^2 + 3x$. The notation f(x) may be used. (Use of the factor theorem is *not* required.)

1c. **Algebra & Functions: Inequalities, Graphs & Transformations**

Sketch graphs (quadratics, simple cubics, reciprocals), asymptote, single transformations, solving linear and quadric inequalities, for graphs understand that f(a) = b corresponds to the coordinates (a, b).

Para 7/9/10

Solution of linear and quadratic inequalities.	For example, $ax + b > cx + d$, $px^2 + qx + r \geq 0$, $px^2 + qx + r < ax + b$.
Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.	Functions to include simple cubic functions and the reciprocal function $y = k/x$ with $x \neq 0$. Knowledge of the term asymptote is expected.
Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.	Candidates should be able to apply one of these transformations to any of the above functions [quadratics, cubics, reciprocal] and sketch the resulting graph. Given the graph of any function $y = f(x)$ candidates should be able to sketch the graph resulting from one of these transformations.

2. **Co-ordinate Geometry:** Straight line equations, parallel/perpendicular, distance between two given coordinates, for graphs understand that f(a) = b corresponds to the coordinates (a, b).

Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax+by+c=0$ Conditions for two straight lines to be parallel or perpendicular to each other.	To include (i) the equation of a line through two given points, (ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line $3x + 4y = 18$ through the point (2, 3) has equation $y - 3 = \frac{4}{3}(x - 2)$.

3. **Sequences and Series:** Sequences from n^{th} term formulas, sequences defined by $x_{n+1} = f(x_n)$, $\sum$ notation, APs.

Sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.	Understanding of $\sum$ notation will be expected.
Arithmetic series, including the formula for the sum of the first n natural numbers.	The general term and the sum to n terms of the series are required. The proof of the sum formula should be known.

4. **Differentiation:** Differentiate x^n, $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ as gradient function, application to tangents & normals to curves, know when to simplify expressions before differentiating for example expanding brackets and including converting surds to fractional indices.

The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.	For example, knowledge that dy/dx is the rate of change of y with respect to x. Knowledge of the chain rule is not required. The notation $f'(x)$ may be used.
Differentiation of x^n, and related sums and differences.	E.g., for $n \neq 1$, the ability to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2+5x-3}{3x^{1/2}}$ is expected.
Applications of differentiation to gradients, tangents and normals.	Use of differentiation to find equations of tangents and normals at specific points on a curve.

5. **Integration:** Integrate x^n, given $f'(x)$ and a point on the curve find $y = f(x)$, know when to simplify expressions before integration for example expanding brackets and including converting surds to fractional indices.

Indefinite integration as the reverse of differentiation.	Candidates should know that a constant of integration is required.
Integration of x^n.	For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}, \frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Given $f'(x)$ and a point on the curve, candidates should be able to find an equation of the curve in the form $y = f(x)$.

Algebra & Functions – Test 1a

EDEXCEL **C1**

Brackets, Indices & Surds

All tests for C1 to be done without a calculator!!

Recommended Time: 15 minutes

1) **Simplify** the following:-

a) $y^3 \times y^2$ b) $\left(\frac{1}{x}\right)^{-1}$ c) $\frac{x^3}{x^3}$

d) $y^{\frac{3}{2}} \times y$ e) $y^3 \times y^{-2}$ f) $x^3 - x^3$

g) $xyz \times xyz^{-1}$ **7**

2) **Expand** and **simplify**:-

a) $x(x+1)$ b) $(x+2)(x+3)$ c) $(x-1)^2$

d) $(x^2+1)(x-1)$ e) $(x^2+3x-4)(x^2+2x-1)$ **5**

3) Which of the following are true?

a) $(x^a)^b = x^{ab}$ b) $x^a \times x^b = x^{a+b}$ c) $x^0 = 1$

d) $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ e) $\frac{\sqrt{x}}{y} = \sqrt{\frac{x}{y^2}}$ f) $y\sqrt{x} = \sqrt{y^2 x}$ **6**

4) Evaluate:-

a) $27^{\frac{2}{3}}$ b) $8^{\frac{2}{3}}$ c) $4^{\frac{5}{2}}$ **3**

Show your working!

5) **Simplify** the following surds:-

a) $\sqrt{12}$ b) $\sqrt{120}$ **2**

6) Write $\sqrt{28} - \sqrt{396}$ in the form $a(\sqrt{7} - b\sqrt{11})$ **3**

7) **Rationalise** the denominator in each case:-

a) $\frac{1}{\sqrt{3}}$ b) $\frac{2}{\sqrt{5}}$ c) $\frac{1}{1-\sqrt{3}}$ d) $\frac{3}{2+\sqrt{5}}$ **4**

{30}

Algebra & Functions – Test 1b

Quadratics, Simultaneous Equations and Factorisation

Recommended Time: 20 minutes

1) **Factorise** (fully) the following:-

a) $x^2 + 12x + 20$ b) $x^2 - 12x + 20$ c) $x^2 - x - 12$
d) $x^3 + x$ e) $x^3 - 3x^2 - 10x$ f) $x^3 - x$ **8**

