EXAMINATION PAPER 1	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed *Where appropriate, give your answers to 3 s.f.*	
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Time Allowed:-1 hour 30 minutes	

1. Express the following expression as a single fraction in its simplest form:

$$\frac{x+1}{(x-1)(x+2)}-\frac{6}{(x-1)(x+3)}$$ **[4]**

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2. $f(x) = x^4 - x - 1$
$f(x) = 0$ has a solution such that $n < x < n + 1$ where n is a positive integer.
a) i) Find a positive value of n such that the inequality is true. **[3]**
ii) Construct a simple logical argument to *prove* that such a solution exists. **[3]**
b) Using an iteration based on the equation $x = \sqrt[4]{1+x}$, find a solution to $f(x) = 0$ to 3 decimal places. **[4]**

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3. $f(x) = (x - 3)^2 + 4$
a) Calculate the equation of the function $g(x)$ where $g(x) = 1 + f(x + 1)$ **[2]**
There is a relationship between the graphs of $y = f(x)$ and $y = g(x)$.
b) i) Clearly define the transformation that takes the graph of $f(x)$ to $g(x)$. **[3]**
ii) Clearly define the transformation that takes the graph of $g(x)$ to $f(x)$. **[1]**
$h(x) = |x+2|-3$
c) Solve the equation $h(x) = 1$ **[3]**
d) Find $fh(-3)$ **[3]**

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4. Given that $2\cos 3x \cos x = \cos 2C + \cos C$
a) Find C in terms of x. **[2]**
b) Let x be 15° and hence, or otherwise find an *exact value* for cos 15°. Leave your answer in *surd form* and *rationalise the denominator* if necessary. **[4]**
c) Hence or otherwise solve the equation $2\cos 3x \cos x = 1$ for $0 < x \leq 180°$.
Give your answers to 1 decimal place. **[6]**

---

5. $f(x) = x^3$, $g(x) = 4x - 2$
a) Find $fg(x)$, $gf(x)$ **[2]**
b) Sketch the graph of $y = g(\sin x)$ and state the coordinates of the minimum point of the graph within the range $0 < x \leq 2\pi$ radians. **[4]**

$h(x) = \dfrac{x+1}{x-1}$ where x is real and $x \neq 1$

c) Find $h^{-1}(x)$ and state its domain and range. **[5]**

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6. $f(x) = \cos x + 2\sin x$
a) Express $f(x)$ in the form $R\cos(x° - \alpha°)$ where $0 \leq \alpha < 90°$ **[4]**
b) Solve the equation $\cos x + 2\sin x = 1$ where $0 \leq x < 360°$ **[4]**
c) For what values of x is $\dfrac{6}{6+\cos x+2\sin x}$ a maximum, where $0 < x < 360°$? **[3]**
d) What is the value of this maximum? **[1]**

---

7. a) Find $\dfrac{dy}{dx}$ when $x = 6$ and $y > 0$ and $x = y^2 - y$. **[5]**
b) i) Find the equation of the tangent to the curve $y = \sin 3x \cos 6x$ when $x = \dfrac{\pi}{3}$ radians. **[5]**
ii) Find the equation of the tangent to the curve $y = \sin 3x \cos 6x$ when $x = \dfrac{\pi}{6}$ radians. **[3]**
iii) Find the equation of the normal to the curve $y = \sin 3x \cos 6x$ when $x = \dfrac{\pi}{6}$ radians. **[1]**

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[75]

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Time Allowed:-1 hour 30 minutes	

1. Solve the simultaneous equations, $e^{3x} = ey$ and $\ln y = 6x - 2$ where e is the exponential constant. **[6]**

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2. a) Simplify the expression: $\dfrac{\tan\phi}{\tan\phi + \cot\phi}$ **[4]**

   b) Hence or otherwise simplify the expression: $\dfrac{\tan^2\phi}{2 + \tan^2\phi + \cot^2\phi}$ **[2]**

---

3. $y = 3e^x$

   a) Sketch this curve, stating where the curve crosses the y-axis. **[2]**

   b) Find the equation of the normal to the curve at the point $(\ln 3, 9)$ **[5]**

---

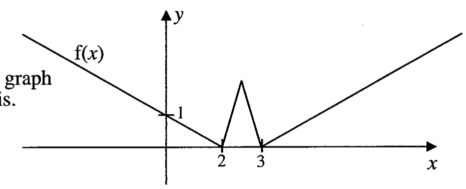
4. Sketch separately the graphs of–

   a) $f(|x|)$ **[2]**

   b) $2f(x + 1)$ **[3]**

   In each sketch clearly show where the graph crosses or touches the x-axis and y-axis.

   c) State the relationship between $f(x)$ and $|f(x)|$. **[1]**



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5. Differentiate the following expressions with respect to x:

   a) $2x^4 \cos^4 x$ **[4]**

   b) $\dfrac{1 + x^3}{e^{3x}}$ **[4]**

   c) $\ln(x^x)$ **[4]**

---

6. $f(x) = 2 + \ln x$ for $x > 0$ with $x \varepsilon \mathbb{R}$ and $g(x) = 2 + e^{2x}$ with $x \varepsilon \mathbb{R}$.

   a) Find $fg(x)$ and $gf(x)$ simplifying your answers where possible. **[5]**

   b) Find $f^{-1}(x)$ and state its range. **[4]**

   c) Find $g^{-1}(x)$ and state its domain. **[4]**

---

7. $f(x) = \sin 3x$ for $x \varepsilon \mathbb{R}$ and $g(x) = \sin x \cos x$ $0 \le x \le \pi/2$ for $x \varepsilon \mathbb{R}$

   a) Show using trigonometric identities that $f(x + \pi/6) = -f(x - \pi/6)$ **[7]**

   b) Show that $g(x)$ is an increasing function for $0 < x < \pi/4$ **[4]**

---

8. a) Show that $10x^3 = \dfrac{1}{1-x}$ has 2 solutions between 0 and 0.9.
   State the range that each solution must lie in. **[5]**

   b) Use the iteration $x_{n+1} = \sqrt[3]{\dfrac{1}{10 - 10x_n}}$ and $x_0 = 0.7$ to find x_1, x_2, x_3, and x_4.
   Give your answers to four decimal places where appropriate. **[4]**

   c) Find $f(0.675)$ where $f(x) = 10x^3 - \dfrac{1}{1-x}$. Give your answer to 3 significant figures **[2]**

   d) Hence using your results from b) and c) find a solution to the equation in a) to 2 decimal places and justify your answer. **[3]**

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[75]

EXAMINATION PAPER 3	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, give your answers to 3 s.f.***	
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Time Allowed:-1 hour 30 minutes	

1. Solve the following equation, leaving your answer exactly:
$e^{10x} - 2e^{5x} - 3 = 0$ **[5]**

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2. a) Finding A and B; write $2\sin 6x \cos 5x$ in the form $\sin Ax + \sin Bx$ **[3]**

b) Show that: $\dfrac{\cot 2\phi \operatorname{cosec} 2\phi}{\tan^2\phi \sec 2\phi + \sec 2\phi} \equiv (\cos\phi \cot 2\phi)^n$ and find n. **[5]**

