Secondary
Maths
Resources

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A LEVEL
TOPIC TESTS
PURE – EDEXCEL – 2004 – C3

1st Edition: Released 16th September 2004

Contents

***Syllabus Reference**

POD - 1149

Teachers and Students Notes

These tests are designed to specifically show what students <u>can do</u> and so are great to encourage confidence and show progress. Each test looks to cover the **different types of problem** that students may encounter and so can be used to identify weaknesses. Additional difficult exam level questions have been added to test the more able. Each test follows the new specifications for **EDEXCEL C3** and a summary of what is tested in shown below. The tests numbers match the numbering in the specification. It is recommended that **students are given a copy of the specification** (or get one from the **internet**!) and are informed of the sections being tested. This should help students to become familiar with their specification. Any important notes about the test are summarised below and it's up to the teacher to decide where to pass this information on to students – possibly by supplying a copy Test Summary below.

Test Summary

*Please use this in conjunction with the specification. Relevant sections of the specification are shown but must **not** be relied upon. Note for example that **'Test 2' tests section 2 of the specification**!*

TEST

1a. **Algebra and Functions: Factorising and Cancelling Expressions**

All aspects tested except for the ref to algebraic division which was covered in C2. Possibly implied here is more complicated algebraic division ie with denominator an irreducible quadratic but this is unclear and is possibly not the main thrust of this section.

Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, eg $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+1}{x^2-1}$.

1b. **Algebra and Functions : Functions**

All aspects tested: All function aspects tested, modulus function and associated modulus functions as per specification

Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f}: x \mapsto \ldots$ and f(x) will be used. Candidates should know that fg will mean 'do g first, then f'. Candidates should know that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{f}\mathrm{f}^{-1}(x) = x$.
The modulus function.	Candidates should be able to sketch the graphs of $y=\lvert ax+b\rvert$, and the graphs of $y=\lvert f(x)\rvert$ and $y=f(\lvert x\rvert)$, given the graph of $y=f(x)$

1c. **Algebra and Functions: Transformations**

All aspects tested.

Combinations of the transformations $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Candidates should be able to sketch the graph of, e.g., $y=2f(3x)$, $y=f(-x)+1$ given the graph of $y=f(x)$ or the graph of, e.g. $y=3+\sin 2x$, $y=-\cos(x+\frac{\pi}{4})$ The graph of $y=f(ax+b)$ will *not* be required.

2. **Trigonometry**

All aspects tested.

Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains. Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$. Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.	Angles beasured in both degrees and radians. To indicate application to half-angles. Knowledge of the $t(\tan\frac{1}{2}\theta)$ formulae will *not* be required. Candidates should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval, and to prove simple idenitites such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$

3. **Exponentials and logarithms**

All aspects tested.

The function e^x and its graph. The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	To include the graph of $y = e^{ax+b} + c$. Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax+b) = q$ is expected.

4. **Differentiation**

All aspects tested.

Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences. Differentiation using the product rule, the quotient rule and the chain rule. The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.	Differentiation of cosec x, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan 2x$. E.g. finding $\frac{dy}{dx}$ for $x = \sin 3y$.

5. **Numerical Methods: Iteration**

All aspects tested: Understand the range the solution lies in and so know when for example when a solution is known to 1dp.

Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous. Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.	Solution of equations by use of iterative procedures for which leads will be given.

6. **Proof**

This follows from the preamble and this test is included for completeness.

Methods of proof, including proof by contradiction and disproof by counter-example, are required.

Algebra and Functions– Test 1a

EDEXCEL **C3**

Factorising and Cancelling Expressions

Recommended Time: 20mins

Express as a single fraction in its simplest form:-

a) $\dfrac{2xy+2zy+2y}{10y}$ b) $\dfrac{x^2-1}{2(x+1)}$ c) $\dfrac{1}{x}+\dfrac{2}{xy}$ (2)(2)(2)

d) $\dfrac{x^2+3x+2}{x^2-x-2}$ e) $\dfrac{x(x+1)(x-2)}{2x(x+1)}+\dfrac{x-2}{4}$ f) $\dfrac{3}{4x}+\dfrac{x+1}{2x}$ (2)(2)(2)

g) $\dfrac{1}{x-1}-\dfrac{x+4}{(x-1)(2x+3)}$ h) $\dfrac{x+1}{x(x-1)}-\dfrac{x+5}{x(x-1)(x+2)}$ (3)(6)

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Algebra and Functions– Test 1a

EDEXCEL **C3**

Factorising and Cancelling Expressions

Recommended Time: 20mins

Express as a single fraction in its simplest form:-

a) $\dfrac{2xy+2zy+2y}{10y}$ b) $\dfrac{x^2-1}{2(x+1)}$ c) $\dfrac{1}{x}+\dfrac{2}{xy}$ (2)(2)(2)

d) $\dfrac{x^2+3x+2}{x^2-x-2}$ e) $\dfrac{x(x+1)(x-2)}{2x(x+1)}+\dfrac{x-2}{4}$ f) $\dfrac{3}{4x}+\dfrac{x+1}{2x}$ (2)(2)(2)

g) $\dfrac{1}{x-1}-\dfrac{x+4}{(x-1)(2x+3)}$ h) $\dfrac{x+1}{x(x-1)}-\dfrac{x+5}{x(x-1)(x+2)}$ (3)(6)

{21}

Algebra and Functions– Test 1a

EDEXCEL **C3**

Factorising and Cancelling Expressions

Recommended Time: 20mins

Express as a single fraction in its simplest form:-

a) $\dfrac{2xy+2zy+2y}{10y}$ b) $\dfrac{x^2-1}{2(x+1)}$ c) $\dfrac{1}{x}+\dfrac{2}{xy}$ (2)(2)(2)

d) $\dfrac{x^2+3x+2}{x^2-x-2}$ e) $\dfrac{x(x+1)(x-2)}{2x(x+1)}+\dfrac{x-2}{4}$ f) $\dfrac{3}{4x}+\dfrac{x+1}{2x}$ (2)(2)(2)

g) $\dfrac{1}{x-1}-\dfrac{x+4}{(x-1)(2x+3)}$ h) $\dfrac{x+1}{x(x-1)}-\dfrac{x+5}{x(x-1)(x+2)}$ (3)(6)

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Algebra and Functions – Test 1b

Assume $x \varepsilon \mathbb{R}$ throughout this test. *Recommended Time: 30 minutes*

Functions: $f(x) = 2x + 4$ $g(x) = 3x - 3$ $h(x) = x^2 + 3$ $k(x) = x^3 + x$ $m(x) = 1 + \sqrt{x}$ $n(x) = (x-1)(x-3)$

1) What is the **range** of h(x)? (1)