2) Solve the following equations (fully). *Give your answer in surd form where appropriate.*

a) $3x + 4 = 0$ b) $\frac{x+2}{4} = 7$ (1, 1)

c) $x + \frac{1}{x} = 3$ (3) **5**

3) Solve the **simultaneous equations**:

a) $3x + 2y = 1$ and $2x + 3y = 2$ (2)
b) $y = x^2 + x - 1$ and $y = x$ (2)
c) $x^2 - 3y^2 = 50$ and $2y + x = 7$ *Give your answer exactly in simplified form.* (5) **9**

4) **Factorise** and hence solve:

a) $x^2 - 2x - 15 = 0$ b) $2x^2 - x - 1 = 0$ (3, 3) **6**

5) Use the method of **completing the square** to solve:

a) $x^2 + 4x = 0$ (2)
b) $x^2 + 4x - 5 = 0$ (2)
c) $16x^2 + 8x - 1 = 0$ *(Leave your final answer in surd form)* (3) **7**

6) The equation $x^2 + bx + c = 0$ has 1 repeated root.
Calculate the discriminant and hence or otherwise write down an equation linking b and c in the form c = f(b). **3**

7) Sketch the graphs of

a) $y = (x-1)(x+2)$ State where this graph crosses the x-axis. (2)
b) $y = -(x-1)(x+2)$ State where this graph crosses the x-axis. (1)
c) $y = x^2 - 2$ State where this graph crosses the y-axis. (1)
d) $y = 2 - x^2$ State where this graph crosses the y-axis. (1)
e) $y = x^2 - x - 6$ State where this graph crosses the x-axis. (2) **7**

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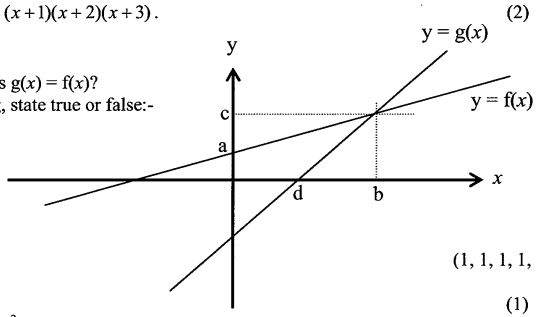
1) a) Sketch the graph of $y = (x - 1)(x - 3)$. State where the graph crosses the x & y-axis. (4)
b) Write the equation in the form $y = (x + b)^2 + c$ (2)
c) Hence or otherwise find the coordinates of the minimum point of the graph. (2) **8**

2) a) Sketch the graph of $y = \frac{5}{x}$ and write down the asymptotes of the graph. (3)

b) Hence or otherwise sketch the graph of $y = \frac{5}{x} + 1$, clearly stating the coordinates of where the graph crosses the x and y -axis and write down the asymptotes of the graph. (4) **7**

3) a) Sketch the graph of $y = x^3 + k$, k a positive constant.
State where the graph crosses the x-axis and y–axis. (3)
b) i) This graph is reflected in the x-axis. State the equation of the new graph. (1)
ii) The graph is now shifted so that it crosses the y-axis at k. Describe the shift. (1)
c) Sketch the graph of $y = (x+1)(x+2)(x+3)$. (2) **7**

4) a) For what value of x does $g(x) = f(x)$?
b) In each of the following, state true or false:-
A) $f(d) < g(d)$
B) $f(d) > g(d)$
C) $f(d) = g(d)$
D) $f(d) \geq g(d)$

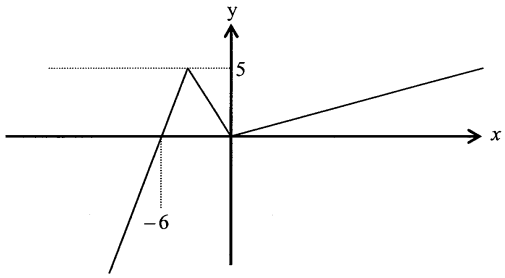


(1, 1, 1, 1, 1) **5**

5) a) **Factorise** $x^2 - 7x + 6$ (1)
b) **Sketch** the graph of $y = x^2 - 7x + 6$ stating where it crosses the x-axis. (1)
c) **Hence** solve $x^2 - 7x + 6 \leq 0$ (2) **4**

6) Solve the following inequalities *giving your answer in surd form where appropriate*:-
a) $-x > 3$ (1)
b) $x^2 \geq 9$ (2)
c) $x^2 + 7x - 7 < 0$ (2) **5**

7) This is a sketch of $y = c(x)$:



a)
i) Copy this sketch and sketch $y = 2c(x)$ on the same grid. (2)
ii) Copy this sketch of $c(x)$ again and sketch $y = c(x) + 2$ on the same grid. (2)
iii) Copy this sketch of $c(x)$ again and sketch $y = c(x + 1)$ on the same grid. (2)
iv) Copy this sketch of $c(x)$ again and sketch $y = c(2x)$ on the same grid. (2)

b) Fully describe each of the transformations above. (4) **12**

In each case indicate known values like those in the sketch of c(x)!
(In question 8 you should have drawn 4 grids with 2 graphs on each.) **{48}**

Co-ordinate Geometry – Test 2

EDEXCEL **C1**

Recommended Time: 25 minutes

1) Four points are A(3, 2), B(7, 11), C(–3, 2) and D(–7, –11).