---

3. $y = 3 - 2e^x$

a) Sketch this curve, stating where the curve crosses the x-axis and y -axis **[4]**

b) Find the equation of the normal to the curve at the point $(1, 3 - 2e)$ **[4]**

---

4. $f(x) = x^6 - x^2 - 1$

$f(x) = 0$ has a solution such that $n < x < n + 1$ where n is a positive integer.

a) Find a positive value of n such that the inequality is true. **[3]**

b) Using an iteration based on the equation $x = \sqrt[6]{1 + x^2}$, find a solution to $f(x) = 0$ to 3 decimal places. **[3]**

c) Calculate $f(-x)$ and hence find a second estimated solution of $f(x) = 0$ **[2]**

---

5. $f(x) = \dfrac{x+16}{x-16}$ where x is real and $x \neq 16$ and $g(x) = x^4$

a) Find $fg(x)$ and $gf(x)$ and state their domains. **[6]**

b) Find $f^{-1}(x)$ and state its domain. **[4]**

---

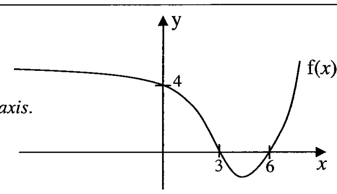
6. Sketch separately the following graphs:

a) $y = |f(x)|$ **[2]**

b) $y = f(|x|)$ **[2]**

c) $y = 2f(3x)$ **[4]**

Write down where each graph crosses the *x and y-axis.*



d) State the relationship between the graphs $y = 2f(3x)$ and $y = -2f(3x)$. **[1]**

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7. Differentiate the following expressions with respect to x:

a) $\sin^3 2x \cos^4 3x$ **[4]**

b) $\dfrac{e^{3x}}{x^5}$ **[4]**

c) Given that $x = \sin 5y$, prove that $\dfrac{dy}{dx} = \dfrac{1}{5\sqrt{1-x^2}}$ **[5]**

---

8. a) Express $6\cos x + 8\sin x$ in the form $R\cos(x^\circ - \alpha^\circ)$ where $0 < \alpha < 90^\circ$.
Give α to two decimal places. **[3]**

b) Solve to 2 decimal places the equation $6\cos 2y + 8\sin 2y = 1$ where $0 < y < 360^\circ$. **[6]**

c) For what values of x is $\dfrac{10}{10 + 6\cos x + 8\sin x}$ a minimum, where $0 < x < 360^\circ$?
Give your answer to two decimal places. **[3]**

d) What is the value of this minimum? **[2]**

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[75]

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Calculators Allowed ***Where appropriate, give your answers to 3 s.f.***	
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Time Allowed:-1 hour 30 minutes	

1. a) Simplify the expression: $1+\dfrac{3x+2}{3x^2-x-2}$ **[4]**

   b) $f(x) = x^3+\dfrac{23}{2}x^2+26x-16$

   Show that $f(x) = 0$ has a solution between 0 and 1. **[3]**

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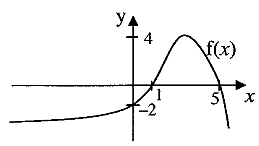
2. $f(x)$ shown, has a maximum value of 4.
   The graph cuts the x-axis at 1 and 5 and cuts the y-axis at – 2.
   Sketch separately the following graphs:



   a) $|\,f(x)\,|$ **[2]**
   b) $f(|\,x\,|)$ **[2]**
   c) $2f(x+1)$ **[3]**

---

3. a) Sketch the curve $y = 3 + 2\ln x$ and state where the curve crosses the x-axis. **[3]**
   b) Find the equation of the tangent to the curve at the point (1, 3) **[4]**

---

4. The temperature of an iron ball is cooled by a 1 second blast of chilled nitrogen. The temperature of the iron ball, T°C, is given by the equation $T = 5(20-e^t),\ for\ 0 < t \le 1$ where t is time in seconds.

   a) Find the value of T at the beginning and end of the air blast giving your answers exactly and if necessary in terms of e, the exponential constant. **[3]**

   b) i) Find $\dfrac{dT}{dt}$ **[1]**

   ii) Hence find when the iron ball is cooling at a rate of 6°C/s giving your answer exactly. **[3]**

   c) i) State the maximum rate of cooling and at what time this occurs. **[2]**

   ii) State the minimum rate of cooling and at what time this occurs. **[2]**

---

5. $f(x) = \dfrac{x^2-49}{x+7}$ where x is real and $x \neq -7$ and $g(x) = x^2 - 2$ where x is real.

   a) Show that $fg(x)$ can be written in the form $(x + A)(x - A)$ and find A. **[4]**

   b) Show that $gf(x)$ can be written in the form $\dfrac{h(x)}{(x+7)^2}$ and find $h(x)$. **[4]**

   The domain of $g(x)$ is now restricted such that $x > 5$.

   c) State the range of $g(x)$. **[1]**

   d) Find $g^{-1}(x)$ and state its domain and range. **[4]**

---

6. a) Expand and simplify the expression $\left(\sqrt{11}+\sqrt{10}\right)\left(\sqrt{11}-\sqrt{10}\right)$ **[1]**

   b) Express $\cos x + 3\sin x$ in the form $R\cos(x^\circ - \alpha^\circ)$ where $0 < \alpha \le 90^\circ$ **[4]**

   c) Solve the equation $\cos x + 3\sin x = 1$ where $0 < x \le 360^\circ$ **[4]**

   d) For what values of x is $\dfrac{1}{\cos x+3\sin x+\sqrt{11}}$ a minimum, where $0 < x \le 360^\circ$? **[2]**

   e) Leaving your answer exactly, calculate this minimum value. **[3]**

---

7. a) Using the identity for sin (A + B), prove the identity $\sin 3x \equiv 3\sin x - 4\sin^3 x$ **[5]**

   b) Using the fact that $\dfrac{d}{dx}(\sin x) = \cos x$, prove that $\dfrac{d}{dx}(\sin ax) = a\cos ax$ **[5]**

   c) By differentiating both sides of the identity in a) find an expression equivalent to $\cos(3x)$ in terms of $\sin x$ and $\cos x$. **[3]**

   d) Without a calculator (or tables) evaluate sin 75° giving your answer exactly. **[3]**

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[75]

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Calculators Allowed *Where appropriate, give your answers to 3 s.f.*	
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Time Allowed:-1 hour 30 minutes	

1. a) Simplify the expression: $1-\dfrac{1}{1+\cot^2\phi}$ **[3]**

   b) Show that : $\cos\phi+\sin\phi\tan 2\phi=\dfrac{\cos\phi}{\cos 2\phi}$ **[4]**

---

2. $f(x)=x^3-2x-3$

   The root α to the equation f(x) = 0 can be estimated using the iterative formula $x_{n+1}=\sqrt{\dfrac{3}{x_n}+2}$ with $x_0=2$.