2) a) What is the largest possible **domain** of m(x)? (1)
b) What is the corresponding **range** of m(x)? (1)

3) a) Which one of the above functions is '**many to one**'? (1)
b) Give an example of 2 such **input** values giving the same **output** value. (1)

4) a) Find the **inverse** of f(x). (1)
b) Sketch (together) f(x) and $f^{-1}(x)$. (1)
c) Draw on a dotted line to show the **mirror line** between the 2 lines. (1)
d) Write down the **equation** of this mirror line. (1)

5) a) If h(x) has the restricted domain $x \geq 0$, then what is the **inverse** of h(x)? (1)
b) State the **domain** of $h^{-1}(x)$. (1)

6) a) Find fh(x). (1)
b) Find hf(x). (1)
c) Find when fh(x) = hf(x). (2)

7) a) **Sketch** $y = 3x - 3$ and $y = |3x - 3|$ on the same grid. (1)
b) **Sketch** on a new grid the graph of $y = |3x| - 3$ (1)

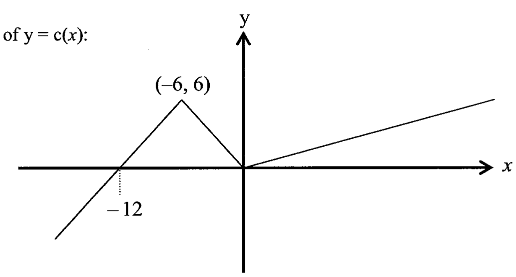
8) Sketch the graphs of: a) i) $f(|x|)$ ii) $|f(x)|$ (2)(2)
b) i) $n(|x|)$ ii) $|n(x)|$ (2)(2)

Clearly state or show where these graphs cross or touch the x-axis and y-axis. **{25}**

Transformations – Test 1c *Recommended Time: 25 minutes*

EDEXCEL **C3**

1. This is a sketch of y = c(x):



a) Copy this **sketch** and **sketch** $y = 2c(3x)$ on the *same grid.* (1)
b) Copy this **sketch** of c(x) *again* and **sketch** $y = c(-x) + 2$ *on the same grid.* (1)
c) Copy this **sketch** of c(x) *again* and **sketch** $y = 2c(x + 1)$ *on the same grid.* (1)
d) Copy this **sketch** of c(x) *again* and **sketch** $y = 1 + c(2x)$ *on the same grid.* (1)

In each case indicate where the graph crosses the x and y-axis. (8)
Similarly indicate the corresponding coordinates to the given coordinates (–6, 6). (4)
(In this question you should have drawn 4 grids with 2 graphs on each.)

2. a) Fully describe the transformation that takes the curve $y = 2\sin x$ to the curve $y = 2\cos x$ (1)
b) Sketch the graph of $y = 2\sin(x - \frac{\pi}{2})$ in the range $0 \leq x \leq 2\pi$ (4)
c) Sketch the graph of $y = 10\cos(2x)$ in the range $0 \leq x \leq 2\pi$ (4)
In sketches b) and c), clearly indicate the functions value when $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$. **{25}**

Trigonometry – Test 2

EDEXCEL **C3**

Recommended Time: 30mins

1) Draw out 3 separate sketch graphs for the following curves in the range $0° \le \phi \le 360°$

a) $y = \sec\phi$ (2)

b) $y = \operatorname{cosec}\phi$ (2)

c) $y = \tan\phi$ (2)

d) State the **periodicity** of these 3 curves. (3)

e) Which if any of the these 3 curves has **symmetry** in the ***x*-axis**? (2)

f) Which if any of the these 3 curves has **symmetry** in the **y-axis**? (2)

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2) a) Sketch the graph of $y = \arcsin x$. *On your sketch clearly show the maximum and minimum values of the curve and where these minimum and maximums occur.* (3)

b) State the restricted domain of the curve. (2)

c) State the range of the curve. (1)

d) Briefly explain why a restricted domain is needed. (1)

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3) Given that $\tan^2\phi = 0.3$

a) What is the value of $\sec^2\phi$? (1)

b) What is the value of $\cos^2\phi$? (2)

c) What are the possible values of $\cos\phi$? (2)

d) Find the possible values of ϕ to 1 decimal place in the range $0° \le \phi \le 360°$ (4)

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4) Solve the following equations to in the range $0° \le \phi \le 360°$.

a) $\tan\phi = \sin\phi$ (4)

b) $\tan\phi = \cos\phi$ *Give your answer to part b) to 1 decimal place.* (4)

8

5) Given that $\tan\phi = 0.3$

Find the **exact** value of $\tan 2\phi$ in the form $\tan 2\phi = \frac{a}{b}$ where a and b are **integers** and such that the fraction $\frac{a}{b}$ will not simplify further. (2)

2

6) a) Simply the expression $\sec\phi\cos\phi$ (1)

b) Simplify the expression: $\dfrac{\sin^3\phi + \cos^2\phi\sin\phi}{\cos\phi}$ (3)

c) Prove that $\sin(2\phi)\sin\phi \equiv 2\cos\phi - 2\cos^3\phi$ (3)

7

7) a) Write $\sin\phi + \sqrt{3}(\cos\phi)$ in the form $R\cos(\phi - \alpha)$ (2)

b) Hence solve the equation $\sin\phi + \sqrt{3}\ (\cos\phi) = 1$ in the range $0° < \phi < 360°$. (3)

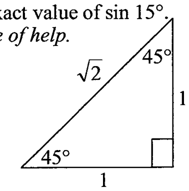
5

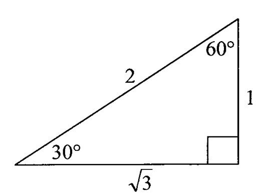
8) a) Using the sin (a + b) formulae, find the exact value of sin 75° (3)

b) Similarly find the exact value of sin 15°. (3)

This triangle may be of help.

6





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Exponentials and Logarithms – Test 3

log is log base 10. $\log_a$ is log base a. ln is natural log base e.

Recommended Time: 25mins

1) a) Draw the line $y = \ln x$ (1)
b) On the same grid draw $y = e^x$ (1)
c) Draw on with a dotted line the mirror line connecting the 2 curves. (1)
d) Give the equation of this mirror line. (1) **4**

2) Solve for x **exactly**. *Leave your answer in terms of e, ln or log where appropriate.*
a) $\ln x = 3$ (1)
b) $\ln\left(\frac{1}{x}\right) - \ln(x) = 5$ (1)
c) $\ln 6x = \ln 6 + \ln 2$ (1)
d) $e^x = 4$ (1)
e) $e^{2x} = 5$ (2)
f) $e^{2x+3} = 4$ (2)
g) $\ln(6x + 2) = 5$ (2)
h) $3\ln(x + 2) = 9$ (2)
i) $e^{2x} - e^x = 0$ (1)
j) $e^{2x} + e^x = 6$ (2)
k) $e^{4x} - e^{2x} = 6$ (2) **17**

3) Solve the **simultaneous** equations:

$x - y = e^5 - e^2$ and $\ln x + \ln y = 7$ **5**

Hint: Your solution must include only positive values for x and y.