a) Find the **distance** between A and B, giving your answers in surd form. (2)
b) Find the **distance** between C and D, giving your answers in surd form. (2)
c) Find the **equation** of the **line**, which passes through A and B. (2)
Write your answer in the form $y = mx + c$
d) Find the **equation** of the **line**, which passes through C and D. (2)
Write your answer in the form $y = mx + c$
e) Calculate where these 2 lines **intersect**. (2)

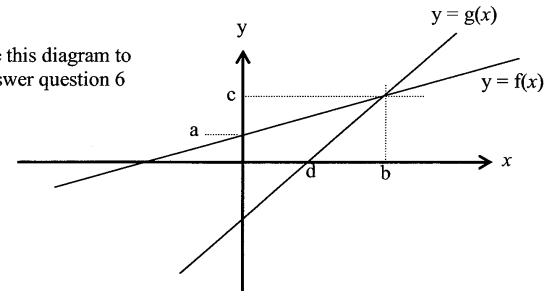
2) Find the **equation** of the **line**, which passes through (2, 3) and has a gradient of 7. (2)
Write your answer in the form $y = mx + c$

3) Find the **equation** of the **line**, which passes through (2, 3) and is **parallel** to $2y = x$ (2)
Write your answer in the form $y = mx + c$

4) Given that $y = 3x + 2$ and $y = mx - 2$ are **parallel**.
Write down the value of m. (1)

5) Given that $y = 3x + 2$ and $y = nx + d$ are **perpendicular**.
Calculate the value of n. (1)

Use this diagram to answer question 6



6) Calculate the **equation** of **lines** f(x) and g(x) in terms of a, b, c and d. (2, 2)

Write your answer in the form $y = mx + c$
(Clearly indicate which is f(x) and which is g(x) and be careful not to confuse the 2 lines)

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Sequences and Series – Test 3

EDEXCEL **C1**

Recommended Time: 30 minutes

1) $x_{n+1} = x_n^2$ and $x_1 = 2$ Write down x_2, and x_3. (1)

2) $x_{n+1} = 2x_n + 1$ and $x_3 = 23$ Write down x_2, and x_1. (2)

3) $x_{n+1} = f(x_n)$ and $x_1 = 3,\ x_2 = 5,\ x_3 = 7,\ x_4 = 9,\ x_5 = 11,\ x_6 = 13$ etc. Find $f(x)$. (2)

4) The n^{th} term of a sequence is given by n^2+1. State the first 3 terms of the sequence. (2)

5) Write in **sigma** notation

a) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ (1)
b) $7 + 12 + 17 + 22 + 27 + 32$ (1)

6) Calculate $\sum_{r=2}^{5}(r+1)^2$ (1)

7) A sequence is such that its n^{th} term is given by: $5n-2$.
a) State the first and tenth terms. (1)
b) Find an expression for the $(n+1)^{th}$ term. (1)
c) Calculate a value for the $(n+1)^{th}$ term $- n^{th}$ term & decide whether the sequence is arithmetic? (1)

8) Calculate the sum of the first 800 odd terms starting 1, 3, 5, 7, … (1)

9) Letters a to f represent numbers and an **arithmetic progression** (AP) begins:-
a, b, c, d, e, f, 21, 31,
a) Find the common difference. (1)
b) Find **a**, the **first term.** (1)
c) Find the sum of the first 100 terms. (1)

10) In an **AP**, the sum of the first 5 terms is 502 and the sum of the first 10 terms is 1009.
a) Find **d**, the **common difference**. (2)
b) Find **a**, the **first term**. (1)

11) In an **arithmetic progression**, the 10^{th} term is 50 and the 20^{th} term is –62.
a) Find **d**, the **common difference**. (1)
b) Find **a**, the **first term**. (1)

12) The first n terms of an **arithmetic progression** is given by:

a, (a + d), (a + 2d), (a + 3d), ... , (a + (n–2)d), (a + (n–1)d).

Prove that the sum of this arithmetic progression is given by: $\frac{n}{2}[2a + (n-1)d]$ (3)

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Differentiation – Test 4

EDEXCEL **C1**

Recommended Time: 30 minutes

1) For each of the following, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$:

a)	$y = x^2$	(1, 1)	
b)	$y = 3x^2$	(1, 1)	
c)	$y = x$	(1, 1)	
d)	$y = (x + 1)^2$	(1, 1)	
e)	$y = (3x + 1)^3$	(2, 1)	
f)	$y = (x+3)(x+1)$	(2, 1)	
g)	$y = \frac{x^5 + 1}{x^2}$	(2, 1)	**17**

2) For each of the following, find $\frac{dy}{dx}$:

a)	$y = \sqrt{x}$	(2)	
b)	$y = \sqrt[3]{x^2}$	(2)	**4**

3) Calculate the **gradient** of each of the following, at the point specified:-

a)	$y = x^2$	when $x = 11$	(2)	
b)	$y = x^2 + \frac{1}{x^2}$	when $x = 2$	(2)	
c)	$y = (x - 2)^2$	when $x = 0$	(2)	**6**

4) Give the **co-ordinates** when $y = x^3$ has a **gradient** of 3. (4) **4**

5) a) Find the equation of the tangent to the curve $y = x^2 - 10x + 10$ at the point (1,1) (3)
b) Find the equation of the normal to the curve $y = x^2 - 10x + 10$ at the point (1,1) (2) **5**