   a) Calculate x_1, x_2, x_3 *and* x_4 giving your answers to 4 significant figures. **[3]**

   b) Prove that, to 4 significant figures, α is 1.893. **[3]**

   John found this iterative formula. He found it by first writing x^3-2x-3 in the form $x(x^2-2)-3$.

   c) Continue the likely algebraic steps that John may have taken to come across this iterative formula. **[3]**

---

3. a) Solve the inequality $|2x+3|>4$ **[3]**

   b) i) Sketch a graph of $y=|(x-1)(x-3)|$ **[2]**

   The coordinates on the graph where the gradient is 1 is (a, b) where $1<a<3$.

   ii) Find the value of a. **[4]**

---

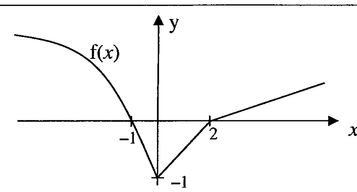
4. Sketch separately the following graphs:

   a) f(| x |) **[2]**

   b) | f(x) | **[2]**

   c) 3f(2x) **[3]**

   In each case write on where each graph crosses or touches the *x and y-axis*.



   d) Given that the curved part of the graph y = f(x) is given by f(x) = k – 3e^{x+2}, $x \le -1$, find the value of k exactly. **[2]**

   e) Find the gradient of the steepest part of the curved part of the graph. **[3]**

---

5. f(x) = x^2 – 1 with $x \varepsilon \mathbb{R}$ and g(x) = 1 – x^2 with $x \varepsilon \mathbb{R}$

   a) Find fg(x) and gf(x) and solve the equation fg(x) = gf(x) **[8]**

   For the inverse of f(x) to exist, it is necessary for the domain of f(x) to be restricted. The domain of the f(x) is now restricted such that $x \ge$ r.

   b) State the largest possible domain of f(x) such that the inverse of f(x) exists. **[2]**

   c) Assuming the domain of f(x) is appropriately restricted, then find the inverse of f(x). **[4]**

---

6. f(x) = ln x and g(x) = ln 2x

   a) Find f '(x) and g '(x) **[2]**

   b) Hence find the tangent to the curve y = f(x) when x = 3. **[3]**

   c) Find the normal to the curve y = g(x) when x = 3. **[4]**

---

7. a) Using a trigonometric identity, simplify the expression: sin 2x cos 4x + cos 2x sin 4x **[2]**

   b) Using your answer to part a) and the identity sin 2x cos 4x ≡ $^1/_2$[sin 6x – sin 2x] prove that 2sin 2x cos 4x + cos 2x sin 4x ≡ $^1/_2$[3sin 6x – sin 2x] **[2]**

   c) Show that the curve $y=e^{-x}\cos x$ has 2 stationary points between 0 < x < 2π and with clear working distinguish if these points are maximum or minimum points. **[11]**

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[75]

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1. It is not necessary to multiply out the denominator.

Obtain common factors in both denominators $\dfrac{\dots}{(x-1)(x+2)(x+3)}$	M1	
Combine to single denominator	M1	
Multiply out numerator to $(x^2+4x+3)-(6x+12)$	M1	
Simplify to $\dfrac{x^2-2x-9}{(x-1)(x+2)(x+3)}$	A1	(4)

2.

a)	i)	If f(any positive integer) attempted	M1	
		Show that f(1) = –1, f(2) = 13	M1	
		Obtain answer $1 < x < 2$, or n = 1	A1	(3)
	ii)	f(x) is continuous	M1	
		If f(1) < 0 and f(2) > 0	M1 for both	
		Then there exists x in the interval $1 < x < 2$ such that f(x) = 0	M1	(3)
		Accept also a generalized solution with n and (n+1) or a good sketch with clear argument!		
b)		Show formula $x_{n+1}=\sqrt[4]{(1+x_n)}$	M1	
		Construct a table showing x_n and x_{n+1}	M1	
		Iterate formula and show values in table	M1	
		Obtain answer x = 1.2207… = 1.221 (3 d.p.)	A1	(4)

3.

a)		g(x) = 1 + f(x + 1)		
		= 1 + $(x + 1 - 3)^2 + 4$	M1	
		= $(x - 2)^2 + 5$ or equivalent	A1	(2)
b)	i)	f(x) to g(x) is a **translation** 1 up and *1 left.*	**A1**A1*A1*	(3)
	ii)	g(x) to f(x) is a translation 1 down and 1 right.	A1	(1)
c)		h(x) = 1 = |x + 2| – 3		
		4 = |x + 2|	M1	
		Therefore x = 2 or –6	A1A1	(3)
d)		h(–3) = |–3 + 2| – 3		
		= |–1| – 3		
		= 1 – 3 = –2	A1	
		f(h(–3)) = f(–2)	M1	
		= $(-2-3)^2 + 4$		
		= 29	A1	(3)

4.

a)	Use formula $\cos A+\cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$	M1	
	Show C = 2x	A1	(2)
b)	Show that 2 cos 45° cos 15° = cos 60° + cos 30°	M1	
	Write down results; cos 45° = $\dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$, cos 60° = $\dfrac{1}{2}$, cos 30° = $\dfrac{\sqrt{3}}{2}$	M1	
	Substitute into equation $\dfrac{2}{\sqrt{2}}\cos 15° = \dfrac{1}{2}+\dfrac{\sqrt{3}}{2}$	M1	