4) Describe the transformation that takes the curve $y = e^x$ to the curve:
a) $y = e^{2x}$ (2)
b) $y = e^{x+2}$ (1)
c) $y = 2e^{x+2}$ (1)
d) $y = 2+2e^x$ (2)

Describe the transformation that takes $y = e^{2x+2}$ to the curve:
e) $y = 3 + e^{2x+2}$ (1) **7**

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Differentiation – Test 4 EDEXCEL **C3**

e is the exponential constant. ln x is log base e.

Recommended Time: 45-55mins

1) Find $\frac{dy}{dx}$ in each of the following:-

a)	$y = \sin x$	b)	$y = 3\sin x$	c)	$y = x + \tan x$	(1)(1)(1)
d)	$y = \frac{\sin x + \cos x}{\cos x}$	e)	$y = 3\cos 3x$	f)	$y = x + e^{3x}$	(1)(1)(1)
g)	$y = \ln x$	h)	$y = 3\ln x$	i)	$y = 2e^x + \ln(5x)$	(1)(1)(2) **10**

2) Find $\frac{dy}{dx}$ in each of the following:-

a)	$y = \frac{\sin x}{x}$	b)	$y = \sin^2 x$	c)	$y = \sin(x^2)$	(2)(2)(2)

d) $y = (x + 1)^3(x + 2)^6$ Write your answer in the form, $(x + 1)^2(x + 2)^5(ax + b)$ with a and b both as **integers.** (4)

e) $y = \frac{(x+2)^2}{(x-1)^4}$ Write your answer as $\frac{f(x)}{(x-1)^5}$ with f(x) a quadratic. (3) **13**

3)
a) Find the **stationary points** of the curve $y = e^x(x + 2)^2$ (5)
Write your answer as co-ordinates.
b) Determine the nature of these **stationary points.** (3)
c) Sketch the graph of $y = e^x(x + 2)^2$ (3) **11**

4) For the curve $x = y^2 + y^3$ find $\frac{dy}{dx}$ as a function of y. (2) **2**

5)
a) Give the **co-ordinates** when $y = e^x$ has a **gradient** of 3. (3)
b) Give the **co-ordinates** when $y = \ln x$ has a **gradient** of 3. (3)
c) Calculate the **equation of the tangent** to the curve $y = e^x$ at the point $(3, e^3)$. (4)
d) Calculate the **equation of the normal** to the curve $y = \ln x$ at the point (3, ln 3). (4) **14**

6)
a) Calculate the ***x* value** when the following curve has a **stationary** point:- (3)
$y = 2ex - e^x$
b) Determine the nature of this **stationary** point. (i.e min. or max.) (1)
c) Calculate the **co-ordinates** of this **stationary** point. (1) **5**

7)
a) Calculate the **co-ordinates** of the **stationary** point of the following curve:-
$y = 2x^2 - \ln x$ *where $x > 0$* (5)
b) **Sketch** the curve. (2) **7**

8) Given that $y = (x + \ln x)^2$
a) Calculate the value of y when $x = 3$ and calculate the value of y when $x = 3.01$ (1)
b) Hence estimate the **gradient** of the curve $y = (x + \ln x)^2$ when $x = 3$ (2) **3**

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Numerical Methods – Iteration – Test 5

Recommended Time: 30mins

1) $x^3 + x - 1 = 0$

a) Show that the above equation has a solution between 0 and 1
Clearly show your reasoning (1)

The above function can be arranged as $x = \dfrac{1}{x^2+1}$

b) Using the iterative sequence $x_{n+1} = \dfrac{1}{{x_n}^2+1}$ where $x_1 = 0.5$.
Calculate x_n for values up to n = 6. (3)

You can assume that your values converge to give a solution to the original equation.

c) **Hence**, give an approximate solution to $x^3 + x - 1 = 0$ to 1 decimal place. (1)

d) By considering your earlier values of x_n, state the **least** value of n for which it is **known** that this is a solution to the original equation to **1 decimal place**. (1)

6

2) a) *Clearly show* how the equation $x^3 - 8x + 1 = 0$
can be rearranged to give both $x = \dfrac{1}{8-x^2}$ and $x = (8x-1)^{\frac{1}{3}}$ (1,1)

b) Let $f(x) = x^3 - 8x + 1$ and calculate f(0), f(1), f(2), f(3), f(10). (2)

From this it can be stated that a **solution** to $x^3 - 8x + 1 = 0$ occurs in the **range a < *x* < b** and another **larger** solution occurs in the range **c < *x* < d**.

c) Using your values of f(0), f(1), f(2), f(3), f(10) give values for **a, b, c** and **d** such that the intervals **a < *x* < b** and **c < *x* < d** are as small as possible. (4)

d) Using the iteration $x_{n+1} = (8x_n - 1)^{\frac{1}{3}}$ find x_2, x_3, and x_4 starting with $x_1 = 3$.
Use your value for x_4 to approximate a solution to $x^3 - 8x + 1 = 0$ to 1 decimal place. (2)

e) Using the same iterative formulae find x_2, x_3, and x_4 starting with $x_1 = -3$.
Use your value for x_4 to approximate a solution to $x^3 - 8x + 1 = 0$ to 1 decimal place. (2)

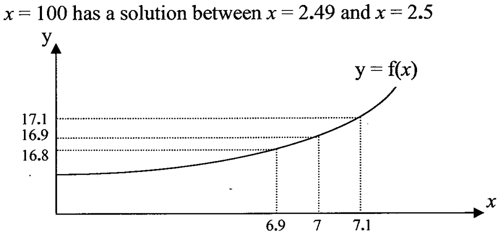
f) Using the iteration $x_{n+1} = \dfrac{1}{8-{x_n}^2}$ find the final solution to 4 decimal places,
start with $x_1 = 0$ (2)

14

3) Show that $x^5 + x = 100$ has a solution between $x = 2.49$ and $x = 2.5$ (2)

2

4)



Given f(s) = 17. Write down an interval for s in the form a < s < b such that a is as large as possible and b is as small as possible. (3)

3

{25}

Proof – (Additional Test based on Preamble) – Test 6 EDEXCEL C3

Calculators not needed.