6) Find the **co-ordinates** of all the points on the curve $y = 3x^3 - \frac{21x^2}{2} + 7x - 1$ when the gradient of the normal to this curve is –1. **6**

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Integration – Test 5

Recommended Time: 20 minutes

1) **Integrate** the following:-

a) $\int x\,dx$ (1)

b) $\int 3x+5\,dx$ (2)

c) $\int \frac{x^3}{x}\,dx$ (2)

d) $\int x^{-\frac{1}{2}}\,dx$ (2)

e) $\int \sqrt[3]{x}\,dx$ (2)

f) $\int \sqrt[3]{x}(\sqrt{x}-\frac{1}{\sqrt{x}})dx$ (3)

g) $\int (x+1)(x-1)\,dx$ (2)

h) $\int 3x^{\frac{2}{3}}\,dx$ (2)

i) $\int \frac{x-2x^3}{x^{\frac{1}{2}}}\,dx$ (3) **19**

2) a) Find y as a function of x, given that $\frac{dy}{dx} = 10x$ and that $x = 1$ when $y = -10$. (3)

b) Find y as a function of x, given that $\frac{dy}{dx} = x^2 - 2$ and that $x = 1$ when $y = 1$. (3) **6**

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1) a) $y^3 \times y^2 = y^{3+2} = y^5$ **A1** b) $\left(\frac{1}{x}\right)^{-1} = x$ **A1**

c) $\frac{x^3}{x^3} = 1$ **A1** d) $y^{\frac{3}{2}} \times y = y^{\frac{3}{2}+1} = y^{\frac{5}{2}}$ **A1**

e) $y^3 \times y^{-2} = y^{3-2} = y$ **A1** f) $x^3 - x^3 = 0$ **A1**

g) $xyz \times xyz^{-1} = x^2y^2$ **A1** (7)

2) a) $x(x+1) = x^2 + x$ **A1**

b) $(x+2)(x+3) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$ **A1**

c) $(x-1)^2 = (x-1)(x-1) = x^2 - x - x + 1 = x^2 - 2x + 1$ **A1**

d) $(x^2+1)(x-1) = x^3 - x^2 + x - 1$ **A1**

e) $(x^2 + 3x - 4)(x^2 + 2x - 1) = x^4 + 2x^3 - x^2 + 3x^3 + 6x^2 - 3x - 4x^2 - 8x + 4$
$= x^4 + 5x^3 + x^2 - 11x + 4$ **A1** (5)

3) All true!! **A1 A1 A1 A1 A1 A1** (6)

4) *With working!*

a) $27^{\frac{2}{3}} = 3^2 = 9$ **A1** b) $8^{\frac{2}{3}} = 2^2 = 4$ **A1** c) $4^{\frac{5}{2}} = 2^5 = 32$ **A1** (3)

5) a) $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ **A1** b) $\sqrt{120} = \sqrt{4 \times 30} = 2\sqrt{30}$ **A1** (2)

6) $\sqrt{28} - \sqrt{396} = \sqrt{4 \times 7} - \sqrt{36 \times 11} = 2\sqrt{7} - 6\sqrt{11} = 2(\sqrt{7} - 3\sqrt{11})$, a = 2, b = 3 **A1A1+A1 for both**(3)

7) a) $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ **A1** b) $\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ **A1**

c) $\frac{1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = -\frac{(1+\sqrt{3})}{2}$ **A1**

d) $\frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = 3\sqrt{5} - 6$ or $3(\sqrt{5} - 2)$ **A1** (4)

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1. a) $x^2 + 12x + 20 = (x + 2)(x + 10)$ **A1**
   b) $x^2 - 12x + 20 = (x - 2)(x - 10)$ **A1**
   c) $x^2 - x - 12 = (x + 3)(x - 4)$ **A1**
   d) $x^3 + x = x(x^2 + 1)$ **A1**
   e) $x^3 - 3x^2 - 10x = x(x^2 - 3x - 10)$ **M1** $= x(x - 5)(x + 2)$ **A1**
   f) $x^3 - x = x(x^2 - 1)$ **M1** $= x(x + 1)(x - 1)$ **A1** **8**

2. a) $3x + 4 = 0 \quad \therefore 3x = -4 \quad x = -\frac{4}{3}$ **A1**

   b) $\frac{x+2}{4} = 7 \quad \therefore x + 2 = 28 \quad \therefore x = 26$ **A1**

   c) $x + \frac{1}{x} = 3 \quad \therefore x^2 + 1 = 3x$ **M1**

   $\therefore x^2 - 3x + 1 = 0$ **A1**

   $x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$ $x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$ **A1** **5**

3. a) $3x + 2y = 1 \quad \times 3$
   $2x + 3y = 2 \quad \times -2$

   $9x + 6y = 3$
   $-4x - 6y = -4$
   adding:
   $5x = -1$
   $x = -\frac{1}{5}$

   Using $3x + 2y = 1$
   $-\frac{3}{5} + 2y = 1$
   $2y = \frac{8}{5}$
   $y = \frac{4}{5}$ $\therefore x = -\frac{1}{5}$ **A1** and $y = \frac{4}{5}$ **A1**

   b) $y = x^2 + x - 1$
   $y = x$
   $\therefore x = x^2 + x - 1$
   $\therefore x^2 - 1 = 0$
   $\therefore x^2 = 1$
   $\therefore x = \pm 1$ **A1 for either or both x values**
   $\therefore x = 1$ and $y = 1$ OR $x = -1$ and $y = -1$