		Simplify to $\cos 15° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$	A1	(4)
	c)	From a) $2\cos 3x\cos x = \cos 4x + \cos 2x$		
		Solve $\cos 4x + \cos 2x = 1$	M1	
		Let $X = 2x$		
		$\cos 2X + \cos X = 1$		
		Using $\cos^2 X + \sin^2 X = 1$ and $\cos^2 X - \sin^2 X = \cos 2X$ to give $\cos 2X = 2\cos^2 X - 1$		
		So $2\cos^2 X + \cos X - 2 = 0$	M1	
		Let $Y = \cos X$		
		Therefore $Y^2 + \dfrac{Y}{2} - 1 = 0$	M1	
		Solve to find $Y = -\dfrac{1}{4} \pm \dfrac{\sqrt{17}}{4}$	A1	
		There $\cos X = -\dfrac{1}{4} \pm \dfrac{\sqrt{17}}{4}$		
		$X = \cos^{-1}\left(-\dfrac{1}{4} \pm \dfrac{\sqrt{17}}{4}\right)$	A1	
		$-1 \le \cos X \le 1$, so we ignore negative root since its value is -1.28		
		$X = 38.668\ldots°$ or $321.331\ldots°$		
		Therefore $2x = 38.668\ldots°,\ 321.33\ldots°,\ 398.66\ldots°,\ 681.33\ldots°$		
		Therefore $x = 19.334\ldots°,\ 160.66\ldots°,\ 199.33\ldots°,\ 340.66\ldots°$		
		$= 19.3°, 160.7°, 199.3°, 340.7°$ (1 d.p.)	A1	(6)
5.	a)	Substitute $g(x)$ into $f(x)$ to obtain $fg(x) = (4x-2)^3$ or $= [8(8x^3 - 8x^2 + 4x - 1)]$	A1	
		Substitute $f(x)$ into $g(x)$ to obtain $gf(x) = 4x^3 - 2$	A1	(2)
	b)	$y = 4\sin x - 2$	A1	
		Max at $y = 2$, min at $y = -6$	A1	
		Single sine shape	A1	
		Minimum point occurs when $x = \dfrac{3\pi}{2}$ and $y = -6$		
		So coordinates of min are $\left(\dfrac{3\pi}{2}, -6\right)$	A1	(4)
	c)	Using equation $y = \dfrac{x+1}{x-1}$		
		Swap variables x and y	M1	
		Rearrange the equation to show $x = \dfrac{y+1}{y-1}$ and state that $h^{-1}(x) = \dfrac{x+1}{x-1}$ i.e. self-inverse	A1A1	
		State the domain; y: $\in \mathbb{R}$, $y \ne 1$	A1	
		State range; $h^{-1}(x)$: $-\infty < h^{-1}(x) < 1$, $1 < h^{-1}(x) < +\infty$	A1	(5)
6.	a)	Use the formula $R = \sqrt{a^2 + b^2}$	M1	
		Obtain the result $R = \sqrt{5}$	A1	
		Use the formula $\alpha = \tan^{-1}\left(\dfrac{b}{a}\right)$	M1	
		Obtain the result $\alpha = \tan^{-1}(2) = 63.434\ldots° = 63.4°$ (3sf)	A1	(4)
	b)	Substitute $R\cos(x - \alpha)$ for $\cos x + 2\sin x$ and equate to 1		
		$\therefore \cos(x - \alpha) = \dfrac{1}{\sqrt{5}}$	M1	
		take $\cos^{-1}$ and add α to obtain the results	M1	

$x - 63.435... = 63.435...°$ or $-63.435...°$
$x = 0°$, $126.86...° = 127°$ (3 s.f). A1A1 (4)

c) Substitute R cos $(x - \alpha)$ for $\cos x + 2\sin x$ into bottom of equation M1
State that the equation is a maximum when $\cos(x - \alpha) = -1$ M1
Obtain the result $x = 243.43...° = 243°$ (3 s.f.) A1 (3)

d) Solve to the result, max $= 6 \div (6 - \sqrt{5}) = 1.5941... = 1.59$ (3 s.f). A1 (1)

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7. a) $\frac{dx}{dy} = 2y - 1$ M1A1

Therefore $\frac{dy}{dx} = \frac{1}{2y-1}$ A1 ft

When $x = 6$, $6 = y^2 - y$, $y > 0$, so $y = 3$ by inspection or other method M1

Therefore $\frac{dy}{dx} = \frac{1}{2\times3-1} = \frac{1}{5}$ A1 (5)

b) i) $\frac{dy}{dx} \sin 3x \frac{d}{dx}(\cos 6x) + \cos 6x \frac{d}{dx}(\sin 3x)$ M1

$= -6\sin3x\sin6x + 3\cos3x\cos6x$ A1

When $x = \frac{\pi}{3}$, $\frac{dy}{dx} = 0 + 3\times -1\times 1 = -3$ A1

Therefore $y = -3x + c$ M1

When $x = \frac{\pi}{3}$, $y = 0$, so $c = \pi$

Therefore $y = -3x + \pi$ A1 (5)

ii) When $x = \frac{\pi}{6}$, $\frac{dy}{dx} = 0$ A1

When $x = \frac{\pi}{6}$, $y = -1$ A1

Therefore tangent is $y = -1$ A1 (3)

iii) The equation of the normal is $x = \frac{\pi}{6}$ A1 (1)

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(75)

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1. $\ln y = 6x - 2$, $e^{3x} = ey \Rightarrow y = \dfrac{e^{3x}}{e}$ M1

Substitute in: $\ln\left(\dfrac{e^{3x}}{e}\right) = 6x - 2$ M1

$3x - 1 = 6x - 2$ (simplify LHS) M1

Obtain result $x = 1/3$ A1

Back substitution; $y = \dfrac{e^{3\times\frac{1}{3}}}{e} = 1$ M1A1(6)

---

2. a) $\dfrac{\frac{\tan\phi}{1}}{\frac{\tan\phi}{1} + \frac{1}{\tan\phi}} = \dfrac{\tan^2\phi}{\tan^2\phi + 1} = \dfrac{\tan^2\phi}{\sec^2\phi} = \tan^2\phi\cos^2\phi = \dfrac{\sin^2\phi}{\cos^2\phi}\cos^2\phi = \sin^2\phi$ M1M1M1A1

(4)

b) Since this equation is the square of the one in part a), the answer is also the square of the answer in part a); $\mathbf{\sin^4\phi}$ **A1*A1*(2)**

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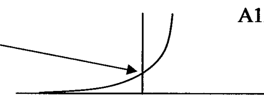
3. a) **Curve sketch which cuts the y-axis at y = 3** **A1A1(2)**



b) $y = 3e^x$, $\dfrac{dy}{dx} = 3e^x$ M1

$3e^{\ln 3} = 9$ = tangent gradient M1

normal gradient = –1/9 M1

$y = mx + c$, $y - 9 = -1/9(x - \ln 3)$, $9y - 81 = \ln 3 - x$, $9y = \ln 3 + 81 - x$ M1

$y = -\dfrac{1}{9}x + \dfrac{\ln 3 + 81}{9}$, c = 9.1220… = 9.12 (3 s.f.) A1 (5)

---

4. a) Crosses y-axis at y = 1 and touches x-axis at $x = -3$, $x = -2$, $x = 2$ and $x = 3$ A1A1(2)

b) Sketch $2f(x + 1)$ A1

Graph is stretched by 2 in the y-direction and translated 1 to left.