Recommended Time: 15-20mins

Difficult

1) The solution to the quadratic $ax^2 + bx + c = 0$ is given by the **quadratic formulae**

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Prove the **quadratic formulae**. (3)

2) Prove that the quadratic $D^2x^2 + Dx + 1 = 0$ has no real solutions, where D is a non-zero real constant. (3)

3*) For what values of D does the quadratic $Dx^2 + 2Dx + 1 = 0$ have no real solutions? *(D is a non-zero real constant)* (4)

Inequality Reminder:
Dividing by a negative or a negative constant requires the inequality sign to be reversed!

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Algebra and Functions – Test 1

No Calculators

EDEXCEL – 2004 – **C3**

a) $\dfrac{2xy+2zy+2y}{10y}=\dfrac{2y(x+z+1)}{10y}=\dfrac{x+z+1}{5}$ **A2** (2)

b) $\dfrac{x^2-1}{2(x+1)}=\dfrac{(x+1)(x-1)}{2(x+1)}$ **M1** $=\dfrac{x-1}{2}$ **A1** (2)

c) $\dfrac{1}{x}+\dfrac{2}{xy}=\dfrac{y+2}{xy}$ **A2** (2)

d) $\dfrac{x^2+3x+2}{x^2-x-2}=\dfrac{(x+1)(x+2)}{(x+1)(x-2)}$ **M1** $=\dfrac{x+2}{x-2}$ **A1** (2)

e) **M1** $\dfrac{\cancel{x(x+1)}(x-2)}{2\cancel{x(x+1)}}+\dfrac{x-2}{4}=\dfrac{x-2}{2}+\dfrac{x-2}{4}=\dfrac{3(x-2)}{4}$ *or* $\dfrac{3x-6}{4}$ **A1** (2)

f) $\dfrac{3}{4x}+\dfrac{x+1}{2x}=\dfrac{3}{4x}+\dfrac{2x+2}{4x}=\dfrac{2x+5}{4x}$ **M1A1** (2)

(2)

g) $\dfrac{1}{x-1}-\dfrac{x+4}{(x-1)(2x+3)}\overset{\textbf{M1}}{=}\dfrac{2x+3}{(x-1)(2x+3)}-\dfrac{x+4}{(x-1)(2x+3)}\overset{\textbf{M1}}{=}\dfrac{x-1}{(x-1)(2x+3)}=\dfrac{1}{2x+3}$ **A1**

(3)

h) $\dfrac{x+1}{x(x-1)}-\dfrac{x+5}{x(x-1)(x+2)}$ **M1** $=\dfrac{(x+1)(x+2)}{x(x-1)(x+2)}-\dfrac{x+5}{x(x-1)(x+2)}$

$=\dfrac{x^2+3x+2-(x+5)}{x(x-1)(x+2)}$ **M1**

Expanded denominators are acceptable but not ideal

$=\dfrac{x^2+3x+2-x-5}{x(x-1)(x+2)}$ **M1** changing +5 to –5

$=\dfrac{x^2+2x-3}{x(x-1)(x+2)}=\dfrac{(x+3)(x-1)}{x(x-1)(x+2)}=\dfrac{x+3}{x(x+2)}$ **M1 A2**

(6)

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Algebra and Functions – Test 1b

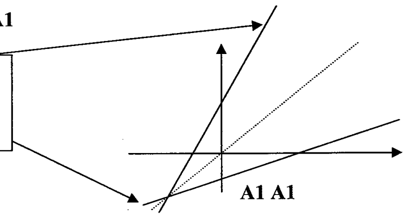
EDEXCEL – 2004 – **C3**

1. $h(x) \geq 3$ **A1** 1

2. a) $x \geq 0$ **A1** b) $m(x) \geq 1$ **A1** 2

3. a) $h(x)$ **A1**
b) $h(1) = 4$, $h(-1) = 4$ **A1** or other valid examples from $h(x) = x^2 + 3$ of the form $h(a)$ and $h(-a)$ 2

4. a) $f^{-1}(x) = \frac{x-4}{2}$ or $(\frac{x}{2}) - 2$ **A1**
b)
c) Graphs of $y = 2x + 4$ and $y = \frac{x}{2} - 2$ and dotted line $y = x$.



d) $y = x$ **A1**

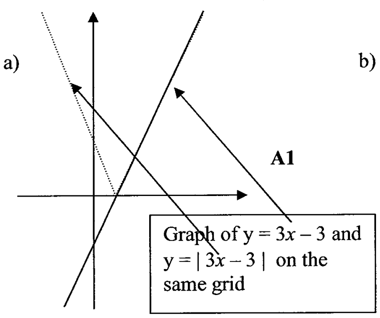
A1 A1 4

5. a) $h^{-1}(x) = \sqrt{x-3}$ **A1** b) $x \geq 3$ **A1** 2

6. a) $f(x^2 + 3) = 2x^2 + 10$ **A1** b) $h(2x + 4) = 4x^2 + 16x + 19$ **A1**
c) $2x^2 + 10 = 4x^2 + 16x + 19 \Rightarrow 2x^2 + 16x + 9 = 0$, $\therefore x = \dfrac{-16 \pm \sqrt{184}}{4} = -0.60883\ldots = -0.609$
or $-7.3911\ldots = -7.39$ to 3 s.f. **A1A1**

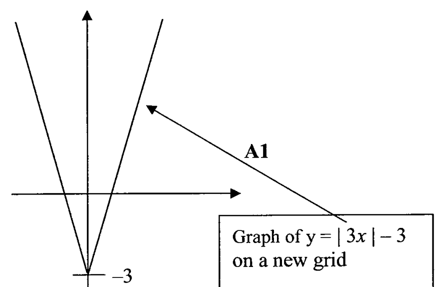
4

7. a)



A1

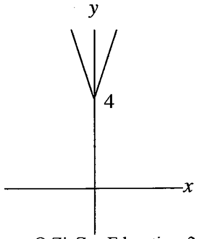
b)



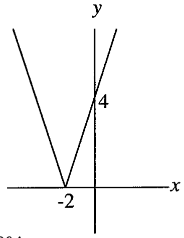
A1 2

8) $f(x) = 2x + 4$ a) i) $f(|x|)$ ii) $|f(x)|$
$n(x) = (x-1)(x-3)$. b) i) $n(|x|)$ ii) $|n(x)|$

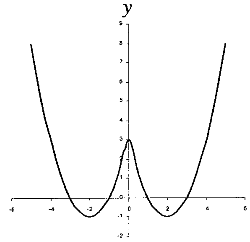
shape **A1**
y-axis value 4 **A1**



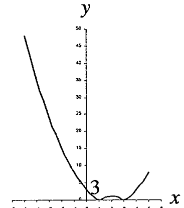
shape **A1**
x-axis value –2 **A1**



shape **A1**
values: x-axis 1, 3, –1,–3, y-axis 3 **A1**



shape **A1**
x-axis values 1, 3 **A1**



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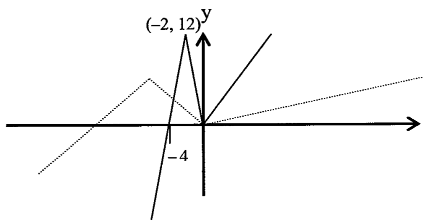
Algebra and Functions – Test 1c

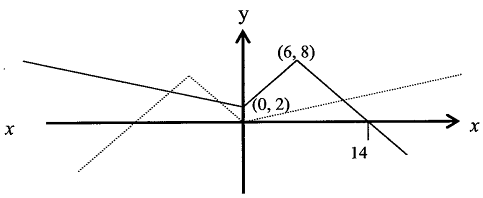
EDEXCEL – 2004 – **C3**

(1 mark for each correct graph sketched plus 1 mark for specifics coordinates or relevant values shown.)