   A1→1 mark for "paired" and correct y-values. only award mark for BOTH y-values!

   c) $x^2 - 3y^2 = 50$
   $2y + x = 7 \quad \Rightarrow \quad x = 7 - 2y$
   sub in
   $\therefore (7 - 2y)^2 - 3y^2 = 50$ **B1 or equivalent in x**
   $\therefore 49 - 14y - 14y + 4y^2 - 3y^2 = 50 \ \therefore y^2 - 28y + 49 - 50 = 0 \ \therefore y^2 - 28y - 1 = 0$
   $\therefore y = \frac{28 \pm \sqrt{784+4}}{2} = 14 \pm \sqrt{197}$
   $y = 14 + \sqrt{197}$ **A1**
   $y = 14 - \sqrt{197}$ **A1**
   $x = 7 - 2y$

   A1 1 mark for "paired" and correct x–values

$x = -21 - 2\sqrt{197}$ *and* $y = 14 + \sqrt{197}$ or $x = -21 + 2\sqrt{197}$ *and* $y = 14 - \sqrt{197}$ **A1** **8**

4. a) $(x-5)(x+3) = 0$ **M1**
$x = 5$ or $x = -3$ **A1 A1**

b) $(2x+1)(x-1) = 0$ **M1**
$x = -0.5$ or $x = 1$ **A1 A1** **6**

5. a) $(x+2)^2 - 4 = 0$ **M1**
$\therefore (x+2)^2 = 4$
$\therefore x + 2 = \pm 2$
$\therefore x = 0$ or $x = -4$ **A1**

b) $x^2 + 4x - 5 = 0$
$\therefore (x+2)^2 - 4 - 5 = 0$ **M1**
$\therefore (x+2)^2 = 9$
$\therefore x + 2 = \pm 3$
$\therefore x = 1$ or $x = -5$ **A1**

c) $16x^2 + 8x - 1 = 0$
$\therefore (4x+1)^2 - 1 - 1 = 0$ **M1**
$\therefore (4x+1)^2 = 2$
$\therefore 4x + 1 = \pm\sqrt{2}$
$\therefore 4x = \pm\sqrt{2} - 1$
$\therefore \quad x = \dfrac{\sqrt{2}-1}{4}$ or $x = \dfrac{-\sqrt{2}-1}{4}$ **A1 A1** **7**

6. Discriminant $= b^2 - 4ac = b^2 - 4c$ $(a = 1)$ **M1**
Equation has 1 repeated root $\Rightarrow$ discriminant $= 0$ **M1**
$\Rightarrow \quad b^2 - 4c = 0$
$\Rightarrow \quad c = \dfrac{b^2}{4}$ **A1** **3**

7) a) $y = (x-1)(x+2)$ U-shaped parabola **A1** through x-axis when $x = 1$ and -2 **A1**

b) $y = -(x-1)(x+2)$ N-shaped parabola through same as i) **A1ft**

c) $y = x^2 - 2$ U-shaped parabola through y-axis when y = -2 **A1**

d) $y = 2 - x^2$ N-shaped parabola through y-axis when y = 2 **A1**

e) $y = x^2 - x - 6 = (x-3)(x+2)$ **M1**
U-shaped parabola through x-axis when $x = 3$ and -2 **A1** **7**

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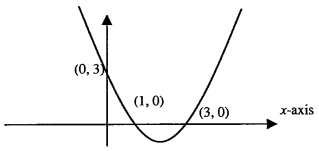
Inequalities, Graphs and Transformations

1. a)



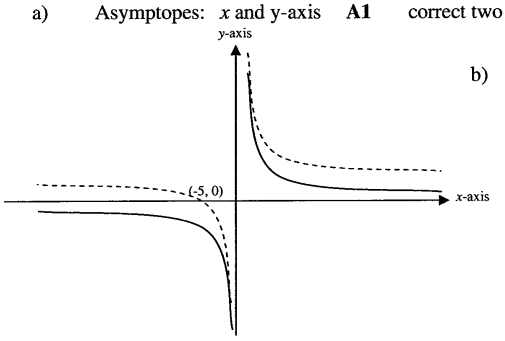
correct values for crossing axes **A1A1A1**
U-shaped parabola **A1**

A1 A1

b) $y = (x-1)(x-3) = x^2 - 4x + 3 = (x-2)^2 - 1$

c) Minimum when $(x-2)^2 - 1$ is the smallest possible value
i.e. when $(x-2)^2$ is smallest
$x = 2$, $y = -1$ **A1A1** **8**

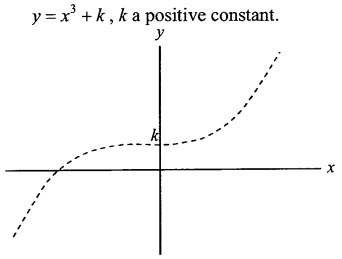
2. a) Asymptopes: x and y-axis **A1** correct two halves **A1A1**



b) Correct shift 1 up. **A1 ft**
Crosses x-axis at –5 **A1**
Asmptopes: y-axis ($x = 0$) & $y = 1$
A1 **A1**