Graph touches x-axis at $x = 1$ and 2. A1

Graph cuts y-axis at y =1 A1 (3)

c) The functions are the same A1 (1)

---

5. a) Using product rule where $u = 2x^4$ $v = \cos^4 x$ M1

$u` = 8x^3$ $v` = -4\cos^3 x\sin x$ A1

$\dfrac{d}{dx}(f(x)) = 8x^3\cos^4 x + -4\cos^3 x\sin x \times 2x^4$ A1

$= 8x^3\cos^4 x - 8x^4\cos^3 x\sin x$ A1 (4)

b) Rearrange to obtain $e^{-3x} + x^3e^{-3x}$ M1

$\dfrac{d}{dx}(f(x)) = -3e^{-3x} + \dfrac{d}{dx}(x^3e^{-3x})$ A1

		Using the product rule:		
		$\frac{d}{dx}(x^3e^{-3x}) = 3x^2e^{-3x} - 3x^3e^{-3x}$	A1	
		$\therefore \frac{d}{dx}(f(x)) = 3e^{-3x}(x^2 - x^3 - 1)$	A1	(4)
	c)	$\ln(x^x) = x\ln x$	M1	
		$\frac{d}{dx}(\ln(x^x)) = \ln x + x\frac{d}{dx}(\ln x)$	A1	
		$= \ln x + \frac{x}{x}$	A1	
		$= 1 + \ln x$	A1	(4)
6.	a)	$fg(x) = 2 + \ln(2 + e^{2x})$	A2	
		$gf(x) = 2 + e^{2(2 + \ln x)}$	M1	
		$= 2 + e^{(4 + 2\ln x)}$	M1	
		$= 2 + e^4x^2$	A1	(5)
	b)	$f(x) = 2 + \ln x \Rightarrow y = 2 + \ln x$		
		$y - 2 = \ln x$	M1	
		$x = e^{(y-2)}$	M1	
		$f^{-1}(x) = e^{(x-2)}$	A1	
		Range: $f^{-1}(x) > 0$	A1	(4)
	c)	$g(x) = 2 + e^{2x} \Rightarrow y = 2 + e^{2x}$		
		$y - 2 = e^{2x}$		
		$2x = \ln(y - 2)$	M1	
		$x = \ln(y - 2)/2$	M1	
		$g^{-1}(x) = \frac{\ln(x-2)}{2}$	A1	
		Domain: $x > 2, x \in \mathbb{R}$	A1	(4)
7.	a)	$f(x) = \sin 3x$	M1	
		$\therefore f(x + \pi/6) = \sin[3x + \pi/2]$	M1	
		$= [\sin 3x\cos(\pi/2) + \cos 3x\sin(\pi/2)]$	M1	
		$= \cos 3x$	A1	
		$f(x - \pi/6) = \sin(3x - \pi/2)$		
		$= (\sin 3x\cos(\pi/2) - \cos 3x\sin(\pi/2))$	M1	
		$= -\cos 3x$	A1	
		$\therefore f(x + \pi/6) = -f(x - \pi/6)$	A1	(7)
	b)	By differentiation or considering $\frac{dy}{dx}$	M1	
		$= \cos^2 x - \sin^2 x$	A1	
		$\cos x > \sin x$ for $0 < x < \frac{\pi}{4}$	M1	
		$\therefore \cos^2 x - \sin^2 x \geq 0$ for $0 < x < \frac{\pi}{4}$	A1	(4)
		OR $g(x) = \cos 2x$ and $\cos 2x \geq 0$ for $0 < x < \frac{\pi}{4}$ OR clear sketch of $g'(x)$ in required region.		
8.	a)	Let $f(x) = 10x^3 - \left(\frac{1}{1-x}\right)$	M1	
		$f(0) = -1$ so $f(0) < 0$: $f(0.9) = -2.71$ so $f(0.9) < 0$	M1	

Look at other x values between extremes i.e. Attempt to find x s.t. $f(x) > 0$, M1
for example; $f(0.7) = 0.096666\ldots = 0.0967$ (3 s.f.), $f(0.8) = 0.12$
[Note: $f(0.6) = -0.34$]
so; $0.6 < x_1 < 0.7$, $0.8 < x_2 < 0.9$ are 2 suitable intervals
- other answers possible A1A1(5)

b) $x_0 = 0.7$, $x_1 = 0.6934$ (0.69336…), $x_2 = 0.6883$ (0.68832…), $x_3 = 0.6846$ (0.68459…), $x_4 = 0.6819$ (0.68188…) (4dp) A1A1A1A1 (4)

c) $f(0.675) = -1.4543\ldots \times 10^{-3} = -1.45 \times 10^{-3}$ (3 s.f.) A2 (2)

d) [note a) equation is linked to the iteration, ie same equation rearranged]
part b) specifies an answer below $0.68188\ldots = 0.6819$ (4 dp) M1
part c) specifies an answer above 0.675, this means that the answer is 0.68
(2 dp as required) M1A1(3)

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(75)

Mark Scheme 3	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, leave your answers to 3 s.f.***	
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1. $e^{10x} - 2e^{5x} - 3 = 0$;
Let $y = e^{5x}$ $\therefore$ $y^2 - 2y - 3 = 0$ or $(e^{5x})^2 - 2e^{5x} - 3 = 0$ — M1
$(y + 1)(y - 3) = 0$ — M1
$y = -1$ is impossible as you cannot have a log of a negative number so $y = 3$ — B1
$y = e^{5x} = 3$; $5x = \ln 3$; $x = \mathbf{(\ln 3)/5}$ $(= 0.2197\ldots)$ — M1**A1**(5)

---

2. a) $2\sin A\cos B = \sin(A + B) + \sin(A - B)$ — M1
$2\sin 6x\cos 5x = \sin(11x) + \sin(x)$ — M1
$A = 11, B = 1$ — A1 (3)

b) $$\frac{\cot 2\phi \operatorname{cosec} 2\phi}{\tan^2\phi \sec 2\phi + \sec 2\phi} = \frac{\cot 2\phi \operatorname{cosec} 2\phi}{\sec 2\phi\left(\tan^2\phi + 1\right)}$$ — M1

Use identity: $\tan^2 A + 1 \equiv \sec^2 A$

$$= \frac{\cot 2\phi \operatorname{cosec} 2\phi}{\sec 2\phi \sec^2\phi}$$

Use identity: $\cot 2A \equiv \frac{\cos 2A}{\sin 2A}$

$$= \frac{\dfrac{\cos 2\phi}{\sin 2\phi} \times \dfrac{1}{\sin 2\phi}}{\dfrac{1}{\cos 2\phi} \times \dfrac{1}{\cos^2\phi}}$$ — M1

Correct manipulation of fractions; — M1

$$= \frac{\cos 2\phi}{\sin^2 2\phi} \div \frac{1}{\cos 2\phi \cos^2\phi}$$

$$= \frac{\cos 2\phi}{\sin^2 2\phi} \times \cos 2\phi \cos^2\phi$$

$$= \frac{\cos^2 2\phi}{\sin^2 2\phi}\cos^2\phi$$

$= \cot^2 2\phi \cos^2\phi = (\cot 2\phi \cos\phi)^2$ $\therefore$ n = 2 or implied — A2 (5)

---

3. a) Sketch of curve — M1
The curve is an *inverted exponential* which crosses the y-axis at **y = 1** — M1**A1**
and the x-axis at $x = \ln(3/2) \approx 0.40546\ldots \approx 0.405$ (3 s.f.) — A1 (4)

b) $\dfrac{dy}{dx} = -2e^x$, — A1

when x = 1, $\dfrac{dy}{dx} = -2e$, $\therefore$ gradient of normal = $\dfrac{1}{2e}$ — A1 ft

Substitute in values; $y = \dfrac{1}{2e}x + c$; $3 - 2e = \dfrac{1}{2e} + c$; $c = 3 - 2e - \dfrac{1}{2e}$ — M1