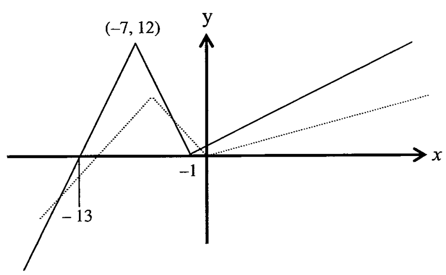
1. a) y = 2c(3x)
A1A1A1 A1 **(for going through the origin)**



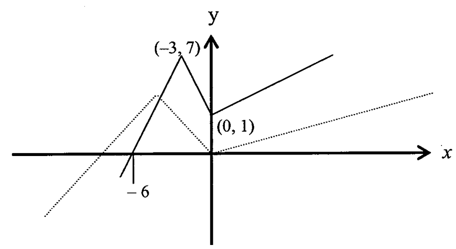
b) y = c(−x) + 2
A1A1A1 A1 **(the value 14 is known due to gradient of −1)**



c) y = 2c(x+1)
A1A1A1A1

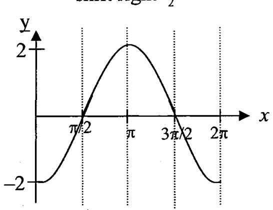


d) y = 1 + c(2x)
A1A1A1A1



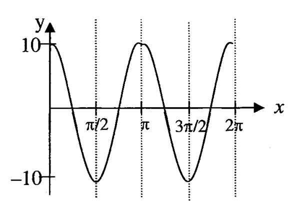
2. a) shift $\frac{\pi}{2}$ left **A1**

b) y = 2sin(x − $\frac{\pi}{2}$)
×2 vertical **M1**
shift right $\frac{\pi}{2}$ **M1**



A1 A1

c) y = 10cos(2x)
×10 vertical **M1**
× ½ horizontal **M1**



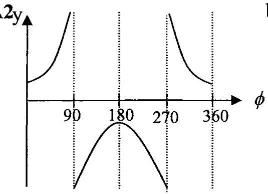
A1 A1

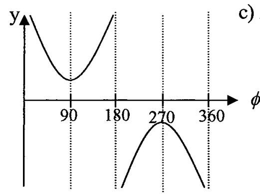
9

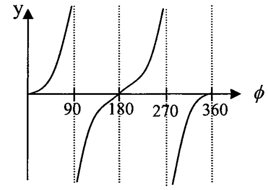
{25}

Trigonometry – Test 2

EDEXCEL – 2004 – **C3**

1.a) **A2** 

b) **A2** 

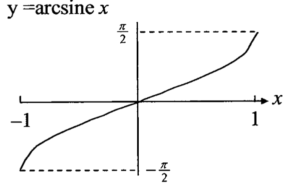
c) **A2** 

d) $\sec\phi$: 360 **A1** $\quad$ $\operatorname{cosec}\phi$: 360 **A1** $\quad$ $\tan\phi$: 180 **A1**

e) None **A2**

f) $\sec\phi$ **A2** **13**

2) a) y =arcsine x



M1 for correct sketch
A1A1 for correct values

b) $-1 \le x \le 1$ **A1 A1**

c) $-\dfrac{\pi}{2} \le f(x) \le \dfrac{\pi}{2}$ **A1**

d) Because f: $x \rightarrow \sin x$ is a many to one mapping or equivalent. **A1**

7

3) a) $1 + \tan^2\phi = \sec^2\phi$ $\quad \therefore \sec^2\phi = 1 + 0.3 = 1.3$ **A1**

b) $\sec\phi = \dfrac{1}{\cos\phi}$ $\quad \therefore \cos^2\phi = \dfrac{1}{\sec^2\phi}$ **M1** $= \dfrac{1}{1.3} = \dfrac{10}{13}$ or 0.7692... = 0.769 to 3 sf **A1**

c) $\therefore \cos\phi = \pm\sqrt{\dfrac{10}{13}}$ or $\pm$ 0.87705… = 0.877 to 3 sf **A1A1**

d) $\therefore \phi$ = 28.710…, 151.28…, 208.71…, 331.28…
= 28.7°, 151.3°, 208.7°, 331.3° (1 d.p. as required) **A1 A1 A1 A1** **9**

4) a) $\tan\phi = \sin\phi$

$\therefore \dfrac{\sin\phi}{\cos\phi} = \sin\phi \qquad \therefore \sin\phi = \sin\phi\cos\phi$

$\div \sin\phi$ **M1**($\sin\phi \ne 0$) $\Rightarrow 1 = \cos\phi \Rightarrow \phi = 0$ or 360°.

OR $\sin\phi = 0$ which is also a solution by inspection.

$\therefore \phi = 180°$. $\therefore \phi = 0, 180°$, or 360°. **A1 A1 A1** (4)

b) $\tan\phi = \cos\phi$

$\therefore \dfrac{\sin\phi}{\cos\phi} = \cos\phi \therefore \sin\phi = \cos^2\phi = 1 - \sin^2\phi$ **M1** (using: $\sin^2\phi + \cos^2\phi = 1$ or equivalent)

$\therefore \sin^2\phi + \sin\phi - 1 = 0$ **M1** (forming a quadratic)

$\therefore\ \sin\phi = \dfrac{-1 \pm \sqrt{1+4}}{2} = 0.6180\ldots \text{ or } -1.6180\ldots$

$\sin\phi = 0.6180\ldots \Rightarrow \phi = 38.172\ldots = 38.2°$ or $141.82\ldots = 141.8°$ (1 d.p. as required)

$\sin\phi = -1.6180\ldots \Rightarrow$ no solutions $\therefore\ \phi = 38.2°$ **A1** or $141.8°$ **A1** (4) **8**