7

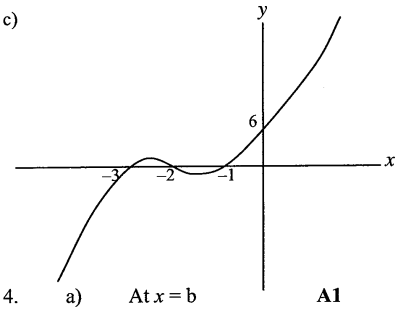
3) a) $y = x^3 + k$, k a positive constant. **A1** for correct sketch



Crosses y-axis at k **A1**
Crosses x-axis ($y = 0$)
$k = -x^3$, $\therefore x = -\sqrt[3]{k}$ **A1**

b) i) $y = -x^3 - k$ **A1**
ii) $+ 2k$ **A1**

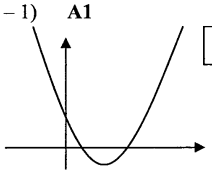
c)



A1 for correct sketch
A1 for correct values **7**

4. a) At $x = b$ **A1**
b) A False **A1** B True **A1** C False **A1** D True **A1** **5**

5. a) $x^2 - 7x + 6 = (x-6)(x-1)$ **A1**

b) 

A1 (U-shaped, cutting x-axis at 1 and 6)

c) $x^2 - 7x + 6 \le 0$

$1 \le x \le 6$

A1 A1

4

6. a) $-x > 3 \quad \therefore x < -3$ **A1**

b) $x^2 \ge 9 \quad \therefore x \ge 3$ or $x \le -3$ **A1 A1**

c) $x^2 + 7x - 7 < 0$

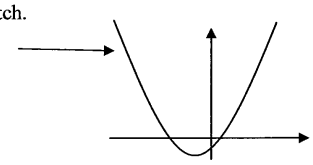
Let $x^2 + 7x - 7 = 0$

$\therefore x = \dfrac{-7 \pm \sqrt{49+28}}{2} = \dfrac{-7 \pm \sqrt{77}}{2}$ $\quad x = \dfrac{-7+\sqrt{77}}{2}$ or $x = \dfrac{-7-\sqrt{77}}{2}$

For less than zero from sketch.

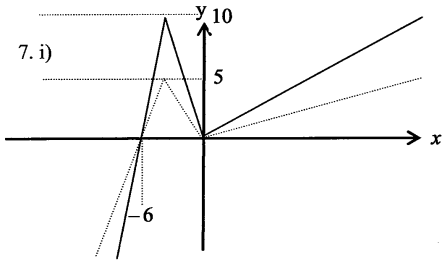
$\dfrac{-7+\sqrt{77}}{2} < x < \dfrac{-7-\sqrt{77}}{2}$

A1 **A1**



5

7. i)

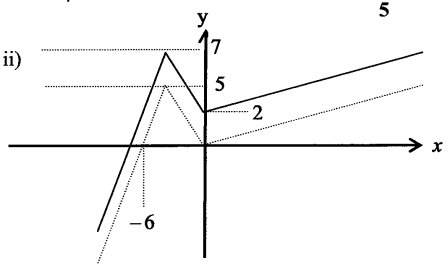


x-axis –6 **A1**

local max 10 **A1**

b) stretch ×2 (or scale factor 2) in vertical direction **A1**

ii)

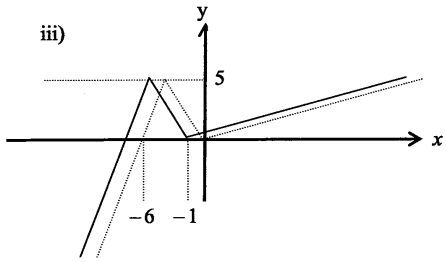


y-axis 2 **A1**

local max 7 **A1**

shift (or translated) **2 up** **A1**

iii)

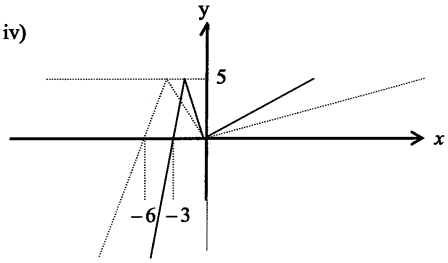


x-axis –1 (and/or –7) **A1**

local max *same height* (5) **A1**

b) shift (or translated) **1 left** **A1**

iv)



x-axis –3 **A1**

local max *same height* (5) **A1**

stretch ×1/2 (or scale factor 1/2) in horizontal dir **A1**

12 {48}

1. a) $AB = \sqrt{(7-3)^2+(11-2)^2} = \sqrt{97}$ **M1 A1**

   b) $CD = \sqrt{(-7--3)^2+(-11-2)^2} = \sqrt{185}$ **M1 A1**

   c) $m = \frac{11-2}{7-3} = \frac{9}{4} \therefore y = \frac{9}{4}x + c$

   Sub in (3, 2) $\Rightarrow c = -\frac{19}{4} \therefore y = \frac{9}{4}x - \frac{19}{4}$ **M1 A1**

   d) $m = \frac{-11-2}{-7+3} = \frac{13}{4} \therefore y = \frac{13}{4}x + c$

   Sub in (–3, 2) $\Rightarrow c = \frac{47}{4} \therefore y = \frac{13}{4}x + \frac{47}{4}$ **M1 A1**

   e) $\frac{13}{4}x + \frac{47}{4} = \frac{9}{4}x - \frac{19}{4} \Rightarrow x = -\frac{33}{2}$ **A1**