$\therefore y = \dfrac{1}{2e}x + 3 - 2e - \dfrac{1}{2e}$ [$\approx 0.18393\ldots x - 2.6205\ldots \approx 0.184x - 2.62$ (3 s.f.)] — A1 (4)

---

4. a) $f(1) = -1$ — M1
$f(2) = 59$ — M1
n = 1 or implied — A1 (3)

b) $x_0 = 1$, $x_1 = 1.1225\ldots$, $x_2 = 1.1456\ldots$, $x_3 = 1.1499\ldots$, $x_4 = 1.1508\ldots$, — M2
$x_5 = 1.1509\ldots$, $x_6 = 1.1510\ldots$, $x_7 = 1.1510\ldots$; so $x = 1.151$ (3 d.p.) — A1 (3)

c) even function or $f(-x) = (-x)^6 - (-x)^2 - 1 = x^6 - x^2 - 1 = f(x)$ — M1

$\therefore x = -1.151$ (3 d.p.) is also a solution — A1 (2)

---

5. a) $fg(x) = \dfrac{x^4+16}{x^4-16}$; domain: $x \in \mathbb{R}, \boldsymbol{x \neq \pm 2}$ — M1A1**A1**

$gf(x) = \left(\dfrac{x+16}{x-16}\right)^4$; domain: $x \in \mathbb{R}, x \neq 16$ — A2**A1**(6)

b) $f(x) = \dfrac{x+16}{x-16} = y$;

Swap variables; $x = \dfrac{y+16}{y-16}$; Attempt to rearrange; — M1

$yx - 16x = y + 16$; $yx - y = 16x + 16$; $y(x-1) = 16(x+1)$; — M1

$y = \dfrac{16(x+1)}{(x-1)} \Rightarrow f^{-1}(x) = \dfrac{16(x+1)}{(x-1)}$ *domain:* $x \in R, x \neq 1$ — *A1*A1(4)

---

6. a) As $f(x)$ except for $3 < x < 6$ which is reflected about the x-axis, — M1
crosses axis at $y = 4$, and touches at $x = 3$ and $x = 6$ — A1 (2)

b) Quadrants 1&4 stay same, quadrants 2&3 reflection of quadrants 1&4 in y-axis, — M1
crosses axis at $y = 4$, $x = 3$, $x = 6$, $x = -3$, $x = -6$ — A1 (2)

c) Stretch $\times 2$ in the y-direction and $\times \frac{1}{3}$ in the x-direction — A1

Crosses axis at $y = 8$, $\boldsymbol{x = 1}$, $x = 2$ — A1**A1**A1 (4)

d) reflected in x-axis — A1 (1)

---

7. a) Using Product rule where $u = \sin^3 2x$ $\quad v = \cos^4 3x$
$u` = 6\sin^2 2x\cos 2x$ $\quad$ $v` = \boldsymbol{-12\cos^3 3x\sin 3x}$ — A1**A1**

$\dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$ — M1

$= 6\sin^2 2x\cos 2x\cos^4 3x - 12\sin^3 2x\cos^3 3x\sin 3x$ — A1 (4)

b) Using Quotient rule where $u = e^{3x}$ $\quad v = x^5$
$u` = 3e^{3x}$ $\quad v` = 5x^4$ — A1*A1*

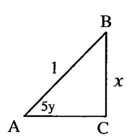
$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ — M1

$\therefore \dfrac{dy}{dx} = \dfrac{3x^5e^{3x} - 5x^4e^{3x}}{x^{10}}$

$= \dfrac{e^{3x}(3x-5)}{x^6}$ — A1 (4)

c) $x = \sin 5y$

$\dfrac{dx}{dy} = 5\cos 5y$ — M1



$\dfrac{dy}{dx} = \dfrac{1}{5\cos 5y}$ — M1

By Pythagoras AC = $\sqrt{1-x^2}$ — M1

$\cos 5y = \dfrac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$ — M1

Therefore $\dfrac{dy}{dx} = \dfrac{1}{5\sqrt{1-x^2}}$ — M1 (5)

8.	a)	$R = \sqrt{6^2 + 8^2} = 10$	M1
		$\tan \alpha = 8/6 \Rightarrow \alpha = 53.130\ldots = 53.13°$ (2 d.p.)	M1
		$\therefore 6 \cos x + 8 \sin x = 10 \cos (x - 53.13)$	A1 (3)
	b)	$6 \cos 2y + 8 \sin 2y = 1$;	
		$10 \cos (2y - 53.130\ldots) = 1$;	M1
		$\cos (2y - 53.130\ldots) = 1/10$	
		$2y - 53.130\ldots = 84.26°, 275.74°, 444.26°, 635.74°$	M1
		$y = 68.695\ldots, 164.43\ldots, 248.67\ldots, 344.43\ldots$	
		$y = 68.70°, 164.43°, 248.68°, 344.43°$ (2 d.p.)	A4 (6)
	c)	$\dfrac{10}{10 + 6\cos x + 8\sin x} = \dfrac{10}{10 + 10\cos(x - 53.130...)}$	M1
		Minimum when $\cos (x - 53.130\ldots) = 1$	M1
		$\therefore x - 53.130\ldots = 0; x = 53.130\ldots = 53.13°$ (2 d.p.)	A1 (3)
	d)	Minimum value $= \dfrac{10}{10 + 10(1)} = \dfrac{1}{2}$	M1A1(2)

(75)

Mark Scheme 4	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, leave your answers to 3 s.f.***	
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1. a) Put over a common denominator — M1

$$\frac{3x^2-x-2+3x+2}{3x^2-x-2}$$

$$=\frac{3x^2+2x}{3x^2-x-2}$$ A1

$$=\frac{x(3x+2)}{(3x+2)(x-1)}$$ factorise denominator correctly — M1

$$=\frac{x}{x-1}$$ A1 (4)

b) Try f(0) and f(1) — M1

$$f(0)=(0)^3+\frac{23}{2}(0)^2+26(0)^2-16=-16<0$$

$$f(1)=(1)^3+\frac{23}{2}(1)^2+26(1)^2-16=22.5>0$$

There is a sign change so there is a solution between 0 and 1 — M1A1(3)

---

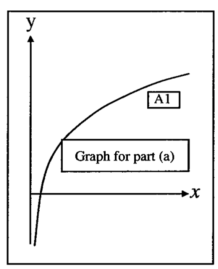
2. a) –ve parts reflected in the x-axis. — M1
Max = 4
Touches x-axis at 1, 5, cuts y-axis at y = 2 — A1 (2)

b) Quadrants 1&4 stay the same, 2&3 are reflected in the y-axis — M1
Max = 4
Cuts x-axis at 1, 5, –1, –5, cuts y-axis at –2 — A1 (2)

c) Stretch ×2 in the y-direction, translate 1 to the left. — M1A1
Max = 8
Cuts x-axis at 0, 4, cuts y-axis at 0 — A1 (3)