5) Given that $\tan\phi = 0.3$

$\tan 2\phi = \dfrac{2\tan\phi}{1-\tan^2\phi}$ **M1**

$\therefore\ \tan 2\phi = \dfrac{2(\frac{3}{10})}{1-\frac{9}{100}} = \dfrac{\frac{6}{10}}{\frac{91}{100}} = \dfrac{60}{91}$ **A1** (2)

2

6) a) $\sec\phi\cos\phi = \dfrac{1}{\cos\phi} \times \cos\phi = 1$ **A1** (1)

b) $\dfrac{\sin^3\phi + \cos^2\phi\sin\phi}{\cos\phi} = \dfrac{\sin\phi(\sin^2\phi + \cos^2\phi)}{\cos\phi}$ **M1** $= \tan\phi$ **M1A1** (3)

c) Prove that $\sin(2\phi)\sin\phi \equiv 2\cos\phi - 2\cos^3\phi$

LHS $= \sin(2\phi)\sin\phi = 2\sin\phi\cos\phi\sin\phi$ **M1** $= 2\sin^2\phi\cos\phi$

$= 2(1-\cos^2\phi)\cos\phi$ **M1** $= 2\cos\phi - 2\cos^3\phi =$ RHS **A1** (3)

7

7) a) $\sin\phi + \sqrt{3}(\cos\phi) = R\cos(\phi - \alpha)$ $R = \sqrt{1+3} = \sqrt{4} = 2$ **A1** $\alpha = \arctan\frac{1}{\sqrt{3}}$ **A1** (2)

b) $2\cos(\phi - \arctan\frac{1}{\sqrt{3}}) = 1 \ \therefore\ \cos(\phi - \arctan\frac{1}{\sqrt{3}}) = \dfrac{1}{2}$ **M1** $\therefore\ \phi - \arctan\frac{1}{\sqrt{3}} = 60, 300.$

$\therefore\ \phi = 90°$ or $330°$ **A1A1**. (3) **5**

8) a) sin (a + b) = sin a cos b + sin b cos a

$\sin 75° = \sin(30+45) = \sin 30\cos 45 + \sin 45\cos 30$ **M1** $= \dfrac{1}{2}\times\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\times\dfrac{\sqrt{3}}{2}$ **M1**

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ gives $\sin 75° = \dfrac{\sqrt{2}}{4}(\sqrt{3}+1)$ **A1** (3)

b) $\sin 15° = \sin(45-30) = \sin 45\cos 30 - \sin 30\cos 45$ **M1** $= \dfrac{1}{2}\times\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\times\dfrac{1}{\sqrt{2}}$ **M1**

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ gives $\sin 15° = \dfrac{\sqrt{2}}{4}(\sqrt{3}-1)$ **A1** (3) **6**

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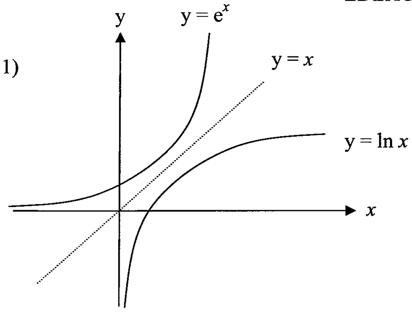
Exponentials and Logarithms – Test 3

EDEXCEL – 2004 – **C3**

1)



a)b)c)d)**A1 A1 A1 A1**

4

2)

a) $\ln x = 3 \therefore x = e^3$ **A1** (1)

b) $\ln\frac{1}{x} - \ln x = 5 \therefore \ln 1 - \ln x - \ln x = -2\ln x = 5 \therefore \ln x = -\frac{5}{2} \therefore x = e^{-\frac{5}{2}}$ **A1** (1)

c) $\ln 6x = \ln 6 + \ln 2 \therefore \ln 6x = \ln 12 \therefore 6x = 12 \therefore x = 2$ **A1** (1)

d) $x = \ln 4$ **A1** (1)

e) $2x = \ln 5 \therefore x = \frac{\ln 5}{2}$ **M1A1** (2)

f) $2x + 3 = \ln 4 \therefore x = \frac{(\ln 4) - 3}{2}$ **M1A1** (2)

g) $6x + 2 = e^5 \therefore x = \frac{e^5 - 2}{6}$ **M1A1** (2)

h) $\ln(x+2) = 3 \therefore x+2 = e^3 \therefore x = e^3 - 2$ **M1A1** (2)

i) $e^{2x} - e^x = 0 \therefore e^{2x} = e^x \therefore 2x = x\ \ x = 0$ **A1** (1)

j) $e^{2x} + e^x = 6$ let $y = e^x$ **M1** note $e^{2x} = (e^x)^2 = y^2 \therefore y^2 + y - 6 = (y+3)(y-2) = 0 \therefore y = -3$ or $+2$
$\therefore e^x = -3 \Rightarrow$ no solution OR $e^x = 2 \Rightarrow x = \ln 2$ **A1** (2)

k) $e^{4x} - e^{2x} = 6$ let $y = e^{2x}$ note $e^{4x} = (e^{2x})^2 = y^2$ **M1** $\therefore y^2 - y - 6 = (y-3)(y+2) = 0 \therefore y = 3$ or -2
$\therefore e^{2x} = -2 \Rightarrow$ no solution OR $e^{2x} = 3 \Rightarrow 2x = \ln 3 \Rightarrow x = \frac{\ln 3}{2}$ **A1** (2) **17**

3)

$x - y = e^5 - e^2$ and $\ln x + \ln y = 7$

$\therefore \ln xy = 7$ (1)

$\therefore xy = e^7$ (1)

$\therefore y = e^7/x$ and sub in $\therefore x - \frac{e^7}{x} = e^5 - e^2$ (1)

$\therefore x^2 - x(e^5 - e^2) - e^7 = 0 \Rightarrow (x - e^5)(x + e^2) = 0$ and this $x = e^5$ or $x = -e^2$. (1)

Now x must be positive and hence $x = e^5$, $y = e^2$. (1)