   Sub in to either equation $\Rightarrow y = -\frac{335}{8}$ **A1** $\therefore$ intersect at $(-\frac{33}{2}, -\frac{335}{8})$ **10**

2. $y = 7x + c$, sub in (2,3) $\Rightarrow c = -11 \therefore y = 7x$ **A1** -11 **A1** **2**

3. $y = \frac{x}{2} + c$, sub in (2,3) $\Rightarrow c = 2 \therefore y = \frac{x}{2}$ **A1** $+2$ **A1** **2**

4. $m = 3$ **A1** **1**

5. $n = -\frac{1}{3}$ **A1** **1**

6. f(x): $y = mx + a$ **A1**, passes through (b, c) $\Rightarrow c = mb + a \Rightarrow m = \dfrac{c-a}{b}$ **A1**

   $\therefore y = \dfrac{x(c-a)}{b} + a$

   g(x): $y = nx + p$, passes though (d, 0) and (b, c) $\Rightarrow n = \dfrac{c}{(b-d)}$ **A1**

   Sub in (d, 0) or (b, c) $\Rightarrow p = \dfrac{cd}{d-b}$ **A1**

   $\therefore y = \dfrac{cx}{b-d} + \dfrac{cd}{d-b}$ **4**

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Sequences and Series – Test 3 – Answers

1. $x_2 = 4$ and $x_3 = 16$ **A1** for both **1**

2. $x_2 = 11$ **A1** and $x_1 = 5$ **A1** **2**

3. $f(x) = x + 2$ **A1A1** **2**

4. 2, 5, 10. A2 **2**

5. a) $\sum_{r=1}^{10}(2r-1)$ **A1**

   b) $\sum_{r=1}^{6}(5r+2)$ **A1** **2**

6. $3^2 + 4^2 + 5^2 + 6^2 = 86$ **A1** **1**

7. 5n–2
   a) first terms: 3 tenth term: 48 **A1**
   b) 5(n+1)–2 OR 5n+3 **A1**
   c) $(n+1)^{th}$ term – n^{th} term = 5n+3 – (5n–2) = **5**, so **YES** is arithmetic (common difference) **A1** **3**

8. $S_n = \frac{n(1st + last)}{2} = \frac{800(1+1599)}{2} = 640{,}000$ **A1** **1**

9. a) 10 **A1**
   b) a = –39 **A1**
   c) $S_n = 50(-78 + (100 - 1)10) = 45600$ **A1** **3**

10. a) $S_5 = 2.5(2a + 4d) = 502 \Rightarrow 10a + 20d = 1004$
    $S_{10} = 5(2a + 9d) = 1009 \Rightarrow 10a + 45d = 1009 \therefore d = \frac{1}{5}$ **M1 A1**
    b) Sub in to any equation from i): a = 100 **A1** **3**

11. a) $10d = -62 - 50 \therefore d = -11.2$ **A1**
    b) $a = 50 - 9(-11.2) \therefore a = 150.8$ **A1** **2**

12. Standard Proof **3**

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1) a) $y = x^2$ $\frac{dy}{dx} = 2x$ **A1** $\frac{d^2y}{dx^2} = 2$ **A1** (1,1)

b) $y = 3x^2$ $\frac{dy}{dx} = 6x$ **A1** $\frac{d^2y}{dx^2} = 6$ **A1** (1,1)

c) $y = x$ $\frac{dy}{dx} = 1$ **A1** $\frac{d^2y}{dx^2} = 0$ **A1** (1,1)

d) $y = (x+1)^2 = x^2 + 2x + 1$ $\frac{dy}{dx} = 2x + 2$ **A1** $\frac{d^2y}{dx^2} = 2$ **A1** (1,1)

e) $y = (3x+1)^3 = (3x+1)(9x^2 + 6x + 1)$ **M1** $= 27x^3 + 27x^2 + 9x + 1$

$\frac{dy}{dx} = 81x^2 + 54x + 9$ **A1** $\frac{d^2y}{dx^2} = 162x + 54$ **A1** (2,1)

f) $y = (x+3)(x+1) = x^2 + 4x + 3$ **M1**, $\frac{dy}{dx} = 2x + 4$ **A1** $\frac{d^2y}{dx^2} = 2$ **A1** (2,1)

g) $y = \frac{x^5 + 1}{x^2} = x^3 + \frac{1}{x^2}$ **M1** $\frac{dy}{dx} = 3x^2 - \frac{2}{x^3}$ **A1** $\frac{d^2y}{dx^2} = 6x + \frac{6}{x^4}$ **A1** (2,1)

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2) a) $y = \sqrt{x} = x^{\frac{1}{2}}$ **M1** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ **A1** (**or** $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$) (2)

b) $y = \sqrt[3]{x^2} = x^{\frac{2}{3}}$ **M1** $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}$ **A1** (**or** $\frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}}$) (2) **4**