---

3. a) Sketch of graph (shape similar to ln x) — A1
Crosses x-axis when y = 0
$\therefore$ 0 = 3 + 2 ln x; ln x = –3/2
$x = e^{-3/2} = 0.22331\ldots$
$= 0.223$ (3 s.f.) — M1A1(3)



b) $\dfrac{dy}{dx}=\dfrac{2}{x}$ — A1
when x = 1, y' = 2/1 = 2 — A1 ft
$\therefore$ y = 2x + c
Substitute in (1,3) — M1
$\therefore$ 3 = 2 + c; c = 1
$\therefore$ y = 2x + 1 — A1 (4)

---

4. a) begin: t = 0
T = 5(20 – e^0) — M1
= 5(20 – 1)
= 95 — A1
end: t = 1
T = 5(20 – e^1)
= 100 – 5e — A1 (3)

	b)	i) $\frac{dT}{dt} = -5e^t$	A1 (1)
		ii) $\frac{dT}{dt} = -6 = -5e^t$	M1
		Therefore $e^t = \frac{6}{5}$	A1
		Therefore $t = \ln\left(\frac{6}{5}\right)$	A1 (3)
	c)	i) max when t = 1, $\frac{dT}{dt} = -5e^1$ = (–5e)°C/s $\therefore$ maximum rate of cooling is 5e°C/s	M1A1(2)
		ii) min when t = 0, $\frac{dT}{dt} = -5e^0$ = –5°C/s $\therefore$ minimum rate of cooling is 5°C/s	M1A1(2)
5.	a)	fg(x) = f(x^2 – 2)	M1
		$= \frac{(x^2-2)^2 - 49}{x^2-2+7}$	A1
		$= \frac{(x^2-9)(x^2+5)}{(x^2+5)}$	M1
		$= (x+3)(x-3)$	A1 (4)
	b)	gf(x) = $g\left(\frac{x^2-49}{x+7}\right)$	M1
		$= \left(\frac{x^2-49}{x+7}\right)^2 - 2$	A1
		$= \frac{(x^2-49)^2 - 2(x+7)^2}{(x+7)^2}$	M1
		$= \frac{x^4 - 100x^2 - 28x + 2303}{(x+7)^2}$	A1 (4)
		$\therefore$ h(x) = $x^4 - 100x^2 - 28x + 2303$	
	c)	g(x) > 23	A1 (1)
	d)	Let y = $x^2 - 2$	M1
		$\therefore y + 2 = x^2 \Rightarrow x = \sqrt{y+2} \quad \therefore g^{-1}(x) = (x+2)^{\frac{1}{2}}$	A1
		Domain: x > 23 Range: $g^{-1}(x) > 5$	A1A1(4)
6.	a)	$11 - \sqrt{11}\sqrt{10} + \sqrt{10}\sqrt{11} - 10 = 1$	A1 (1)
	b)	R = $\sqrt{a^2+b^2} = \sqrt{1^2+3^2} = \sqrt{10}$	A1
		$\alpha = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(3)$ = 71.565… = 71.6° (3 s.f.)	M1A1
		cos x + 3 sin x = ($\sqrt{10}$)cos(x – 71.565…)	A1 (4)
	c)	cos x + 3 sin x = 1 $\Rightarrow$ ($\sqrt{10}$)cos(x – 71.565…) = 1	M1
		cos(x – 71.565…) = $\frac{1}{\sqrt{10}}$	M1

		$x - 71.565\ldots = 0°, 71.565\ldots° \therefore x = 143.13\ldots° = 143°, 360°$ (3 s.f.)	A1A1(4)
	d)	Minimum occurs when $\cos(x - 71.565\ldots) = 1$	M1
		$\therefore x = 71.565\ldots = 71.6°$ (3 s.f.)	A1 (2)
	e)	Minimum value $= \left(\dfrac{1}{\sqrt{10}+\sqrt{11}}\right)$	A1
		$\left(\dfrac{1}{\sqrt{10}+\sqrt{11}}\right) \times \left(\dfrac{\sqrt{11}-\sqrt{10}}{\sqrt{11}-\sqrt{10}}\right) = \underline{\sqrt{11}-\sqrt{10}}$	M1$\underline{\text{A1}}$(3)
7.	a)	$\sin(A + B) = \sin A\cos B + \sin B\cos A$	M1
		$\sin 3x = \sin(2x + x) = \sin 2x\cos x + \sin x\cos 2x$	M1
		$\sin 2A = 2\cos A\sin A$ and $\cos 2A = \cos^2 A - \sin^2 A$	M1
		Therefore $\sin 3x = 2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x$	
		$= 3\sin x\cos^2 x - \sin^3 x$	M1
		$\cos^2 A = 1 - \sin^2 A$	
		Therefore $\sin 3x = 3\sin x(1 - \sin^2 x) - \sin^3 x$	
		$= 3\sin x - 4\sin^3 x$	A1 (5)
	b)	Let $y = \sin(ax)$, let $u = ax$, therefore $y = \sin(u)$	M1
		$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$	M1
		$\dfrac{dy}{du} = \cos u$	M1
		$\dfrac{du}{dx} = a$	M1
		Therefore $\dfrac{dy}{dx} = a\cos u = a\cos(ax)$	A1 (5)
	c)	$\dfrac{d}{dx}(\sin 3x) = \dfrac{d}{dx}(3\sin x - 4\sin^3 x)$	
		$3\cos 3x = 3\cos x - 12\sin^2 x\cos x$	A2
		$\cos 3x = \cos x - 4\sin^2 x\cos x$	A1 (3)
	d)	$\sin(75°) = \sin(30° + 45°)$	M1
		$= \sin 30°\cos 45° + \sin 45°\cos 30°$	
		$= \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\dfrac{\sqrt{3}}{2}$	M1
		$= \dfrac{\sqrt{2}+\sqrt{6}}{4}$	A1 (3)

(75)

Mark Scheme 5	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, leave your answers to 3 s.f.***	
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1. a) $1-\dfrac{1}{1+\cot^2\phi}$

$=1-\dfrac{1}{\text{cosec}^2\phi}$ M1

$=1-\sin^2\phi$ M1

$=\cos^2\phi$ A1 (3)

b) L.H.S. $=\cos\phi+\sin\phi\tan 2\phi=\cos\phi+\dfrac{\sin\phi\sin 2\phi}{\cos 2\phi}$ using $\tan 2A=\dfrac{\sin 2A}{\cos 2A}$ M1

$=\dfrac{\cos\phi\cos 2\phi+\sin\phi\sin 2\phi}{\cos 2\phi}$ M1

$=\dfrac{\cos\phi}{\cos 2\phi}=$ R.H.S. [using $\cos(A-B)=\cos A\cos B+\sin A\sin B$] A2 (4)

---

2. a) $x_1=\sqrt{\dfrac{3}{2}+2}=1.8708\ldots=1.871$ (4 s.f.) A1

$x_2=\sqrt{\dfrac{3}{1.87\ldots}+2}=1.8983\ldots=1.898$ (4 s.f.) A1

$x_3=\sqrt{\dfrac{3}{1.89\ldots}+2}=1.8921\ldots=1.892$ (4 s.f.)