5

4)

a) stretch scale factor ½ (or ×½) **A1** **in horizontal** **A1** (2)

b) shift (translation) left 2 **A1** (1)

c) stretch scale factor ½ (or ×½) in horizontal **and** (then) shift left 2 (either transformation can be done first!) **A1** (1)

d) stretch scale factor 2 (or ×2) in vertical **A1** **AND THEN shift up 2** **A1** (2)
[Alternatively **shift up 1 and then** stretch scale factor 2 (or ×2) in vertical]

e) shift (translation) up 3 **A1** (1) **7**

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Differentiation – Test 4

EDEXCEL – 2004 – **C3**

1) a) y = $\sin x$ $\quad \frac{dy}{dx} = \cos x$ **A1** (1)

b) y = $3\sin x$ $\quad \frac{dy}{dx} = 3\cos x$ **A1** (1)

c) y = $x + \tan x$ $\quad \frac{dy}{dx} = 1 + \sec^2 x$ **A1** (1)

d) $y = \dfrac{\sin x + \cos x}{\cos x} = \tan x + 1$ $\quad \frac{dy}{dx} = \sec^2 x$ **A1** (1)

e) y = $3\cos 3x$ $\quad \frac{dy}{dx} = -9\sin 3x$ **A1** (1)

f) y = $x + e^{3x}$ $\quad \frac{dy}{dx} = 1 + 3e^{3x}$ **A1** (1)

g) y = $\ln x$ $\quad \frac{dy}{dx} = \frac{1}{x}$ **A1** (1)

h) $3 \ln x$ $\quad \frac{dy}{dx} = \frac{3}{x}$ **A1** (1)

i) y = $2e^x + \ln(5x)$ $\quad$ y = $2e^x + \ln 5 + \ln x$ $\therefore \frac{dy}{dx} = 2e^x + \frac{1}{x}$ **M1A1** (2)

2) a) $y = \dfrac{\sin x}{x}$ Quotient rule let u = sin x u' = cosx **M1**

let v = x v' = 1

$\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$ or equivalent **A1** (2)

b) y = $\sin^2 x$ Product Rule let u = $\sin x$ u' = $\cos x$ **M1** (2)

let v = $\sin x$ v' = $\cos x$

$\dfrac{dy}{dx} = \sin x \cos x + \sin x \cos x = 2 \sin x \cos x$ **A1**

c) y = $\sin (x^2)$ $\quad \dfrac{dy}{dx} = 2x \cos x^2$ **A1 A1** (2)

d) y = $(x + 1)^3(x + 2)^6$ Product Rule let u = $(x + 1)^3$ u' = $3(x + 1)^2$ **M1**

let v = $(x + 2)^6$ v' = $6(x + 2)^5$

$\dfrac{dy}{dx} = (x + 1)^3 6(x + 2)^5 + (x + 2)^6 3(x + 1)^2$ **A1**

factorising: $\dfrac{dy}{dx} = (x+1)^2(x+2)^5\,[6(x+1) + 3(x+2)] = (x + 1)^2(x + 2)^5[9x + 12]$ **A1 A1**(4)

e) $y = \dfrac{(x+2)^2}{(x-1)^4}$ Quotient rule let u = $(x + 2)^2$ u' = $2(x + 2)$ **M1**

let v = (x – 1)4 v' = 4(x – 1)3

$$\frac{dy}{dx} = \frac{(x-1)^4 2(x+2) - 4(x-1)^3(x+2)^2}{(x-1)^8}$$ **A1**

$$\frac{dy}{dx} = \frac{(x-1)2(x+2) - 4(x+2)^2}{(x-1)^5} \Rightarrow \frac{dy}{dx} = \frac{-2x^2 - 14x - 20}{(x-1)^5}$$ **A1** (3) **13**

3) a) $y = e^x(x+2)^2$ Product Rule let $u = e^x$ $u' = e^x$ **M1**
let $v = (x+2)^2$ $v' = 2(x+2)$

$\frac{dy}{dx} = e^x 2(x+2) + e^x(x+2)^2 = e^x(2(x+2) + (x+2)^2) = e^x(x^2 + 6x + 8)$ **A1**

for stationary point $\frac{dy}{dx} = 0 \therefore e^x(x^2 + 6x + 8) = 0$ **M1** $\therefore x^2 + 6x + 8 = 0$

$\therefore (x+4)(x+2) = 0 \therefore x = -2$ or $x = -4$ **A1**

when $x = -2$ $y = e^{-2}(-2+2)^2 = 0$ $\therefore (-2, 0)$

when $x = -4$ $y = e^{-4}(-4+2)^2 = e^{-4}(-4+2)^2 = 4e^{-4} = 0.073262\ldots = 0.0733$ to 3sf

$\therefore (-4, 0.0733)$

$\therefore (-2, 0)$ and $(-4, 0.0733)$ are the stationary co-ordinates **A1** (5)

b) Determine the nature of these **stationary points**.

when $x = -1.9$, $y > 0$ when $x = -2.1$, $y > 0$

$\therefore (-2, 0)$ is a minimum point **A1** (or by graphical means or other suitable explanation)

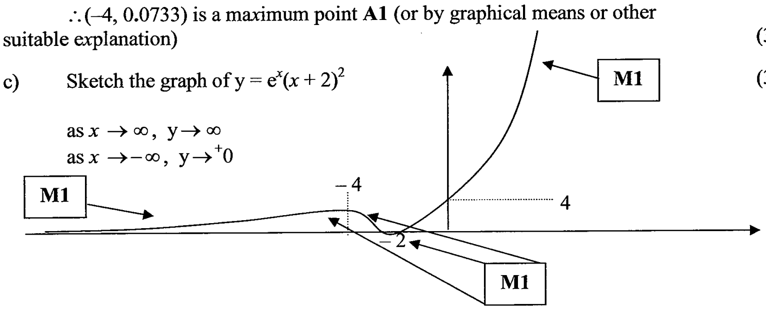
when $x = -3.9$, $y = 0.073073\ldots$ when $x = -4.1$ $y = 0.073085\ldots$ **M1**

$\therefore (-4, 0.0733)$ is a maximum point **A1** (or by graphical means or other suitable explanation) (3)

c) Sketch the graph of $y = e^x(x+2)^2$ (3)

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to {}^{+}0$



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4) For the curve $x = y^2 + y^3$ find $\frac{dy}{dx}$ as a function of y. (2)

$\frac{dx}{dy} = 2y + 3y^2 \therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{2y + 3y^2}$ **M1A1** 2

5) a) Give the **co-ordinates** when $y = e^x$ has a **gradient** of 3. (3)

$\frac{dy}{dx} = e^x \therefore 3 = e^x$ **M1** $\therefore x = \ln 3$ **A1** and $y = 3$ **A1**

b) Give the **co-ordinates** when $y = \ln x$ has a **gradient** of 3. (3)

$\frac{dy}{dx} = \frac{1}{x} \therefore 3 = \frac{1}{x}$ **M1** $\therefore x = \frac{1}{3}$ **A1** and $y = \ln\frac{1}{3}$ or $-\ln 3$ **A1**

c) Calculate the **equation of the tangent** to the curve $y = e^x$ at the point $(3, e^3)$. (4)

$\frac{dy}{dx} = e^x$ when $x = 3$ $\frac{dy}{dx} = e^3$ **M1** $\therefore y = e^3x + C$ sub in **M1** gives $e^3 = 3e^3 + C \therefore C = -2e^3$

$\therefore y = e^3x - 2e^3$ **A1A1**

d) Calculate the **equation of the normal** to the curve $y = \ln x$ at the point $(3, \ln 3)$.