3) a) $y = x^2$, $\frac{dy}{dx} = 2x$ **M1** when $x = 11$, $\frac{dy}{dx} = 2 \times 11 = 22$ **A1** (2)

b) $y = x^2 + \frac{1}{x^2}$ $\frac{dy}{dx} = 2x - \frac{2}{x^3}$ **M1** when $x = 2$, $\frac{dy}{dx} = 3.75$ **A1** (2)

c) $y = (x-2)^2 = x^2 - 4x + 4$ $\frac{dy}{dx} = 2x - 4$ **M1** when $x = 0$, $\frac{dy}{dx} = -4$ **A1** (2) **6**

4) $y = x^3$ $\frac{dy}{dx} = 3x^2$ **M1** $\therefore 3 = 3x^2$ **M1** $\therefore x^2 = 1$ $\therefore x = \pm 1$ $\therefore$ co-ordinates are (1,1) **A1** (–1,–1) **A1**(4)

4

5) a) $\frac{dy}{dx} = 2x - 10$ **B1** when $x = 1$ $\frac{dy}{dx} = -8$ **M1** $\therefore y = -8x + c$ sub in (1,1) $\therefore c = 9$

$\therefore y = -8x + 9$ **A1** (3)

b) $m_1 m_2 = -1$ $\therefore$ grad of normal is $\frac{1}{8}$ **M1** $\therefore y = \frac{x}{8} + c$ sub in (1,1) $\therefore c = \frac{7}{8}$

$\therefore y = \frac{x}{8} + \frac{7}{8}$ **A1** (2) **5**

6) $\frac{dy}{dx} = 9x^2 - 21x + 7$ **M1**

$m_1m_2 = -1 \therefore$ grad of normal is $-\frac{1}{9x^2 - 21x + 7}$ **M1**

$\therefore -\frac{1}{9x^2 - 21x + 7} = -1 \therefore 9x^2 - 21x + 7 = 1 \therefore 9x^2 - 21x + 6 = 0 \therefore (3x - 1)(x - 2) = 0$ **M1**

$x = \frac{1}{3}$ or $x = 2$ **A1**

Substitute into original equation:

$y = 3\left(\frac{1}{3}\right)^3 - \frac{21}{2}\left(\frac{1}{3}\right)^2 + 7\left(\frac{1}{3}\right) - 1 = \frac{5}{18}$ and $y = 3(2^3) - \frac{21(2^2)}{2} + 7(2) - 1 = -5$

Coordinates are $\left(\frac{1}{3}, \frac{5}{18}\right)$ and $(2, -5)$ **A2** **6**

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1) a) $\int x\,dx = \frac{x^2}{2} + C$ **A1** (1)

Deduct 1 mark if constants have not been added.

b) $\int 3x + 5\,dx = \frac{3x^2}{2}$ **A1** $+ 5x$ **A1** $+ C$ (2)

c) $\int \frac{x^3}{x}\,dx = \int x^2\,dx = \frac{x^3}{3}$ **A2** $+ C$ (2)

d) $\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}$ **A2** $+ C$ (2)

e) $\int \sqrt[3]{x} = \int x^{\frac{1}{3}}dx$ **M1** $= \frac{3x^{\frac{4}{3}}}{4} + C$ **A1** (2)

f $\int \sqrt[3]{x}(\sqrt{x} - \frac{1}{\sqrt{x}})dx = \int x^{\frac{1}{3}}(x^{\frac{1}{2}} - x^{-\frac{1}{2}})dx$ **M1** $= \int x^{\frac{5}{6}} - x^{-\frac{1}{6}}dx$ **M1**$= \frac{6}{11}x^{\frac{11}{6}} - \frac{6}{5}x^{\frac{5}{6}} + C$ **A1** (3)

g) $\int (x+1)(x-1)\,dx = \int (x^2 - 1)\,dx$ **M1** $= \frac{x^3}{3} - x + C$ **A1** (2)

h) $\int 3x^{\frac{2}{3}}\,dx = \frac{3}{5} \times 3x^{\frac{5}{3}} = \frac{9x^{\frac{5}{3}}}{5}$ **A2** $+ C$ (2)

i) $\int \frac{x - 2x^3}{x^{\frac{1}{2}}}\,dx = \int x^{\frac{1}{2}} - 2x^{\frac{5}{2}}\,dx$ **M1** $= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{7} \times 2x^{\frac{7}{2}} = \frac{2}{3}x^{\frac{3}{2}}$ **A1** $- \frac{4}{7}x^{\frac{7}{2}}$ **A1** (3) **19**

2) a) $y = \int 10x\,dx = 5x^2 + C$ **M1A1** sub in $\therefore -10 = 5.1^2 + C \therefore C = -15$

$\therefore y = 5x^2 - 15$ **A1** (3)

b) $y = \int x^2 - 2\,dx = \frac{x^3}{3} - 2x + C$ **M1A1** sub in $\therefore 1 = \frac{1}{3} - 2 + C \therefore C = 2\frac{2}{3}$

$\therefore y = \frac{x^3}{3} - 2x + 2\frac{2}{3}$ **A1** (3)

6

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