$x_4=\sqrt{\dfrac{3}{1.89\ldots}+2}=1.8935\ldots=1.894$ (4 s.f.) A1 (3)

b) $x_4=1.8935$ (5 s.f)
$x_5=1.8932$ (5 s.f)
$x_6=1.8933$ (5 s.f)

$f(1.8932)<0$ M1
$f(1.8933)>0$ M1
Therefore as $f(x)$ is continuous then there exists a solution $f(n)=0$ with $1.8532<n<1.8933$.
Therefore $n=1.893$ (4 s.f.) A1 (3)

c) $0=x^3-2x^2-3$
$0=x(x^2-2)-3$
$3=x(x^2-2)$

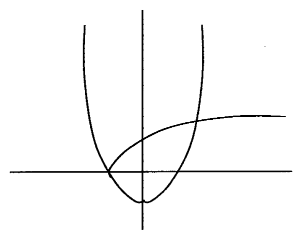
$\dfrac{3}{x}=x^2-2$ M1

$\dfrac{3}{x}+2=x^2$ M1

$x=\sqrt{\dfrac{3}{x}+2}$ M1 (3)

3.	a)	$\lvert 2x + 3\rvert > 4$	
		With $x > -\frac{3}{2}$, $2x + 3 > 4 \rightarrow x > \frac{1}{2}$	M1
		With $x < -\frac{3}{2}$, $2x + 3 < -4 \rightarrow x < -\frac{7}{2}$	
		Therefore $x > \frac{1}{2}$ or $x < -\frac{7}{2}$	A1A1(3)
	b)	i) Sketch of z = $(x-1)(x-3)$	M1
		All points that lie below the x-axis are reflected to the +ve y-axis	
		Sketch of y = $\lvert(x-1)(x-3)\rvert$	A1 (2)
		ii) For $1 < x < 3$, y $= -(x-1)(x-3)$	M1
		$= -x^2 + 4x - 3$	
		$\frac{dy}{dx} = -2x + 4$	A1
		When $\frac{dy}{dx} = 1$, $-2x + 4 = 1$, so $x = \frac{3}{2}$ $\quad\therefore a = 3/2$	M1A1(4)
4.	a)	Quadrants 1 and 4 remain the same. Quadrants 2 and 3 reflected in y-axis.	A1
		Cuts x-axis at 2 and –2, cuts y-axis at –1.	**A1** (2)
	b)	Section between –1 and 2 is reflected in the x-axis.	A1
		Touches x-axis at –1 and 2, cuts y-axis at 1.	**A1** (2)
	c)	Stretch ×3 in y-direction and $\times\frac{1}{2}$ in the x-direction	M1
		Cuts x-axis at $-\frac{1}{2}$ and 1, cuts y-axis at –3	A2 (3)
	d)	f(–1) = 0. Therefore 0 = k – 3e	
		Therefore k = 3e	M1A1(2)
	e)	$\frac{dy}{dx} = -3e^{x+2}$	A1
		Steepest when $x = -1$.	
		Therefore $\frac{dy}{dx} = -3e^{1} = -3e$	M1A1(3)

5.	a)	$fg(x) = (1 - x^2)^2 - 1$	M1
		$= 1 - 2x^2 + x^4 - 1$	
		$= x^4 - 2x^2$	A1
		$gf(x) = 1 - (x^2 - 1)^2$	M1
		$= 1 - 1 - x^4 + 2x$	
		$= 2x^2 - x^4$	A1
		$f(g(x)) = g(f(x))$	
		$x^4 - 2x^2 = 2x^2 - x^4$	M1
		$2x^4 = 4x^2$	
		$x^4 = 2x^2$	
		$x^2 = 2$	
		$x = +\sqrt{2}$ or $-\sqrt{2}$	A1A1
		or $x = 0$	A1 (8)
	b)	From sketch the required domain is $x \geq 0$	M1A1(2)



	c)	$f(x) = x^2 - 1$ Let $y = x^2 - 1$	M1
		$x = y^2 - 1$ <switch variables>	M1
		$y^2 = x + 1$	
		$y = \sqrt{x+1}$	A1
		$f^{-1}(x) = \sqrt{x+1}$	A1 (4)

6.	a)	$f(x) = \ln x \therefore f'(x) = \frac{1}{x}$	A1
		$g(x) = \ln 2x \therefore g'(x) = \frac{1}{x}$	A1 (2)
	b)	Gradient of $f'(x) = \frac{1}{3}$	M1
		$\therefore$ when $x = 3$, $y = \ln 3$	M1
		Tangent to curve is $y - \ln 3 = \frac{x}{3} - 1$	
		$\therefore y = \frac{x}{3} + \ln 3 - 1$	A1 (3)
	c)	Gradient of normal of $g(x) = -3$	M1
		Co-ords to the normal $= (3, \ln 6)$	M1
		$\therefore y - \ln 6 = -3(x - 3)$	M1
		$y = \ln 6 - 3x + 9$	A1 (4)

7.	a)	Using $\sin(A + B) = \sin A \cos B + \cos A \sin B \Rightarrow A = 2x, B = 4x$	M1
		$\sin 2x \cos 4x + \cos 2x \sin 4x \equiv \sin 6x$	A1 (2)
	b)	$2 \sin 2x \cos 4x + \cos 2x \sin 4x \equiv \sin 2x \cos 4x + \sin 2x \cos 4x + \cos 2x \sin 4x$	M1
		$\equiv \frac{1}{2}(\sin(6x) + \sin(-2x)) + \sin 6x = \frac{1}{2}(3\sin 6x - \sin 2x)$	A1 (2)
	c)	$y = e^{-x}\cos x$	
		$\frac{dy}{dx} = -e^{-x}\cos x - e^{-x}\sin x$	A1A1M1
		Let $\frac{dy}{dx} = 0$	M1
		So $-e^{-x}\cos x - e^{-x}\sin x = 0$	
		Therefore $e^{-x}(\cos x + \sin x) = 0$	
		e^{-x} is never zero, so $\cos x + \sin x = 0$	
		$\cos x = -\sin x$, or $\tan x = -1$	A1
		Therefore $x = \frac{3\pi}{4}$ (2.3561… = 2.36 (3 s.f.))	A1
		or $x = \frac{3\pi}{4} + \pi = \frac{7\pi}{4}$ (5.4977…= 5.50 (3 s.f.))	A1
		$\frac{d^2y}{dx^2} = e^{-x}(\cos x + \sin x) - e^{-x}(-\sin x + \cos x) = 2e^{-x}\sin x$	
		When $x = \frac{3\pi}{4}$, $\frac{d^2y}{dx^2} > 0$. Therefore minimum point	M1A1
		When $x = \frac{7\pi}{4}$, $\frac{d^2y}{dx^2} < 0$. Therefore maximum point	M1A1(11)

(75)