$\frac{dy}{dx} = \frac{1}{x}$ when $x = 3$ $\frac{dy}{dx} = \frac{1}{3}$

$m_1 m_2 = -1 \therefore m_2 = -3$ **M1** $\therefore y = -3x + C$ sub in **M1** gives $\ln 3 = -3.3 + C \therefore C = \ln 3 + 9$

$\therefore y = -3x + \ln 3 + 9$ **A1A1** (4) 14

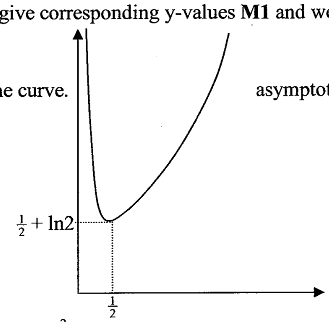
6) a) $y = 2ex - e^x$ $\frac{dy}{dx} = 2e - e^x$ **M1** let $\frac{dy}{dx} = 0$ **M1** $\therefore 2e - e^x = 0 \therefore 2e = e^x \therefore x = \ln(2e)$ **A1** (3)

b) $\frac{d^2y}{dx^2} = -e^x$ when $x = \ln(2e)$ $\frac{d^2y}{dx^2} < 0 \therefore$ maximum point. **M1** or by graph sketch or other suitable explanation. (1)

c) $(\ln(2e), 2e\ln(2e) - 2e)$ or more simply $(1+\ln(2), 2e\ln(2))$ **A1** (1) **5**

7) a) Calculate the **co-ordinates** of the **stationary** point of the following curve:-
$y = 2x^2 - \ln x$ where $x > 0$ (5)
$\frac{dy}{dx} = 4x - \frac{1}{x}$ **M1** let $\frac{dy}{dx} = 0$ **M1**
$\therefore 4x - \frac{1}{x} = 0 \quad \therefore 4x = \frac{1}{x} \therefore x^2 = \frac{1}{4} \quad \therefore x = \pm\frac{1}{2}$ ie $x = \frac{1}{2}$ **A1**
sub in to give corresponding y-values **M1** and we have $(\frac{1}{2}, \frac{1}{2} + \ln 2)$ **A1**

b) **Sketch** the curve. asymptote ($x = 0$) and U-shaped **A1A1** (2) **7**



8) Given that $y = (x + \ln x)^2$
a) when $x = 3$ $y = 16.798\ldots = 16.8$ to 3sf, and when $x = 3.01$ $y = 16.908\ldots$
$= 16.9$ to 3sf **A1** for both (1)
b) Hence estimate the **gradient** of the curve $y = (x + \ln x)^2$ when $x = 3$
$\therefore \frac{dy}{dx}$ when $x = 3 \approx \Delta y/\Delta x$ **M1**$= (16.908\ldots - 16.798\ldots)/(3.01-3) = 10.942\ldots = 10.9$ (3sf) **A1**(2)
3 {65}

Numerical Methods – Iteration – Test 5

EDEXCEL – 2004 – **C3**

1) $x^3 + x - 1 = 0$

a) f(0) = –1, f(1) = 1, sign change ⇒ a soln. **A1 (continuous function)** (1)

b)
$x_2 = 0.8$ ---------------------------- $\boxed{\textbf{B1}}$
$x_3 = 0.609756097$ 0.609...
$x_4 = 0.728967909$ 0.728... $\boxed{\textbf{B1}}$
$x_5 = 0.652999724$ 0.652...
$x_6 = 0.701061373$ 0.701... **B1** (3)

c) $x_6 = 0.7$ to 1dp **A1** (1)

d) n = 5 **A1** (1)

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2) a) $x^3 - 8x = -1 \therefore x(x^2 - 8) = -1 \therefore x = \dfrac{-1}{x^2 - 8} = \dfrac{1}{8 - x^2}$ **A1**

$x^3 = 8x - 1 \quad \therefore x = (8x - 1)^{\frac{1}{3}}$ **A1** (1, 1)

b) f(0) = 1 f(1) = –6 f(2) = –7 f(3) = 4 f(10) = 921
A1 for any correct **+A1** for all correct. (2)

c) $0 < x < 1$ **A1A1**
$2 < x < 3$ **A1A1** (4)

d) $(x_1 = 3)$
$x_2 = 2.84386698$ 2.843...
$x_3 = 2.79142512$ 2.791...
$x_4 = 2.77336135$ 2.773...
$\therefore x = 2.8$ to 1dp **M1A1** (2)

e) $(x_1 = -3)$
$x_2 = -2.924017738$ –2.924...
$x_3 = -2.900124525$ –2.900...
$x_4 = -2.892529173$ –2.892...
$\therefore x = -2.9$ to 1dp **M1A1** (2)

f) $(x_1 = 0)$
$x_2 = 0.125$
$x_3 = 0.125244618$
$x_4 = 0.125245578$
$\therefore x = 0.1252$ to 4dp **M1A1** (2)

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3) $x^5 + x = 100$ has a solution between $x = 2.49$ and $x = 2.5$ (2)
$2.49^5 + 2.49 = 98.208\ldots$
$2.5^5 + 2.5 = 100.15\ldots$ $98.208\ldots < 100 < 100.15\ldots$ ∴ a solution lies between 2.49 and 2.5. **M1A1**
Or rearrange to give $x^5 + x - 100 = 0$ and look for a sign change.

2

4) $7 < s < 7.1$ **A1A1 +A1** for both (3)

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Proof – Test 6

EDEXCEL – 2004 – **C3**

No Calculators *Difficult

1) Standard Proof (3)

2) Prove that the quadratic $D^2x^2+Dx+1=0$ has no real solutions.
Discriminant is $b^2-4ac=D^2-4.D^2.1=-3D^2$ **M1A1**
$-3D^2<0$ (for all non-zero real D) $\therefore$ $D^2x^2+Dx+1=0$ has no real solutions **A1**
(3)

3*) For what values of D does the quadratic $Dx^2+2Dx+1=0$ have no real solutions? (4)
Discriminant is $b^2-4ac=(2D)^2-4.D.1=4D^2-4D$ **A1**
No real solutions when Discriminant <0
$\therefore$ require $4D^2-4D<0$ **M1** $\therefore$ $D^2<D$
Assume $D>0$ **M1** and $\div D$ $\therefore$ $D<1$ $\therefore$ $0<D<1$
Assume $D<0$ and $\div D$ $\therefore$ $D>1$ *contradiction*, and therefore gives no solutions.
$\therefore$ $0<D<1$ to give no real solutions. **A1**

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